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Homework to prepare for tomorrow

We specify below the basic elements of a financial market with T periods:

- A finite probability space $\Omega = \{\omega_1, \ldots, \omega_k\}$ with k elements.
- A probability measure $\mathbb{P}$ on Ω, such that $\mathbb{P}(\omega) > 0$ for all $\omega \in \Omega$.
- A riskless asset (a saving account) $S_t^0, t \in \{0, 1, 2, \ldots N\}$ such that $S_0^0 = 1$ with a constant interest rate r.
- A d–dimensional price process S_t, $t \in \{0, 1, 2, \ldots N\}$ where $S_t = (S_t^0, S_t^1, \ldots, S_t^d)$ and S_t^i stands for the price of the asset i at time t.

1. Consider the following model $k = 3$, $d = 1$, $r = \frac{1}{9}$

n	S_n^0	S_n^1		
		ω_1	ω_2	ω_3
0	1	5	5	5
1	$\frac{10}{9}$	$\frac{20}{3}$	$\frac{40}{9}$	$\frac{30}{9}$

   Question: Is this model arbitrage free ?

2. Consider now, the following model: given by $k = 3$, $d = 2$, $r = \frac{1}{9}$ and the discounted price

n	S_n^0	$\widetilde{S}_n^1$			$\widetilde{S}_n^2$		
		ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0	1	5	5	5	10	10	10
1	$\frac{10}{9}$	6	6	3	12	8	8

   Question: Is this model arbitrage free ?

3. Consider the following model $\Omega := \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and that the volatility is given by

$$\sigma(\omega) = \begin{cases} h & \text{if } \ \omega \in \{\omega_1, \omega_2\} \\ l & \text{if } \ \omega \in \{\omega_3, \omega_4\} \end{cases}$$

   where $0 < l < h < 1$ and l stands for low volatility whereas h stands for high volatility. The stock price S_1 is then modeled by:

$$S_1(\omega) = \begin{cases} S_0\,(1+\sigma) & \text{if } \ \omega \in \{\omega_1, \omega_3\} \\ S_0\,(1-\sigma) & \text{if } \ \omega \in \{\omega_2, \omega_4\} \end{cases}$$

   where S_0 denotes the initial stock price.
   The riskless asset is model by $S_0^0 = 1$ and $S_1^0 = 1 + r$.
   Question: Is this model arbitrage free ?