Student Solutions Manual

to accompany

Applied Linear Statistical Models

Fifth Edition

Michael H. Kutner
Emory University
Christopher J. Nachtsheim
University of Minnesota
John Neter
University of Georgia
William Li
University of Minnesota

2005
McGraw-Hill/Irwin
Chicago, IL
Boston, MA

PREFACE

This Student Solutions Manual gives intermediate and final numerical results for all starred (*) end-of-chapter Problems with computational elements contained in *Applied Linear Statistical Models*, 5th edition. No solutions are given for Exercises, Projects, or Case Studies.

In presenting calculational results we frequently show, for ease in checking, more digits than are significant for the original data. Students and other users may obtain slightly different answers than those presented here, because of different rounding procedures. When a problem requires a percentile (e.g. of the t or F distributions) not included in the Appendix B Tables, users may either interpolate in the table or employ an available computer program for finding the needed value. Again, slightly different values may be obtained than the ones shown here.

The data sets for all Problems, Exercises, Projects and Case Studies are contained in the compact disk provided with the text to facilitate data entry. It is expected that the student will use a computer or have access to computer output for all but the simplest data sets, where use of a basic calculator would be adequate. For most students, hands-on experience in obtaining the computations by computer will be an important part of the educational experience in the course.

While we have checked the solutions very carefully, it is possible that some errors are still present. We would be most grateful to have any errors called to our attention. Errata can be reported via the website for the book: http://www.mhhe.com/KutnerALSM5e.

We acknowledge with thanks the assistance of Lexin Li and Yingwen Dong in the checking of Chapters 1-14 of this manual. We, of course, are responsible for any errors or omissions that remain.

Michael H. Kutner
Christopher J. Nachtsheim
John Neter
William Li

Contents

Chapter 1

LINEAR REGRESSION WITH ONE PREDICTOR VARIABLE

1.20. a. $\hat{Y} = -0.5802 + 15.0352X$

d. $\hat{Y}_h = 74.5958$

1.21. a. $\hat{Y} = 10.20 + 4.00X$

b. $\hat{Y}_h = 14.2$

c. 4.0

d. $(\bar{X}, \bar{Y}) = (1, 14.2)$

1.24. a.

i:	1	2	...	44	45
e_i:	-9.4903	0.4392	...	1.4392	2.4039

$\sum e_i^2 = 3416.377$

Min $Q = \sum e_i^2$

b. $MSE = 79.45063$, $\sqrt{MSE} = 8.913508$, minutes

1.25. a. $e_1 = 1.8000$

b. $\sum e_i^2 = 17.6000$, $MSE = 2.2000$, σ^2

1.27. a. $\hat{Y} = 156.35 - 1.19X$

b. (1) $b_1 = -1.19$, (2) $\hat{Y}_h = 84.95$, (3) $e_8 = 4.4433$,

(4) $MSE = 66.8$

Chapter 2

INFERENCES IN REGRESSION AND CORRELATION ANALYSIS

2.5. a. $t(.95; 43) = 1.6811$, $15.0352 \pm 1.6811(.4831)$, $14.2231 \leq \beta_1 \leq 15.8473$

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $t^* = (15.0352 - 0)/.4831 = 31.122$. If $|t^*| \leq 1.681$ conclude H_0, otherwise H_a. Conclude H_a. P-value= 0+

c. Yes

d. H_0: $\beta_1 \leq 14$, H_a: $\beta_1 > 14$. $t^* = (15.0352 - 14)/.4831 = 2.1428$. If $t^* \leq 1.681$ conclude H_0, otherwise H_a. Conclude H_a. P-value= .0189

2.6. a. $t(.975; 8) = 2.306$, $b_1 = 4.0$, $s\{b_1\} = .469$, $4.0 \pm 2.306(.469)$,

$2.918 \leq \beta_1 \leq 5.082$

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $t^* = (4.0 - 0)/.469 = 8.529$. If $|t^*| \leq 2.306$ conclude H_0, otherwise H_a. Conclude H_a. P-value= .00003

c. $b_0 = 10.20$, $s\{b_0\} = .663$, $10.20 \pm 2.306(.663)$, $8.671 \leq \beta_0 \leq 11.729$

d. H_0: $\beta_0 \leq 9$, H_a: $\beta_0 > 9$. $t^* = (10.20 - 9)/.663 = 1.810$. If $t^* \leq 2.306$ conclude H_0, otherwise H_a. Conclude H_0. P-value= .053

e. H_0: $\beta_1 = 0$: $\delta = |2 - 0|/.5 = 4$, power = .93

H_0: $\beta_0 \leq 9$: $\delta = |11 - 9|/.75 = 2.67$, power = .78

2.14. a. $\hat{Y}_h = 89.6313$, $s\{\hat{Y}_h\} = 1.3964$, $t(.95;\ 43) = 1.6811$, $89.6313 \pm 1.6811(1.3964)$, $87.2838 \leq E\{Y_h\} \leq 91.9788$

b. $s\{\text{pred}\} = 9.0222$, $89.6313 \pm 1.6811(9.0222)$, $74.4641 \leq Y_{h(\text{new})} \leq 104.7985$, yes, yes

c. $87.2838/6 = 14.5473$, $91.9788/6 = 15.3298$, $14.5473 \leq$ Mean time per machine ≤ 15.3298

d. $W^2 = 2F(.90;\ 2, 43) = 2(2.4304) = 4.8608$, $W = 2.2047$, $89.6313 \pm 2.2047(1.3964)$, $86.5527 \leq \beta_0 + \beta_1 X_h \leq 92.7099$, yes, yes

2.15. a. $X_h = 2$: $\hat{Y}_h = 18.2$, $s\{\hat{Y}_h\} = .663$, $t(.995;\ 8) = 3.355$, $18.2 \pm 3.355(.663)$, $15.976 \leq E\{Y_h\} \leq 20.424$

$X_h = 4$: $\hat{Y}_h = 26.2$, $s\{\hat{Y}_h\} = 1.483$, $26.2 \pm 3.355(1.483)$, $21.225 \leq E\{Y_h\} \leq 31.175$

b. $s\{\text{pred}\} = 1.625$, $18.2 \pm 3.355(1.625)$, $12.748 \leq Y_{h(\text{new})} \leq 23.652$

c. $s\{\text{predmean}\} = 1.083$, $18.2 \pm 3.355(1.083)$, $14.567 \leq \bar{Y}_{h(\text{new})} \leq 21.833$, $44 = 3(14.567) \leq$ Total number of broken ampules $\leq 3(21.833) = 65$

d. $W^2 = 2F(.99; 2, 8) = 2(8.649) = 17.298$, $W = 4.159$

$X_h = 2$: $18.2 \pm 4.159(.663)$, $15.443 \leq \beta_0 + \beta_1 X_h \leq 20.957$

$X_h = 4$: $26.2 \pm 4.159(1.483)$, $20.032 \leq \beta_0 + \beta_1 X_h \leq 32.368$

yes, yes

2.24. a.

Source	SS	df	MS
Regression	76,960.4	1	76,960.4
Error	3,416.38	43	79.4506
Total	80,376.78	44	

Source	SS	df	MS
Regression	76,960.4	1	76,960.4
Error	3,416.38	43	79.4506
Total	80,376.78	44	
Correction for mean	261,747.2	1	
Total, uncorrected	342,124	45	

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $F^* = 76,960.4/79.4506 = 968.66$, $F(.90; 1, 43) = 2.826$. If $F^* \leq 2.826$ conclude H_0, otherwise H_a. Conclude H_a.

c. 95.75% or 0.9575, coefficient of determination

d. +.9785

e. R^2

2.25. a.

Source	SS	df	MS
Regression	160.00	1	160.00
Error	17.60	8	2.20
Total	177.60	9	

b. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $F^* = 160.00/2.20 = 72.727$, $F(.95; 1, 8) = 5.32$. If $F^* \leq 5.32$ conclude H_0, otherwise H_a. Conclude H_a.

c. $t^* = (4.00 - 0)/.469 = 8.529$, $(t^*)^2 = (8.529)^2 = 72.7 = F^*$

d. $R^2 = .9009$, $r = .9492$, 90.09%

2.27. a. H_0: $\beta_1 \geq 0$, H_a: $\beta_1 < 0$. $s\{b_1\} = 0.090197$,

$t^* = (-1.19 - 0)/.090197 = -13.193$, $t(.05; 58) = -1.67155$.
If $t^* \geq -1.67155$ conclude H_0, otherwise H_a. Conclude H_a.
P-value= 0+

c. $t(.975; 58) = 2.00172$, $-1.19 \pm 2.00172(.090197)$, $-1.3705 \leq \beta_1 \leq -1.0095$

2.28. a. $\hat{Y}_h = 84.9468$, $s\{\hat{Y}_h\} = 1.05515$, $t(.975; 58) = 2.00172$,
$84.9468 \pm 2.00172(1.05515)$, $82.835 \leq E\{Y_h\} \leq 87.059$

b. $s\{Y_{h(\text{new})}\} = 8.24101$, $84.9468 \pm 2.00172(8.24101)$, $68.451 \leq Y_{h(\text{new})} \leq 101.443$

c. $W^2 = 2F(.95; 2, 58) = 2(3.15593) = 6.31186$, $W = 2.512342$,
$84.9468 \pm 2.512342(1.05515)$, $82.296 \leq \beta_0 + \beta_1 X_h \leq 87.598$, yes, yes

2.29. a.

i:	1	2	...	59	60
$Y_i - \hat{Y}_i$:	0.823243	-1.55675	...	-0.666887	8.09309
$\hat{Y}_i - \bar{Y}$:	20.2101	22.5901	...	-14.2998	-19.0598

b.

Source	SS	df	MS
Regression	11,627.5	1	11,627.5
Error	3,874.45	58	66.8008
Total	15,501.95	59	

c. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$. $F^* = 11,627.5/66.8008 = 174.0623$,
$F(.90;\ 1, 58) = 2.79409$. If $F^* \leq 2.79409$ conclude H_0, otherwise H_a. Conclude H_a.

d. 24.993% or .24993

e. $R^2 = 0.750067$, $r = -0.866064$

2.42. b. .95285, ρ_{12}

c. $H_0 : \rho_{12} = 0$, $H_a : \rho_{12} \neq 0$. $t^* = (.95285\sqrt{13})/\sqrt{1 - (.95285)^2} = 11.32194$,
$t(.995; 13) = 3.012$. If $|t^*| \leq 3.012$ conclude H_0, otherwise H_a. Conclude H_a.

d. No

2.44. a. $H_0 : \rho_{12} = 0$, $H_a : \rho_{12} \neq 0$. $t^* = (.87\sqrt{101})/\sqrt{1 - (.87)^2} = 17.73321$, $t(.95; 101) = 1.663$. If $|t^*| \leq 1.663$ conclude H_0, otherwise H_a. Conclude H_a.

b. $z' = 1.33308$, $\sigma\{z'\} = .1$, $z(.95) = 1.645$, $1.33308 \pm 1.645(.1)$, $1.16858 \leq \zeta \leq 1.49758$, $.824 \leq \rho_{12} \leq .905$

c. $.679 \leq \rho_{12}^2 \leq .819$

2.47. a. -0.866064,

b. $H_0 : \rho_{12} = 0$, $H_a : \rho_{12} \neq 0$. $t^* = (-0.866064\sqrt{58})/\sqrt{1 - (-0.866064)^2} = -13.19326$, $t(.975; 58) = 2.00172$. If $|t^*| \leq 2.00172$ conclude H_0, otherwise H_a. Conclude H_a.

c. -0.8657217

d. H_0 : There is no association between X and Y
H_a : There is an association between X and Y
$t^* = \dfrac{-0.8657217\sqrt{58}}{\sqrt{1 - (-0.8657217)^2}} = -13.17243$. $t(0.975,\ 58) = 2.001717$. If $|t^*| \leq 2.001717$, conclude H_0, otherwise, conclude H_a. Conclude H_a.

Chapter 3

DIAGNOSTICS AND REMEDIAL MEASURES

3.4.c and d.

i:	1	2	...	44	45
$\hat{Y}_i$:	29.49034	59.56084	...	59.56084	74.59608
e_i:	-9.49034	0.43916	...	1.43916	2.40392

e.

Ascending order:	1	2	...	44	45
Ordered residual:	-22.77232	-19.70183	...	14.40392	15.40392
Expected value:	-19.63272	-16.04643	...	16.04643	19.63272

H_0: Normal, H_a: not normal. $r = 0.9891$. If $r \geq .9785$ conclude H_0, otherwise H_a. Conclude H_0.

g. $SSR^* = 15,155$, $SSE = 3416.38$, $X^2_{BP} = (15,155/2) \div (3416.38/45)^2 = 1.314676$, $\chi^2(.95;1) = 3.84$. If $X^2_{BP} \leq 3.84$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

3.5. c.

i:	1	2	3	4	5	6	7	8	9	10
e_i:	1.8	-1.2	-1.2	1.8	-.2	-1.2	-2.2	.8	.8	.8

e.

Ascending Order:	1	2	3	4	5	6	7	8	9	10
Ordered residual:	-2.2	-1.2	-1.2	-1.2	-.2	.8	.8	.8	1.8	1.8
Expected value:	-2.3	-1.5	-1.0	-.6	-.2	.2	.6	1.0	1.5	2.3

H_0: Normal, H_a: not normal. $r = .961$. If $r \geq .879$ conclude H_0, otherwise H_a. Conclude H_0.

g. $SSR^* = 6.4$, $SSE = 17.6$, $X^2_{BP} = (6.4/2) \div (17.6/10)^2 = 1.03$, $\chi^2(.90;1) = 2.71$. If $X^2_{BP} \leq 2.71$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

Yes.

3.7.b and c.

i:	1	2	...	59	60
e_i:	0.82324	-1.55675	...	-0.66689	8.09309
$\hat{Y}_i$:	105.17676	107.55675	...	70.66689	65.90691

d.

Ascending order:	1	2	...	59	60
Ordered residual:	-16.13683	-13.80686	...	13.95312	23.47309
Expected value:	-18.90095	-15.75218	...	15.75218	18.90095

H_0: Normal, H_a: not normal. $r = 0.9897$. If $r \geq 0.984$ conclude H_0, otherwise H_a. Conclude H_0.

e. $SSR^* = 31,833.4$, $SSE = 3,874.45$,

$X^2_{BP} = (31,833.4/2) \div (3,874.45/60)^2 = 3.817116$, $\chi^2(.99; 1) = 6.63$. If $X^2_{BP} \leq 6.63$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant. Yes.

3.13. a. H_0: $E\{Y\} = \beta_0 + \beta_1 X$, H_a: $E\{Y\} \neq \beta_0 + \beta_1 X$

b. $SSPE = 2797.66$, $SSLF = 618.719$, $F^* = (618.719/8) \div (2797.66/35) = 0.967557$, $F(.95; 8, 35) = 2.21668$. If $F^* \leq 2.21668$ conclude H_0, otherwise H_a. Conclude H_0.

3.17. b.

λ:	.3	.4	.5	.6	.7
SSE:	1099.7	967.9	916.4	942.4	1044.2

c. $\hat{Y}' = 10.26093 + 1.07629X$

e.

i:	1	2	3	4	5
e_i:	-.36	.28	.31	-.15	.30
$\hat{Y}'_i$:	10.26	11.34	12.41	13.49	14.57
Expected value:	-.24	.14	.36	-.14	.24

i:	6	7	8	9	10
e_i:	-.41	.10	-.47	.47	-.07
$\hat{Y}'_i$:	15.64	16.72	17.79	18.87	19.95
Expected value:	-.36	.04	-.56	.56	-.04

f. $\hat{Y} = (10.26093 + 1.07629X)^2$

Chapter 4

SIMULTANEOUS INFERENCES AND OTHER TOPICS IN REGRESSION ANALYSIS

4.3. a. Opposite directions, negative tilt

b. $B = t(.9875; 43) = 2.32262$, $b_0 = -0.580157$, $s\{b_0\} = 2.80394$, $b_1 = 15.0352$, $s\{b_1\} = 0.483087$

$-0.580157 \pm 2.32262(2.80394)$ $\quad -7.093 \leq \beta_0 \leq 5.932$

$15.0352 \pm 2.32262(0.483087)$ $\quad 13.913 \leq \beta_1 \leq 16.157$

c. Yes

4.4. a. Opposite directions, negative tilt

b. $B = t(.9975; 8) = 3.833$, $b_0 = 10.2000$, $s\{b_0\} = .6633$, $b_1 = 4.0000$, $s\{b_1\} = .4690$

$10.2000 \pm 3.833(.6633)$ $\quad 7.658 \leq \beta_0 \leq 12.742$

$4.0000 \pm 3.833(.4690)$ $\quad 2.202 \leq \beta_1 \leq 5.798$

4.6. a. $B = t(.9975; 14) = 2.91839$, $b_0 = 156.347$, $s\{b_0\} = 5.51226$, $b_1 = -1.190$,$s\{b_1\} = 0.0901973$

$156.347 \pm 2.91839(5.51226)$ $\quad 140.260 \leq \beta_0 \leq 172.434$

$-1.190 \pm 2.91839(0.0901973)$ $\quad -1.453 \leq \beta_1 \leq -0.927$

b. Opposite directions

c. No

4.7. a. $F(.90; 2, 43) = 2.43041$, $W = 2.204727$

$X_h = 3$: $44.5256 \pm 2.204727(1.67501)$ $\quad 40.833 \leq E\{Y_h\} \leq 48.219$

$X_h = 5$: $74.5961 \pm 2.204727(1.32983)$ $\quad 71.664 \leq E\{Y_h\} \leq 77.528$

$X_h = 7$: $104.667 \pm 2.204727(1.6119)$ $\quad 101.113 \leq E\{Y_h\} \leq 108.221$

b. $F(.90; 2, 43) = 2.43041$, $S = 2.204727$; $B = t(.975; 43) = 2.01669$; Bonferroni

c. $X_h = 4$: $59.5608 \pm 2.01669(9.02797)$ $\quad 41.354 \leq Y_{h(\text{new})} \leq 77.767$

$X_h = 7$: $104.667 \pm 2.01669(9.05808)$ $\quad 86.3997 \leq Y_{h(\text{new})} \leq 122.934$

4.8. a. $F(.95; 2, 8) = 4.46$, $W = 2.987$

$X_h = 0$: $10.2000 \pm 2.987(.6633)$ $\quad 8.219 \leq E\{Y_h\} \leq 12.181$

$X_h = 1$: $14.2000 \pm 2.987(.4690)$ $\quad 12.799 \leq E\{Y_h\} \leq 15.601$

$X_h = 2$: $18.2000 \pm 2.987(.6633)$ $\quad 16.219 \leq E\{Y_h\} \leq 20.181$

b. $B = t(.99167; 8) = 3.016$, yes

c. $F(.95; 3, 8) = 4.07$, $S = 3.494$

$X_h = 0$: $10.2000 \pm 3.494(1.6248)$ $\quad 4.523 \leq Y_{h(\text{new})} \leq 15.877$

$X_h = 1$: $14.2000 \pm 3.494(1.5556)$ $\quad 8.765 \leq Y_{h(\text{new})} \leq 19.635$

$X_h = 2$: $18.2000 \pm 3.494(1.6248)$ $\quad 12.523 \leq Y_{h(\text{new})} \leq 23.877$

d. $B = 3.016$, yes

4.10. a. $F(.95; 2, 58) = 3.15593$, $W = 2.512342$

$X_h = 45$: $102.797 \pm 2.512342(1.71458)$ $\quad 98.489 \leq E\{Y_h\} \leq 107.105$

$X_h = 55$: $90.8968 \pm 2.512342(1.1469)$ $\quad 88.015 \leq E\{Y_h\} \leq 93.778$

$X_h = 65$: $78.9969 \pm 2.512342(1.14808)$ $\quad 76.113 \leq E\{Y_h\} \leq 81.881$

b. $B = t(.99167; 58) = 2.46556$, no

c. $B = 2.46556$

$X_h = 48$: $99.2268 \pm 2.46556(8.31158)$ $\quad 78.734 \leq Y_{h(\text{new})} \leq 119.720$

$X_h = 59$: $86.1368 \pm 2.46556(8.24148)$ $\quad 65.817 \leq Y_{h(\text{new})} \leq 106.457$

$X_h = 74$: $68.2869 \pm 2.46556(8.33742)$ $\quad 47.730 \leq Y_{h(\text{new})} \leq 88.843$

d. Yes, yes

4.16. a. $\hat{Y} = 14.9472X$

b. $s\{b_1\} = 0.226424$, $t(.95; 44) = 1.68023$, $14.9472 \pm 1.68023(0.226424)$, $14.567 \leq \beta_1 \leq 15.328$

c. $\hat{Y}_h = 89.6834$, $s\{\text{pred}\} = 8.92008$, $89.6834 \pm 1.68023(8.92008)$, $74.696 \leq Y_{h(\text{new})} \leq 104.671$

4.17. b.

i:	1	2	...	44	45
e_i:	-9.89445	0.21108	...	1.2111	2.2639

No

c. H_0: $E\{Y\} = \beta_1 X$, H_a: $E\{Y\} \neq \beta_1 X$. $SSLF = 622.12$, $SSPE = 2797.66$, $F^* = (622.12/9) \div (2797.66/35) = 0.8647783$, $F(.99; 9, 35) = 2.96301$. If $F^* \leq 2.96301$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= 0.564$

Chapter 5

MATRIX APPROACH TO SIMPLE LINEAR REGRESSION ANALYSIS

5.4. (1) 503.77 (2) $\begin{bmatrix} 5 & 0 \\ 0 & 160 \end{bmatrix}$ (3) $\begin{bmatrix} 49.7 \\ -39.2 \end{bmatrix}$

5.6. (1) 2,194 (2) $\begin{bmatrix} 10 & 10 \\ 10 & 20 \end{bmatrix}$ (3) $\begin{bmatrix} 142 \\ 182 \end{bmatrix}$

5.12. $\begin{bmatrix} .2 & 0 \\ 0 & .00625 \end{bmatrix}$

5.14. a. $\begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 25 \\ 12 \end{bmatrix}$

b. $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 4.5 \\ 1 \end{bmatrix}$

5.18. a. $\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix}$

b. $\mathbf{E}\left\{\begin{bmatrix} W_1 \\ W_2 \end{bmatrix}\right\} = \begin{bmatrix} \frac{1}{4}[E\{Y_1\} + E\{Y_2\} + E\{Y_3\} + E\{Y_4\}] \\ \frac{1}{2}[E\{Y_1\} + E\{Y_2\} - E\{Y_3\} - E\{Y_4\}] \end{bmatrix}$

c. $\boldsymbol{\sigma}^2\{\mathbf{W}\} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \sigma^2\{Y_1\} & \sigma\{Y_1, Y_2\} & \sigma\{Y_1, Y_3\} & \sigma\{Y_1, Y_4\} \\ \sigma\{Y_2, Y_1\} & \sigma^2\{Y_2\} & \sigma\{Y_2, Y_3\} & \sigma\{Y_2, Y_4\} \\ \sigma\{Y_3, Y_1\} & \sigma\{Y_3, Y_2\} & \sigma^2\{Y_3\} & \sigma\{Y_3, Y_4\} \\ \sigma\{Y_4, Y_1\} & \sigma\{Y_4, Y_2\} & \sigma\{Y_4, Y_3\} & \sigma^2\{Y_4\} \end{bmatrix}$

$\times \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$

Using the notation σ_1^2 for $\sigma^2\{Y_1\}, \sigma_{12}$ for $\sigma\{Y_1, Y_2\}$, etc., we obtain:

$\sigma^2\{W_1\} = \frac{1}{16}(\sigma_1^2 + +\sigma_2^2 + \sigma_3^2 + \sigma_4^2 + 2\sigma_{12} + 2\sigma_{13} + 2\sigma_{14} + 2\sigma_{23} + 2\sigma_{24} + 2\sigma_{34})$

$$\sigma^2\{W_2\} = \frac{1}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 + 2\sigma_{12} - 2\sigma_{13} - 2\sigma_{14} - 2\sigma_{23} - 2\sigma_{24} + 2\sigma_{34})$$
$$\sigma\{W_1, W_2\} = \frac{1}{8}(\sigma_1^2 + \sigma_2^2 - \sigma_3^2 - \sigma_4^2 + 2\sigma_{12} - 2\sigma_{34})$$

5.19. $\begin{bmatrix} 3 & 5 \\ 5 & 17 \end{bmatrix}$

5.21. $5Y_1^2 + 4Y_1Y_2 + Y_2^2$

5.23. a. (1) $\begin{bmatrix} 9.940 \\ -.245 \end{bmatrix}$ (2) $\begin{bmatrix} -.18 \\ .04 \\ .26 \\ .08 \\ -.20 \end{bmatrix}$ (3) 9.604 (4) .148

(5) $\begin{bmatrix} .00987 & 0 \\ 0 & .000308 \end{bmatrix}$ (6) 11.41 (7) .02097

c. $\begin{bmatrix} .6 & .4 & .2 & 0 & -.2 \\ .4 & .3 & .2 & .1 & 0 \\ .2 & .2 & .2 & .2 & .2 \\ 0 & .1 & .2 & .3 & .4 \\ -.2 & 0 & .2 & .4 & .6 \end{bmatrix}$

d. $\begin{bmatrix} .01973 & -.01973 & -.00987 & .00000 & .00987 \\ -.01973 & .03453 & -.00987 & -.00493 & .00000 \\ -.00987 & -.00987 & .03947 & -.00987 & -.00987 \\ .00000 & -.00493 & -.00987 & .03453 & -.01973 \\ .00987 & .00000 & -.00987 & -.01973 & .01973 \end{bmatrix}$

5.25. a. (1) $\begin{bmatrix} .2 & -.1 \\ -.1 & .1 \end{bmatrix}$ (2) $\begin{bmatrix} 10.2 \\ 4.0 \end{bmatrix}$ (3) $\begin{bmatrix} 1.8 \\ -1.2 \\ -1.2 \\ 1.8 \\ -.2 \\ -1.2 \\ -2.2 \\ .8 \\ .8 \\ .8 \end{bmatrix}$

(4) $\begin{bmatrix} .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & 0 & .2 & 0 & .3 & .1 & 0 & .1 & .2 & 0 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & -.1 & .3 & -.1 & .5 & .1 & -.1 & .1 & .3 & -.1 \\ .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & 0 & .2 \\ .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 & .1 \\ .1 & .0 & .2 & 0 & .3 & .1 & 0 & .1 & .2 & 0 \\ .1 & .2 & 0 & .2 & -.1 & .1 & .2 & .1 & .0 & .2 \end{bmatrix}$

(5) 17.60 (6) $\begin{bmatrix} .44 & -.22 \\ -.22 & .22 \end{bmatrix}$ (7) 18.2 (8) .44

b. (1) .22 (2) $-.22$ (3) .663

c. $$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & -.1 & .1 & -.1 & .2 & 0 & -.1 & 0 & .1 & -.1 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & -.2 & .2 & -.2 & .4 & 0 & -.2 & 0 & .2 & -.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.1 & .1 & -.1 & .2 & 0 & -.1 & 0 & .1 & -.1 \\ 0 & .1 & -.1 & .1 & -.2 & 0 & .1 & 0 & -.1 & .1 \end{bmatrix}$$

Chapter 6

MULTIPLE REGRESSION – I

6.9. c.

$$\begin{matrix} Y \\ X_1 \\ X_2 \\ X_3 \end{matrix} \begin{bmatrix} 1.0000 & .2077 & .0600 & .8106 \\ & 1.0000 & .0849 & .0457 \\ & & 1.0000 & .1134 \\ & & & 1.0000 \end{bmatrix}$$

6.10. a. $\hat{Y} = 4149.89 + 0.000787X_1 - 13.166X_2 + 623.554X_3$

b&c.

i:	1	2	...	51	52
e_i:	-32.0635	169.2051	...	-184.8776	64.5168
Expected Val.:	-24.1737	151.0325	...	-212.1315	75.5358

e. $n_1 = 26$, $\bar{d}_1 = 145.0$, $n_2 = 26$, $\bar{d}_2 = 77.4$, $s = 81.7$,
$t^*_{BF} = (145.0 - 77.4)/[81.7\sqrt{(1/26) + (1/26)}] = 2.99$, $t(.995; 50) = 2.67779$. If $|t^*_{BF}| \leq 2.67779$ conclude error variance constant, otherwise error variance not constant. Conclude error variance not constant.

