The equation (*) becomes:

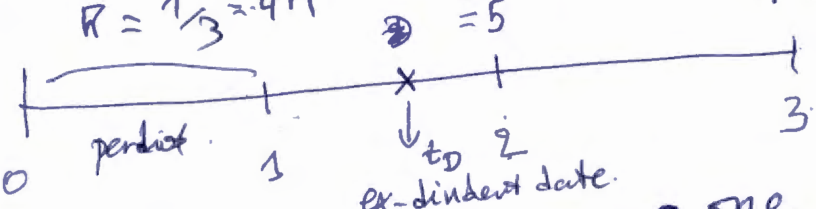
$$\begin{cases} \alpha e^{rh} + \Delta\left(S_t^u + D\right) = C^u & (1) \\ \alpha e^{rh} + \Delta\left(S_t^d + D\right) = C^d & (2) \end{cases}$$

$$(1) - (2) \Rightarrow \Delta = \frac{C^u - C^d}{S_t^u - S_t^d}$$

and $$\alpha = e^{-rh}\left[\frac{S_t^u C^d - S_t^d C^u}{S_t^u - S_t^d}\right] - \Delta D e^{-rh}$$

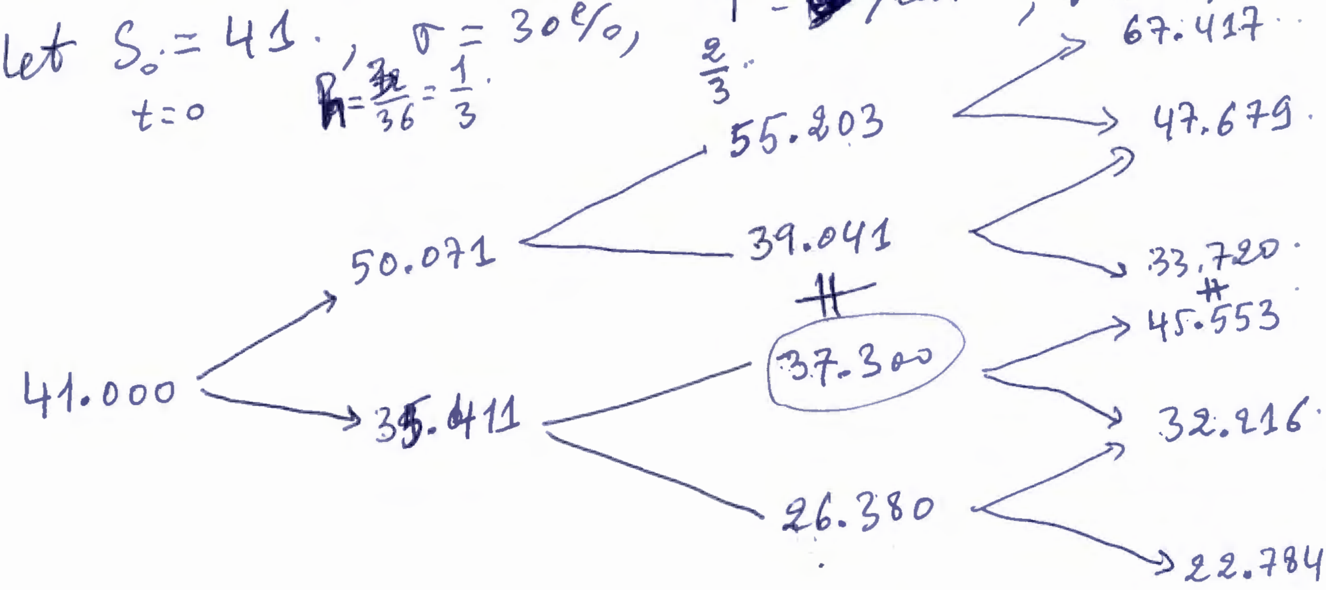
How to construct the tree in this case:

$T = 1$ year $\quad \frac{4}{12} = \frac{1}{3}$



let $S_0 = 41$, $\sigma = 30\%$, $T =$ one year $\frac{2}{3}$, $r = 8\%$

$t = 0$ $\quad h = \frac{12}{36} = \frac{1}{3}$



$$S_h^u = 41\, e^{0.08 \times h + \sigma\sqrt{h}} = 50.071$$

$$S_h^d = 41\, e^{rh - \sigma\sqrt{h}} = 35.411$$

<u>Remark:</u> The tree do not recombine.

