

Q.1) Find the distance d between the point $(1, -4, -2)$ and the plane $4x - 2y + 6z = 3$.

Answer:

$$A(x_0 = 1, y_0 = -4, z_0 = -2)$$

$$\text{Plane (P): } 4x - 2y + 6z - 3 = 0 \quad ; \quad a = 4, \quad b = -2, \quad c = 6, \quad d = -3$$

The distance d between A and the Plane (P) is given by

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|4 + 8 - 12 - 3|}{\sqrt{16 + 4 + 36}} = \frac{3}{\sqrt{56}} = \frac{3}{2\sqrt{14}}$$

Q.2) Let $\vec{r}(t) = (\ln t)\vec{i} + e^{-4t}\vec{j} + \frac{\cos t}{\sqrt{1-t}}\vec{k}$. Find the domain of \vec{r} .

Answer:

$$\text{The domain of } \vec{r} \text{ is } \left\{ t \in \mathbb{R} \mid \begin{array}{l} t > 0 \text{ and} \\ 1 - t > 0 \end{array} \right\} = (0, 1)$$

Q.3) Find the tangential component of acceleration at time t for the curve C determined by the vector of position:

$$\vec{r}(t) = 2t^3\vec{i} + 3\sin t\vec{j} + 2\cos t\vec{k}$$

Answer:

$$a_T = \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{\|\vec{r}'(t)\|}$$

$$\vec{r}'(t) = 6t^2\vec{i} + 3\cos t\vec{j} - 2\sin t\vec{k}$$

$$\vec{r}''(t) = 12t\vec{i} - 3\sin t\vec{j} - 2\cos t\vec{k}$$

$$\vec{r}'(t) \cdot \vec{r}''(t) = 72t^3 - 9\sin t\cos t + 4\sin t\cos t$$

$$\|\vec{r}'(t)\| = \sqrt{(6t^2)^2 + (3\cos t)^2 + (-2\sin t)^2}$$

$$\text{We get } a_T = \frac{72t^3 - 5\sin t\cos t}{\sqrt{36t^4 + 9\cos^2 t + 4\sin^2 t}}$$

End of quiz