6.11. a. H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all $\beta_k = 0$ ($k = 1$, 2,3). $MSR = 725,535$, $MSE = 20,531.9$, $F^* = 725,535/20,531.9 = 35.337$, $F(.95; 3, 48) = 2.79806$. If $F^* \leq 2.79806$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+.

b. $s\{b_1\} = .000365$, $s\{b_3\} = 62.6409$, $B = t(.9875; 48) = 2.3139$

$0.000787 \pm 2.3139(.000365)$ $\quad -.000058 \leq \beta_1 \leq 0.00163$

$623.554 \pm 2.3139(62.6409)$ $\quad 478.6092 \leq \beta_3 \leq 768.4988$

c. $SSR = 2,176,606$, $SSTO = 3,162,136$, $R^2 = .6883$

6.12. a. $F(.95; 4, 48) = 2.56524$, $W = 3.2033$; $B = t(.995; 48) = 2.6822$

X_{h1}	X_{h2}	X_{h3}		
302,000	7.2	0:	$4292.79 \pm 2.6822(21.3567)$	$4235.507 \leq E\{Y_h\} \leq 4350.073$
245,000	7.4	0:	$4245.29 \pm 2.6822(29.7021)$	$4165.623 \leq E\{Y_h\} \leq 4324.957$
280,000	6.9	0:	$4279.42 \pm 2.6822(24.4444)$	$4213.855 \leq E\{Y_h\} \leq 4344.985$
350,000	7.0	0:	$4333.20 \pm 2.6822(28.9293)$	$4255.606 \leq E\{Y_h\} \leq 4410.794$
295,000	6.7	1:	$4917.42 \pm 2.6822(62.4998)$	$4749.783 \leq E\{Y_h\} \leq 5085.057$

b.Yes, no

6.13. $F(.95;4,48) = 2.5652$, $S = 3.2033$; $B = t(.99375;48) = 2.5953$

X_{h1}	X_{h2}	X_{h3}		
230,000	7.5	0:	$4232.17 \pm 2.5953(147.288)$	$3849.913 \leq Y_{h(\text{new})} \leq 4614.427$
250,000	7.3	0:	$4250.55 \pm 2.5953(146.058)$	$3871.486 \leq Y_{h(\text{new})} \leq 4629.614$
280,000	7.1	0:	$4276.79 \pm 2.5953(145.134)$	$3900.124 \leq Y_{h(\text{new})} \leq 4653.456$
340,000	6.9	0:	$4326.65 \pm 2.5953(145.930)$	$3947.918 \leq Y_{h(\text{new})} \leq 4705.382$

6.14. a. $\hat{Y}_h = 4278.37$, $s\{\text{predmean}\} = 85.82262$, $t(.975;48) = 2.01063$,

$4278.37 \pm 2.01063(85.82262)$, $4105.812 \leq \bar{Y}_{h(\text{new})} \leq 4450.928$

b. $12317.44 \leq$ Total labor hours ≤ 13352.78

6.15. b.

$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \end{array} \begin{bmatrix} 1.000 & -.7868 & -.6029 & -.6446 \\ & 1.000 & .5680 & .5697 \\ & & 1.000 & .6705 \\ & & & 1.000 \end{bmatrix}$$

c. $\hat{Y} = 158.491 - 1.1416X_1 - 0.4420X_2 - 13.4702X_3$

d&e.

i:	1	2	...	45	46
e_i:	.1129	−9.0797	...	−5.5380	10.0524
Expected Val.:	−0.8186	−8.1772	...	−5.4314	8.1772

f. No

g. $SSR^* = 21,355.5$, $SSE = 4,248.8$, $X^2_{BP} = (21,355.5/2) \div (4,248.8\ /46)^2 = 1.2516$, $\chi^2(.99;3) = 11.3449$. If $X^2_{BP} \leq 11.3449$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

6.16. a. H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all $\beta_k = 0$ $(k = 1,2,3)$.

$MSR = 3,040.2$, $MSE = 101.2$, $F^* = 3,040.2/101.2 = 30.05$, $F(.90;3,42) = 2.2191$. If $F^* \leq 2.2191$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0.4878$

b. $s\{b_1\} = .2148$, $s\{b_2\} = .4920$, $s\{b_3\} = 7.0997$, $B = t(.9833;42) = 2.1995$

$-1.1416 \pm 2.1995(.2148)$ $\quad -1.6141 \leq \beta_1 \leq -0.6691$

$-.4420 \pm 2.1995(.4920)$ $\quad -1.5242 \leq \beta_2 \leq 0.6402$

$-13.4702 \pm 2.1995(7.0997)$ $\quad -29.0860 \leq \beta_3 \leq 2.1456$

c. $SSR = 9,120.46$, $SSTO = 13,369.3$, $R = .8260$

6.17. a. $\hat{Y}_h = 69.0103$, $s\{\hat{Y}_h\} = 2.6646$, $t(.95;42) = 1.6820$, $69.0103 \pm 1.6820(2.6646)$, $64.5284 \leq E\{Y_h\} \leq 73.4922$

b. $s\{\text{pred}\} = 10.405$, $69.0103 \pm 1.6820(10.405)$, $51.5091 \leq Y_{h(\text{new})} \leq 86.5115$

Chapter 7

MULTIPLE REGRESSION – II

7.4. a. $SSR(X_1) = 136,366$, $SSR(X_3|X_1) = 2,033,566$, $SSR(X_2|X_1, X_3) = 6,674$, $SSE(X_1, X_2, X_3) = 985,530$, df: 1, 1, 1,48.

b. H_0: $\beta_2 = 0$, H_a: $\beta_2 \neq 0$. $SSR(X_2|X_1, X_3) = 6,674$, $SSE(X_1, X_2, X_3) = 985,530$, $F^* = (6,674/1) \div (985,530/48) = 0.32491$, $F(.95; 1, 17) = 4.04265$. If $F^* \leq 4.04265$ conclude H_0, otherwise H_a. Conclude H_0. P-value = 0.5713.

c. Yes, $SSR(X_1)+SSR(X_2|X_1) = 136,366+5,726 = 142,092$, $SSR(X_2)+SSR(X_1|X_2) = 11,394.9 + 130,697.1 = 142,092$.

Yes.

7.5. a. $SSR(X_2) = 4,860.26$, $SSR(X_1|X_2) = 3,896.04$, $SSR(X_3|X_2, X_1) = 364.16$, $SSE(X_1, X_2, X_3) = 4,248.84$, df: 1, 1, 1, 42

b. H_0: $\beta_3 = 0$, H_a: $\beta_3 \neq 0$. $SSR(X_3|X_1, X_2) = 364.16$, $SSE(X_1, X_2, X_3) =4,248.84$, $F^* = (364.16/1)\div(4,248.84/42) = 3.5997$, $F(.975; 1, 42) = 5.4039$. If $F^* \leq 5.4039$ conclude H_0, otherwise H_a . Conclude H_0. P-value = 0.065.

7.6. H_0: $\beta_2 = \beta_3 = 0$, H_a: not both β_2 and $\beta_3 = 0$. $SSR(X_2, X_3|X_1) = 845.07$, $SSE(X_1, X_2, X_3) = 4,248.84$, $F^* = (845.07/2)\div(4,248.84/42) = 4.1768$, $F(.975; 2, 42) = 4.0327$. If $F^* \leq 4.0327$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0.022.

7.9. H_0: $\beta_1 = -1.0$, $\beta_2 = 0$; H_a: not both equalities hold. Full model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$, reduced model: $Y_i + X_{i1} = \beta_0 + \beta_3 X_{i3} + \varepsilon_i$. $SSE(F) = 4,248.84$, $df_F = 42$, $SSE(R) = 4,427.7$, $df_R = 44$, $F^* = [(4427.7-4248.84)/2]\div(4,248.84/42) = .8840$, $F(.975; 2, 42) = 4.0327$. If $F^* \leq 4.0327$ conclude H_0, otherwise H_a. Conclude H_0.

7.13. $R^2_{Y1} = .0431$, $R^2_{Y2} = .0036$, $R^2_{12} = .0072$, $R^2_{Y1|2} = 0.0415$, $R^2_{Y2|1} = 0.0019$, $R^2_{Y2|13} = .0067$ $R^2 = .6883$

7.14. a. $R^2_{Y1} = .6190$, $R^2_{Y1|2} = .4579$, $R^2_{Y1|23} = .4021$

b. $R^2_{Y2} = .3635$, $R^2_{Y2|1} = .0944$, $R^2_{Y2|13} = .0189$

7.17. a. $\hat{Y}^* = .17472X_1^* - .04639X_2^* + .80786X_3^*$

b. $R^2_{12} = .0072$, $R^2_{13} = .0021$,$R^2_{23} = .0129$

c. $s_Y = 249.003,\ s_1 = 55274.6,\ s_2 = .87738,\ s_3 = .32260\ b_1 = \frac{249.003}{55274.6}(.17472) = .00079,\ b_2 = \frac{249.003}{.87738}(-.04639) = -13.16562,\ b_3 = \frac{249.003}{5.32260}(.80786) = 623.5572,$
$b_0 = 4363.04 - .00079(302,693) + 13.16562(7.37058) - 623.5572(0.115385) = 4149.002.$

7.18. a. $\hat{Y}^* = -.59067X_1^* - .11062X_2^* - .23393X_3^*$

b. $R_{12}^2 = .32262,\ R_{13}^2 = .32456,\ R_{23}^2 = .44957$

c. $s_Y = 17.2365,\ s_1 = 8.91809,\ s_2 = 4.31356,\ s_3 = .29934,\ b_1 = \frac{17.2365}{8.91809}(-.59067) = -1.14162,\ b_2 = \frac{17.2365}{4.31356}(-.11062) = -.44203,\ b_3 = \frac{17.2365}{.29934}(-.23393) = -13.47008,$
$b_0 = 61.5652 + 1.14162(38.3913) + .44203(50.4348) + 13.47008(2.28696) = 158.4927$

7.25. a. $\hat{Y} = 4079.87 + 0.000935X_2$

c. No, $SSR(X_1) = 136,366,\ SSR(X_1|X_2) = 130,697$

d. $r_{12} = .0849$

7.26. a. $\hat{Y} = 156.672 - 1.26765X_1 - 0.920788X_2$

c. No, $SSR(X_1) = 8,275.3,\ SSR(X_1|X_3) = 3,483.89$

No, $SSR(X_2) = 4,860.26,\ SSR(X_2|X_3) = 708$

d. $r_{12} = .5680,\ r_{13} = .5697,\ r_{23} = .6705$

Chapter 8

MODELS FOR QUANTITATIVE AND QUALITATIVE PREDICTORS

8.4. a. $\hat{Y} = 82.9357 - 1.18396x + .0148405x^2$, $R^2 = .76317$

b. H_0: $\beta_1 = \beta_{11} = 0$, H_a: not both β_1 and $\beta_{11} = 0$. $MSR = 5915.31$, $MSE = 64.409$, $F^* = 5915.31/64.409 = 91.8398$, $F(.95; 2, 57) = 3.15884$. If $F^* \leq 3.15884$ conclude H_0, otherwise H_a. Conclude H_a.

c. $\hat{Y}_h = 99.2546$, $s\{\hat{Y}_h\} = 1.4833$, $t(.975; 57) = 2.00247$, $99.2546 \pm 2.00247(1.4833)$, $96.2843 \leq E\{Y_h\} \leq 102.2249$

d. $s\{\text{pred}\} = 8.16144$, $99.2546 \pm 2.00247(8.16144)$, $82.91156 \leq Y_{h(\text{new})} \leq 115.5976$

e. H_0: $\beta_{11} = 0$, H_a: $\beta_{11} \neq 0$. $s\{b_{11}\} = .00836$, $t^* = .0148405/.00836 = 1.7759$, $t(.975; 57) = 2.00247$. If $|t^*| \leq 2.00247$ conclude H_0, otherwise H_a. Conclude H_0. Alternatively, $SSR(x^2|x) = 203.1$, $SSE(x, x^2) = 3671.31$, $F^* = (203.1/1) \div (3671.31/57) = 3.15329$, $F(.95; 1, 57) = 4.00987$. If $F^* \leq 4.00987$ conclude H_0, otherwise H_a. Conclude H_0.

f. $\hat{Y} = 207.350 - 2.96432X + .0148405X^2$

g. $r_{X,X^2} = .9961$, $r_{x,x^2} = -.0384$

8.5. a.

i:	1	2	3	...	58	59	60
e_i:	-1.3238	-4.7592	-3.8091	...	-11.7798	-.8515	6.22023

b. H_0: $E\{Y\} = \beta_0 + \beta_1 x + \beta_{11}x^2$, H_a: $E\{Y\} \neq \beta_0 + \beta_1 x + \beta_{11}x^2$. $MSLF = 62.8154$, $MSPE = 66.0595$, $F^* = 62.8154/66.0595 = 0.95$, $F(.95; 29, 28) = 1.87519$. If $F^* \leq 1.87519$ conclude H_0, otherwise H_a. Conclude H_0.

c. $\hat{Y} = 82.92730 - 1.26789x + .01504x^2 + .000337x^3$

H_0: $\beta_{111} = 0$, H_a: $\beta_{111} \neq 0$. $s\{b_{111}\} = .000933$, $t^* = .000337/.000933 = .3612$, $t(.975; 56) = 2.00324$. If $|t^*| \leq 2.00324$ conclude H_0, otherwise H_a. Conclude H_0. Yes. Alternatively, $SSR(x^3|x, x^2) = 8.6$, $SSE(x, x^2, x^3) = 3662.78$, $F^* = (8.6/1) \div (3662.78/56) = .13148$, $F(.95; 1, 56) = 4.01297$. If $F^* \leq 4.01297$ conclude H_0, otherwise H_a. Conclude H_0. Yes.

8.19. a. $\hat{Y} = 2.81311 + 14.3394X_1 - 8.14120X_2 + 1.77739X_1X_2$

b. $H_0 : \beta_3 = 0$, $H_a : \beta_3 \neq 0$. $s\{b_3\} = .97459$, $t^* = 1.77739/.97459 = 1.8237$, $t(.95; 41) = 1.68288$. If $|t^*| \leq 1.68288$ conclude H_0, otherwise H_a. Conclude H_a. Alternatively, $SSR(X_1X_2|X_1, X_2) = 255.9$, $SSE(X_1, X_2, X_1X_2) = 3154.44$, $F^* = (255.9/1) \div (3154.44/41) = 3.32607$, $F(.90; 1, 41) = 2.83208$. If $F^* \leq 2.83208$ conclude H_0, otherwise H_a. Conclude H_a.

Chapter 9

BUILDING THE REGRESSION MODEL I: MODEL SELECTION AND VALIDATION

9.9.

Variables in Model	R_p^2	AIC_p	C_p	$PRESS_p$
None	0	262.916	88.16	13,970.10
X_1	.6190	220.529	8.35	5,569.56
X_2	.3635	244.131	42.11	9,254.49
X_3	.4155	240.214	35.25	8,451.43
X_1, X_2	.6550	217.968	5.60	5,235.19
X_1, X_3	.6761	215.061	2.81	4,902.75
X_2, X_3	.4685	237.845	30.25	8,115.91
X_1, X_2, X_3	.6822	216.185	4.00	5,057.886

9.10. b.

$$\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{bmatrix} 1 & .102 & .181 & .327 \\ & 1 & .519 & .397 \\ & & 1 & .782 \\ & & & 1 \end{bmatrix}$$

c. $\hat{Y} = -124.3820 + .2957X_1 + .0483X_2 + 1.3060X_3 + .5198X_4$

9.11. a.

Subset	$R_{a,p}^2$
X_1, X_3, X_4	.9560
X_1, X_2, X_3, X_4	.9555
X_1, X_3	.9269
X_1, X_2, X_3	.9247

9.17. a. X_1, X_3

b. .10

c. X_1, X_3

d. X_1, X_3

9.18. a. X_1, X_3, X_4

9.21. $PRESS = 760.974$, $SSE = 660.657$

9.22. a.

$$\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{bmatrix} 1 & .011 & .177 & .320 \\ & 1 & .344 & .221 \\ & & 1 & .871 \\ & & & 1 \end{bmatrix}$$

b.

	Model-building data set	Validation data set
b_0:	-127.596	-130.652
$s\{b_0\}$:	12.685	12.189
b_1:	.348	.347
$s\{b_1\}$:	.054	.048
b_3:	1.823	1.848
$s\{b_3\}$:	.123	.122
MSE:	27.575	21.446
R^2:	.933	.937

c. $MSPR = 486.519/25 = 19.461$

d. $\hat{Y} = -129.664 + .349X_1 + 1.840X_3$, $s\{b_0\} = 8.445$, $s\{b_1\} = .035$, $s\{b_3\} = .084$

Chapter 10

BUILDING THE REGRESSION MODEL II: DIAGNOSTICS

10.10.a&f.

i:	1	2	...	51	52
t_i:	$-.224$	1.225	...	-1.375	.453
D_i:	.0003	.0245	...	.0531	.0015

$t(.9995192; 47) = 3.523$. If $|t_i| \leq 3.523$ conclude no outliers, otherwise outliers. Conclude no outliers.

b. $2p/n = 2(4)/52 = .15385$. Cases 3, 5, 16, 21, 22, 43, 44, and 48.

c. $\mathbf{X}'_{\text{new}} = [\ 1 \quad 300,000 \quad 7.2 \quad 0\ ]$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 1.8628 & -.0000 & -.1806 & .0473 \\ & .0000 & -.0000 & -.0000 \\ & & .0260 & -.0078 \\ & & & .1911 \end{bmatrix}$$

$h_{\text{new, new}} = .01829$, no extrapolation

d.

		DFBETAS				
	DFFITS	b_0	b_1	b_2	b_3	D
Case 16:	$-.554$	$-.2477$	$-.0598$	.3248	$-.4521$	.0769
Case 22:	.055	.0304	$-.0253$	$-.0107$	.0446	.0008
Case 43:	.562	$-.3578$	.1338	.3262	.3566	.0792
Case 48:	$-.147$	.0450	$-.0938$	.0090	$-.1022$	.0055
Case 10:	.459	.3641	$-.1044$	$-.3142$	$-.0633$	.0494
Case 32:	$-.651$	.4095	.0913	$-.5708$	.1652	.0998
Case 38:	.386	$-.0996$	$-.0827$	.2084	$-.1270$	.0346
Case 40:	.397	.0738	$-.2121$	.0933	$-.1110$	.0365

e. Case 16: .161%, case 22: .015%, case 43: .164%, case 48: .042%, case 10: .167%, case 32: .227%, case 38: .152%, case 40: .157%.

10.11.a&f.

i:	1	2	...	45	46
t_i:	.0116	$-.9332$	...	$-.5671$	1.0449
D_i:	.000003	.015699	...	.006400	.024702

$t(.998913; 41) = 3.27$. If $|t_i| \leq 3.27$ conclude no outliers, otherwise outliers. Conclude no outliers.

b. $2p/n = 2(4)/46 = .1739$. Cases 9, 28, and 39.

c. $\mathbf{X}'_{\text{new}} = [\ 1 \quad 30 \quad 58 \quad 2.0\ ]$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 3.24771 & .00922 & -.06793 & -.06730 \\ & .00046 & -.00032 & -.00466 \\ & & .00239 & -.01771 \\ & & & .49826 \end{bmatrix}$$

$h_{\text{new, new}} = .3267$, extrapolation

d.

	$DFFITS$	$DFBETAS$ b_0	b_1	b_2	b_3	D
Case 11:	.5688	.0991	−.3631	−.1900	.3900	.0766
Case 17:	.6657	−.4491	−.4711	.4432	.0893	.1051
Case 27:	−.6087	−.0172	.4172	−.2499	.1614	.0867

e. Case 11: 1.10%, case 17: 1.32% , case 27: 1.12%.

10.16. b. $(VIF)_1 = 1.0086$, $(VIF)_2 = 1.0196$, $(VIF)_3 = 1.0144$.

10.17. b. $(VIF)_1 = 1.6323$, $(VIF)_2 = 2.0032$, $(VIF)_3 = 2.0091$

10.21. a. $(VIF)_1 = 1.305$, $(VIF)_2 = 1.300$, $(VIF)_3 = 1.024$

b&c.

i:	1	2	3	$\cdots$	32	33
e_i:	13.181	−4.042	3.060	$\cdots$	14.335	1.396
$e(Y \mid X_2, X_3)$:	26.368	−2.038	−31.111	$\cdots$	6.310	5.845
$e(X_1 \mid X_2, X_3)$:	−.330	−.050	.856	$\cdots$	.201	.111
$e(Y \mid X_1, X_3)$:	18.734	−17.470	8.212	$\cdots$	12.566	−8.099
$e(X_2 \mid X_1, X_3)$:	−7.537	18.226	−6.993	$\cdots$	2.401	12.888
$e(Y \mid X_1, X_2)$:	11.542	−7.756	15.022	$\cdots$	6.732	−15.100
$e(X_3 \mid X_1, X_2)$:	−2.111	−4.784	15.406	$\cdots$	−9.793	−21.247
Exp. value:	11.926	−4.812	1.886	$\cdots$	17.591	−.940

10.22. a. $\hat{Y}' = -2.0427 - .7120X'_1 + .7474X'_2 + .7574X'_3$, where $Y' = \log_e Y$, $X'_1 = \log_e X_1$, $X'_2 = \log_e(140 - X_2)$, $X'_3 = \log_e X_3$

b.

i:	1	2	3	$\cdots$	31	32	33
e_i:	−.0036	.0005	−.0316	$\cdots$	−.1487	.2863	.1208
Exp. value:	.0238	.0358	−.0481	$\cdots$	−.1703	.2601	.1164

c. $(VIF)_1 = 1.339$, $(VIF)_2 = 1.330$, $(VIF)_3 = 1.016$

d&e.

i:	1	2	3	$\cdots$	31	32	33
h_{ii}:	.101	.092	.176	$\cdots$	.058	.069	.149
t_i:	−.024	.003	−.218	$\cdots$	−.975	1.983	.829

$t(.9985; 28) = 3.25$. If $|t_i| \leq 3.25$ conclude no outliers, otherwise outliers. Conclude no outliers.

f.

		DFBETAS				
Case	*DFFITS*	b_0	b_1	b_2	b_3	D
28	.739	.530	−.151	−.577	−.187	.120
29	−.719	−.197	−.310	−.133	.420	.109

Chapter 11

BUILDING THE REGRESSION MODEL III: REMEDIAL MEASURES

11.7. a. $\hat{Y} = -5.750 + .1875X$

i:	1	2	3	4	5	6
e_i:	-3.75	5.75	-13.50	-16.25	-9.75	7.50

i:	7	8	9	10	11	12
e_i:	-10.50	26.75	14.25	-17.25	-1.75	18.50

b. $SSR^* = 123,753.125$, $SSE = 2,316.500$,

$X^2_{BP} = (123,753.125/2)/(2,316.500/12)^2 = 1.66$, $\chi^2(.90; 1) = 2.71$. If $X^2_{BP} \leq 2.71$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

d. $\hat{v} = -180.1 + 1.2437X$

i:	1	2	3	4	5	6
weight:	.01456	.00315	.00518	.00315	.01456	.00518

i:	7	8	9	10	11	12
weight:	.00518	.00315	.01456	.00315	.01456	.00518

e. $\hat{Y} = -6.2332 + .1891X$

f.

	Unweighted	Weighted
$s\{b_0\}$:	16.7305	13.1672
$s\{b_1\}$:	.0538	.0506

g. $\hat{Y} = -6.2335 + .1891X$

11.10. a. $\hat{Y} = 3.32429 + 3.76811X_1 + 5.07959X_2$

d. $c = .07$

e. $\hat{Y} = 6.06599 + 3.84335X_1 + 4.68044X_2$

11.11. a. $\hat{Y} = 1.88602 + 15.1094X$ (47 cases)

$\hat{Y} = -.58016 + 15.0352X$ (45 cases)

b.

i:	1	2	...	46	47
u_i:	-1.4123	-.2711	...	4.6045	10.3331

smallest weights: .13016 (case 47), .29217 (case 46)

c. $\hat{Y} = -.9235 + 15.13552X$

d. 2nd iteration: $\hat{Y} = -1.535 + 15.425X$

3rd iteration: $\hat{Y} = -1.678 + 15.444X$

smallest weights: .12629 (case 47), .27858 (case 46)

Chapter 12

AUTOCORRELATION IN TIME SERIES DATA

12.6. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 2.4015$, $d_L = 1.29$, $d_U = 1.38$. If $D > 1.38$ conclude H_0, if $D < 1.29$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

12.9. a. $\hat{Y} = -7.7385 + 53.9533X$, $s\{b_0\} = 7.1746$, $s\{b_1\} = 3.5197$

t:	1	2	3	4	5	6	7	8
e_t:	$-.0737$	$-.0709$	.5240	.5835	.2612	$-.5714$	-1.9127	$-.8276$

t:	9	10	11	12	13	14	15	16
e_t:	$-.6714$	.9352	1.803	.4947	.9435	.3156	$-.6714$	-1.0611

c. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = .857$, $d_L = 1.10$, $d_U = 1.37$. If $D > 1.37$ conclude H_0, if $D < 1.10$ conclude H_a, otherwise the test is inconclusive. Conclude H_a.

12.10. a. $r = .5784$, $2(1 - .5784) = .8432$, $D = .857$

b. $b'_0 = -.69434$, $b'_1 = 50.93322$

$\hat{Y}' = -.69434 + 50.93322X'$

$s\{b'_0\} = 3.75590$, $s\{b'_1\} = 4.34890$

c. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 1.476$, $d_L = 1.08$, $d_U = 1.36$. If $D > 1.36$ conclude H_0, if $D < 1.08$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

d. $\hat{Y} = -1.64692 + 50.93322X$

$s\{b_0\} = 8.90868$, $s\{b_1\} = 4.34890$

f. $F_{17} = -1.64692 + 50.93322(2.210) + .5784(-.6595) = 110.534$, $s\{\text{pred}\} = .9508$, $t(.975; 13) = 2.160$, $110.534 \pm 2.160(.9508)$, $108.48 \leq Y_{17(\text{new})} \leq 112.59$

g. $t(.975; 13) = 2.160$, $50.93322 \pm 2.160(4.349)$, $41.539 \leq \beta_1 \leq 60.327$.

12.11. a.