W8954

the elegant method of constructing a tree of dividend paying stock. that solves problems encountered with the previous tree is proposed by Schroder (1988) which is the following:

Assume that the stock ~~pays~~ will pay a dividend D at time $T_D < T$.

(a) For $t < T_D$, the stock price is the sum of the prepaid forward price and the present value of the dividend: $S_t = F^P_{t,T} + D e^{-r(T_D - t)}$.

or equivalently $F^P_{t,T} = S_t - D e^{-r(T_D - t)}$.

As before $u = e^{rh + \sigma\sqrt{h}}$ and $d = e^{rh - \sigma\sqrt{h}}$.

(b). The observed stock price at time $t + h < T_D$ is then

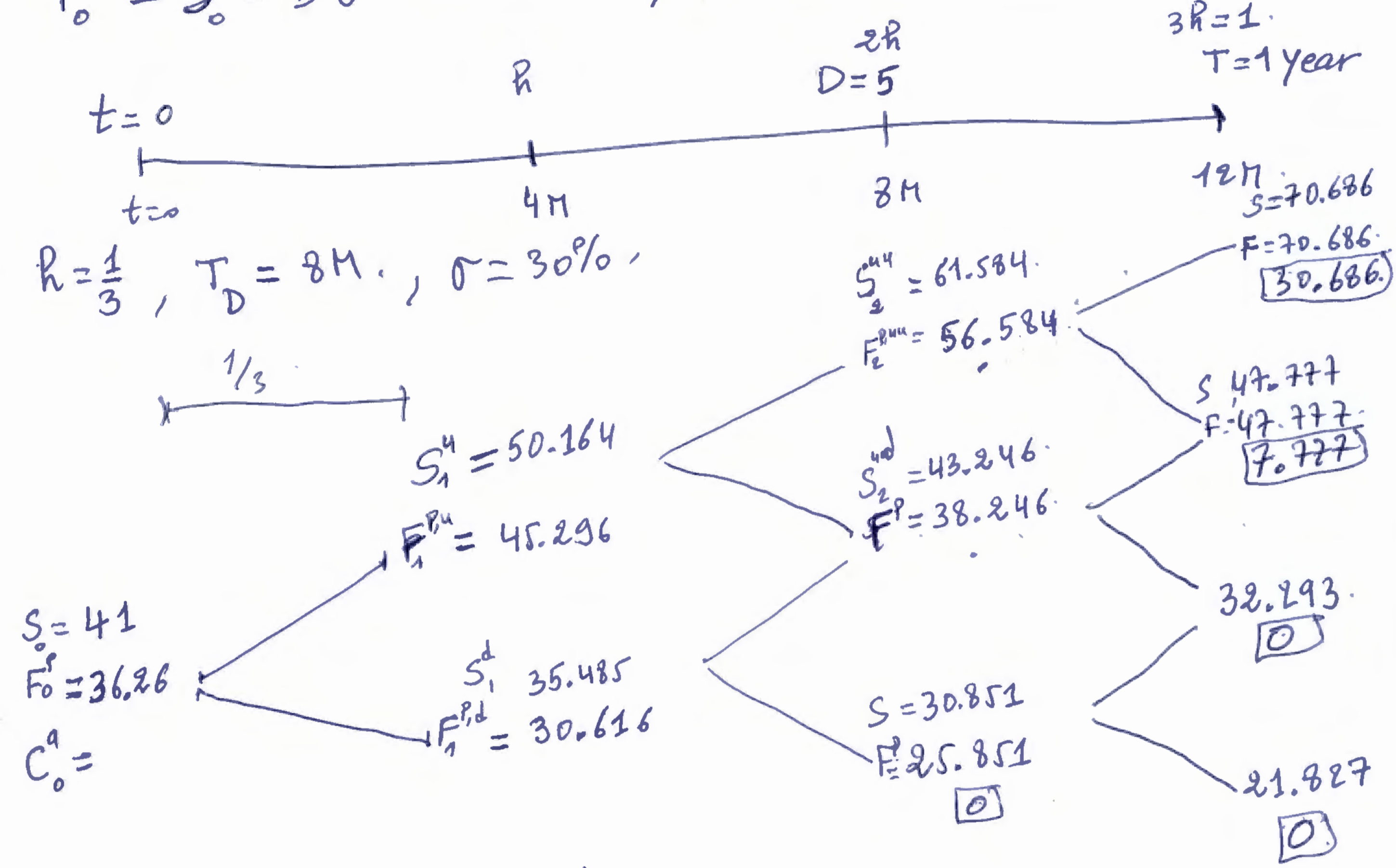
$$S_{t+h} = F^P_{t,T} e^{rh \pm \sigma\sqrt{h}} + D e^{-r(T_D - (t+h))}.$$

