Answer Sheet (Dr Borhen)

Semester I-1444H / MATH-204 / Quiz-I **Name:**... **Id:**.................... **Attendance Number :**.........	جامعة الملك سعود King Saud University

Question 1 [4 Marks]:

Determine the largest local region for which the following initial value problem admits a unique solution

$$\begin{cases} \ln(x-2)\dfrac{dy}{dx} = \sqrt{y-2} \\ y\left(\frac{5}{2}\right) = 4 \end{cases}$$

and sketch it.

Answer:

$$\ln(x-2)\frac{dy}{dx} = \sqrt{y-2} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{y-2}}{\ln(x-2)} = f(x,y)$$

1 $$\frac{\partial f}{\partial y}(x,y) = \frac{1}{\ln(x-2)} \cdot \frac{1}{2\sqrt{y-2}}$$

Then f & $\frac{\partial f}{\partial y}$ are continuous on the region

1 $$R = \{(x,y) \in \mathbb{R}^2 \,/\, x>2,\ x\neq 3,\ y>2\}$$
$$= \{(x,y) \,/\, 2<x<3,\ y>2\} \cup \{(x,y) \,/\, x>3,\ y>2$$

1 As $\left(\frac{5}{2}, 4\right) \in R_1 = \{(x,y) \,/\, 2<x<3 \ \&\ y>2\}$

Then the largest region for which the IVP has a ~~unique~~ unique solution is $R_1 = \{(x,y) \,/\, 2<x<3 \ \&\ y>2\}$

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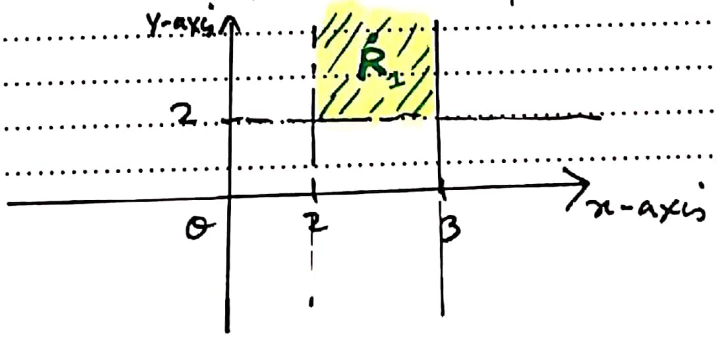

Question 2 [3 Marks]:

Solve the following differential equation

$$2x(y^2+y)dx + (x^2-1)ydy = 0;\ \ y \neq 0.$$

Answer: $2x\,(y^2+y)\,dx + (x^2-1)\,y\,dy = 0 \;;\; y \neq 0$

$2x\cancel{y}(y+1)\,dx = (1-x^2)\cancel{y}\,dy$

Can be separated as:

(1) $$\frac{2x}{x^2-1}\,dx = \frac{-1}{1+y}\,dy$$

Integrating both sides,

(1) $$\int \frac{2x}{x^2-1}\,dx = -\int \frac{dy}{1+y}$$

$$\ln|x^2-1| = -\ln|1+y| + c$$

$$\ln|x^2-1| + \ln|1+y| = c$$

(1) So $(x^2-1)(1+y) = a$ with $a \in \mathbb{R}\setminus\{0\}$.

$$1+y = \frac{a}{x^2-1} \Rightarrow \boxed{y(x) = -1 + \frac{a}{x^2-1}} \quad x \neq \pm 1$$

Question 3 [3 Marks]:

Find the general solution of the differential equation $x\,\frac{dy}{dx} - y = \sqrt{x^2+y^2}\,; x > 0$.

Answer: (*) $x\frac{dy}{dx} - y = \sqrt{x^2+y^2} \;;\; x>0$

$$\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x} = f(x,y)$$

(1) Put $y = ux$, $f(x,ux) = \frac{ux+\sqrt{x^2+u^2x^2}}{x} = u + \sqrt{1+u^2} = F(u)$

So (*) is homogeneous DE.

(1) $$\frac{du}{F(u)-u} = \frac{dx}{x}$$

$$\frac{du}{\sqrt{1+u^2}} = \frac{dx}{x}$$

Integrating both sides, $\int \frac{du}{\sqrt{1+u^2}} = \int \frac{dx}{x}$

$$\sinh^{-1}(u) = \ln\left[u+\sqrt{1+u^2}\right] = \ln x + c$$

(1) $\Rightarrow u + \sqrt{1+u^2} = Ax$ with $A>0$, $x>0$

As $u = y/x$ so $\frac{y}{x} + \sqrt{1+(y/x)^2} = Ax$

$$\boxed{y + \sqrt{y^2+x^2} = Ax^2} \quad \text{with } A>0,\; x>0$$

The general solution of (*)