ρ:	.1	.2	.3	.4	.5
SSE:	11.5073	10.4819	9.6665	9.0616	8.6710

ρ:	.6	.7	.8	.9	1.0
SSE:	8.5032	8.5718	8.8932	9.4811	10.3408

$\rho = .6$

b. $\hat{Y}' = -.5574 + 50.8065X'$, $s\{b_0'\} = 3.5967$, $s\{b_1'\} = 4.3871$

c. $H_0 : \rho = 0$, $H_a : \rho > 0$. $D = 1.499$, $d_L = 1.08$, $d_U = 1.36$. If $D > 1.36$ conclude H_0, if $D < 1.08$ conclude H_a, otherwise test is inconclusive. Conclude H_0.

d. $\hat{Y} = -1.3935 + 50.8065X$, $s\{b_0\} = 8.9918$, $s\{b_1\} = 4.3871$

f. $F_{17} = -1.3935 + 50.8065(2.210) + .6(-.6405) = 110.505$, $s\{\text{pred}\} = .9467$, $t(.975; 13) = 2.160$, $110.505 \pm 2.160(.9467)$, $108.46 \leq Y_{17(\text{new})} \leq 112.55$

12.12. a. $b_1 = 49.80564$, $s\{b_1\} = 4.77891$

b. $H_0 : \rho = 0$, $H_a : \rho \neq 0$. $D = 1.75$ (based on regression with intercept term), $d_L = 1.08$, $d_U = 1.36$. If $D > 1.36$ and $4 - D > 1.36$ conclude H_0, if $D < 1.08$ or $4 - D < 1.08$ conclude H_a, otherwise the test is inconclusive. Conclude H_0.

c. $\hat{Y} = .71172 + 49.80564X$, $s\{b_1\} = 4.77891$

e. $F_{17} = .71172 + 49.80564(2.210) - .5938 = 110.188$, $s\{\text{pred}\} = .9078$, $t(.975; 14) = 2.145$, $110.188 \pm 2.145(.9078)$, $108.24 \leq Y_{17(\text{new})} \leq 112.14$

f. $t(.975; 14) = 2.145$, $49.80564 \pm 2.145(4.77891)$, $39.555 \leq \beta_1 \leq 60.056$

Chapter 13

INTRODUCTION TO NONLINEAR REGRESSION AND NEURAL NETWORKS

13.1. a. Intrinsically linear

$\log_e f(\mathbf{X}, \boldsymbol{\gamma}) = \gamma_0 + \gamma_1 X$

b. Nonlinear

c. Nonlinear

13.3. b. 300, 3.7323

13.5. a. $b_0 = -.5072512$, $b_1 = -0.0006934571$, $g_0^{(0)} = 0$, $g_1^{(0)} = .0006934571$, $g_2^{(0)} = .6021485$

b. $g_0 = .04823$, $g_1 = .00112$, $g_2 = .71341$

13.6. a. $\hat{Y} = .04823 + .71341\exp(-.00112X)$

City A

i:	1	2	3	4	5
$\hat{Y}_i$:	.61877	.50451	.34006	.23488	.16760
e_i:	.03123	−.04451	−.00006	.02512	.00240
Exp. value:	.04125	−.04125	−.00180	.02304	.00180

i:	6	7	8
$\hat{Y}_i$:	.12458	.07320	.05640
e_i:	.02542	−.01320	−.01640
Exp. value:	.02989	−.01777	−.02304

City B

i:	9	10	11	12	13
$\hat{Y}_i$:	.61877	.50451	.34006	.23488	.16760
e_i:	.01123	−.00451	−.04006	.00512	.02240
Exp. value:	.01327	−.00545	−.02989	.00545	.01777

i:	14	15	16
$\hat{Y}_i$:	.12458	.07320	.05640
e_i:	$-.00458$	.00680	$-.00640$
Exp. value:	$-.00923$	.00923	$-.01327$

13.7. $H_0 : E\{Y\} = \gamma_0 + \gamma_2 \exp(-\gamma_1 X)$, $H_a : E\{Y\} \neq \gamma_0 + \gamma_2 \exp(-\gamma_1 X)$.

$SSPE = .00290$, $SSE = .00707$, $MSPE = .00290/8 = .0003625$,

$MSLF = (.00707 - .00290)/5 = .000834$, $F^* = .000834/.0003625 = 2.30069$, $F(.99; 5, 8) = 6.6318$. If $F^* \leq 6.6318$ conclude H_0, otherwise H_a. Conclude H_0.

13.8. $s\{g_0\} = .01456$, $s\{g_1\} = .000092$, $s\{g_2\} = .02277$, $z(.9833) = 2.128$

$.04823 \pm 2.128(.01456)$ $\quad$ $.01725 \leq \gamma_0 \leq .07921$

$.00112 \pm 2.128(.000092)$ $\quad$ $.00092 \leq \gamma_1 \leq .00132$

$.71341 \pm 2.128(.02277)$ $\quad$ $.66496 \leq \gamma_2 \leq .76186$

13.9. a. $g_0 = .04948$, $g_1 = .00112$, $g_2 = .71341$, $g_3 = -.00250$

b. $z(.975) = 1.96$, $s\{g_3\} = .01211$, $-.00250 \pm 1.96(.01211)$, $-.02624 \leq \gamma_3 \leq .02124$, yes, no.

13.13. $g_0 = 100.3401$, $g_1 = 6.4802$, $g_2 = 4.8155$

13.14. a. $\hat{Y} = 100.3401 - 100.3401/[1 + (X/4.8155)^{6.4802}]$

b.

i:	1	2	3	4	5	6	7
$\hat{Y}_i$:	.0038	.3366	4.4654	11.2653	11.2653	23.1829	23.1829
e_i:	.4962	1.9634	-1.0654	.2347	$-.3653$	.8171	2.1171
Expected Val.:	.3928	1.6354	-1.0519	$-.1947$	$-.5981$	.8155	2.0516

i:	8	9	10	11	12	13	14
$\hat{Y}_i$:	39.3272	39.3272	56.2506	56.2506	70.5308	70.5308	80.8876
e_i:	.2728	-1.4272	-1.5506	.5494	.2692	-2.1308	1.2124
Expected Val.:	.1947	-1.3183	-1.6354	.5981	.0000	-2.0516	1.0519

i:	15	16	17	18	19
$\hat{Y}_i$:	80.8876	87.7742	92.1765	96.7340	98.6263
e_i:	$-.2876$	1.4258	2.6235	$-.5340$	-2.2263
Expected Val.:	$-.3928$	1.3183	2.7520	$-.8155$	-2.7520

13.15. $H_0 : E\{Y\} = \gamma_0 - \gamma_0/[1 + (X/\gamma_2)^{\gamma_1}]$, $H_a : E\{Y\} \neq \gamma_0 - \gamma_0/[1 + (X/\gamma_2)^{\gamma_1}]$.

$SSPE = 8.67999$, $SSE = 35.71488$, $MSPE = 8.67999/6 = 1.4467$, $MSLF = (35.71488 - 8.67999)/10 = 2.7035$, $F^* = 2.7035/1.4467 = 1.869$,$F(.99; 10, 6) = 7.87$. If $F^* \leq 7.87$ conclude H_0, otherwise H_a. Conclude H_0.

13.16. $s\{g_0\} = 1.1741$, $s\{g_1\} = .1943$, $s\{g_2\} = .02802$, $z(.985) = 2.17$

$100.3401 \pm 2.17(1.1741)$ $\quad$ $97.7923 \leq \gamma_0 \leq 102.8879$

$6.4802 \pm 2.17(.1943)$	$6.0586 \leq \gamma_1 \leq 6.9018$
$4.8155 \pm 2.17(.02802)$	$4.7547 \leq \gamma_2 \leq 4.8763$

Chapter 14

LOGISTIC REGRESSION, POISSON REGRESSION,AND GENERALIZED LINEAR MODELS

14.5. a. $E\{Y\} = [1 + \exp(-20 + .2X)]^{-1}$

b. 100

c. $X = 125: \quad \pi = .006692851, \; \pi/(1-\pi) = .006737947$

$X = 126: \quad \pi = .005486299, \; \pi/(1-\pi) = .005516565$

$005516565/.006737947 = .81873 = \exp(-.2)$

14.7. a. $b_0 = -4.80751$, $b_1 = .12508$, $\hat{\pi} = [1 + \exp(4.80751 - .12508X)]^{-1}$

c. 1.133

d. .5487

e. 47.22

14.11. a.

j:	1	2	3	4	5	6
p_j:	.144	.206	.340	.592	.812	.898

b. $b_0 = -2.07656$, $b_1 = .13585$

$\hat{\pi} = [1 + \exp(2.07656 - .13585X)]^{-1}$

d. 1.1455

e. .4903

f. 23.3726

14.14. a. $b_0 = -1.17717$, $b_1 = .07279$, $b_2 = -.09899$, $b_3 = .43397$

$\hat{\pi} = [1 + \exp(1.17717 - .07279X_1 + .09899X_2 - .43397X_3)]^{-1}$

b. $\exp(b_1) = 1.0755$, $\exp(b_2) = .9058$, $\exp(b_3) = 1.5434$

c. .0642

14.15. a. $z(.95) = 1.645$, $s\{b_1\} = .06676$, $\exp[.12508 \pm 1.645(.06676)]$,

$1.015 \leq \exp(\beta_1) \leq 1.265$

b. $H_0: \beta_1 = 0$, $H_a: \beta_1 \neq 0$. $b_1 = .12508$, $s\{b_1\} = .06676$, $z^* = .12508/.06676 = 1.8736$. $z(.95) = 1.645$, $|z^*| \leq 1.645$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value=.0609.

c. $H_0: \beta_1 = 0$, $H_a: \beta_1 \neq 0$. $G^2 = 3.99$, $\chi^2(.90; 1) = 2.7055$. If $G^2 \leq 2.7055$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value=.046

14.17. a. $z(.975) = 1.960$, $s\{b_1\} = .004772$, $.13585 \pm 1.960(.004772)$,

$$.1265 \leq \beta_1 \leq .1452, \qquad 1.1348 \leq \exp(\beta_1) \leq 1.1563.$$

b. $H_0: \beta_1 = 0$, $H_a: \beta_1 \neq 0$. $b_1 = .13585$, $s\{b_1\} = .004772$, $z^* = .13585/.004772 = 28.468$. $z(.975) = 1.960$, $|z^*| \leq 1.960$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value= 0+.

c. $H_0: \beta_1 = 0$, $H_a: \beta_1 \neq 0$. $G^2 = 1095.99$, $\chi^2(.95; 1) = 3.8415$. If $G^2 \leq 3.8415$, conclude H_0, otherwise conclude H_a. Conclude H_a. P-value= 0+.

14.20. a. $z(1-.1/[2(2)]) = z(.975) = 1.960$, $s\{b_1\} = .03036$, $s\{b_2\} = .03343$, $\exp\{30[.07279 \pm 1.960(.03036)]\}$, $1.49 \leq \exp(30\beta_1) \leq 52.92$, $\exp\{25[-.09899 \pm 1.960(.03343)]\}$,$.016 \leq \exp(2\beta_2) \leq .433$.

b. $H_0: \beta_3 = 0$, $H_a: \beta_3 \neq 0$. $b_3 = .43397$, $s\{b_3\} = .52132$, $z^* = .43397/.52132 = .8324$. $z(.975) = 1.96$, $|z^*| \leq 1.96$, conclude H_0, otherwise conclude H_a. Conclude H_0. P-value= .405.

c. $H_0: \beta_3 = 0$, $H_a: \beta_3 \neq 0$. $G^2 = .702$, $\chi^2(.95; 1) = 3.8415$. If $G^2 \leq 3.8415$, conclude H_0, otherwise conclude H_a. Conclude H_0.

d. $H_0: \beta_3 = \beta_4 = \beta_5 = 0$, H_a: not all $\beta_k = 0$, for $k = 3, 4, 5$. $G^2 = 1.534$, $\chi^2(.95; 3) = 7.81$. If $G^2 \leq 7.81$, conclude H_0, otherwise conclude H_a. Conclude H_0.

14.22. a. X_1 enters in step 1; X_2 enters in step 2;

no variables satisfy criterion for entry in step 3.

b. X_{11} is deleted in step 1; X_{12} is deleted in step 2; X_3 is deleted in step 3; X_{22} is deleted in step 4; X_1 and X_2 are retained in the model.

c. The best model according to the AIC_p criterion is based on X_1 and X_2. $AIC_3 = 111.795$.

d. The best model according to the SBC_p criterion is based on X_1 and X_2. $SBC_3 = 121.002$.

14.23.

j:	1	2	3	4	5	6
O_{j1}:	72	103	170	296	406	449
E_{j1}:	71.0	99.5	164.1	327.2	394.2	440.0
O_{j0}:	428	397	330	204	94	51
E_{j0}:	429.0	400.5	335.9	172.9	105.8	60.0

$H_0: E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$,

$H_a: E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}$.

$X^2 = 12.284$, $\chi^2(.99; 4) = 13.28$. If $X^2 \leq 13.28$ conclude H_0, otherwise H_a. Conclude H_0.

14.25. a.

Class j	$\hat{\pi}'$ Interval	Midpoint	n_j	p_j
1	-1.1 - under $-.4$	$-.75$	10	.3
2	$-.4$ - under .6	.10	10	.6
3	.6 - under 1.5	1.05	10	.7

b.

i:	1	2	3	$\cdots$	28	29	30
r_{SP_i}:	$-.6233$	1.7905	$-.6233$	$\cdots$	.6099	.5754	-2.0347

14.28. a.

j:	1	2	3	4	5	6	7	8
O_{j1}:	0	1	0	2	1	8	2	10
E_{j1}:	.2	.5	1.0	1.5	2.4	3.4	4.7	10.3
O_{j0}:	19	19	20	18	19	12	18	10
E_{j0}:	18.8	19.5	19.0	18.5	17.6	16.6	15.3	9.7

b. $H_0: E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1}$,

$H_a: E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1}$.

$X^2 = 12.116$, $\chi^2(.95; 6) = 12.59$. If $X^2 \leq 12.59$, conclude H_0, otherwise conclude H_a. Conclude H_0. P-value = .0594.

c.

i:	1	2	3	$\cdots$	157	158	159
dev_i:	$-.5460$	$-.5137$	1.1526	$\cdots$	.4248	.8679	1.6745

14.29 a.

i:	1	2	3	$\cdots$	28	29	30
h_{ii}:	.1040	.1040	.1040	$\cdots$	.0946	.1017	.1017

b.

i:	1	2	3	$\cdots$	28	29	30
ΔX_i^2:	.3885	3.2058	.3885	$\cdots$	4.1399	.2621	.2621
Δdev_i:	.6379	3.0411	.6379	$\cdots$	3.5071	.4495	.4495
D_i:	.0225	.1860	.0225	$\cdots$	.2162	.0148	.0148

14.32 a.

i:	1	2	3	$\cdots$	157	158	159
h_{ii}:	.0197	.0186	.0992	$\cdots$	.0760	.1364	.0273

b.

i:	1	2	3	$\cdots$	157	158	159
ΔX_i^2:	.1340	.1775	1.4352	$\cdots$	.0795	.6324	2.7200
Δdev_i:	.2495	.3245	1.8020	$\cdots$	.1478	.9578	2.6614
D_i:	.0007	.0008	.0395	$\cdots$	.0016	.0250	.0191

14.33. a. $z(.95) = 1.645$, $\hat{\pi}'_h = .19561$, $s^2\{b_0\} = 7.05306$, $s^2\{b_1\} = .004457$, $s\{b_0, b_1\} = -.175353$, $s\{\hat{\pi}'_h\} = .39428$, $.389 \leq \pi_h \leq .699$

b.

Cutoff	Renewers	Nonrenewers	Total
.40	18.8	50.0	33.3
.45	25.0	50.0	36.7
.50	25.0	35.7	30.0
.55	43.8	28.6	36.7
.60	43.8	21.4	33.3

c. Cutoff = .50. Area = .70089.

14.36. a. $\hat{\pi}'_h = -1.3953$, $s^2\{\hat{\pi}'_h\} = .1613$, $s\{\pi'_h\} = .4016$, $z(.95) = 1.645$. $L = -1.3953 - 1.645(.4016) = -2.05597$, $U = -1.3953 + 1.645(.4016) = -.73463$.
$L^* = [1 + \exp(2.05597)]^{-1} = .11345$, $U^* = [1 + \exp(.73463)]^{-1} = .32418$.

b.

Cutoff	Received	Not receive	Total
.05	4.35	62.20	66.55
.10	13.04	39.37	52.41
.15	17.39	26.77	44.16
.20	39.13	15.75	54.88

c. Cutoff = .15. Area = .82222.

14.38. a. $b_0 = 2.3529$, $b_1 = .2638$, $s\{b_0\} = .1317$, $s\{b_1\} = .0792$, $\hat{\mu} = \exp(2.3529 + .2638X)$.

b.

i:	1	2	3	$\cdots$	8	9	10
dev_i:	.6074	$-.4796$	$-.1971$	$\cdots$	.3482	.2752	.1480

c.

X_h:	0	1	2	3
Poisson:	10.5	13.7	17.8	23.2
Linear:	10.2	14.2	18.2	22.2

e. $\hat{\mu}_h = \exp(2.3529) = 10.516$

$$P(Y \leq 10 \mid X_h = 0) = \sum_{Y=0}^{10} \frac{(10.516)^Y \exp(-10.516)}{Y!}$$
$$= 2.7 \times 10^{-5} + \cdots + .1235 = .5187$$

f. $z(.975) = 1.96$, $.2638 \pm 1.96(.0792)$, $.1086 \leq \beta_1 \leq .4190$

Chapter 15

INTRODUCTION TO THE DESIGN OF EXPERIMENTAL AND OBSERVATIONAL STUDIES

15.9. a. Observational.

b. Factor: expenditures for research and development in the past three years.

Factor levels: low, moderate, and high.

c. Cross-sectional study.

d. Firm.

15.14. a. Experimental.

b. Factor 1: ingredient 1, with three levels (low, medium, high).

Factor 2: ingredient 2, with three levels (low, medium, high).

There are 9 factor-level combinations.

d. Completely randomized design.

e. Volunteer.

15.20. a. 2^3 factorial design with two replicates.

c. Rod.

15.23. a. H_0: $\bar{W} = 0$, H_a: $\bar{W} \neq 0$. $t^* = -.1915/.0112 = -17.10$, $t(.975, 19) = 2.093$. If $|t^*| > 2.093$ conclude H_0, otherwise conclude H_a. Conclude H_a. P-value = 0+.

Agree with results on page 670. They should agree.

b. H_0: $\beta_2 = \cdots = \beta_{20} = 0$, H_a: not all β_k $(k = 2, 3, \ldots, 20)$ equal zero. $F^* = [(.23586 - .023828)/(38 - 19)] \div [.023828/19] = 8.90$, $F(.95; 19, 19) = 2.17$. If $F^* > 2.17$ conclude H_0, otherwise conclude H_a. Conclude H_a. P-value = 0+.

Not of primary interest because blocking factor was used here to increase the precision.

Chapter 16

SINGLE-FACTOR STUDIES

16.7. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = 6.87778$, $\hat{Y}_{2j} = \bar{Y}_{2.} = 8.13333$, $\hat{Y}_{3j} = \bar{Y}_{3.} = 9.20000$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	.772	1.322	−.078	−1.078	.022	−.278
2	−1.433	−.033	1.267	.467	−.333	−.433
3	−.700	.500	.900	−1.400	.400	.300

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	−.578	.822	−.878			
2	.767	−.233	.167	.567	−1.033	.267

Yes

d.

Source	SS	df	MS
Between levels	20.125	2	10.0625
Error	15.362	24	.6401
Total	35.487	26	

e. H_0: all μ_i are equal ($i = 1, 2, 3$), H_a: not all μ_i are equal. $F^* = 10.0625/.6401 = 15.720$, $F(.95; 2, 24) = 3.40$. If $F^* \leq 3.40$ conclude H_0, otherwise H_a. Conclude H_a.

f. P-value = 0+

16.10. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = 21.500$, $\hat{Y}_{2j} = \bar{Y}_{2.} = 27.750$, $\hat{Y}_{3j} = \bar{Y}_{3.} = 21.417$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	1.500	3.500	−.500	.500	−.500	.500
2	.250	−.750	−.750	1.250	−1.750	1.250
3	1.583	−1.417	3.583	−.417	.583	1.583

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	−1.500	1.500	−2.500	.500	−2.500	−.500
2	−.750	2.250	.250	−.750	−1.750	1.250
3	−.417	−1.417	−2.417	−1.417	.583	−.417

d.

Source	SS	df	MS
Between ages	316.722	2	158.361
Error	82.167	33	2.490
Total	398.889	35	

e. H_0: all μ_i are equal ($i = 1, 2, 3$), H_a: not all μ_i are equal. $F^* = 158.361/2.490 = 63.599$, $F(.99; 2, 33) = 5.31$. If $F^* \leq 5.31$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

16.11. b. $\hat{Y}_{1j} = \bar{Y}_{1.} = .0735$, $\hat{Y}_{2j} = \bar{Y}_{2.} = .1905$, $\hat{Y}_{3j} = \bar{Y}_{3.} = .4600$, $\hat{Y}_{4j} = \bar{Y}_{4.} = .3655$,

$\hat{Y}_{5j} = \bar{Y}_{5.} = .1250$, $\hat{Y}_{6j} = \bar{Y}_{6.} = .1515$

c. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	−.2135	.1265	−.0035	.1065	.3065
2	.2695	−.0805	−.0705	.2795	.0495
3	−.2500	.3200	−.1400	−.0100	−.2400
4	.1245	.2145	.1545	−.0755	−.0955
5	−.3150	.1450	−.0650	−.0150	.1050
6	−.1015	−.2015	.1285	.3185	−.0315

i	$j=6$	$j=7$	$j=8$	$j=9$	$j=10$
1	.0265	−.1135	−.3435	.1965	−.2835
2	−.1305	−.3105	.1395	−.1305	−.2205
3	−.1100	.0800	−.2200	.0100	.1600
4	.1845	.0345	−.2255	.1145	−.0255
5	.0250	−.1150	.0950	.1650	.0150
6	.1185	−.0715	.0185	.2785	−.2215

i	$j=11$	$j=12$	$j=13$	$j=14$	$j=15$
1	.3165	−.1435	−.0935	.2065	.0165
2	−.1405	.3395	.2295	.0995	.1695
3	.0100	.0900	.1300	.2500	−.0100
4	−.3555	−.0355	−.1855	−.2355	.1145
5	.0750	.1750	−.2350	.1450	−.3250
6	.0485	−.1415	−.0515	.0085	−.2115

i	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	.0565	.1865	−.0035	−.0835	−.2635
2	−.1505	−.0205	−.1705	−.0805	−.0705
3	.0200	−.0200	.0400	−.2600	.1500
4	.1745	.1445	.0545	.0845	−.1655
5	.1150	.0750	.0150	.2250	−.3050
6	−.0215	.2785	.1985	−.2415	−.1015

Yes

d.

Source	SS	df	MS
Between machines	2.28935	5	.45787
Error	3.53060	114	.03097
Total	5.81995	119	

e. H_0: all μ_i are equal ($i = 1, ..., 6$), H_a: not all μ_i are equal. $F^* = .45787/.03097 = 14.78$, $F(.95; 5, 114) = 2.29$. If $F^* \leq 2.29$ conclude H_0, otherwise H_a. Conclude H_a.

f. P-value = 0+

16.18. a.

$$\mathbf{Y} = \begin{bmatrix} 7.6 \\ 8.2 \\ 6.8 \\ 5.8 \\ 6.9 \\ 6.6 \\ 6.3 \\ 7.7 \\ 6.0 \\ 6.7 \\ 8.1 \\ 9.4 \\ 8.6 \\ 7.8 \\ 7.7 \\ 8.9 \\ 7.9 \\ 8.3 \\ 8.7 \\ 7.1 \\ 8.4 \\ 8.5 \\ 9.7 \\ 10.1 \\ 7.8 \\ 9.6 \\ 9.5 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} \mu_. \\ \tau_1 \\ \tau_2 \end{bmatrix}$$

b.

$$\mathbf{X}\boldsymbol{\beta} = \begin{bmatrix} \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_1 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. + \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \\ \mu_. - \tau_1 - \tau_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_1 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_2 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \\ \mu_3 \end{bmatrix}$$

c. $\hat{Y} = 8.07037 - 1.19259X_1 + .06296X_2$, $\mu_.$ defined in (16.63)

d.

Source	SS	df	MS
Regression	20.125	2	10.0625
Error	15.362	24	.6401
Total	35.487	26	

e. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero. $F^* = 10.0625/.6401 = 15.720$, $F(.95; 2, 24) = 3.40$. If $F^* \leq 3.40$ conclude H_0, otherwise H_a. Conclude H_a.

16.21. a. $\hat{Y} = 23.55556 - 2.05556X_1 + 4.19444X_2$, $\mu_.$ defined in (16.63)

b.

Source	SS	df	MS
Regression	316.722	2	158.361
Error	82.167	33	2.490
Total	398.889	35	

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero. $F^* = 158.361/2.490 = 63.599$, $F(.99; 2, 33) = 5.31$. If $F^* \leq 5.31$ conclude H_0, otherwise H_a. Conclude H_a.

16.25. $\mu_{.} = 7.889$, $\phi = 2.457$, $1 - \beta \cong .95$

16.27 $\mu_{.} = 24$, $\phi = 6.12$, $1 - \beta > .99$

16.29 a.

$\triangle$:	10	15	20	30
n :	51	23	14	7

b.