We measure σ by observing movement in S_t but σ is used in this equation to characterize movements in F^P_t. We want the total volatility of the prepaid forward to equal that of the stock.

$$\boxed{\sigma_F = \sigma_S \frac{S}{F^P}}. \quad F_0^P ?$$

WB P55

$F_0^P = S_0 - De^{-r(T_D - 0)}$, $r = 8\%$.



we get a recombining tree.

$$F_1^{P,u} = F_0^P u_F = F_0^P \cdot e^{rh + \sigma_F \sqrt{h}} \quad \left(\sigma_F = \sigma_S \frac{S_0}{F_0^P} \right) = 0.3392.$$

$$S_1^u = F_1^{P,u} + De^{-r(T_D - \frac{1}{3})}$$

Now consider 40- American call option maturing in 1Y.

Find the tree of the call option:

Now, we need the RNPM $Q = (q, 1-q)$. We have constructed the ~~forwd~~ forward tree using up and down movements based on the prepaid forward contract.

$$F_{t+h}^P = \begin{cases} F_t^P u_F \\ F_t^P d_F \end{cases} \qquad q = \frac{e^{(r-\delta)h} - d_F}{u_F - d_F} \quad (\delta = 0).$$

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$$C_{2h}^{a,uu} = \max\left(\max(S_{2h}^{uu} - 40, 0); \left(q\, C_{3h}^{a,uuu} + (1-q)\, C_{3h}^{a,uud}\right) e^{-rh}\right)$$

Options on futures contracts:

We assume the forward and futures prices are the same. We can build the tree of the forward price using. $u = e^{\sigma\sqrt{h}}$ and $d = e^{-\sigma\sqrt{h}}$.

Consider a payoff with values $\begin{cases} C^u \\ C^d \end{cases}$:

The replicating portfolio (α, Δ) is given by.

$$\begin{cases} \alpha e^{rh} + \Delta F_0 u = C^u \\ \alpha e^{rh} + \Delta F_0 d = C^d \end{cases}$$

$$\Delta = \frac{C^u - C^d}{F_0 u - F_0 d}, \quad F_0 \text{ initial forward price.}$$

$$\alpha = (C^d - \Delta F_0 d)\, e^{-rh}.$$

$$V_0 = \alpha + \Delta F_0 = e^{-rh}\left[\frac{1-d}{u-d} C^u + \frac{u-1}{u-d} C^d\right].$$

$$q = \frac{1-d}{u-d}$$ (forward price with no dividend)

$$\begin{tikzcd}[row sep=small]
 & & F_0 u^2 \\
 & F_0 u \arrow[ur] \arrow[dr] & \\
F_0 \arrow[ur] \arrow[dr] & & F_0 u d \\
 & F_0 d \arrow[ur] \arrow[dr] & \\
 & & F_0 d^2
\end{tikzcd}$$

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