KING SAUD UNIVERSITY
College of Sciences, Mathematics Department
Bachelor AFM, Riyadh

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Course: QMF Actu. 468
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Homework 2 Multi–period binomial model

Problem 1. Consider a two–period binomial model for a non–dividend paying asset with price S_t, $t = 0, 1, 2$ with $S_0 = 50$ and $u = \frac{1}{d} = 2$. Let $r = \frac{1}{4}$ denote the effective interest rate per period. You need to price a European put option on the underlying which expires at the end of the two periods and has the strike $K = 70$.

1. Find the values of the given option at all the nodes in the binomial tree. In particular, find the no–arbitrage price at time 0 of this option.
2. Find the number of shares ? one needs to invest in at every node in the tree in order to replicate the option.
3. You are given the following information: $S_0 = \$1.20/_{€}$, $r = 5\%$; $r_{€} = 9\%$; $u = 1.06716 = \frac{1}{d}$ Using a three-period binomial tree, calculate the price of a nine months European call on the euro, denominated in dollars, with a strike price of \$1.10.
4. If the option were American, would there be early exercise?

Problem 2.

1. Find the current price of a 60–strike 1.5–year (18–month) European call option on one share of an underlying dividend–paying stock. Let $S_0 = 60$; $r = 0.03$; $\sigma = 0.25$; $\delta = 0.03$; and $h = 0.50$ ($u = e^{\sigma\sqrt{h}} = \frac{1}{d}$).
2. Consider a two-period binomial model. Show that the current price of a call option is given by the formula

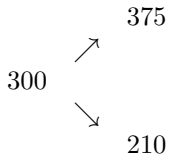
$$C_0 = \left(q^2 C^{uu} + 2q\,(1-q)\,C^{ud} + (1-q)^2\,C^{dd}\right) e^{-2hr}$$

3. Consider the following information about a European call option on stock XYZ:
   The strike price is \$95, the current stock price is \$100, the time to expiration is two years, the annual continuously-compounded risk-free rate is 5%, the stock pays non dividends, the price is calculated using two-step binomial model where each step is one year in length.
   (a) Build the stock and option trees.
   (b) Find the premium of the option and its replicating portefolio.

Problem 3.

Consider the following three–period binomial tree model for a stock that pays dividends continuously at a rate proportional to its price. The length of each period is 1 year, the continuously compounded risk–free interest rate is 10%, and the continuous dividend yield on the stock is 6.5%.

1. Given that the first period of the tree is given by



   calculate the price of a 3–year at–the–money American put option on the stock.
2. Find the replicating portefolio for this put.

Problem 4.

Consider a one–period binomial model with $h = 1$, $S_0 = 100$, $r = 0.08$, $\sigma = 0.3$, $\delta = 0.08$ and $u = 1.25 = \frac{1}{d}$.

1. Find the expression C_0^a for the time–0 price of the American call option on the stock with strike K and maturity at the end of the period.
2. Determine the condition for the strike K to be such that early exercise occurs?
3. In particular, is there early exercise for $K = 70$?