$\triangle$:	10	15	20	30
n :	39	18	11	6

16.34. a. $\Delta/\sigma = .15/.15 = 1.0$, $n = 22$

b. $\phi = \frac{1}{.15}\left[\frac{22}{6}(.02968)\right]^{1/2} = 2.199$, $1 - \beta \geq .97$

c. $(.10\sqrt{n})/.15 = 3.1591$, $n = 23$

Chapter 17

ANALYSIS OF FACTOR LEVEL MEANS

17.8. a. $\bar{Y}_{1.} = 6.878$, $\bar{Y}_{2.} = 8.133$, $\bar{Y}_{3.} = 9.200$

b. $s\{\bar{Y}_{3.}\} = .327$, $t(.975; 24) = 2.064$, $9.200 \pm 2.064(.327), 8.525 \leq \mu_3 \leq 9.875$

c. $\hat{D} = \bar{Y}_{2.} - \bar{Y}_{1.} = 1.255$, $s\{\hat{D}\} = .353$, $t(.975; 24) = 2.064, 1.255 \pm 2.064(.353)$, $.526 \leq D \leq 1.984$

d. $\hat{D}_1 = \bar{Y}_{3.} - \bar{Y}_{2.} = 1.067$, $\hat{D}_2 = \bar{Y}_{3.} - \bar{Y}_{1.} = 2.322$, $\hat{D}_3 = \bar{Y}_{2.} - \bar{Y}_{1.} = 1.255$, $s\{\hat{D}_1\} = .400$, $s\{\hat{D}_2\} = .422$, $s\{\hat{D}_3\} = .353$, $q(.90; 3, 24) = 3.05$, $T = 2.157$

$1.067 \pm 2.157(.400)$	$.204 \leq D_1 \leq 1.930$
$2.322 \pm 2.157(.422)$	$1.412 \leq D_2 \leq 3.232$
$1.255 \pm 2.157(.353)$	$.494 \leq D_3 \leq 2.016$

e. $F(.90; 2, 24) = 2.54$, $S = 2.25$

$B = t(.9833; 24) = 2.257$

Yes

17.11. a. $\bar{Y}_{1.} = 21.500$, $\bar{Y}_{2.} = 27.750$, $\bar{Y}_{3.} = 21.417$

b. $MSE = 2.490$, $s\{\bar{Y}_{1.}\} = .456$, $t(.995; 33) = 2.733$, $21.500 \pm 2.733(.456)$, $20.254 \leq \mu_1 \leq 22.746$

c. $\hat{D} = \bar{Y}_{3.} - \bar{Y}_{1.} = -.083$, $s\{\hat{D}\} = .644$, $t(.995; 33) = 2.733$, $-.083 \pm 2.733(.644)$, $-1.843 \leq D \leq 1.677$

d. $H_0: 2\mu_2 - \mu_1 - \mu_3 = 0$, $H_a: 2\mu_2 - \mu_1 - \mu_3 \neq 0$. $F^* = (12.583)^2/1.245 = 127.17$, $F(.99; 1, 33) = 7.47$. If $F^* \leq 7.47$ conclude H_0, otherwise H_a. Conclude H_a.

e. $\hat{D}_1 = \bar{Y}_{3.} - \bar{Y}_{1.} = -.083$, $\hat{D}_2 = \bar{Y}_{3.} - \bar{Y}_{2.} = -6.333$, $\hat{D}_3 = \bar{Y}_{2.} - \bar{Y}_{1.} = 6.250$, $s\{\hat{D}_i\} = .644$ $(i = 1, 2, 3)$, $q(.90; 3, 33) = 3.01$, $T = 2.128$

$-.083 \pm 2.128(.644)$	$-1.453 \leq D_1 \leq 1.287$
$-6.333 \pm 2.128(.644)$	$-7.703 \leq D_2 \leq -4.963$
$6.250 \pm 2.128(.644)$	$4.880 \leq D_3 \leq 7.620$

f. $B = t(.9833; 33) = 2.220$, no

17.12. a. $\bar{Y}_{1.} = .0735$, $\bar{Y}_{2.} = .1905$, $\bar{Y}_{3.} = .4600, \bar{Y}_{4.} = .3655$, $\bar{Y}_{5.} = .1250$, $\bar{Y}_{6.} = .1515$

b. $MSE = .03097$, $s\{\bar{Y}_{1.}\} = .0394$, $t(.975; 114) = 1.981$, $.0735 \pm 1.981(.0394)$, $-.005 \leq \mu_1 \leq .152$

c. $\hat{D} = \bar{Y}_{2.} - \bar{Y}_{1.} = .1170$, $s\{\hat{D}\} = .0557$, $t(.975; 114) = 1.981$, $.1170 \pm 1.981(.0557)$, $.007 \leq D \leq .227$

e. $\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{4.} = -.2920$, $\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{5.} = -.0515$, $\hat{D}_3 = \bar{Y}_{4.} - \bar{Y}_{5.} = .2405$, $s\{\hat{D}_i\} = .0557$ $(i = 1, 2, 3)$, $B = t(.9833; 114) = 2.178$

Test i	Comparison i	t_i^*	Conclusion
1	D_1	-5.242	H_a
2	D_2	$-.925$	H_0
3	D_3	4.318	H_a

f. $q(.90; 6, 114) = 3.71$, $T = 2.623$, no

17.14. a. $\hat{L} = (\bar{Y}_{1.} + \bar{Y}_{2.})/2 - \bar{Y}_{3.} = (6.8778 + 8.1333)/2 - 9.200 = -1.6945$,
$s\{\hat{L}\} = .3712$, $t(.975; 24) = 2.064$, $-1.6945 \pm 2.064(.3712)$, $-2.461 \leq L \leq -.928$

b. $\hat{L} = (3/9)\bar{Y}_{1.} + (4/9)\bar{Y}_{2.} + (2/9)\bar{Y}_{3.} = 7.9518$, $s\{\hat{L}\} = .1540$, $t(.975; 24) = 2.064$, $7.9518 \pm 2.064(.1540)$, $7.634 \leq L \leq 8.270$

c. $F(.90;\ 2,\ 24) = 2.54$, $S = 2.254$; see also part (a) and Problem 17.8.

$1.067 \pm 2.254(.400)$	$.165 \leq D_1 \leq 1.969$
$2.322 \pm 2.254(.422)$	$1.371 \leq D_2 \leq 3.273$
$1.255 \pm 2.254(.353)$	$.459 \leq D_3 \leq 2.051$
$-1.6945 \pm 2.254(.3712)$	$-2.531 \leq L_1 \leq -.858$

17.16. a. $\hat{L} = (\bar{Y}_{3.} - \bar{Y}_{2.}) - (\bar{Y}_{2.} - \bar{Y}_{1.}) = \bar{Y}_{3.} - 2\bar{Y}_{2.} + \bar{Y}_{1.} = 21.4167-$
$2(27.7500) + 21.500 = -12.5833$, $s\{\hat{L}\} = 1.1158$, $t(.995; 33) = 2.733$,
$-12.5833 \pm 2.733(1.1158)$, $-15.632 \leq L \leq -9.534$

b. $\hat{D}_1 = \bar{Y}_{2.} - \bar{Y}_{1.} = 6.2500$, $\hat{D}_2 = \bar{Y}_{3.} - \bar{Y}_{2.} = -6.3333$, $\hat{D}_3 = \bar{Y}_{3.} - \bar{Y}_{1.} = -.0833$,
$\hat{L}_1 = \hat{D}_2 - \hat{D}_1 = -12.5833$, $s\{\hat{D}_i\} = .6442$ $(i = 1, 2, 3)$, $s\{\hat{L}_1\} = 1.1158$, $F(.90; 2, 33) = 2.47$, $S = 2.223$

$6.2500 \pm 2.223(.6442)$	$4.818 \leq D_1 \leq 7.682$
$-6.3333 \pm 2.223(.6442)$	$-7.765 \leq D_2 \leq -4.901$
$-.0833 \pm 2.223(.6442)$	$-1.515 \leq D_3 \leq 1.349$
$-12.5833 \pm 2.223(1.1158)$	$-15.064 \leq L_1 \leq -10.103$

17.17. a. $\hat{L} = (\bar{Y}_{1.} + \bar{Y}_{2.})/2 - (\bar{Y}_{3.} + \bar{Y}_{4.})/2 = (.0735 + .1905)/2 - (.4600 + .3655)/2$
$= -.28075$, $s\{\hat{L}\} = .03935$, $t(.975;\ 114) = 1.981$, $-.28075 \pm 1.981(.03935)$, $-.3587 \leq L \leq -.2028$

b. $\hat{D}_1 = -.1170$, $\hat{D}_2 = .0945$, $\hat{D}_3 = -.0265$, $\hat{L}_1 = -.28075$, $\hat{L}_2 = -.00625$, $\hat{L}_3 = -.2776$, $\hat{L}_4 = .1341$, $s\{\hat{D}_i\} = .0557$ $(i = 1, 2, 3)$, $s\{\hat{L}_1\} = s\{\hat{L}_2\} = .03935$, $s\{\hat{L}_3\} = s\{\hat{L}_4\} = .03408$, $B = t(.99286; 114) = 2.488$

$-.1170 \pm 2.488(.0557)$	$-.2556 \leq D_1 \leq .0216$

$.0945 \pm 2.488(.0557)$	$-.0441 \leq D_2 \leq .2331$
$-.0265 \pm 2.488(.0557)$	$-.1651 \leq D_3 \leq .1121$
$-.28075 \pm 2.488(.03935)$	$-.3787 \leq L_1 \leq -.1828$
$-.00625 \pm 2.488(.03935)$	$-.1042 \leq L_2 \leq .0917$
$-.2776 \pm 2.488(.03408)$	$-.3624 \leq L_3 \leq -.1928$
$.1341 \pm 2.488(.03408)$	$.0493 \leq L_4 \leq .2189$

17.19. a. $L_1 = \mu_1 - \mu_.$ $L_2 = \mu_2 - \mu_.$

$L_3 = \mu_3 - \mu_.$ $L_4 = \mu_4 - \mu_.$

$L_5 = \mu_5 - \mu_.$ $L_6 = \mu_6 - \mu_.$

$\hat{L}_1 = .0735 - .2277 = -.1542,\ \hat{L}_2 = .1905 - .2277 = -.0372$

$\hat{L}_3 = .4600 - .2277 = .2323,\ \hat{L}_4 = .3655 - .2277 = .1378$

$\hat{L}_5 = .1250 - .2277 = -.1027,\ \hat{L}_6 = .1515 - .2277 = -.0762$

$$s\{\hat{L}_i\} = \sqrt{\frac{.03097}{20}\left(\frac{25}{36}\right) + \frac{.03097}{36}\left(\frac{5}{20}\right)} = .0359$$

$B = t(.99583; 114) = 2.685$

$-.1542 \pm 2.685(.0359)$	$.2506 \leq L_1 \leq -.0578$
$-.0372 \pm 2.685(.0359)$	$-.1336 \leq L_2 \leq .0592$
$.2323 \pm 2.685(.0359)$	$.1359 \leq L_3 \leq .3287$
$.1378 \pm 2.685(.0359)$	$.0414 \leq L_4 \leq .2342$
$-.1027 \pm 2.685(.0359)$	$-.1991 \leq L_5 \leq -.0063$
$-.0762 \pm 2.685(.0359)$	$-.1726 \leq L_6 \leq .0202$

Conclude not all μ_i are equal.

17.25. Bonferroni, $n = 45$

17.29. a. $\hat{Y} = .18472 + .06199x + .01016x^2$

b. e_{ij} :

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
1	−.2310	.1090	−.0210	.0890	.2890	.0090	−.1310
2	.2393	−.1107	−.1007	.2493	.0193	−.1607	−.3407
3	−.2440	.3260	−.1340	−.0040	−.2340	−.1040	.0860
4	.1268	.2168	.1568	−.0732	−.0932	.1868	.0368
5	−.2969	.1631	−.0469	.0031	.1231	.0431	−.0969
6	−.0802	−.1802	.1498	.3398	−.0102	.1398	−.0502

i	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$
1	−.3610	.1790	−.3010	.2990	−.1610	−.1110	.1890
2	.1093	−.1607	−.2507	−.1707	.3093	.1993	.0693
3	−.2140	.0160	.1660	.0160	.0960	.1360	.2560
4	−.2232	.1168	−.0232	−.3532	−.0332	−.1832	−.2332
5	.1131	.1831	.0331	.0931	.1931	−.2169	.1631
6	.0398	.2998	−.2002	.0698	−.1202	−.0302	.0298

i	$j=15$	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	−.0010	.0390	.1690	−.0210	−.1010	−.2810
2	.1393	−.1807	−.0507	−.2007	−.1107	−.1007
3	−.0040	.0260	−.0140	.0460	−.2540	.1560
4	.1168	.1768	.1468	.0568	.0868	−.1632
5	−.3069	.1331	.0931	.0331	.2431	−.2869
6	−.1902	−.0002	.2998	.2198	−.2202	−.0802

c. $H_0: E\{Y\} = \beta_0 + \beta_1 x + \beta_{11} x^2$, $H_a: E\{Y\} \neq \beta_0 + \beta_1 x + \beta_{11} x^2$. $SSPE = 3.5306$, $SSLF = .0408$, $F^* = (.0408/3) \div (3.5306/114) = .439, F(.99; 3, 114) = 3.96$. If $F^* \leq 3.96$ conclude H_0, otherwise H_a. Conclude H_0.

d. $H_0: \beta_{11} = 0$, $H_a: \beta_{11} \neq 0$. $s\{b_{11}\} = .00525, t^* = .01016/.00525 = 1.935, t(.995; 117) = 2.619$. If $|t^*| \leq 2.619$ conclude H_0, otherwise H_a. Conclude H_0.

Chapter 18

ANOVA DIAGNOSTICS AND REMEDIAL MEASURES

18.4. a. See Problem 16.7c.

b. $r = .992$

c. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	.9557	1.8377	−.1010	−1.4623	.0288	−.3615
2	−1.9821	−.0426	1.7197	.6011	−.4277	−.5575
3	−.9568	.6768	1.2464	−2.0391	.5395	.4035

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	−.7592	1.0945	−1.1728			
2	1.0009	−.2988	.2132	.7326	−1.3737	.3417

H_0: no outliers, H_a: at least one outlier. $t(.999815; 23) = 4.17$.

If $| t_{ij} | \leq 4.17$ conclude H_0, otherwise H_a. Conclude H_0.

18.7. a. See Problem 16.10c.

b. $r = .984$

d. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	.9927	2.4931	−.3265	.3265	−.3265	.3265
2	.1630	−.4907	−.4907	.8234	−1.1646	.8234
3	1.0497	−.9360	2.5645	−.2719	.3811	1.0497

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	−.9927	.9927	−1.7017	.3265	−1.7017	−.3265
2	−.4907	1.5185	.1630	−.4907	−1.1646	.8234
3	−.2719	−.9360	−1.6401	−.9360	.3811	−.2719

H_0: no outliers, H_a: at least one outlier. $t(.99965; 32) = 3.75$.

If $|t_{ij}| \leq 3.75$ conclude H_0, otherwise H_a. Conclude H_0.

18.8. a. See Problem 16.11c.

b. $r = .992$

d. t_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$
1	−1.2477	.7360	−.0203	.6192	1.8045	.1538	−.6601
2	1.5815	−.4677	−.4095	1.6415	.2874	−.7594	−1.8287
3	−1.4648	1.8864	−.8150	−.0580	−1.4052	−.6396	.4648
4	.7243	1.2537	.9000	−.4386	−.5551	1.0764	.2003
5	−1.8560	.8443	−.3775	−.0871	.6105	.1451	−.6688
6	−.5901	−1.1767	.7477	1.8773	−.1829	.6893	−.4153

i	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$
1	−2.0298	1.1472	−1.6656	1.8651	−.8355	−.5434	1.2063
2	.8121	−.7594	−1.2892	−.8179	2.0053	1.3427	.5784
3	−1.2863	.0580	.9323	.0580	.5230	.7565	1.4648
4	−1.3189	.6659	−.1480	−2.1035	−.2061	−1.0823	−1.3784
5	.5522	.9616	.0871	.4357	1.0204	−1.3754	.8443
6	.1074	1.6355	−1.2952	.2816	−.8238	−.2990	.0493

i	$j=15$	$j=16$	$j=17$	$j=18$	$j=19$	$j=20$
1	.0958	.3281	1.0882	−.0203	−.4852	−1.5455
2	.9881	−.8765	−.1190	−.9940	−.4677	−.4095
3	−.0580	.1161	−.1161	.2322	−1.5246	.8736
4	.6659	1.0175	.8414	.3165	.4910	−.9646
5	−1.9168	.6688	.4357	.0871	1.3160	−1.7954
6	−1.2359	−.1248	1.6355	1.1590	−1.4141	−.5901

H_0 : no outliers, H_a : at least one outlier. $t(.9999417; 113) = 4.08$.

If $|t_{ij}| \leq 4.08$ conclude H_0, otherwise H_a. Conclude H_0.

18.11. H_0: all σ_i^2 are equal ($i = 1, 2, 3$), H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 6.80$, $\tilde{Y}_2 = 8.20$, $\tilde{Y}_3 = 9.55$, $MSTR = .0064815$, $MSE = .26465$,

$F_L^* = .0064815/.26465 = .02$, $F(.95; 2, 24) = 3.40$. If $F_L^* \leq 3.40$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .98

18.13. a. H_0: all σ_i^2 are equal ($i = 1, 2, 3$), H_a: not all σ_i^2 are equal.

$s_1 = 1.7321$, $s_2 = 1.2881$, $s_3 = 1.6765$, $n_i \equiv 12$, $H^* = (1.7321)^2/(1.2881)^2 = 1.808$,

$H(.99; 3, 11) = 6.75$. If $H^* \leq 6.75$ conclude H_0, otherwise H_a. Conclude H_0. P-value > .05

b. $\tilde{Y}_1 = 21.5$, $\tilde{Y}_2 = 27.5$, $\tilde{Y}_3 = 21.0$, $MSTR = .19444$, $MSE = .93434$,

$F_L^* = .19444/.93434 = .21$, $F(.99; 2, 33) = 5.31$. If $F_L^* \leq 5.31$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .81

18.14. a. H_0: all σ_i^2 are equal ($i = 1, ..., 6$), H_a: not all σ_i^2 are equal.

$s_1 = .1925$, $s_2 = .1854$, $s_3 = .1646$, $s_4 = .1654$, $s_5 = .1727$, $s_6 = .1735$, $n_i \equiv 20$, $H^* = (.1925)^2/(.1646)^2 = 1.3677$, $H(.99;\ 6,\ 19) = 5.2$. If $H^* \leq 5.2$ conclude H_0, otherwise H_a. Conclude H_0. P-value > .05

b. $\tilde{Y}_1 = .08$, $\tilde{Y}_2 = .12$, $\tilde{Y}_3 = .47$, $\tilde{Y}_4 = .41$, $\tilde{Y}_5 = .175$, $\tilde{Y}_6 = .125$, $MSTR = .002336$, $MSE = .012336$, $F_L^* = .002336/.012336 = .19$, $F(.99; 5, 114) = 3.18$. If $F_L^* \leq 3.18$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .97

18.17. a. $\bar{Y}_{1.} = 3.5625$, $\bar{Y}_{2.} = 5.8750$, $\bar{Y}_{3.} = 10.6875$, $\bar{Y}_{4.} = 15.5625$

e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	.4375	−.5625	−1.5625	−.5625	.4375	.4375
2	1.1250	.1250	−1.8750	.1250	1.1250	−3.8750
3	1.3125	−4.6875	3.3125	1.3125	−.6875	−1.6875
4	.4375	−1.5625	−9.5625	3.4375	−3.5625	−5.5625

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	−.5625	2.4375	1.4375	.4375	−1.5625	.4375
2	3.1250	−.8750	−.8750	3.1250	−2.8750	2.1250
3	1.3125	6.3125	−3.6875	−4.6875	1.3125	.3125
4	−.5625	8.4375	−5.5625	7.4375	1.4375	4.4375

i	$j=13$	$j=14$	$j=15$	$j=16$
1	.4375	−1.5625	−.5625	.4375
2	.1250	−1.8750	1.1250	.1250
3	−4.6875	2.3125	−.6875	3.3125
4	−.5625	2.4375	−7.5625	6.4375

c. H_0: all σ_i^2 are equal $(i = 1, 2, 3, 4)$, H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = 4.0$, $\tilde{Y}_2 = 6.0$, $\tilde{Y}_3 = 11.5$, $\tilde{Y}_4 = 16.5$, $MSTR = 37.1823$, $MSE = 3.8969$, $F_L^* = 37.1823/3.8969 = 9.54$, $F(.95;\ 3,\ 60) = 2.76$. If $F_L^* \leq 2.76$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d.

i	$\bar{Y}_{i.}$	s_i
1	3.5625	1.0935
2	5.8750	1.9958
3	10.6875	3.2397
4	16.5625	5.3786

e.

λ	SSE
−1.0	1,038.26
−.8	790.43
−.6	624.41
−.4	516.16
−.2	450.16
−.1	429.84
0	416.84

λ	SSE
.1	410.65
.2	410.92
.4	430.49
.6	476.68
.8	553.64
1.0	669.06

Yes

18.18. a. $\bar{Y}'_{1.} = .5314$, $\bar{Y}'_{2.} = .7400$, $\bar{Y}'_{3.} = 1.0080$, $\bar{Y}'_{4.} = 1.1943$

e'_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$
1	.071	−.054	−.230	−.054	.071	.071	−.054	.247
2	.105	.038	−.138	.038	.105	−.439	.214	−.041
3	.071	−.230	.138	.071	−.008	−.054	.071	.222
4	.036	−.018	−.349	.107	−.080	−.153	.010	.204

i	$j=9$	$j=10$	$j=11$	$j=12$	$j=13$	$j=14$	$j=15$	$j=16$
1	.168	.071	−.230	.071	.071	−.230	−.054	.071
2	−.041	.214	−.263	.163	.038	−.138	.105	.038
3	−.163	−.230	.071	.033	−.230	.106	−.008	.138
4	−.153	.186	.061	.128	.010	.085	−.240	.167

b. $r = .971$

c. H_0: all σ_i^2 are equal $(i = 1, 2, 3, 4)$, H_a: not all σ_i^2 are equal.

$\tilde{Y}_1 = .6021$, $\tilde{Y}_2 = .7782$, $\tilde{Y}_3 = 1.0603$, $\tilde{Y}_4 = 1.2173$, $MSTR = .001214$,

$MSE = .01241$, $F_L^* = .001214/.01241 = .10$, $F(.95;\ 3,\ 60) = 2.76$.

If $F_L^* \leq 2.76$ conclude H_0, otherwise H_a. Conclude H_0.

18.20.

i:	1	2	3	4
s_i:	1.09354	1.99583	3.23973	5.37858
w_i:	.83624	.25105	.09528	.034567

H_0: all μ_i are equal $(i = 1, 2, 3, 4)$, H_a: not all μ_i are equal.

$SSE_w(F) = 60$, $df_F = 60$, $SSE_w(R) = 213.9541$, $df_R = 63$,

$F_w^* = [(213.9541 - 60)/3] \div (60/60) = 51.32$, $F(.99;\ 3,\ 60) = 4.13$.

If $F_w^* \leq 4.13$ conclude H_0, otherwise H_a. Conclude H_a.

18.23. a. H_0: all μ_i are equal $(i = 1, 2, 3, 4)$, H_a: not all μ_i are equal.

$MSTR = 470.8125$, $MSE = 28.9740$, $F_R^* = 470.8125/28.9740 = 16.25$,

$F(.95; 2, 24) = 3.40$. If $F_R^* \leq 3.40$ conclude H_0, otherwise H_a. Conclude H_a.

b. P-value = 0+

e. $\bar{R}_{1.} = 6.50$, $\bar{R}_{2.} = 15.50$, $\bar{R}_{3.} = 22.25$, $B = z(.9833) = 2.13$

Comparison	Testing Limits	
1 and 2	$-9.00 \pm 2.13(3.500)$	-16.455 and -1.545
1 and 3	$15.75 \pm 2.13(4.183)$	-24.660 and -9.840
2 and 3	$-6.75 \pm 2.13(3.969)$	-15.204 and 1.704

Group 1	**Group 2**
Low Level $i=1$	Moderate level $i=2$
	High level $i=3$

18.24. a. H_0: all μ_i are equal $(i = 1, 2, 3)$, H_a: not all μ_i are equal.

$MSTR = 1,297.0000$, $MSE = 37.6667$, $F_R^* = 1,297.0000/37.6667 = 34.43$,

$F(.99; 2, 33) = 5.31$. If $F_R^* \leq 5.31$ conclude H_0, otherwise H_a. Conclude H_a.

b. P-value = 0+

e. $\bar{R}_{1.} = 12.792$, $\bar{R}_{2.} = 30.500$, $\bar{R}_{3.} = 12.208$, $B = z(.9833) = 2.128$

Comparison	Testing Limits	
1 and 2	$-17.708 \pm 2.128(4.301)$	-26.861 and -8.555
1 and 3	$.584 \pm 2.128(4.301)$	-8.569 and 9.737
2 and 3	$18.292 \pm 2.128(4.301)$	9.140 and 27.445

Group 1	**Group 2**
Young $i = 1$	Middle $i = 2$
Elderly $i = 3$	

Chapter 19

TWO-FACTOR STUDIES WITH EQUAL SAMPLE SIZES

19.4. a. $\mu_{1.} = 31$, $\mu_{2.} = 37$

b. $\alpha_1 = \mu_{1.} - \mu_{..} = 31 - 34 = -3$, $\alpha_2 = \mu_{2.} - \mu_{..} = 37 - 34 = 3$

19.7. a. $E\{MSE\} = 1.96$, $E\{MSA\} = 541.96$

19.10. a. $\bar{Y}_{11.} = 21.66667$, $\bar{Y}_{12.} = 21.33333$, $\bar{Y}_{21.} = 27.83333$,

$\bar{Y}_{22.} = 27.66667$, $\bar{Y}_{31.} = 22.33333$, $\bar{Y}_{32.} = 20.50000$

b. e_{ijk}:

i	$j=1$	$j=2$	i	$j=1$	$j=2$	i	$j=1$	$j=2$
1	$-.66667$	$-.33333$	2	2.16667	-1.66667	3	2.66667	2.50000
	1.33333	.66667		1.16667	1.33333		$-.33333$	-1.50000
	-2.66667	-1.33333		-1.83333	$-.66667$		.66667	$-.50000$
	.33333	$-.33333$		.16667	.33333		-1.33333	.50000
	.33333	-2.33333		$-.83333$	$-.66667$		$-.33333$	$-.50000$
	1.33333	3.66667		$-.83333$	1.33333		-1.33333	$-.50000$

d. $r = .986$

19.11. b.

Source	SS	df	MS
Treatments	327.222	5	65.444
A (age)	316.722	2	158.361
B (gender)	5.444	1	5.444
AB interactions	5.056	2	2.528
Error	71.667	30	2.389
Total	398.889	35	

Yes, factor A (age) accounts for most of the total variability.

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.

$F^* = 2.528/2.389 = 1.06$, $F(.95; 2, 30) = 3.32$.

If $F^* \leq 3.32$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .36$

d. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero.

$F^* = 158.361/2.389 = 66.29$, $F(.95; 2, 30) = 3.32$.

If $F^* \leq 3.32$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = 5.444/2.389 = 2.28$, $F(.95; 1, 30) = 4.17$.

If $F^* \leq 4.17$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .14

e. $\alpha \leq .143$

g. $SSA = SSTR$, $SSB + SSAB + SSE = SSE$, yes

19.14. a. $\bar{Y}_{11.} = 2.475$, $\bar{Y}_{12.} = 4.600$, $\bar{Y}_{13.} = 4.575$, $\bar{Y}_{21.} = 5.450$, $\bar{Y}_{22.} = 8.925$, $\bar{Y}_{23.} = 9.125$, $\bar{Y}_{31.} = 5.975$, $\bar{Y}_{32.} = 10.275$, $\bar{Y}_{33.} = 13.250$

b. e_{ijk}:

i	$j=1$	$j=2$	$j=3$
1	−.075	0	.225
	.225	−.400	−.075
	−.175	.300	−.175
	.025	.100	.025

i	$j=1$	$j=2$	$j=3$
2	.350	−.025	−.025
	−.250	.175	.175
	.050	−.225	−.425
	−.150	.075	.275

i	$j=1$	$j=2$	$j=3$
3	.125	−.375	.250
	−.275	.225	−.250
	−.075	.325	.050
	.225	−.175	−.050

d. $r = .988$

19.15. b.

Source	SS	df	MS
Treatments	373.105	8	46.638
A (ingredient 1)	220.020	2	110.010
B (ingredient 2)	123.660	2	61.830
AB interactions	29.425	4	7.356
Error	1.625	27	.0602
Total	374.730	35	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 7.356/.0602 = 122.19$, $F(.95; 4, 27) = 2.73$. If $F^* \leq 2.73$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero. $F^* = 110.010/.0602 = 1,827.41$, $F(.95; 2, 27) = 3.35$. If $F^* \leq 3.35$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all β_j equal zero ($j = 1, 2, 3$), H_a: not all β_j equal zero. $F^* = 61.830/.0602 = 1,027.08$, $F(.95; 2, 27) = 3.35$. If $F^* \leq 3.35$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\alpha \leq .143$

19.20. a. $\bar{Y}_{11.} = 222.00$, $\bar{Y}_{12.} = 106.50$, $\bar{Y}_{13.} = 60.50$, $\bar{Y}_{21.} = 62.25$, $\bar{Y}_{22.} = 44.75$, $\bar{Y}_{23.} = 38.75$

b. e_{ijk}:

i	$j=1$	$j=2$	$j=3$
1	18	3.5	−4.5
	−16	11.5	−.5
	−5	−3.5	7.5
	3	−11.5	−2.5

i	$j=1$	$j=2$	$j=3$
2	8.75	2.25	−1.75
	−9.25	7.25	−5.75
	5.75	−13.75	1.25
	−5.25	4.25	6.25

d. $r = .994$

19.21. b.

