

### Homework 1 One period binomial model

#### **Problem 1.**

Assume that  $T = 1$  and let  $(S_t)_{t \in \{0,1\}}$  be the price of a stock with initial price  $S_0 = 100$  SAR and has two possible values a time  $T = 1$  :

$$S_1 = \begin{cases} 200 \text{ SAR with probability } p \\ 75 \text{ SAR with probability } 1 - p. \end{cases}$$

Assume that the stock pays no-dividend. Denote by  $F$  the payoff of an European put option with strike price  $K = 150$  SAR.

1. Give the value of  $F$  at time  $T = 1$ .
2. Verify the non arbitrage conditions
3. Find the price  $C_0$  at time zero for the call option
4. Find the replicating portfolio and give the interpretation of the quantities hold on stock and the riskless asset
5. What will be your position if an investor offer a call with a price different from  $C_0$ ? Discuss the two possible cases
6. Deduce the price of a corresponding put option using call-put parity.
7. Find arbitrage strategies when  $r$ ,  $d$  and  $u$  do not satisfy the non arbitrage condition.

#### **Problem 2.**

Find the current price of a one-year, \$110-strike European put option on a non-dividend-paying stock whose current price is  $S_0 = \$100$ . Assume that the continuously compounded interest rate equals  $r = 0.06$ .

1. Give the value of the put option in one year
2. Use a one-period binomial tree with  $u = 1.23$ , and  $d = 0.86$ . Verify the non arbitrage conditions
3. Calculate the price  $P_0$  of the put option.
4. Find the replicating portfolio and give the interpretation of the quantities hold on stock and the riskless asset
5. What will be your position if an investor offer a call with a price different from  $P_0$ ? Discuss the two possible cases
6. Deduce the price of a corresponding call option using call-put parity.