Source	SS	df	MS
Treatments	96,024.37500	5	19,204.87500
A (type)	39,447.04167	1	39,447.04167
B (years)	36,412.00000	2	18,206.00000
AB interactions	20,165.33333	2	10,082.66667
Error	1,550.25000	18	86.12500
Total	97,574.62500	23	

c. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = 10,082.66667/86.12500 = 117.07$, $F(.99; 2, 18) = 6.01$. If $F^* \leq 6.01$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero. $F^* = 39,447.04167/86.12500 = 458.02$, $F(.99; 1, 18) = 8.29$. If $F^* \leq 8.29$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all β_j equal zero ($j = 1, 2, 3$), H_a: not all β_j equal zero. $F^* = 18,206.00000/86.12500 = 211.39$, $F(.99; 2, 18) = 6.01$. If $F^* \leq 6.01$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\alpha \leq .030$

19.27. a. $B = t(.9975; 75) = 2.8925$, $q(.95; 5, 75) = 3.96$, $T = 2.800$

b. $B = t(.99167; 27) = 2.552$, $q(.95; 3, 27) = 3.51$, $T = 2.482$

19.30. a. $s\{\bar{Y}_{11.}\} = .631$, $t(.975; 30) = 2.042$, $21.66667 \pm 2.042(.631)$, $20.378 \leq \mu_{11} \leq 22.955$

b. $\bar{Y}_{.1.} = 23.94$, $\bar{Y}_{.2.} = 23.17$

c. $\hat{D} = .77$, $s\{\hat{D}\} = .5152$, $t(.975; 30) = 2.042$, $.77 \pm 2.042(.5152)$, $-.282 \leq D \leq 1.822$

d. $\bar{Y}_{1..} = 21.50$, $\bar{Y}_{2..} = 27.75$, $\bar{Y}_{3..} = 21.42$

e. $\hat{D}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -6.25$, $\hat{D}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = .08$, $\hat{D}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = 6.33$, $s\{\hat{D}_i\} = .631$ ($i = 1, 2, 3$), $q(.90; 3, 30) = 3.02$, $T = 2.1355$

$$-6.25 \pm 2.1355(.631) \qquad -7.598 \leq D_1 \leq -4.902$$
$$.08 \pm 2.1355(.631) \qquad -1.268 \leq D_2 \leq 1.428$$
$$6.33 \pm 2.1355(.631) \qquad 4.982 \leq D_3 \leq 7.678$$

f. Yes

g. $\hat{L} = -6.29$, $s\{\hat{L}\} = .5465$, $t(.976; 30) = 2.042$, $-6.29 \pm 2.042(.5465)$, $-7.406 \leq L \leq -5.174$

h. $L = .3\mu_{12} + .6\mu_{22} + .1\mu_{32}$, $\hat{L} = 25.05000$, $s\{\hat{L}\} = .4280$, $t(.975; 30) = 2.042$, $25.05000 \pm 2.042(.4280)$, $24.176 \leq L \leq 25.924$

19.32. a. $s\{\bar{Y}_{23.}\} = .1227$, $t(.975; 27) = 2.052$, $9.125 \pm 2.052(.1227)$, $8.873 \leq \mu_{23} \leq 9.377$

b. $\hat{D} = 2.125$, $s\{\hat{D}\} = .1735$, $t(.975; 27) = 2.052$, $2.125 \pm 2.052(.1735)$, $1.769 \leq D \leq 2.481$

c. $\hat{L}_1 = 2.1125$, $\hat{L}_2 = 3.5750$, $\hat{L}_3 = 5.7875$, $\hat{L}_4 = 1.4625$, $\hat{L}_5 = 3.6750$, $\hat{L}_6 = 2.2125$, $s\{\hat{L}_i\} = .1502$ $(i = 1, 2, 3)$, $s\{\hat{L}_i\} = .2125$ $(i = 4, 5, 6)$, $F(.90; 8, 27) = 1.90$, $S = 3.899$

$2.1125 \pm 3.899(.1502)$	$1.527 \leq L_1 \leq 2.698$
$3.5750 \pm 3.899(.1502)$	$2.989 \leq L_2 \leq 4.161$
$5.7875 \pm 3.899(.1502)$	$5.202 \leq L_3 \leq 6.373$
$1.4625 \pm 3.899(.2125)$	$.634 \leq L_4 \leq 2.291$
$3.6750 \pm 3.899(.2125)$	$2.846 \leq L_5 \leq 4.504$
$2.2125 \pm 3.899(.2125)$	$1.384 \leq L_6 \leq 3.041$

d. $s\{\hat{D}_i\} = .1735$, $q(.90; 9, 27) = 4.31$, $T = 3.048$, $Ts\{\hat{D}_i\} = .529$, $\bar{Y}_{33.} = 13.250$

e.

i	j	$1/\bar{Y}_{ij.}$	$\sqrt{\bar{Y}_{ij.}}$
1	1	.404	1.573
1	2	.217	2.145
1	3	.219	2.139
2	1	.183	2.335
2	2	.112	2.987
2	3	.110	3.021
3	1	.167	2.444
3	2	.097	3.205
3	3	.075	3.640

19.35. a. $s\{\bar{Y}_{23.}\} = 4.6402$, $t(.995;\ 18) = 2.878$,

$38.75 \pm 2.878(4.6402)$, $25.3955 \leq \mu_{23} \leq 52.1045$

b. $\hat{D} = 46.00$, $s\{\hat{D}\} = 6.5622$, $t(.995; 18) = 2.878$,

$46.00 \pm 2.878(6.5622)$, $27.114 \leq D \leq 64.886$

c. $F(.95; 5, 18) = 2.77$, $S = 3.7216$, $B = t(.99583; 18) = 2.963$

d. $\hat{D}_1 = 159.75$, $\hat{D}_2 = 61.75$, $\hat{D}_3 = 21.75$, $\hat{L}_1 = 98.00$, $\hat{L}_2 = 138.00$, $\hat{L}_3 = 40.00$, $s\{\hat{D}_i\} = 6.5622$ $(i = 1, 2, 3)$, $s\{\hat{L}_i\} = 9.2804$ $(i = 1, 2, 3)$, $B = t(.99583; 18) = 2.963$

$$159.75 \pm 2.963(6.5622) \qquad 140.31 \le D_1 \le 179.19$$
$$61.75 \pm 2.963(6.5622) \qquad 42.31 \le D_2 \le 81.19$$
$$21.75 \pm 2.963(6.5622) \qquad 2.31 \le D_3 \le 41.19$$
$$98.00 \pm 2.963(9.2804) \qquad 70.50 \le L_1 \le 125.50$$
$$138.00 \pm 2.963(9.2804) \qquad 110.50 \le L_2 \le 165.50$$
$$40.00 \pm 2.963(9.2804) \qquad 12.50 \le L_3 \le 67.50$$

e. $q(.95; 6, 18) = 4.49$, $T = 3.1749$, $s\{\hat{D}\} = 6.5622$, $Ts\{\hat{D}\} = 20.834$, $\bar{Y}_{23.} = 38.75$, $\bar{Y}_{22.} = 44.75$

f. $B = t(.9875; 18) = 2.445$, $s\{\bar{Y}_{ij.}\} = 4.6402$

$$44.75 \pm 2.445(4.6402) \qquad 33.405 \le \mu_{22} \le 56.095$$
$$38.75 \pm 2.445(4.6402) \qquad 27.405 \le \mu_{23} \le 50.095$$

g.

i	j	$1/\bar{Y}_{ij.}$	$\log_{10}\bar{Y}_{ij.}$
1	1	.00450	2.346
1	2	.00939	2.027
1	3	.01653	1.782
2	1	.01606	1.794
2	2	.02235	1.651
2	3	.02581	1.588

19.38. $\Delta/\sigma = 2$, $2n = 8$, $n = 4$

19.40. $.5\sqrt{n}/.29 = 4.1999$, $n = 6$

19.42. $8\sqrt{n}/9.1 = 3.1591$, $n = 13$

Chapter 20

TWO-FACTOR STUDIES – ONE CASE PER TREATMENT

20.2. b.

Source	SS	df	MS
Location	37.0050	3	12.3350
Week	47.0450	1	47.0450
Error	.3450	3	.1150
Total	84.3950	7	

H_0: all α_i equal zero ($i = 1, ..., 4$), H_a: not all α_i equal zero.

$F^* = 12.3350/.1150 = 107.26$, $F(.95; 3, 3) = 9.28$. If $F^* \leq 9.28$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0015

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$F^* = 47.0450/.1150 = 409.09$, $F(.95; 1, 3) = 10.1$. If $F^* \leq 10.1$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0003. $\alpha \leq .0975$

c. $\hat{D}_1 = \bar{Y}_{1.} - \bar{Y}_{2.} = 18.95 - 14.55 = 4.40$, $\hat{D}_2 = \bar{Y}_{1.} - \bar{Y}_{3.} = 18.95 - 14.60 = 4.35$, $\hat{D}_3 = \bar{Y}_{1.} - \bar{Y}_{4.} = 18.95 - 18.80 = .15$, $\hat{D}_4 = \bar{Y}_{2.} - \bar{Y}_{3.} = -.05$, $\hat{D}_5 = \bar{Y}_{2.} - \bar{Y}_{4.} = -4.25$, $\hat{D}_6 = \bar{Y}_{3.} - \bar{Y}_{4.} = -4.20$, $\hat{D}_7 = \bar{Y}_{.1} - \bar{Y}_{.2} = 14.30 - 19.15 = -4.85$, $s\{\hat{D}_i\} = .3391$ ($i = 1, ..., 6$), $s\{\hat{D}_7\} = .2398$, $B = t(.99286; 3) = 5.139$

$$4.40 \pm 5.139(.3391) \qquad 2.66 \leq D_1 \leq 6.14$$
$$4.35 \pm 5.139(.3391) \qquad 2.61 \leq D_2 \leq 6.09$$
$$.15 \pm 5.139(.3391) \qquad -1.59 \leq D_3 \leq 1.89$$
$$-.05 \pm 5.139(.3391) \qquad -1.79 \leq D_4 \leq 1.69$$
$$-4.25 \pm 5.139(.3391) \qquad -5.99 \leq D_5 \leq -2.51$$
$$-4.20 \pm 5.139(.3391) \qquad -5.94 \leq D_6 \leq -2.46$$
$$-4.85 \pm 5.139(.2398) \qquad -6.08 \leq D_7 \leq -3.62$$

20.3. a. $\hat{\mu}_{32} = \bar{Y}_{3.} + \bar{Y}_{.2} - \bar{Y}_{..} = 14.600 + 19.150 - 16.725 = 17.025$

b. $s^2\{\hat{\mu}_{32}\} = .071875$

c. $s\{\hat{\mu}_{32}\} = .2681$, $t(.975; 3) = 3.182$, $17.025 \pm 3.182(.2681)$, $16.172 \leq \mu_{32} \leq 17.878$

21.4. $\hat{D} = (-4.13473)/(18.5025)(11.76125) = -.019$, $SSAB^* = .0786, SSRem^* = .2664$.

H_0: $D = 0$, H_a: $D \neq 0$. $F^* = (.0786/1) \div (.2664/2) = .59$, $F(.975; 1, 2) = 38.5$. If $F^* \leq 38.5$ conclude H_0, otherwise H_a. Conclude H_0.

Chapter 21

RANDOMIZED COMPLETE BLOCK DESIGNS

21.5. b. e_{ij}:

i	$j=1$	$j=2$	$j=3$
1	-2.50000	1.50000	1.00000
2	1.50000	$-.50000$	-1.00000
3	2.16667	$-.83333$	-1.33333
4	.16667	$-.83333$	.66667
5	4.16667	-4.83333	.66667
6	1.50000	$-.50000$	-1.00000
7	-1.50000	-1.50000	3.00000
8	-2.83333	3.16667	$-.33333$
9	-1.50000	2.50000	-1.00000
10	-1.16667	1.83333	$-.66667$

$r = .984$

d. H_0: $D = 0$, H_a: $D \neq 0$. $SSBL.TR^* = .13$, $SSRem^* = 112.20$,

$F^* = (.13/1) \div (112.20/17) = .02$, $F(.99; 1, 17) = 8.40$. If $F^* \leq 8.40$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .89

21.6. a.

Source	SS	df	MS
Blocks	433.36667	9	48.15185
Training methods	1,295.00000	2	647.50000
Error	112.33333	18	6.24074
Total	1,840.70000	29	

b. $\bar{Y}_{.1} = 70.6$, $\bar{Y}_{.2} = 74.6$, $\bar{Y}_{.3} = 86.1$

c. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero.

$F^* = 647.50000/6.24074 = 103.754$, $F(.95; 2, 18) = 3.55$. If $F^* \leq 3.55$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $\hat{D}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = -4.0$, $\hat{D}_2 = \bar{Y}_{.1} - \bar{Y}_{.3} = -15.5$, $\hat{D}_3 = \bar{Y}_{.2} - \bar{Y}_{.3} = -11.5$, $s\{\hat{D}_i\} = 1.1172$ ($i = 1, 2, 3$), $q(.90; 3, 18) = 3.10$, $T = 2.192$

$$-4.0 \pm 2.192(1.1172) \qquad -6.45 \leq D_1 \leq -1.55$$
$$-15.5 \pm 2.192(1.1172) \qquad -17.95 \leq D_2 \leq -13.05$$
$$-11.5 \pm 2.192(1.1172) \qquad -13.95 \leq D_3 \leq -9.05$$

e. H_0: all ρ_i equal zero ($i = 1, ..., 10$), H_a: not all ρ_i equal zero.

$F^* = 48.15185/6.24074 = 7.716$, $F(.95; 9, 18) = 2.46$.

If $F^* \leq 2.46$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .0001

21.12. b. $\bar{Y}_{.1..} = 7.25$, $\bar{Y}_{.2..} = 12.75$, $\hat{L} = \bar{Y}_{.1..} - \bar{Y}_{.2..} = -5.50$, $s\{\hat{L}\} = 1.25$,

$t(.995; 8) = 3.355$, $-5.50 \pm 3.355(1.25)$, $-9.69 \leq L \leq -1.31$

21.14. $\phi = \frac{1}{2.5}\sqrt{\frac{10(18)}{3}} = 3.098$, $\nu_1 = 2$, $\nu_2 = 27$, $1 - \beta > .99$

21.16. $n = 49$ blocks

21.18. $\hat{E} = 3.084$

Chapter 22

ANALYSIS OF COVARIANCE

22.7. a. e_{ij} :

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	$-.5281$	.4061	.0089	.4573	$-.1140$	$-.1911$
2	$-.2635$	$-.2005$	.3196	.2995	$-.1662$	.0680
3	$-.1615$	.2586	$-.0099$	$-.3044$	.0472	.1700

i	$j=7$	$j=8$	$j=9$	$j=10$	$j=11$	$j=12$
1	.0660	$-.0939$	$-.0112$			
2	$-.0690$	$-.1776$	$-.0005$	.0653	.0251	.0995

b. $r = .988$

c. $Y_{ij} = \mu_{.} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \beta_1 I_{ij1} x_{ij} + \beta_2 I_{ij2} x_{ij} + \varepsilon_{ij}$

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_1 and β_2 equal zero.

$SSE(F) = .9572$, $SSE(R) = 1.3175$,

$F^* = (.3603/2) \div (.9572/21) = 3.95$, $F(.99; 2, 21) = 5.78$.

If $F^* \leq 5.78$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .035

d. Yes, 5

22.8. b. Full model: $Y_{ij} = \mu_{.} + \tau_1 I_{ij1} + \tau_2 I_{ij2} + \gamma x_{ij} + \varepsilon_{ij}$, $(\bar{X}_{..} = 9.4)$.

Reduced model: $Y_{ij} = \mu_{.} + \gamma x_{ij} + \varepsilon_{ij}$.

c. Full model: $\hat{Y} = 7.80627 + 1.65885 I_1 - .17431 I_2 + 1.11417x$, $SSE(F) = 1.3175$

Reduced model: $\hat{Y} = 7.95185 + .54124x$, $SSE(R) = 5.5134$

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (4.1959/2) \div (1.3175/23) = 36.625$, $F(.95; 2, 23) = 3.42$.

If $F^* \leq 3.42$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $MSE(F) = .0573$, $MSE = .6401$

e. $\hat{Y} = \hat{\mu}_{.} + \hat{\tau}_2 - .4\hat{\gamma} = 7.18629$, $s^2\{\hat{\mu}_{.}\} = .00258$, $s^2\{\hat{\tau}_2\} = .00412$, $s^2\{\hat{\gamma}\} = .00506$, $s\{\hat{\mu}_{.}, \hat{\tau}_2\} = -.00045$, $s\{\hat{\tau}_2, \hat{\gamma}\} = -.00108$, $s\{\hat{\mu}_{.}, \hat{\gamma}\} = -.00120$, $s\{\hat{Y}\} = .09183$, $t(.975; 23) = 2.069$, $7.18629 + 2.069(.09183)$, $6.996 \leq \mu_{.} + \tau_2 - .4\gamma \leq 7.376$

f. $\hat{D}_1 = \hat{\tau}_1 - \hat{\tau}_2 = 1.83316$, $\hat{D}_2 = \hat{\tau}_1 - \hat{\tau}_3 = 2\hat{\tau}_1 + \hat{\tau}_2 = 3.14339$, $\hat{D}_3 = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = 1.31023$, $s^2\{\hat{\tau}_1\} = .03759$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.00418$, $s\{\hat{D}_1\} = .22376$, $s\{\hat{D}_2\} = .37116$, $s\{\hat{D}_3\} = .19326$, $F(.90; 2, 23) = 2.55$, $S = 2.258$

$1.83316 \pm 2.258(.22376)$	$1.328 \leq D_1 \leq 2.338$
$3.14339 \pm 2.258(.37116)$	$2.305 \leq D_2 \leq 3.981$
$1.31023 \pm 2.258(.19326)$	$.874 \leq D_3 \leq 1.747$

22.15. a. e_{ijk}:

i	$j=1$	$j=2$	i	$j=1$	$j=2$	i	$j=1$	$j=2$
1	−.1184	−.3510	2	−.6809	.2082	3	.9687	.6606
	−.3469	−.0939		.8660	−.1877		−.0150	.0565
	.0041	.0286		−.1177	.2531		.8789	−.1109
	−.6041	.0735		.2905	−.2327		−1.1211	−.0660
	1.2000	−.0163		−.3912	−.2776		.0912	−.4293
	−.1347	.3592		.0333	.2367		−.8027	−.1109

b. $r = .974$

c. $$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3} + (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \delta_1 I_{ijk1} x_{ijk} + \delta_2 I_{ijk2} x_{ijk} + \delta_3 I_{ijk3} x_{ijk} + \delta_4 I_{ijk1} I_{ijk3} x_{ijk} + \delta_5 I_{ijk2} I_{ijk3} x_{ijk} + \epsilon_{ijk}$$

H_0: all δ_i equal zero ($i = 1, ..., 5$), H_a: not all δ_i equal zero.

$SSE(R) = 8.2941$, $SSE(F) = 6.1765$,

$F^* = (2.1176/5) \div (6.1765/24) = 1.646$, $F(.99; 5, 24) = 3.90$.

If $F^* \leq 3.90$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .19

22.16. a. $$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3} + (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$$

$$I_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk2} = \begin{cases} 1 & \text{if case from level 2 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$I_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 2 for factor } B \end{cases}$$

$x_{ijk} = X_{ijk} - \bar{X}_{...}$ $\quad (\bar{X}_{...} = 3.4083)$

$\hat{Y} = 23.55556 - 2.15283 I_1 + 3.68152 I_2 + .20907 I_3 - .06009 I_1 I_3 - .04615 I_2 I_3 + 1.06122 x$

$SSE(F) = 8.2941$

b. <u>Interactions</u>:

$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + \beta_1 I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 23.55556 - 2.15400 I_1 + 3.67538 I_2 + .20692 I_3 + 1.07393 x$

$SSE(R) = 8.4889$

Factor A:

$Y_{ijk} = \mu_{..} + \beta_1 I_{ijk3} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3} + (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$

$\hat{Y} = 23.55556 + .12982 I_3 + .01136 I_1 I_3 + .06818 I_2 I_3 + 1.52893x$

$SSE(R) = 240.7835$

Factor B:

$$Y_{ijk} = \mu_{..} + \alpha_1 I_{ijk1} + \alpha_2 I_{ijk2} + (\alpha\beta)_{11} I_{ijk1} I_{ijk3}$$
$$+ (\alpha\beta)_{21} I_{ijk2} I_{ijk3} + \gamma x_{ijk} + \epsilon_{ijk}$$

$\hat{Y} = 23.55556 - 2.15487 I_1 + 3.67076 I_2 - .05669 I_1 I_3 - .04071 I_2 I_3 + 1.08348x$

$SSE(R) = 9.8393$

c. H_0: $(\alpha\beta)_{11} = (\alpha\beta)_{21} = 0$, H_a: not both $(\alpha\beta)_{11}$ and $(\alpha\beta)_{21}$ equal zero.

$F^* = (.1948/2) \div (8.2941/29) = .341$, $F(.95; 2, 29) = 3.33$.

If $F^* \leq 3.33$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .714

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero.

$F^* = (232.4894/2) \div (8.2941/29) = 406.445$, $F(.95; 2, 29) = 3.33$.

If $F^* \leq 3.33$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$.

$F^* = (1.5452/1) \div (8.2941/29) = 5.403$, $F(.95; 1, 29) = 4.18$.

If $F^* \leq 4.18$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .027

f. $\hat{D}_1 = \hat{\alpha}_1 - \hat{\alpha}_2 = -5.83435$, $\hat{D}_2 = \hat{\alpha}_1 - \hat{\alpha}_3 = 2\hat{\alpha}_1 + \hat{\alpha}_2 = -.62414$,

$\hat{D}_3 = \hat{\alpha}_2 - \hat{\alpha}_3 = 2\hat{\alpha}_2 + \hat{\alpha}_1 = 5.21021$, $\hat{D}_4 = \hat{\beta}_1 - \hat{\beta}_2 = 2\hat{\beta}_1 = .41814$,

$s^2\{\hat{\alpha}_1\} = .01593$, $s^2\{\hat{\alpha}_2\} = .01708$, $s\{\hat{\alpha}_1, \hat{\alpha}_2\} = -.00772$, $s^2\{\hat{\beta}_1\} = .00809$,

$s\{\hat{D}_1\} = .22011$, $s\{\hat{D}_2\} = .22343$, $s\{\hat{D}_3\} = .23102$, $s\{\hat{D}_4\} = .17989$,

$B = t(.9875; 29) = 2.364$

$-5.83435 \pm 2.364(.22011)$	$-6.355 \leq D_1 \leq -5.314$
$-.62414 \pm 2.364(.22343)$	$-1.152 \leq D_2 \leq -.096$
$5.21021 \pm 2.364(.23102)$	$4.664 \leq D_3 \leq 5.756$
$.41814 \pm 2.364(.17989)$	$-.007 \leq D_4 \leq .843$

22.19. b. $$Y_{ij} = \mu_{..} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \rho_4 I_{ij4} + \rho_5 I_{ij5} + \rho_6 I_{ij6}$$
$$+ \rho_7 I_{ij7} + \rho_8 I_{ij8} + \rho_9 I_{ij9} + \tau_1 I_{ij10} + \tau_2 I_{ij11} + \gamma x_{ij} + \epsilon_{ij}$$

$$I_{ij1} = \begin{array}{l} 1 \text{ if experimental unit from block 1} \\ -1 \text{ if experimental unit from block 10} \\ 0 \text{ otherwise} \end{array}$$

$I_{ij2}, \ldots, I_{ij9}$ are defined similarly

$$I_{ij10} = \begin{array}{l} 1 \text{ if experimental unit received treatment 1} \\ -1 \text{ if experimental unit received treatment 3} \\ 0 \text{ otherwise} \end{array}$$

$$I_{ij11} = \begin{array}{l} 1 \text{ if experimental unit received treatment 2} \\ -1 \text{ if experimental unit received treatment 3} \\ 0 \text{ otherwise} \end{array}$$

$x_{ij} = X_{ij} - \bar{X}_{..} \qquad (\bar{X}_{..} = 80.033333)$

c. $\hat{Y} = 77.10000 + 4.87199I_1 + 3.87266I_2 + 2.21201I_3 + 3.22003I_4$
$+1.23474I_5 + .90876I_6 - 1.09124I_7 - 3.74253I_8 - 4.08322I_9$
$-6.50033I_{10} - 2.49993I_{11} + .00201x$

$SSE(F) = 112.3327$

d. $Y_{ij} = \mu_{..} + \rho_1 I_{ij1} + \rho_2 I_{ij2} + \rho_3 I_{ij3} + \rho_4 I_{ij4} + \rho_5 I_{ij5} + \rho_6 I_{ij6}$
$+\rho_7 I_{ij7} + \rho_8 I_{ij8} + \rho_9 I_{ij9} + \gamma x_{ij} + \epsilon_{ij}$

$\hat{Y} = 77.10000 + 6.71567I_1 + 5.67233I_2 + 3.61567I_3 + 4.09567I_4$
$+1.14233I_5 + .33233I_6 - 1.66767I_7 - 5.33100I_8 - 5.18767I_9 - .13000x$

$SSE(R) = 1,404.5167$

e. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1,292.18/2) \div (112.3327/17) = 97.777$, $F(.95; 2, 17) = 3.59$.

If $F^* \leq 3.59$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f. $\hat{\tau}_1 = -6.50033$, $\hat{\tau}_2 = -2.49993$, $\hat{L} = -4.0004$, $L^2\{\hat{\tau}_1\} = .44162$, $s^2\{\hat{\tau}_2\} = .44056$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.22048$, $s\{\hat{L}\} = 1.1503$, $t(.975; 17) = 2.11$,

$-4.0004 \pm 2.11(1.1503)$, $-6.43 \leq L \leq -1.57$

22.21. a.

Source	SS	df	MS
Between treatments	25.5824	2	12.7912
Error	1.4650	24	.0610
Total	27.0474	26	

b. Covariance: $MSE = .0573$, $\hat{\gamma} = 1.11417$

Chapter 23

TWO-FACTOR STUDIES – UNEQUAL SAMPLE SIZES

23.4. a. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4}$

$+(\alpha\beta)_{11} X_{ijk1} X_{ijk3} + (\alpha\beta)_{12} X_{ijk1} X_{ijk4}$

$+(\alpha\beta)_{21} X_{ijk2} X_{ijk3} + (\alpha\beta)_{22} X_{ijk2} X_{ijk4} + \epsilon_{ijk}$

$X_{ijk1} =$ 1 if case from level 1 for factor A
−1 if case from level 3 factor A
0 otherwise

$X_{ijk2} =$ 1 if case from level 2 for factor A
−1 if case from level 3 for factor A
0 otherwise

$X_{ijk3} =$ 1 if case from level 1 for factor B
−1 if case from level 3 for factor B
0 otherwise

$X_{ijk4} =$ 1 if case from level 2 for factor B
−1 if case from level 3 for factor B
0 otherwise

b. **Y** entries: in order Y_{111}, ..., Y_{114}, Y_{121}, ..., Y_{124}, Y_{131}, ..., $Y_{134,}Y_{211}$, ...

$\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, α_2, β_1, β_2, $(\alpha\beta)_{11}$, $(\alpha\beta)_{12}$, $(\alpha\beta)_{21}$, $(\alpha\beta)_{22}$

X entries:

A	B	Freq.		X_1	X_2	X_3	X_4	X_1X_3	X_1X_4	X_2X_3	X_2X_4
1	1	4	1	1	0	1	0	1	0	0	0
1	2	4	1	1	0	0	1	0	1	0	0
1	3	4	1	1	0	−1	−1	−1	−1	0	0
2	1	4	1	0	1	1	0	0	0	1	0
2	2	4	1	0	1	0	1	0	0	0	1
2	3	4	1	0	1	−1	−1	0	0	−1	−1
3	1	4	1	−1	−1	1	0	−1	0	−1	0
3	2	4	1	−1	−1	0	1	0	−1	0	−1
3	3	4	1	−1	−1	−1	−1	1	1	1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
1	3	$\mu_{..} + \alpha_1 - \beta_1 - \beta_2 - (\alpha\beta)_{11} - (\alpha\beta)_{12} = \mu_{..} + \alpha_1 + \beta_3 + (\alpha\beta)_{13}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
2	3	$\mu_{..} + \alpha_2 - \beta_1 - \beta_2 - (\alpha\beta)_{21} - (\alpha\beta)_{22} = \mu_{..} + \alpha_2 + \beta_3 + (\alpha\beta)_{23}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$
3	3	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22}$ $= \mu_{..} + \alpha_3 + \beta_3 + (\alpha\beta)_{33}$

d. $\hat{Y} = 7.18333 - 3.30000X_1 + .65000X_2 - 2.55000X_3 + .75000X_4$

$+1.14167X_1X_3 - .03333X_1X_4 + .16667X_2X_3 + .34167X_2X_4$

$\alpha_1 = \mu_{1.} - \mu_{..}$

e.

Source	SS	df
Regression	373.125	8
X_1	212.415	1] A
$X_2 \mid X_1$	7.605	1] A
$X_3 \mid X_1, X_2$	113.535	1] B
$X_4 \mid X_1, X_2, X_3$	10.125	1] B
$X_1X_3 \mid X_1, X_2, X_3, X_4$	26.7806	1] AB
$X_1X_4 \mid X_1, X_2, X_3, X_4, X_1X_3$	.2269	1] AB
$X_2X_3 \mid X_1, X_2, X_3, X_4, X_1X_3, X_1X_4$	1.3669	1] AB
$X_2X_4 \mid X_1, X_2, X_3, X_4, X_1X_3, X_1X_4, X_2X_3$	1.0506	1] AB
Error	1.625	27
Total	374.730	35

Yes.

f. See Problem 19.15c and d.

23.6. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + (\alpha\beta)_{11} X_{ijk1} X_{ijk3}$

$+(\alpha\beta)_{21}X_{ijk2}X_{ijk3} + \epsilon_{ijk}$

$$X_{ijk1} = \begin{cases} 1 & \text{if case from level 1 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1 & \text{if case from level 2 for factor } A \\ -1 & \text{if case from level 3 for factor } A \\ 0 & \text{otherwise} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1 & \text{if case from level 1 for factor } B \\ -1 & \text{if case from level 2 for factor } B \end{cases}$$

b. $\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, α_2, β_1, $(\alpha\beta)_{11}$, $(\alpha\beta)_{21}$

X entries:

A	B	Freq.		X_1	X_2	X_3	X_1X_3	X_2X_3
1	1	6	1	1	0	1	1	0
1	2	6	1	1	0	−1	−1	0
2	1	5	1	0	1	1	0	1
2	2	6	1	0	1	−1	0	−1
3	1	6	1	−1	−1	1	−1	−1
3	2	5	1	−1	−1	−1	1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 - \beta_1 - (\alpha\beta)_{11} = \mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 - \beta_1 - (\alpha\beta)_{21} = \mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 + (\alpha\beta)_{11} + (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$

d. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \epsilon_{ijk}$

e. Full model:

$\hat{Y} = 23.56667 - 2.06667X_1 + 4.16667X_2 + .36667X_3 - .20000X_1X_3 - .30000X_2X_3$,

$SSE(F) = 71.3333$

Reduced model:

$\hat{Y} = 23.59091 - 2.09091X_1 + 4.16911X_2 + .36022X_3$,

$SSE(R) = 75.5210$

H_0: $(\alpha\beta)_{11} = (\alpha\beta)_{21} = 0$, H_a: not both $(\alpha\beta)_{11}$ and $(\alpha\beta)_{21}$ equal zero.

$F^* = (4.1877/2) \div (71.3333/28) = .82$, $F(.95; 2, 28) = 3.34$.

If $F^* \leq 3.34$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .45

f. A effects:

$\hat{Y} = 23.50000 + .17677X_3 - .01010X_1X_3 - .49495X_2X_3$,

$SSE(R) = 359.9394$

H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_1 and α_2 equal zero.

$F^* = (288.6061/2) \div (71.3333/28) = 56.64$, $F(.95; 2, 28) = 3.34$.

If $F^* \leq 3.34$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

<u>B effects</u>:

$\hat{Y} = 23.56667 - 2.06667X_1 + 4.13229X_2 - .17708X_1X_3 - .31146X_2X_3$,

$SSE(R) = 75.8708$

H_0: $\beta_1 = 0$, H_a: $\beta_1 \neq 0$.

$F^* = (4.5375/1) \div (71.3333/28) = 1.78$, $F(.95; 1, 28) = 4.20$.

If $F^* \leq 4.20$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .19

g. $\hat{D}_1 = \hat{\alpha}_1 - \hat{\alpha}_2 = -6.23334$, $\hat{D}_2 = \hat{\alpha}_1 - \hat{\alpha}_3 = 2\hat{\alpha}_1 + \hat{\alpha}_2 = .03333$, $\hat{D}_3 = \hat{\alpha}_2 - \hat{\alpha}_3 = 2\hat{\alpha}_2 + \hat{\alpha}_1 = 6.26667$, $s^2\{\hat{\alpha}_1\} = .14625$, $s^2\{\hat{\alpha}_2\} = .15333$, $s\{\hat{\alpha}_1, \hat{\alpha}_2\} = -.07313$, $s\{\hat{D}_1\} = .6677$, $s\{\hat{D}_2\} = .6677$, $s\{\hat{D}_3\} = .6834$, $q(.90; 3, 28) = 3.026$, $T = 2.140$

$$-6.23334 \pm 2.140(.6677) \quad -7.662 \leq D_1 \leq -4.804$$
$$.03333 \pm 2.140(.6677) \quad -1.396 \leq D_2 \leq 1.462$$
$$6.26667 \pm 2.140(.6834) \quad 4.804 \leq D_3 \leq 7.729$$

h. $\hat{L} = .3\bar{Y}_{12.} + .6\bar{Y}_{22.} + .1\bar{Y}_{32.} = .3(21.33333) + .6(27.66667) + .1(20.60000) = 25.06000$, $s\{\hat{L}\} = .4429$, $t(.975; 28) = 2.048$, $25.06000 \pm 2.048(.4429)$, $24.153 \leq L \leq 25.967$

23.7. a. $Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$

$$Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4}$$
$$+ (\alpha\beta)_{11} X_{ijk1} X_{ijk3} + (\alpha\beta)_{12} X_{ijk1} X_{ijk4} + (\alpha\beta)_{21} X_{ijk2} X_{ijk3}$$
$$+ (\alpha\beta)_{22} X_{ijk2} X_{ijk4} + \epsilon_{ijk}$$

$$X_{ijk1} = \begin{array}{l} 1 \text{ if case from level 1 for factor } A \\ -1 \text{ if case from level 3 for factor } A \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk2} = \begin{array}{l} 1 \text{ if case from level 2 for factor } A \\ -1 \text{ if case from level 3 for factor } A \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk3} = \begin{array}{l} 1 \text{ if case from level 1 for factor } B \\ -1 \text{ if case from level 3 for factor } B \\ 0 \text{ otherwise} \end{array}$$

$$X_{ijk4} = \begin{array}{l} 1 \text{ if case from level 2 for factor } B \\ -1 \text{ if case from level 3 for factor } B \\ 0 \text{ otherwise} \end{array}$$

b. $\boldsymbol{\beta}$ entries: $\mu_{..}$, α_1, α_2, β_1, β_2, $(\alpha\beta)_{11}$, $(\alpha\beta)_{12}$, $(\alpha\beta)_{21}$, $(\alpha\beta)_{22}$

X entries:

A	B	Freq.		X_1	X_2	X_3	X_4	X_1X_3	X_1X_4	X_2X_3	X_2X_4
1	1	3	1	1	0	1	0	1	0	0	0
1	2	4	1	1	0	0	1	0	1	0	0
1	3	4	1	1	0	-1	-1	-1	-1	0	0
2	1	4	1	0	1	1	0	0	0	1	0
2	2	2	1	0	1	0	1	0	0	0	1
2	3	4	1	0	1	-1	-1	0	0	-1	-1
3	1	4	1	-1	-1	1	0	-1	0	-1	0
3	2	4	1	-1	-1	0	1	0	-1	0	-1
3	3	4	1	-1	-1	1	-1	1	1	1	1

c. $\mathbf{X}\boldsymbol{\beta}$ entries:

A	B	
1	1	$\mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11}$
1	2	$\mu_{..} + \alpha_1 + \beta_2 + (\alpha\beta)_{12}$
1	3	$\mu_{..} + \alpha_1 - \beta_1 - \beta_2 - (\alpha\beta)_{11} - (\alpha\beta)_{12} = \mu_{..} + \alpha_1 + \beta_3 + (\alpha\beta)_{13}$
2	1	$\mu_{..} + \alpha_2 + \beta_1 + (\alpha\beta)_{21}$
2	2	$\mu_{..} + \alpha_2 + \beta_2 + (\alpha\beta)_{22}$
2	3	$\mu_{..} + \alpha_2 - \beta_1 - \beta_2 - (\alpha\beta)_{21} - (\alpha\beta)_{22} = \mu_{..} + \alpha_2 + \beta_3 + (\alpha\beta)_{23}$
3	1	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21} = \mu_{..} + \alpha_3 + \beta_1 + (\alpha\beta)_{31}$
3	2	$\mu_{..} - \alpha_1 - \alpha_2 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22} = \mu_{..} + \alpha_3 + \beta_2 + (\alpha\beta)_{32}$
3	3	$\mu_{..} - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22} = \mu_{..} + \alpha_3 + \beta_3 + (\alpha\beta)_{33}$

d. $Y_{ijk} = \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \beta_1 X_{ijk3} + \beta_2 X_{ijk4} + \epsilon_{ijk}$

e. Full model:

$$\hat{Y} = 7.18704 - 3.28426X_1 + .63796X_2 - 2.53426X_3 + .73796X_4$$
$$+1.16481X_1X_3 - .04074X_1X_4 + .15926X_2X_3 + .33704X_2X_4,$$

$SSE(F) = 1.5767$

Reduced model:

$$\hat{Y} = 7.12711 - 3.33483X_1 + .62861X_2 - 2.58483X_3 + .72861X_4,$$

$SSE(R) = 29.6474$

H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero.

$F^* = (28.0707/4) \div (1.5767/24) = 106.82$, $F(.95; 4, 24) = 2.78$.

If $F^* \leq 2.78$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f. $\bar{Y}_{11.} = 2.5333$, $\bar{Y}_{12.} = 4.6000$, $\bar{Y}_{13.} = 4.57500$, $\bar{Y}_{21.} = 5.45000$, $\bar{Y}_{22.} = 8.90000$, $\bar{Y}_{23.} = 9.12500$, $\bar{Y}_{31.} = 5.97500$, $\bar{Y}_{32.} = 10.27500$, $\bar{Y}_{33.} = 13.25000$, $\hat{L}_1 = 2.0542$, $\hat{L}_2 = 3.5625$, $\hat{L}_3 = 5.7875$, $\hat{L}_4 = 1.5083$, $\hat{L}_5 = 3.7333$, $\hat{L}_6 = 2.2250$, $s\{\hat{L}_1\} = .1613$, $s\{\hat{L}_2\} = .1695$, $s\{\hat{L}_3\} = .1570$, $s\{\hat{L}_4\} = .2340$, $s\{\hat{L}_5\} = .2251$, $s\{\hat{L}_6\} = .2310$, $F(.90; 8, 24) = 1.94$, $S = 3.9395$

$$\begin{array}{ll}
2.0542 \pm 3.9395(.1613) & 1.419 \leq L_1 \leq 2.690 \\
3.5625 \pm 3.9395(.1695) & 2.895 \leq L_2 \leq 4.230 \\
5.7875 \pm 3.9395(.1570) & 5.169 \leq L_3 \leq 6.406 \\
1.5083 \pm 3.9395(.2340) & .586 \leq L_4 \leq 2.430 \\
3.7333 \pm 3.9395(.2251) & 2.846 \leq L_5 \leq 4.620 \\
2.2250 \pm 3.9395(.2310) & 1.315 \leq L_6 \leq 3.135
\end{array}$$

23.12. a. See Problem 19.14a. $\hat{D}_1 = \bar{Y}_{13.} - \bar{Y}_{11.} = 2.100$, $\hat{D}_2 = \bar{Y}_{23.} - \bar{Y}_{21.} = 3.675$,

$\hat{D}_3 = \bar{Y}_{33.} - \bar{Y}_{31.} = 7.275$, $\hat{L}_1 = \hat{D}_1 - \hat{D}_2 = -1.575$, $\hat{L}_2 = \hat{D}_1 - \hat{D}_3 = -5.175$, $MSE = .06406$, $s\{\hat{D}_i\} = .1790$ $(i = 1, 2, 3)$, $s\{\hat{L}_i\} = .2531$ $(i = 1, 2)$, $B = t(.99; 24) = 2.492$

$$\begin{array}{ll}
2.100 \pm 2.492(.1790) & 1.654 \leq D_1 \leq 2.546 \\
3.675 \pm 2.492(.1790) & 3.229 \leq D_2 \leq 4.121 \\
7.275 \pm 2.492(.1790) & 6.829 \leq D_3 \leq 7.721 \\
-1.575 \pm 2.492(.2531) & -2.206 \leq L_1 \leq -.944 \\
-5.175 \pm 2.492(.2531) & -5.806 \leq L_2 \leq -4.544
\end{array}$$

b. H_0: $\mu_{12} - \mu_{13} = 0$, H_a: $\mu_{12} - \mu_{13} \neq 0$. $\hat{D} = \bar{Y}_{12.} - \bar{Y}_{13.} = .025$, $s\{\hat{D}\} = .1790$, $t^* = .025/.1790 = .14$, $t(.99; 24) = 2.492$. If $|t^*| \leq 2.492$ conclude H_0, otherwise H_a. Conclude H_0.

H_0: $\mu_{32} - \mu_{33} = 0$, H_a: $\mu_{32} - \mu_{33} \neq 0$. $\hat{D} = \bar{Y}_{32.} - \bar{Y}_{33.} = -2.975$, $s\{\hat{D}\} = .1790$, $t^* = -2.975/.1790 = -16.62$, $t(.99; 24) = 2.492$. If $|t^*| \leq 2.492$ conclude H_0, otherwise H_a. Conclude H_a. $\alpha \leq .04$

23.14. a. See Problem 19.20a. $\hat{D}_1 = \bar{Y}_{12.} - \bar{Y}_{13.} = 46.0$, $\hat{D}_2 = \bar{Y}_{22.} - \bar{Y}_{23.} = 6.0$,

$\hat{L}_1 = \hat{D}_1 - \hat{D}_2 = 40.0$, $MSE = 88.50$, $s\{\hat{D}_1\} = s\{\hat{D}_2\} = 6.652$, $s\{\hat{L}_1\} = 9.407$, $B = t(.99167; 15) = 2.694$

$$\begin{array}{ll}
46.0 \pm 2.694(6.652) & 28.080 \leq D_1 \leq 63.920 \\
6.0 \pm 2.694(6.652) & -11.920 \leq D_2 \leq 23.920 \\
40.0 \pm 2.694(9.407) & 14.658 \leq L_1 \leq 65.342
\end{array}$$

b. H_0: $\mu_{22} - \mu_{23} \leq 0$, H_a: $\mu_{22} - \mu_{23} > 0$. $\hat{D} = \bar{Y}_{22.} - \bar{Y}_{23.} = 6.0$, $s\{\hat{D}\} = 6.652$, $t^* = 6.0/6.652 = .90$, $t(.95; 15) = 1.753$. If $t^* \leq 1.753$ conclude H_0 otherwise H_a. Conclude H_0. P-value $= .19$

23.16. a. $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$

$$+\rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \tau_1 X_{ij10} + \tau_2 X_{ij11} + \epsilon_{ij}$$

$$X_{ij1} = \begin{cases} 1 \text{ if experimental unit from block 1} \\ -1 \text{ if experimental unit from block 10} \\ 0 \text{ otherwise} \end{cases}$$

X_{ij2}, ..., X_{ij9} are defined similarly

$$X_{ij10} = \begin{cases} 1 \text{ if experimental unit received treatment 1} \\ -1 \text{ if experimental unit received treatment 3} \\ 0 \text{ otherwise} \end{cases}$$

$$X_{ij11} = \begin{cases} 1 \text{ if experimental unit received treatment 2} \\ -1 \text{ if experimental unit received treatment 3} \\ 0 \text{ otherwise} \end{cases}$$

b. $\hat{Y} = 77.10000 + 4.90000X_1 + 3.90000X_2 + 2.23333X_3 + 3.23333X_4 + 1.23333X_5$
$+.90000X_6 - 1.10000X_7 - 3.76667X_8 - 4.10000X_9 - 6.50000X_{10} - 2.50000X_{11}$

c.

Source	SS	df	MS
Regression	1,728.3667	1	157.1242
$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$	433.3667	9	48.1519
$X_{10}, X_{11} \| X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$	1,295.0000	2	647.5000
Error	112.3333	18	6.2407
Total	1,840.7000	29	

d. H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1,295.0000/2 \div (112.3333/18) = 103.754$, $F(.95; 2, 18) = 3.55$.

If $F^* \leq 3.55$ conclude H_0, otherwise H_a. Conclude H_a.

23.18. a. $Y_{ij} = \mu_{..} + \rho_i + \tau_j + \epsilon_{ij}$

$Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + \rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$
$+\rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \tau_1 X_{ij10} + \tau_2 X_{ij11} + \epsilon_{ij}$

$$X_{ij1} = \begin{cases} 1 \text{ if experimental unit from block 1} \\ -1 \text{ if experimental unit from block 10} \\ 0 \text{ otherwise} \end{cases}$$

$X_{ij2}, \ldots, X_{ij9}$ are defined similarly

$$X_{ij10} = \begin{cases} 1 \text{ if experimental unit received treatment 1} \\ -1 \text{ if experimental unit received treatment 3} \\ 0 \text{ otherwise} \end{cases}$$

$$X_{ij11} = \begin{cases} 1 \text{ if experimental unit received treatment 2} \\ -1 \text{ if experimental unit received treatment 3} \\ 0 \text{ otherwise} \end{cases}$$

b. $Y_{ij} = \mu_{..} + \rho_1 X_{ij1} + \rho_2 X_{ij2} + +\rho_3 X_{ij3} + \rho_4 X_{ij4} + \rho_5 X_{ij5} + \rho_6 X_{ij6}$

$+\rho_7 X_{ij7} + \rho_8 X_{ij8} + \rho_9 X_{ij9} + \epsilon_{ij}$

c. Full model: $\hat{Y} = 77.15556 + 4.84444X_1 + 4.40000X_2 + 2.17778X_3$
$+3.17778X_4 + 1.17778X_5 + .84444X_6 - 1.15556X_7$
$-3.82222X_8 - 4.15556X_9 - 6.55556X_{10} - 2.55556X_{11}$

$SSE(F) = 110.6667$

Reduced model: $\hat{Y} = 76.70000 + 5.30000X_1 + .30000X_2 + 2.63333X_3$
$+3.63333X_4 + 1.63333X_5 + 1.30000X_6 - .70000X_7$
$-3.36667X_8 - 3.70000X_9$

$SSE(R) = 1,311.3333$

H_0: $\tau_1 = \tau_2 = 0$, H_a: not both τ_1 and τ_2 equal zero.

$F^* = (1,200.6666/2) \div (110.6667/17) = 92.22$, $F(.95; 2, 17) = 3.59$.

If $F^* \leq 3.59$ conclude H_0, otherwise H_a. Conclude H_a.

d. $\hat{L} = \hat{\tau}_2 - \hat{\tau}_3 = 2\hat{\tau}_2 + \hat{\tau}_1 = -11.66667$, $s^2\{\hat{\tau}_i\} = .44604$ $(i = 1, 2)$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.20494$, $s\{\hat{L}\} = 1.1876$, $t(.975; 17) = 2.11$,

$-11.66667 \pm 2.11(1.1876)$, $-14.17 \leq L \leq -9.16$

23.20. See Problem 19.10a. $L_1 = .3\mu_{11} + .6\mu_{21} + .1\mu_{31}$, $L_2 = .3\mu_{12} + .6\mu_{22} + .1\mu_{32}$.

H_0: $L_1 = L_2$, H_a: $L_1 \neq L_2$.

$\hat{L}_1 - \hat{L}_2 = 25.43332 - 25.05001 = .38331$, $MSE = 2.3889$, $s\{\hat{L}_1 - \hat{L}_2\} = .6052$,

$t^* = .38331/.6052 = .63$, $t(.975; 30) = 2.042$.

If $|t^*| \leq 2.042$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .53

23.23. H_0: $\frac{4\mu_{11}+4\mu_{12}+2\mu_{13}}{10} = \frac{4\mu_{21}+4\mu_{22}+3\mu_{23}}{11}$, H_a: equality does not hold.

$\bar{Y}_{...} = 93.714$, $\bar{Y}_{1..} = 143$, $\bar{Y}_{2..} = 48.91$

$SSA = 10(143 - 93.714)^2 + 11(48.91 - 93.714)^2 = 46,372$

$F^* = (46,372/1) \div (1,423.1667/15) = 488.8$, $F(.99; 1, 15) = 8.68$.

If $F^* \leq 8.68$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

Chapter 24

MULTIFACTOR STUDIES

24.6. a. e_{ijkm}:

	$k=1$			$k=2$	
i	$j=1$	$j=2$	i	$j=1$	$j=2$
1	3.7667	1.1667	1	$-.5000$	-1.0333
	-3.9333	-1.6333		.4000	1.3667
	.1667	.4667		.1000	$-.3333$
2	-1.7000	$-.8333$	2	1.1333	$-.5667$
	1.1000	1.3667		-1.6667	1.7333
	.6000	$-.5333$		.5333	-1.1667

b. $r = .973$

24.7. a. $\bar{Y}_{111.} = 36.1333$, $\bar{Y}_{112.} = 56.5000$, $\bar{Y}_{121.} = 52.3333$, $\bar{Y}_{122.} = 71.9333$, $\bar{Y}_{211.} = 46.9000$, $\bar{Y}_{212.} = 68.2667$, $\bar{Y}_{221.} = 64.1333$, $\bar{Y}_{222.} = 83.4667$

b.

Source	SS	df	MS
Between treatments	4,772.25835	7	681.75119
A (chemical)	788.90667	1	788.90667
B (temperature)	1,539.20167	1	1,539.20167
C (time)	2,440.16667	1	2,440.16667
AB interactions	.24000	1	.24000
AC interactions	.20167	1	.20167
BC interactions	2.94000	1	2.94000
ABC interactions	.60167	1	.60167
Error	53.74000	16	3.35875
Total	4,825.99835	23	

c. H_0: all $(\alpha\beta\gamma)_{ijk}$ equal zero, H_a: not all $(\alpha\beta\gamma)_{ijk}$ equal zero. $F^* = .60167/3.35875 = .18$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .68$

d. H_0: all $(\alpha\beta)_{ij}$ equal zero, H_a: not all $(\alpha\beta)_{ij}$ equal zero. $F^* = .24000/3.35875 = .07$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .79$

H_0: all $(\alpha\gamma)_{ik}$ equal zero, H_a: not all $(\alpha\gamma)_{ik}$ equal zero. $F^* = .20167/3.35875 = .06$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .81

H_0: all $(\beta\gamma)_{jk}$ equal zero, H_a: not all $(\beta\gamma)_{jk}$ equal zero. $F^* = 2.94000/3.35875 = .875$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .36

e. H_0: all α_i equal zero ($i = 1, 2$), H_a: not all α_i equal zero. $F^* = 788.90667/3.35875 = 234.88$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all β_j equal zero ($j = 1, 2$), H_a: not all β_j equal zero. $F^* = 1{,}539.20167/3.35875 = 458.27$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

H_0: all γ_k equal zero ($k = 1, 2$), H_a: not all γ_k equal zero. $F^* = 2{,}440.1667/3.35875 = 726.51$, $F(.975; 1, 16) = 6.12$. If $F^* \leq 6.12$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

f. $\alpha \leq .1624$

24.8. a. $\hat{D}_1 = 65.69167 - 54.22500 = 11.46667$, $\hat{D}_2 = 67.96667 - 51.95000 = 16.01667$,

$\hat{D}_3 = 70.04167 - 49.87500 = 20.16667$, $MSE = 3.35875$,

$s\{\hat{D}_i\} = .7482$ $(i = 1, 2, 3)$, $B = t(.99167; 16) = 2.673$

$$
\begin{array}{ll}
11.46667 \pm 2.673(.7482) & 9.467 \leq D_1 \leq 13.467 \\
16.01667 \pm 2.673(.7482) & 14.017 \leq D_2 \leq 18.017 \\
20.16667 \pm 2.673(.7482) & 18.167 \leq D_3 \leq 22.167
\end{array}
$$

b. $\bar{Y}_{222.} = 83.46667$, $s\{\bar{Y}_{222.}\} = 1.0581$, $t(.975;\ 16) = 2.120$,

$83.46667 \pm 2.120(1.0581)$, $81.2235 \leq \mu_{222} \leq 85.7098$

24.15. a. $$Y_{ijkm} = \mu_{...} + \alpha_1 X_{ijkm1} + \beta_1 X_{ijkm2} + \gamma_1 X_{ijkm3} + (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2} + (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm3} + (\beta\gamma)_{11} X_{ijkm2} X_{ijkm3} + (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm3} + \epsilon_{ijkm}$$

$$X_{ijk1} = \begin{array}{l} 1 \text{ if case from level 1 for factor } A \\ -1 \text{ if case from level 2 for factor } A \end{array}$$

$$X_{ijk2} = \begin{array}{l} 1 \text{ if case from level 1 for factor } B \\ -1 \text{ if case from level 2 for factor } B \end{array}$$

$$X_{ijk3} = \begin{array}{l} 1 \text{ if case from level 1 for factor } C \\ -1 \text{ if case from level 2 for factor } C \end{array}$$

b. $$Y_{ijkm} = \mu_{...} + \beta_1 X_{ijkm2} + \gamma_1 X_{ijkm3} + (\alpha\beta)_{11} X_{ijkm1} X_{ijkm2} + (\alpha\gamma)_{11} X_{ijkm1} X_{ijkm3} + (\beta\gamma)_{11} X_{ijkm2} X_{ijkm3} + (\alpha\beta\gamma)_{111} X_{ijkm1} X_{ijkm2} X_{ijkm3} + \epsilon_{ijkm}$$

c. Full model:

$$\hat{Y} = 60.01667 - 5.67500 X_1 - 8.06667 X_2 - 10.02500 X_3 + .04167 X_1 X_2 + .15000 X_1 X_3 - .40833 X_2 X_3 + .10000 X_1 X_2 X_3,$$

$SSE(F) = 49.4933$

Reduced model:

$\hat{Y} = 61.15167 - 9.20167X_2 - 8.89000X_3 - 1.09333X_1X_2 + 1.28500X_1X_3$
$\quad -1.54333X_2X_3 - 1.03500X_1X_2X_3,$

$SSE(R) = 667.8413$

H_0: $\alpha_1 = 0$, H_a: $\alpha_1 \neq 0$.

$F^* = (618.348/1) \div (49.4933/14) = 174.91$, $F(.975; 1, 14) = 6.298$.

If $F^* \leq 6.298$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $\hat{D} = \hat{\mu}_{2..} - \hat{\mu}_{1..} = \hat{\alpha}_2 - \hat{\alpha}_1 = -2\hat{\alpha}_1 = 11.35000$, $s^2\{\hat{\alpha}_1\} = .18413$, $s\{\hat{D}\} = .8582$, $t(.975;\, 14) = 2.145$,

$11.35000 \pm 2.145(.8582)$, $9.509 \leq D \leq 13.191$

24.17. $\dfrac{2\sqrt{n}}{1.8} = 4.1475$, $n = 14$

Chapter 25

RANDOM AND MIXED EFFECTS MODELS

25.5. b. H_0: $\sigma^2_\mu = 0$, H_a: $\sigma^2_\mu > 0$. $F^* = .45787/.03097 = 14.78$, $F(.95; 5, 114) = 2.29$.

If $F^* \leq 2.29$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{..} = .22767$, $n_T = 120$, $s\{\bar{Y}_{..}\} = .06177$, $t(.975; 5) = 2.571$,

$.22767 \pm 2.571(.06177)$, $.0689 \leq \mu_. \leq .3865$

25.6. a. $F(.025; 5, 114) = .1646$, $F(.975; 5, 114) = 2.680$, $L = .22583$, $U = 4.44098$

$$.1842 \leq \frac{\sigma^2_\mu}{\sigma^2_\mu + \sigma^2} \leq .8162$$

b. $\chi^2(.025; 114) = 90.351$, $\chi^2(.975; 114) = 145.441$, $.02427 \leq \sigma^2 \leq .03908$

c. $s^2_\mu = .02135$

d. Satterthwaite:

$$df = (ns^2_\mu)^2 \div [(MSTR)^2/(r-1) + (MSE)^2/r(n-1)]$$
$$= [20(.02135)]^2 \div [(.45787)^2/5 + (.03907)^2/6(19)] = 4.35,$$

$\chi^2(.025; 4) = .484$, $\chi^2(.975; 4) = 11.143$

$$.0083 = \frac{4.35(.02135)}{11.143} \leq \sigma^2_\mu \leq \frac{4.35(.02135)}{.484} = .192$$

MLS: $c_1 = .05$, $c_2 = -.05$, $MS_1 = .45787$, $MS_2 = .03907$, $df_1 = 5$, $df_2 = 114$, $F_1 = F(.975; 5, \infty) = 2.57$, $F_2 = F(.975; 114, \infty) = 1.28$, $F_3 = F(.975; \infty, 5) = 6.02$, $F_4 = F(.975; \infty, 114) = 1.32$, $F_5 = F(.975; 5, 114) = 2.68$, $F_6 = F(.975; 114, 5) = 6.07$, $G_1 = .6109$, $G_2 = .2188$, $G_3 = .0147$, $G_4 = -.2076$, $H_L = .014$, $H_U = .115$, $.02135 - .014$, $.02135 + .115$, $.0074 \leq \sigma^2_\mu \leq .1364$

25.16. a. H_0: $\sigma^2_{\alpha\beta} = 0$, H_a: $\sigma^2_{\alpha\beta} > 0$. $F^* = 303.822/52.011 = 5.84$, $F(.99; 4, 36) = 3.89$.

If $F^* \leq 3.89$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .001

b. $s^2_{\alpha\beta} = 50.362$

c. H_0: $\sigma^2_\alpha = 0$, H_a: $\sigma^2_\alpha > 0$. $F^* = 12.289/52.011 = .24$, $F(.99; 2, 36) = 5.25$.

If $F^* \leq 5.25$ conclude H_0, otherwise H_a. Conclude H_0.

d. H_0: all β_j equal zero $(j = 1, 2, 3)$, H_a: not all β_j equal zero.

$F^* = 14.156/303.822 = .047$, $F(.99; 2, 4) = 18.0$.

If $F^* \leq 18.0$ conclude H_0, otherwise H_a. Conclude H_0.

e. $\bar{Y}_{.1.} = 56.133$, $\bar{Y}_{.2.} = 56.600$, $\bar{Y}_{.3.} = 54.733$, $\hat{D}_1 = \bar{Y}_{.1.} - \bar{Y}_{.2.} = -.467$, $\hat{D}_2 = \bar{Y}_{.1.} - \bar{Y}_{.3.} = -1.400$, $\hat{D}_3 = \bar{Y}_{.2.} - \bar{Y}_{.3.} = 1.867$, $s\{\hat{D}_i\} = 6.3647$ $(i = 1, 2, 3)$, $q(.95; 3, 4) = 5.04$, $T = 3.5638$

$$-.467 \pm 3.5638(6.3647) \quad -23.150 \leq D_1 \leq 22.216$$
$$-1.400 \pm 3.5638(6.3647) \quad -24.083 \leq D_2 \leq 21.283$$
$$1.867 \pm 3.5638(6.3647) \quad -20.816 \leq D_3 \leq 24.550$$

f. $\hat{\mu}_{.1} = 56.1333$, $MSA = 12.28889$, $MSAB = 303.82222$, $s^2\{\hat{\mu}_{.1}\} = (2/45)(303.82222)+ (1/45)(12.28889) = 13.7763$, $s\{\hat{u}_{.1}\} = 3.712$, $df = (13.7763)^2 \div \{[(2/45)(303.82222)]^2/4 + [(1/45)(12.28889)]^2/2\} = 4.16$, $t(.995; 4) = 4.60$,

$56.1333 \pm 4.60(3.712)$, $39.06 \leq \mu_{.1} \leq 73.21$

g. $MSA = 12.28889$, $MSE = 52.01111$, $s^2_\alpha = (MSA - MSE)/nb = -2.648$, $c_1 = 1/15$, $c_2 = -1/15$, $df_1 = 2$, $df_2 = 36$, $F_1 = F(.995; 2, \infty) = 5.30$, $F_2 = F(.995; 36, \infty) = 1.71$, $F_3 = F(.995; \infty, 2) = 200$, $F_4 = F(.995; \infty, 36) = 2.01$, $F_5 = F(.995; 2, 36) = 6.16$, $F_6 = F(.995; 36, 2) = 199.5$, $G_1 = .8113$, $G_2 = .4152$, $G_3 = .1022$, $G_4 = -35.3895$, $H_L = 3.605$, $H_U = 162.730$, $-2.648 - 3.605$, $-2.648 + 162.730$, $-6.253 \leq \sigma^2_\alpha \leq 160.082$

25.19. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	−.175	−1.300	.325	−2.050	3.200
2	.025	4.900	−3.475	1.150	−2.600
3	−.575	−1.700	.925	−1.450	2.800
4	.025	1.900	−1.475	2.150	−2.600
5	.025	−1.100	.525	−.850	1.400
6	.025	−1.100	2.525	−.850	−.600
7	−.175	−.300	−.675	2.950	−1.800
8	.825	−1.300	1.325	−1.050	.200

$r = .985$

c. H_0: $D = 0$, H_a: $D \neq 0$. $SSBL.TR^* = 27.729$, $SSRem^* = 94.521$,

$F^* = (27.729/1) \div (94.521/27) = 7.921$, $F(.995; 1, 27) = 9.34$.

If $F^* \leq 9.34$ conclude H_0, otherwise H_a. Conclude H_0.

25.20. a.

Source	SS	df	MS
Blocks	4,826.375	7	689.48214
Paint type	531.350	4	132.83750
Error	122.250	28	4.36607
Total	5,479.975	39	

b. H_0: all τ_j equal zero $(j = 1, ..., 5)$, H_a: not all τ_j equal zero.

$F^* = 132.83750/4.36607 = 30.425,\ F(.95; 4, 28) = 2.71.$

If $F^* \leq 2.71$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{.1} = 20.500,\ \bar{Y}_{.2} = 23.625,\ \bar{Y}_{.3} = 19.000,\ \bar{Y}_{.4} = 29.375,\ \bar{Y}_{.5} = 21.125,\ \hat{L}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = -3.125,\ \hat{L}_2 = \bar{Y}_{.1} - \bar{Y}_{.3} = 1.500,\ \hat{L}_3 = \bar{Y}_{.1} - \bar{Y}_{.4} = -8.875,\ \hat{L}_4 = \bar{Y}_{.1} - \bar{Y}_{.5} = -.625,\ s\{\hat{L}_i\} = 1.0448\ (i = 1, ..., 4),\ B = t(.9875; 28) = 2.369$

$$-3.125 \pm 2.369(1.0448) \qquad -5.60 \leq L_1 \leq -.65$$
$$1.500 \pm 2.369(1.0448) \qquad -.98 \leq L_2 \leq 3.98$$
$$-8.875 \pm 2.369(1.0448) \qquad -11.35 \leq L_3 \leq -6.40$$
$$-.625 \pm 2.369(1.0448) \qquad -3.10 \leq L_4 \leq 1.85$$

d. $\hat{L} = \frac{1}{3}(\bar{Y}_{.1}+\bar{Y}_{.3}+\bar{Y}_{.5}) - \frac{1}{2}(\bar{Y}_{.2}+\bar{Y}_{.4}) = -6.29167,\ s\{\hat{L}\} = .6744,\ t(.975; 28) = 2.048,$ $-6.29167 \pm 2.048(.6744),\ -7.67 \leq L \leq -4.91$

25.23. a. H_0: $\sigma^2_{\alpha\beta\gamma} = 0$, H_a: $\sigma^2_{\alpha\beta\gamma} > 0$. $F^* = MSABC/MSE = 1.49/2.30 = .648,$

$F(.975; 8, 60) = 2.41$. If $F^* \leq 2.41$ conclude H_0, otherwise H_a. Conclude H_0. P-value=.27.

b. H_0: $\sigma^2_{\alpha\beta} = 0$, H_a: $\sigma^2_{\alpha\beta} > 0$. $F^* = MSAB/MSABC = 2.40/1.49 = 1.611,$

$F(.99; 2, 8) = 8.65$. If $F^* \leq 8.65$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: $\sigma^2_\beta = 0$, H_a: $\sigma^2_\beta > 0$. $F^{**} = MSB/(MSAB + MSBC - MSABC) = 4.20/(2.40+3.13-1.49) = 1.04$, $df = 16.32161/5.6067 = 2.91$, $F(.99; 1, 3) = 34.1$. If $F^{**} \leq 34.1$ conclude H_0, otherwise H_a. Conclude H_0.

d. $s^2_\alpha = (MSA - MSAB - MSAC + MSABC)/nbc = .126,$

$$df = [(8.650/30) - (2.40/30) - (3.96/30) + (1.49/30)]^2$$
$$\div \left[\frac{(8.65/30)^2}{2} + \frac{(2.40/30)^2}{2} + \frac{(3.96/30)^2}{8} + \frac{(1.49/30)^2}{8}\right] = .336$$

$\chi^2(.025; 1) = .001,\ \chi^2(.975; 1) = 5.02$

$$.008 = \frac{.336(.126)}{5.02} \leq \sigma^2_\alpha \leq \frac{.336(.126)}{.001} = 42.336$$

25.26. a. $\hat{\mu}_{..} = 55.593,\ \hat{\beta}_1 = .641,\ \hat{\beta}_2 = .218,\ \hat{\sigma}^2_\alpha = 5.222,\ \hat{\sigma}^2_{\alpha\beta} = 15.666, \hat{\sigma}^2 = 55.265$, no (Note: Unrestricted estimators are same except that variance component for random effect A is zero.)

b. Estimates remain the same.

c. H_0: $\sigma^2_{\alpha\beta} = 0$, H_a: $\sigma^2_{\alpha\beta} > 0$. $z(.99) = 2.326$, $s\{\hat{\sigma}^2_{\alpha\beta}\} = 13.333$, $z^* = 15.666/13.333 = 1.175$. If $z^* \leq 2.326$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .12.

d. H_0: $\beta_1 = \beta_2 = \beta_3 = 0$, H_a: not all $\beta_j = 0$ $(j = 1, 2, 3)$. $-2\log_e L(R) = 295.385$, $-2\log_e L(F) = 295.253$, $X^2 = 295.385 - 295.253 = .132$, $\chi^2(.99; 2) = 9.21$. If $X^2 \leq 9.21$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .94.

e. $z(.995) = 2.576,\ 15.666 \pm 2.576(13.333),\ -18.680 \leq \alpha^2_{\alpha\beta} \leq 50.012$

Chapter 26

NESTED DESIGNS, SUBSAMPLING, AND PARTIALLY NESTED DESIGNS

26.9. a. e_{ijk}:

	$i=1$		
k	$j=1$	$j=2$	$j=3$
1	1.8	−12.8	−9.6
2	15.8	−.8	7.4
3	−5.2	3.2	16.4
4	−.2	−3.8	−14.6
5	−12.2	14.2	.4

	$i=2$		
k	$j=1$	$j=2$	$j=3$
1	−7.2	−2.6	8.8
2	3.8	−15.6	−8.2
3	−15.2	6.4	−10.2
4	7.8	11.4	11.8
5	10.8	.4	−2.2

	$i=3$		
k	$j=1$	$j=2$	$j=3$
1	−5.8	−9.8	−12.0
2	11.2	12.2	0.0
3	−.8	−.8	17.0
4	−12.8	3.2	2.0
5	8.2	−4.8	−7.0

$r = .987$

26.10. a.

Source	SS	df	MS
States (A)	6,976.84	2	3,488.422
Cities within states [$B(A)$]	167.60	6	27.933
Error (E)	3,893.20	36	108.144
Total	11,037.64	44	

b. H_0: all α_i equal zero ($i = 1, 2, 3$), H_a: not all α_i equal zero. $F^* = 3,488.422/108.144 = 32.257$, $F(.95; 2, 36) = 3.26$. If $F^* \leq 3.26$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. H_0: all $\beta_{j(i)}$ equal zero, H_a: not all $\beta_{j(i)}$ equal zero. $F^* = 27.933/108.144 = .258$, $F(.95; 6, 36) = 2.36$. If $F^* \leq 2.36$ conclude H_0, otherwise H_a. Conclude H_0.

P-value $= .95$

d. $\alpha \leq .10$

26.11. a. $\bar{Y}_{11.} = 40.2$, $s\{\bar{Y}_{11.}\} = 4.6507$, $t(.975; 36) = 2.0281$,

$40.2 \pm 2.0281(4.6507)$, $30.77 \leq \mu_{11} \leq 49.63$

b. $\bar{Y}_{1..} = 40.8667$, $\bar{Y}_{2..} = 57.3333$, $\bar{Y}_{3..} = 26.8667$, $s\{\bar{Y}_{i..}\} = 2.6851$ $(i = 1, 2, 3)$, $t(.995; 36) = 2.7195$

$40.8667 \pm 2.7195(2.6851)$	$33.565 \leq \mu_{1.} \leq 48.169$
$57.3333 \pm 2.7195(2.6851)$	$50.031 \leq \mu_{2.} \leq 64.635$
$26.8667 \pm 2.7195(2.6851)$	$19.565 \leq \mu_{3.} \leq 34.169$

c. $\hat{L}_1 = \bar{Y}_{1..} - \bar{Y}_{2..} = -16.4666$, $\hat{L}_2 = \bar{Y}_{1..} - \bar{Y}_{3..} = 14.0000$, $\hat{L}_3 = \bar{Y}_{2..} - \bar{Y}_{3..} = 30.4666$, $s\{\hat{L}_i\} = 3.7973$ $(i = 1, 2, 3)$, $q(.90; 3, 36) = 2.998$, $T = 2.120$

$-16.4666 \pm 2.120(3.7973)$	$-24.52 \leq L_1 \leq -8.42$
$14.0000 \pm 2.120(3.7973)$	$5.95 \leq L_2 \leq 22.05$
$30.4666 \pm 2.120(3.7973)$	$22.42 \leq L_3 \leq 38.52$

d. $\hat{L} = 12.4$, $s\{\hat{L}\} = 6.5771$, $t(.975; 36) = 2.0281$, $12.4 \pm 2.0281(6.5771)$, $-.94 \leq L \leq 25.74$

26.12. a. $\beta_{j(i)}$ are independent $N(0, \sigma_\beta^2)$; $\beta_{j(i)}$ are independent of $\epsilon_{k(j)}$.

b. $\hat{\sigma}_\beta^2 = 0$, yes.

c. H_0: $\sigma_\beta^2 = 0$, H_a: $\sigma_\beta^2 > 0$. $F^* = 27.933/108.144 = .258$, $F(.90; 6, 36) = 1.94$.

If $F^* \leq 1.94$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .95$

d. H_0: all α_i equal zero $(i = 1, 2, 3)$, H_a: not all α_i equal zero. $F^* = 3,488.422/27.933 = 124.885$, $F(.90; 2, 6) = 3.46$. If $F^* \leq 3.46$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

e. See Problem 26.11c. $s\{\hat{L}_i\} = 1.9299$ $(i = 1, 2, 3)$, $q(.90; 3, 6) = 3.56$, $T = 2.5173$

$-16.4666 \pm 2.5173(1.9299)$	$-21.32 \leq L_1 \leq -11.61$
$14.0000 \pm 2.5173(1.9299)$	$9.14 \leq L_2 \leq 18.86$
$30.4666 \pm 2.5173(1.9299)$	$25.61 \leq L_3 \leq 35.32$

f. H_0: all $\sigma^2\{\beta_{j(i)}\}$ are equal $(i = 1, 2, 3)$, H_a: not all $\sigma^2\{\beta_{j(i)}\}$ are equal.

$H^* = 37.27/16.07 = 2.32$, $H(.95; 3, 2) = 87.5$.

If $H^* \leq 87.5$ conclude H_0, otherwise H_a. Conclude H_0.

26.13. a. α_i are independent $N(0, \sigma_\alpha^2)$; $\beta_{j(i)}$ are independent $N(0, \sigma_\beta^2)$;

α_i, $\beta_{j(i)}$, and $\epsilon_{k(ij)}$ are independent.

b. $\hat{\sigma}_\beta^2 = 0$, $\hat{\sigma}_\alpha^2 = 230.699$

c. H_0: $\sigma_\alpha^2 = 0$, H_a: $\sigma_\alpha^2 > 0$. $F^* = 3,488.422/27.933 = 124.885$, $F(.99; 2, 6) = 10.9$.

If $F^* \leq 10.9$ conclude H_0, otherwise H_a. Conclude H_a. P-value $= 0+$

d. $c_1 = 1/15$, $c_2 = -1/15$, $MS_1 = 3488.422$, $MS_2 = 27.933$, $df_1 = 2$, $df_2 = 6$, $F_1 = F(.995; 2, \infty) = 5.30$, $F_2 = F(.995; 6, \infty) = 3.09$, $F_3 = F(.995; \infty, 2) = 200$, $F_4 = F(.995; \infty, 6) = 8.88$, $F_5 = F(.995; 2, 6) = 14.5$, $F_6 = F(.995; 6, 2) = 199$, $G_1 = .8113$, $G_2 = .6764$, $G_3 = -1.2574$, $G_4 = -93.0375$, $H_L = 187.803$, $H_U = 46,279.30$, $230.699 - 187.803$, $230.699 + 46,279.30$, $42.90 \leq \sigma_\alpha^2 \leq 46,510.00$

e. $\bar{Y}_{...} = 41.6889$, $s\{\bar{Y}_{...}\} = 8.8046$, $t(.995; 2) = 9.925$, $41.6889 \pm 9.925(8.8046)$, $-45.70 \leq \mu_{..} \leq 129.07$

26.19. e_{ijk}:

	$i = 1$				$i = 2$		
k	$j = 1$	$j = 2$	$j = 3$	k	$j = 1$	$j = 2$	$j = 3$
1	−.4000	.0333	−.3667	1	.0667	.4333	−.2000
2	.0000	.3333	.0333	2	−.2333	.0667	.3000
3	.4000	−.3667	.3333	3	.1667	−.3667	−.1000

	$i = 3$				$i = 4$		
k	$j = 1$	$j = 2$	$j = 3$	k	$j = 1$	$j = 2$	$j = 3$
1	−.4333	−.1333	−.3667	1	−.0667	−.3000	.4000
2	.1667	.4667	.3333	2	.4333	.2000	.0000
3	.2667	−.3333	−.0667	3	−.3667	.1000	−.4000

$r = .972$

26.20. a.

Source	SS	df	MS
Plants	343.1789	3	114.3930
Leaves, within plants	187.4533	8	23.4317
Observations, within leaves	3.0333	24	.1264
Total	533.6655	35	

b. H_0: $\sigma_\tau^2 = 0$, H_a: $\sigma_\tau^2 > 0$. $F^* = 114.3930/23.4317 = 4.88$, $F(.95; 3, 8) = 4.07$.

If $F^* \leq 4.07$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .03

c. H_0: $\sigma^2 = 0$, H_a: $\sigma^2 > 0$. $F^* = 23.4317/.1264 = 185.38$, $F(.95; 8, 24) = 2.36$.

If $F^* \leq 2.36$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

d. $\bar{Y}_{...} = 14.26111$, $s\{\bar{Y}_{...}\} = 1.7826$, $t(.975; 3) = 3.182$,

$14.26111 \pm 3.182(1.7826)$, $8.59 \leq \mu_{..} \leq 19.93$

e. $\hat{\sigma}_\tau^2 = 10.1068$, $\hat{\sigma}^2 = 7.7684$, $\hat{\sigma}_\eta^2 = .1264$

f. $c_1 = 1/9 = .1111$, $c_2 = -1/9 = -.1111$, $MS_1 = 114.3930$, $MS_2 = 23.4317$, $df_1 = 3$, $df_2 = 8$, $F_1 = F(.95; 3, \infty) = 2.60$, $F_2 = F(.95; 8, \infty) = 1.94$, $F_3 = F(.95; \infty, 3) = 8.53$, $F_4 = F(.95; \infty, 8) = 2.93$, $F_5 = F(.95; 3, 8) = 4.07$, $F_6 = F(.95; 8, 3) = 8.85$, $G_1 = .6154$, $G_2 = .4845$, $G_3 = -.1409$, $G_4 = -1.5134$, $H_L = 9.042$, $H_U = 95.444$, $10.1068 - 9.042$, $10.1068 + 95.444$, $1.065 \leq \sigma_\tau^2 \leq 105.551$

Chapter 27

REPEATED MEASURES AND RELATED DESIGNS

27.6. a. e_{ij}:

i	$j=1$	$j=2$	$j=3$
1	-1.2792	$-.2417$	1.5208
2	$-.8458$	.6917	.1542
3	.6208	.0583	$-.6792$
4	.5542	.1917	$-.7458$
5	.5208	$-.3417$	$-.1792$
6	$-.1458$	.3917	$-.2458$
7	.9875	$-.7750$	$-.2125$
8	$-.4125$	.0250	.3875

$r = .992$

d. H_0: $D = 0$, H_a: $D \neq 0$. $SSTR.S = 9.5725$, $SSTR.S^* = 2.9410$, $SSRem^* = 6.6315$, $F^* = (2.9410/1) \div (6.6315/13) = 5.765$, $F(.99; 1, 13) = 9.07$. If $F^* \leq 9.07$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .032

27.7. a.

Source	SS	df	MS
Stores	745.1850	7	106.4550
Prices	67.4808	2	33.7404
Error	9.5725	14	.68375
Total	822.2383	23	

b. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero. $F^* = 33.7404/.68375 = 49.346$, $F(.95; 2, 14) = 3.739$. If $F^* \leq 3.739$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

c. $\bar{Y}_{.1} = 55.4375$, $\bar{Y}_{.2} = 53.6000$, $\bar{Y}_{.3} = 51.3375$, $\hat{L}_1 = \bar{Y}_{.1} - \bar{Y}_{.2} = 1.8375$, $\hat{L}_2 = \bar{Y}_{.1} - \bar{Y}_{.3} = 4.1000$, $\hat{L}_3 = \bar{Y}_{.2} - \bar{Y}_{.3} = 2.2625$, $s\{\hat{L}_i\} = .413446$ ($i = 1, 2, 3$), $q(.95; 3, 14) = 3.70$, $T = 2.616$

$1.8375 \pm 2.616(.413446)$ $\quad .756 \leq L_1 \leq 2.919$

$4.1000 \pm 2.616(.413446)$ $\quad 3.018 \leq L_2 \leq 5.182$

$2.2625 \pm 2.616(.413446)$ $\quad 1.181 \leq L_3 \leq 3.344$

d. $\hat{E} = 48.08$

27.9. H_0: all τ_j equal zero ($j = 1, 2, 3$), H_a: not all τ_j equal zero. $MSTR = 8$, $MSTR.S = 0$, $F_R^* = 8/0$. Note: Nonparametric F test results in $SSTR.S = 0$ and therefore should not be used.

27.13. a. e_{ijk}:

		$k=1$	$k=2$	$k=3$	$k=4$
	$i=1$	9.250	−8.750	1.250	−1.750
	$i=2$	−11.750	−2.750	15.250	−.750
$j=1$	$i=3$	7.750	−5.250	5.750	−8.250
	$i=4$	−5.250	16.750	−22.250	10.750
	$i=1$	3.625	−3.125	−13.875	13.375
$j=2$	$i=2$	15.375	6.625	7.875	−29.875
	$i=3$	−8.375	−3.125	−3.875	15.375
	$i=4$	−10.625	−.375	9.875	1.125

$r = .981$

27.14. a. H_0: $\sigma^2\{\rho_{i(1)}\} = \sigma^2\{\rho_{i(2)}\}$, H_a: $\sigma^2\{\rho_{i(1)}\} \neq \sigma^2\{\rho_{i(2)}\}$.

$SSS(A_1) = 1,478,757.00$, $SSS(A_2) = 1,525,262.25$,

$H^* = (1,525,262.25/3) \div (1,478,757.00/3) = 1.03$, $H(.99; 2, 3) = 47.5$.

If $H^* \leq 47.5$ conclude H_0, otherwise H_a. Conclude H_0.

b. H_0: $\sigma^2\{\epsilon_{1jk}\} = \sigma^2\{\epsilon_{2jk}\}$, H_a: $\sigma^2\{\epsilon_{1jk}\} \neq \sigma^2\{\epsilon_{2jk}\}$.

$SSB.S(A_1) = 1,653.00$, $SSB.S(A_2) = 2,172.25$,

$H^* = (2,172.25/9) \div (1,653.00/9) = 1.31$, $H(.99; 2, 9) = 6.54$.

If $H^* \leq 6.54$ conclude H_0, otherwise H_a. Conclude H_0.

27.15. a.

Source	SS	df	MS
A (type display)	266,085.1250	1	266,085.1250
$S(A)$	3,004,019.2500	6	500,669.8750
B (time)	53,321.6250	3	17,773.8750
AB interactions	690.6250	3	230.2083
Error	3,825.2500	18	212.5139
Total	3,327,941.8750	31	

b. $\bar{Y}_{.11} = 681.500$, $\bar{Y}_{.12} = 696.500$, $\bar{Y}_{.13} = 671.500$, $\bar{Y}_{.14} = 785.500$,

$\bar{Y}_{.21} = 508.500$, $\bar{Y}_{.22} = 512.250$, $\bar{Y}_{.23} = 496.000$, $\bar{Y}_{.24} = 588.750$

c. H_0: all $(\alpha\beta)_{jk}$ equal zero, H_a: not all $(\alpha\beta)_{jk}$ equal zero.

$F^* = 230.2083/212.5139 = 1.08$, $F(.975; 3, 18) = 3.95$.

If $F^* \leq 3.95$ conclude H_0, otherwise H_a. Conclude H_0. P-value $= .38$

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_j equal zero.

$F^* = 266,085.1250/500,669.8750 = .53$, $F(.975; 1, 6) = 8.81$.

If $F^* \leq 8.81$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .49

H_0: all β_k equal zero ($k = 1, ..., 4$), H_a: not all β_k equal zero.

$F^* = 17,773.8750/212.5139 = 83.636$, $F(.975; 3, 18) = 3.95$.

If $F^* \leq 3.95$ conclude H_0, otherwise H_a. Conclude H_a. P-value = 0+

e. $\bar{Y}_{.1.} = 708.750$, $\bar{Y}_{.2.} = 526.375$, $\bar{Y}_{..1} = 595.000$, $\bar{Y}_{..2} = 604.375$, $\bar{Y}_{..3} = 583.750$, $\bar{Y}_{..4} = 687.125$, $\hat{L}_1 = 182.375$, $\hat{L}_2 = -9.375$, $\hat{L}_3 = 20.625$, $\hat{L}_4 = -103.375$, $s\{\hat{L}_1\} = 250.1674$, $s\{\hat{L}_i\} = 7.2889$ $(i = 2, 3, 4)$, $B_1 = t(.9875; 6) = 2.969$, $B_i = t(.9875; 18) = 2.445$ $(i = 2, 3, 4)$

$$182.375 \pm 2.969(250.1674) \qquad -560.372 \leq L_1 \leq 925.122$$
$$-9.375 \pm 2.445(7.2889) \qquad -27.196 \leq L_2 \leq 8.446$$
$$20.625 \pm 2.445(7.2889) \qquad 2.804 \leq L_3 \leq 38.446$$
$$-103.375 \pm 2.445(7.2889) \qquad -121.196 \leq L_4 \leq -85.554$$

27.18. a. e_{ijk}:

	$j=1$		$j=2$	
i	$k=1$	$k=2$	$k=1$	$k=2$
1	−.045	.045	.045	−.045
2	−.120	.120	.120	−.120
3	.080	−.080	−.080	.080
4	−.045	.045	.045	−.045
5	.080	−.080	−.080	.080
6	.055	−.055	−.055	.055
7	.030	−.030	−.030	.030
8	−.045	.045	.045	−.045
9	.055	−.055	−.055	.055
10	−.045	.045	.045	−.045

$r = .973$

27.19. a.

Source	SS	df	MS
Subjects	154.579	9	17.175
A	3.025	1	3.025
B	14.449	1	11.449
AB	.001	1	.001
AS	2.035	9	.226
BS	5.061	9	.562
ABS	.169	9	.019
Total	176.319	39	

b. $\bar{Y}_{.11} = 3.93$, $\bar{Y}_{.12} = 5.01$, $\bar{Y}_{.21} = 4.49$, $\bar{Y}_{.22} = 5.55$

c. H_0: all $(\alpha\beta)_{jk}$ equal zero, H_a: not all $(\alpha\beta)_{jk}$ equal zero.

$F^* = .001/.019 = .05$, $F(.995; 1, 9) = 13.6$.

If $F^* \leq 13.6$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .82

d. H_0: $\alpha_1 = \alpha_2 = 0$, H_a: not both α_j equal zero.

$F^* = 3.025/.226 = 13.38$, $F(.995; 1, 9) = 13.6$.

If $F^* \leq 13.6$ conclude H_0, otherwise H_a. Conclude H_0. P-value = .005

H_0: $\beta_1 = \beta_2 = 0$, H_a: not both β_k equal zero.

$F^* = 11.449/.562 = 20.36$, $F(.995; 1, 9) = 13.6$.

If $F^* \leq 13.6$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .001

e. $\hat{L}_1 = .56$, $\hat{L}_2 = 1.08$, $\hat{L}_3 = -.52$, $\hat{L}_4 = 1.62$,

$s\{\hat{L}_i\} = .0613$ $(i = 1, ..., 4)$, $B = t(.99375; 27) = 3.11$

$.56 \pm 3.11(.0613)$	$.37 \leq L_1 \leq .75$
$1.08 \pm 3.11(.0613)$	$.89 \leq L_2 \leq 1.27$
$-.52 \pm 3.11(.0613)$	$-.71 \leq L_3 \leq -.33$
$1.62 \pm 3.11(.0613)$	$1.43 \leq L_4 \leq 1.81$

Chapter 28

BALANCED INCOMPLETE BLOCK, LATIN SQUARE, AND RELATED DESIGNS

28.8. e_{ij}:

i	$j=1$	$j=2$	$j=3$	$j=4$
1	13.2083	8.8333	−22.0417	
2	−7.9167	4.7083		3.2083
3	−5.2917		−1.5417	6.8333
4		−13.5417	23.5833	−10.0417

$r = .996$

28.9. a. $\hat{\mu}_{..} = 297.667$, $\hat{\tau}_1 = -45.375$, $\hat{\tau}_2 = -41.000$, $\hat{\tau}_3 = 30.875$, $\hat{\tau}_4 = 55.550$

$\hat{\mu}_{.1} = 252.292$, $\hat{\mu}_{.2} = 256.667$, $\hat{\mu}_{.3} = 328.542$, $\hat{\mu}_{.4} = 353.167$

b. H_0: $\tau_1 = \tau_2 = \tau_3 = 0$, H_a: not all τ_j equal zero. $SSE(F) = 1750.9$, $SSE(R) = 22480$, $F^* = (20729.1/3) \div (1750.9/5) = 19.73$, $F(.95; 3, 5) = 5.41$. If $F^* \leq 5.41$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .003

c. H_0: $\rho_1 = \rho_2 = \rho_3 = 0$, H_a: not all ρ_i equal zero. $SSE(F) = 14.519$, $SSE(R) = 22789$, $F^* = (21038.1/3) \div (1750.9/5) = 20.03$, $F(.95; 3, 5) = 5.41$. If $F^* \leq 5.41$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .003

d. $\hat{\mu}_{.1} = 252.292$, $s^2(\hat{\mu}_{.1}) = s^2(\hat{\mu}_{..}) + s^2(\hat{\tau}_1) = (.08333 + .28125)350.2 = 127.68$, $B = t(.975; 5) = 2.571$, $252.292 \pm 2.571(11.30)$, $223.240 \leq \mu_{.1} \leq 281.344$

e.

95% C.I.	lower	center	upper
$\mu_{.1} - \mu_{.2}$	−64.19	−4.375	55.44
$\mu_{.1} - \mu_{.3}$	−136.07	−76.250	−16.43
$\mu_{.1} - \mu_{.4}$	−160.69	−100.875	−41.06
$\mu_{.2} - \mu_{.3}$	−131.70	−71.87	−12.06
$\mu_{.2} - \mu_{.4}$	−156.30	−96.50	−36.68
$\mu_{.3} - \mu_{.4}$	−84.44	−24.63	35.19

28.10. $r = 4$, and $r_b = 3$, $df_e = 4n - 4 - 4n/3 + 1 = 8n/3 - 3$.

Since $n_p = n(3-1)/(4-1) = 2n/3$, $\sigma^2\{\hat{D}_j\} = 2\sigma^2(3)/(4n_p) = 9\sigma^2/(4n)$

$$T\sigma\{\hat{D}_j\} = \frac{1}{\sqrt{2}}q[.95; 4, 8n/3-3]\sqrt{\frac{9\sigma^2}{4n}}$$

For $\sigma^2 = 2.0$ and $T\sigma\{\hat{D}_j\} \leq 1.5$, so we need to iterate to find n so that

$$n \geq 2q^2[.95; 4, 8n/3-3]$$

We iteratively find $n \geq 28$. Since design 2 in Table 28.1 has $n = 3$, we require that design 2 be repeated 10 times. Thus, $n = 30$, and $n_b = 40$.

28.14. e_{ijk}:

i	$j=1$	$j=2$	$j=3$	$j=4$
1	$-.1375$	.0875	$-.0125$	.0625
2	$-.0125$	$-.0125$	.1625	$-.1375$
3	.1375	$-.0875$	$-.0625$	.0125
4	.0125	.0125	$-.0875$	.0625

$r = .986$

28.15. a. $\bar{Y}_{..1} = 1.725$, $\bar{Y}_{..2} = 1.900$, $\bar{Y}_{..3} = 2.175$, $\bar{Y}_{..4} = 2.425$

b.

Source	SS	df	MS
Rows (sales volumes)	5.98187	3	1.99396
Columns (locations)	.12188	3	.04062
Treatments (prices)	1.13688	3	.37896
Error	.11875	6	.01979
Total	7.35938	15	

H_0: all τ_k equal zero ($k = 1, ..., 4$), H_a: not all τ_k equal zero. $F^* = .37896/.01979 = 19.149$, $F(.95; 3, 6) = 4.76$. If $F^* \leq 4.76$ conclude H_0, otherwise H_a. Conclude H_a. P-value = .002

c. $\hat{L}_1 = \bar{Y}_{..1} - \bar{Y}_{..2} = -.175$, $\hat{L}_2 = \bar{Y}_{..1} - \bar{Y}_{..3} = -.450$, $\hat{L}_3 = \bar{Y}_{..1} - \bar{Y}_{..4} = -.700$, $\hat{L}_4 = \bar{Y}_{..2} - \bar{Y}_{..3} = -.275$, $\hat{L}_5 = \bar{Y}_{..2} - \bar{Y}_{..4} = -.525$, $\hat{L}_6 = \bar{Y}_{..3} - \bar{Y}_{..4} = -.250$, $s\{\hat{L}_i\} = .09947$ ($i = 1, ..., 6$), $q(.90; 4, 6) = 4.07$, $T = 2.8779$

$-.175 \pm 2.8779(.09947)$ $\quad -.461 \leq L_1 \leq .111$
$-.450 \pm 2.8779(.09947)$ $\quad -.736 \leq L_2 \leq -.164$
$-.700 \pm 2.8779(.09947)$ $\quad -.986 \leq L_3 \leq -.414$
$-.275 \pm 2.8779(.09947)$ $\quad -.561 \leq L_4 \leq .011$
$-.525 \pm 2.8779(.09947)$ $\quad -.811 \leq L_5 \leq -.239$
$-.250 \pm 2.8779(.09947)$ $\quad -.536 \leq L_6 \leq .036$

28.16. a. $\hat{E}_1 = 21.1617$, $\hat{E}_2 = 1.2631$, $\hat{E}_3 = 25.9390$

28.20. $\phi = 3.399$, $1 - \beta \cong .99$

28.24. a. $Y_{ijk} = \mu_{...} + \rho_1 X_{ijk1} + \rho_2 X_{ijk2} + \rho_3 X_{ijk3} + \kappa_1 X_{ijk4} + \kappa_2 X_{ijk5}$
$\quad + \kappa_3 X_{ijk6} + \tau_1 X_{ijk7} + \tau_2 X_{ijk8} + \tau_3 X_{ijk9} + \epsilon_{(ijk)}$

$$X_{ijk1} = \begin{cases} 1 & \text{if experimental unit from row blocking class 1} \\ -1 & \text{if experimental unit from row blocking class 4} \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk2} and X_{ijk3} are defined similarly

$$X_{ijk4} = \begin{cases} 1 & \text{if experimental unit from column blocking class 1} \\ -1 & \text{if experimental unit from column blocking class 4} \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk5} and X_{ijk6} are defined similarly

$$X_{ijk7} = \begin{cases} 1 & \text{if experimental unit received treatment 1} \\ -1 & \text{if experimental unit received treatment 4} \\ 0 & \text{otherwise} \end{cases}$$

X_{ijk8} and X_{ijk9} are defined similarly

b. <u>Full model</u>:

$$\hat{Y} = 2.05625 - .70625X_1 - .45625X_2 + .34375X_3 + .14375X_4$$
$$-.05625X_5 - .00625X_6 - .33125X_7 - .15625X_8 + .11875X_9$$

$SSE(F) = .1188$

<u>Reduced model</u>:

$$\hat{Y} = 2.05625 - .70625X_1 - .45625X_2 + .34375X_3 + .14375X_4 - .05625X_5 - .00625X_6$$

$SSE(R) = 1.2556$

H_0: all τ_k equal zero ($k = 1, 2, 3$), H_a: not all τ_k equal zero. $F^* = (1.1368/3) \div (.1188/6) = 19.138$, $F(.95; 3, 6) = 4.76$. If $F^* \leq 4.76$ conclude H_0, otherwise H_a. Conclude H_a.

c. $\hat{L} = \hat{\tau}_3 - (-\hat{\tau}_1 - \hat{\tau}_2 - \hat{\tau}_3) = 2\hat{\tau}_3 + \hat{\tau}_1 + \hat{\tau}_2 = -.250$, $s^2\{\hat{\tau}_i\} = .00371$ $(i = 1, 2, 3)$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = s\{\hat{\tau}_1, \hat{\tau}_3\} = s\{\hat{\tau}_2, \hat{\tau}_3\} = -.00124$, $s\{\hat{L}\} = .09930$, $t(.975; 6) = 2.447$, $-.250 \pm 2.447(.09930)$, $-.493 \leq L \leq -.007$

d. (i) <u>Full model</u>:

$$\hat{Y} = 2.02917 - .67917X_1 - .53750X_2 + .37083X_3 + .17083X_4 - .02917X_5$$
$$-.08750X_6 - .30417X_7 - .23750X_8 + .14583X_9$$

$SSE(F) = .0483$

<u>Reduced model</u>:

$$\hat{Y} = 2.05556 - .70556X_1 - .45833X_2 + .34444X_3 + .14444X_4 - .05556X_5 - .00833X_6$$

$SSE(R) = 1.2556$

H_0: all τ_k equal zero ($k = 1, 2, 3$), H_a: not all τ_k equal zero. $F^* = (1.2073/3) \div (.0483/5) = 41.66$, $F(.95; 3, 5) = 5.41$. If $F^* \leq 5.41$ conclude H_0, otherwise H_a. Conclude H_a.

(ii) $\hat{L} = \hat{\tau}_1 - \hat{\tau}_2 = -.06667$, $s^2\{\hat{\tau}_1\} = .00191$, $s^2\{\hat{\tau}_2\} = .00272$, $s\{\hat{\tau}_1, \hat{\tau}_2\} = -.00091$, $s\{\hat{L}\} = .0803$, $t(.975; 5) = 2.571$, $-.06667 \pm 2.571(.0803)$, $-.273 \leq L \leq .140$

Chapter 29

EXPLORATORY EXPERIMENTS – TWO-LEVEL FACTORIAL AND FRACTIONAL FACTORIAL DESIGNS

29.3. a. Six factors, two levels, 64 trials

b. No

29.6. a. $\sigma^2\{b_1\} = \sigma^2/n_T = 5^2/64 = .391$. Yes, yes

b. $z(.975) = 1.96$, $n_T = [1.96(5)/(.5)]^2 = 384.16$, $384.16/64 = 6$ replicates

29.7. a. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_5 X_{i5} + \beta_{12} X_{i12} + \cdots + \beta_{45} X_{i45} + \beta_{123} X_{i123}$

$$+ \cdots + \beta_{345} X_{i345} + \beta_{1234} X_{i1234} + \cdots + \beta_{2345} X_{i2345} + \beta_{12345} X_{i12345} + \epsilon_i$$

Coef.	b_q	Coef.	b_q	Coef.	b_q	Coef.	b_q
b_0	6.853	b_{14}	−.239	b_{123}	.070	b_{245}	.076
b_1	1.606	b_{15}	.611	b_{124}	.020	b_{345}	−.576
b_2	−.099	b_{23}	−.134	b_{125}	−.118	b_{1234}	.062
b_3	1.258	b_{24}	−.127	b_{134}	−.378	b_{1235}	.323
b_4	−1.151	b_{25}	−.045	b_{135}	−.138	b_{1245}	.357
b_5	−1.338	b_{34}	−.311	b_{145}	−.183	b_{1345}	−.122
b_{12}	−.033	b_{35}	.912	b_{234}	.233	b_{2345}	−.292
b_{13}	.455	b_{45}	−.198	b_{235}	.055	b_{12345}	.043

29.8. a. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_5 X_{i5} + \beta_{12} X_{i12} + \cdots + \beta_{45} X_{i45} + \epsilon_i$

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	6.853		b_{14}	$-.239$	.340
b_1	1.606	.000	b_{15}	.611	.023
b_2	$-.099$	.689	b_{23}	$-.134$	.589
b_3	1.258	.000	b_{24}	$-.127$	.610
b_4	-1.151	.000	b_{25}	$-.045$	.855
b_5	-1.338	.000	b_{34}	$-.311$	.219
b_{12}	$-.033$	.892	b_{35}	.912	.002
b_{13}	.455	.080	b_{45}	$-.198$	.426

b. H_0: Normal, H_a: not normal. $r = .983$. If $r \geq .9656$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = .2432$. If P-value $\geq .0034$ conclude H_0, otherwise H_a. Active effects (see part a): β_1, β_3, β_4, β_5, β_{35}

29.15. Defining relation: $0 = 123 = 245 = 1345$

Confounding scheme:

$$
\begin{array}{lllllll}
0 & = & 123 & = & 245 & = & 1345 \\
1 & = & 23 & = & 1245 & = & 345 \\
2 & = & 13 & = & 45 & = & 12345 \\
3 & = & 12 & = & 2345 & = & 145 \\
4 & = & 1234 & = & 25 & = & 135 \\
5 & = & 1235 & = & 24 & = & 134 \\
14 & = & 234 & = & 125 & = & 35 \\
15 & = & 235 & = & 124 & = & 34
\end{array}
$$

Resolution = III, no

29.18. a. Defining relation: $0 = 1235 = 2346 = 1247 = 1456 = 3457 = 1367 = 2567$,

resolution = IV, no

b. Omitting four-factor and higher-order interactions:

$$
\begin{array}{lllllllll}
1 & = & 235 & = & 247 & = & 367 & = & 456 \\
2 & = & 135 & = & 147 & = & 346 & = & 567 \\
3 & = & 125 & = & 167 & = & 246 & = & 457 \\
4 & = & 127 & = & 156 & = & 236 & = & 357 \\
5 & = & 123 & = & 146 & = & 267 & = & 347 \\
6 & = & 137 & = & 145 & = & 234 & = & 257 \\
7 & = & 124 & = & 136 & = & 256 & = & 345 \\
12 & = & 35 & = & 47 & & & & \\
13 & = & 25 & = & 67 & & & & \\
14 & = & 27 & = & 56 & & & & \\
15 & = & 23 & = & 46 & & & & \\
16 & = & 37 & = & 45 & & & & \\
17 & = & 24 & = & 36 & & & & \\
26 & = & 34 & = & 57 & & & &
\end{array}
$$

c. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_7 X_{i7} + \beta_{12} X_{i12} + \beta_{13} X_{i13} + \beta_{14} X_{i14}$

$+\beta_{15}X_{i15} + \beta_{16}X_{i16} + \beta_{17}X_{i17} + \beta_{26}X_{i26} + \epsilon_i$

Coef.	b_q	Coef.	b_q	Coef.	b_q
b_0	8.028	b_5	.724	b_{14}	$-.316$
b_1	.127	b_6	$-.467$	b_{15}	.318
b_2	.003	b_7	$-.766$	b_{16}	.117
b_3	.021	b_{12}	.354	b_{17}	.021
b_4	-2.077	b_{13}	$-.066$	b_{26}	$-.182$

e. H_0: $\beta_{12} = \cdots = \beta_{17} = \beta_{26} = 0$, H_a: not all $\beta_q = 0$. $F^* = (6.046/7) \div (.1958/1) = 4.41$, $F(.99; 7, 1) = 5,928$. If $F^* \leq 5,928$ conclude H_0, otherwise H_a. Conclude H_0.

29.19. a.

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	8.028		b_4	-2.077	.000
b_1	.127	.581	b_5	.724	.011
b_2	.003	.989	b_6	$-.467$	.067
b_3	.021	.928	b_7	$-.766$	.008

b. H_0: Case i not an outlier, H_a: case i an outlier ($i = 3, 14$). $t_3 = 2.70$, $t_{14} = -4.09$, $t(.99844; 7) = 4.41$. If $| t_i | \leq 4.41$ conclude H_0, otherwise H_a. Conclude H_0 for both cases.

c. H_0: Normal, H_a: not normal. $r = .938$. If $r \geq .929$ conclude H_0, otherwise H_a. Conclude H_0.

d. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = .2208$. If P-value $\geq .02$ conclude H_0, otherwise H_a. Active effects (see part a): β_4, β_5, β_7

e. Set $X_4 = -1$, $X_5 = 1$, $X_7 = -1$ to maximize extraction.

29.26. b. The seven block effects are confounded with the following interaction terms: β_{135}, β_{146}, β_{236}, β_{245}, β_{1234}, β_{1256}, β_{3456}

No, no

c. $Y_i = \beta_0 X_{i0} + \beta_1 X_{i1} + \cdots + \beta_6 X_{i6} + \beta_{12} X_{i12} + \cdots + \beta_{56} X_{i56} + \beta_{123} X_{i123}$

$$+ \cdots + \beta_{456} X_{i456} + \beta_{1235} X_{i1235} + \cdots + \beta_{2456} X_{i2456} + \beta_{12345} X_{i12345}$$

$$+ \cdots + \beta_{23456} X_{i23456} + \beta_{123456} X_{i123456} + \alpha_1 Z_{i1} + \cdots + \alpha_7 Z_{i7} + \epsilon_i$$

where α_1, ..., α_7 are the block effects

Coef.	b_q	Coef.	b_q	Coef.	b_q	Coef.	b_q
b_0	63.922	b_{34}	.297	b_{246}	−.391	b_{2356}	.766
b_1	2.297	b_{35}	.266	b_{256}	.078	b_{2456}	.203
b_2	5.797	b_{36}	.984	b_{345}	−.672	b_{12345}	−.297
b_3	2.172	b_{45}	−.422	b_{346}	.734	b_{12346}	−.391
b_4	2.359	b_{46}	−.141	b_{356}	−.734	b_{12356}	−.734
b_5	2.828	b_{56}	.516	b_{456}	−.234	b_{12456}	−.422
b_6	2.922	b_{123}	.422	b_{1235}	.578	b_{13456}	−.109
b_{12}	.547	b_{124}	.172	b_{1236}	.922	b_{23456}	.203
b_{13}	−.266	b_{125}	1.391	b_{1245}	.453	b_{123456}	.016
b_{14}	−.203	b_{126}	.984	b_{1246}	.109	Block 1	−4.172
b_{15}	−.797	b_{134}	.297	b_{1345}	−.797	Block 2	−.422
b_{16}	−.141	b_{136}	−.641	b_{1346}	.547	Block 3	1.203
b_{23}	−.641	b_{145}	−.109	b_{1356}	−1.109	Block 4	6.703
b_{24}	−1.141	b_{156}	−.547	b_{1456}	−.109	Block 5	−.797
b_{25}	.891	b_{234}	.234	b_{2345}	.328	Block 6	−1.047
b_{26}	.047	b_{235}	.266	b_{2346}	−.578	Block 7	−9.547

29.27. a.

Coef.	b_q	P-value	Coef.	b_q	P-value
b_0	63.922		b_{26}	.047	.935
b_1	2.297	.000	b_{34}	.297	.607
b_2	5.797	.000	b_{35}	.266	.645
b_3	2.172	.001	b_{36}	.984	.094
b_4	2.359	.000	b_{45}	−.422	.466
b_5	2.828	.000	b_{46}	−.141	.807
b_6	2.922	.000	b_{56}	.516	.373
b_{12}	.547	.346	Block 1	−4.172	.009
b_{13}	−.266	.645	Block 2	−.422	.782
b_{14}	−.203	.725	Block 3	1.203	.432
b_{15}	−.797	.172	Block 4	6.703	.000
b_{16}	−.141	.807	Block 5	−.797	.602
b_{23}	−.641	.270	Block 6	−1.047	.494
b_{24}	−1.141	.054	Block 7	−9.547	.000
b_{25}	.891	.128			

b. H_0: Normal, H_a: not normal. $r = .989$. If $r \geq .9812$ conclude H_0, otherwise H_a. Conclude H_0.

c. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{\hat{\alpha}_i\} = 1.513$ for block effects, $s\{b_q\} = .5719$ for factor effects. If P-value $\geq .01$ conclude H_0, otherwise H_a. Active effects (see part a): Block effects 1, 4, 7, all main effects

29.28. a. See Problem 29.27a for estimated factor and block effects. (These do not change with subset model.)

b. Maximum team effectiveness is accomplished by setting each factor at its high level.

c. $\hat{Y}_h = 82.297$, $s\{\text{pred}\} = 4.857$, $t(.975; 50) = 2.009$, $82.297 \pm 2.009(4.857)$, $72.54 \leq Y_{h(new)} \leq 92.05$

29.32. a.

i	1	2	3	4	5	6	7	8
s_i^2	.0164	.0173	.0804	.1100	.0010	.0079	.0953	.1134
$\log_e s_i^2$	−4.109	−4.058	−2.521	−2.207	−6.949	−4.838	−2.351	−2.176

b. $\widehat{\log_e s_i^2} = -3.651 + .331X_{i1} + 1.337X_{i2} - .427X_{i3} - .275X_{i4} - .209X_{i12} + .240X_{i13} + .477X_{i14}$.

X_2 appears to be active.

c. $\hat{v}_i = .006819$ (for $i = 1, 2, 5, 6$)

$\hat{v}_i = .09887$ (for $i = 3, 4, 7, 8$)

d. $\hat{Y}_i = 7.5800 + .0772X_{i1}$

e. From the location model: $X_1 = +1$; from the dispersion model: $X_2 = -1$

f. From dispersion model: $\hat{s^2} = \exp[-3.651 + 1.337(-1)]) = .006819$,

and a 97.5% P.I. is $[\exp(-6.16), \exp(-3.82)]$,or (.00211, .0219).

g. $\widehat{MSE} = .006819 + (8 - 7.659)^2 = .124$

Chapter 30

RESPONSE SURFACE METHODOLOGY

30.11. b.

Coef.	b_q	P-value
b_0	1.868	
b_1	.190	.007
b_2	.195	.006
b_3	−.120	.039
b_{12}	.162	.020

Coef.	b_q	P-value
b_{13}	−.038	.471
b_{23}	−.062	.251
b_{11}	.228	.044
b_{22}	−.047	.602
b_{33}	.028	.757

d. H_0: $\beta_q = 0$, H_a: $\beta_q \neq 0$. $s\{b_q\} = .0431$ (for linear effects), $s\{b_q\} = .0481$ (for interaction effects), $s\{b_q\} = .0849$ (for quadratic effects). If P-value $\geq .05$ conclude H_0, otherwise H_a. Active effects (see part b): β_1, β_2, β_3, β_{12}, β_{11}

30.12. a.

Coef.	b_q
b_0	1.860
b_1	.190
b_2	.195

Coef.	b_q
b_3	−.120
b_{12}	.162
b_{11}	.220

b. H_0: Normal, H_a: not normal. $r = .947$. If $r \geq .938$ conclude H_0, otherwise H_a. Conclude H_0.

30.14. a.

Design Matrix:

X_1	X_2
$-.707$	$-.707$
$.707$	$-.707$
$-.707$	$.707$
$.707$	$.707$
-1	0
1	0
0	-1
0	1
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0

Corner Points:

X_1	X_2
$-.707$	$-.707$
$.707$	$-.707$
$-.707$	$.707$
$.707$	$.707$

b.

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} .125 & 0 & 0 & -.125 & -.125 & 0 \\ 0 & .250 & 0 & 0 & 0 & 0 \\ 0 & 0 & .250 & 0 & 0 & 0 \\ -.125 & 0 & 0 & .5 & 0 & 0 \\ -.125 & 0 & 0 & 0 & .5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

30.16. a.

$$\mathbf{b}^* = \begin{bmatrix} -2.077 \\ .724 \end{bmatrix} \qquad s = 2.200$$

b.

t	X_1	X_2
1.5	-1.416	.494
2.5	-2.361	.823
3.5	-3.304	1.152