Answer

Semester 432 / MATH-244 / Quiz-II Name:.. Id:.................. Attendance Number :.........		جامعة الملك سعود King Saud University

Question 1 [Marks: 1.5]:

Show that $\{(x, y) \in \mathbb{R}^2 | x^4 + y^2 \le 0\}$ is a *vector subspace* of *Euclidean space* $\mathbb{R}^2$.

Answer:

$S = \{(x, y) \in \mathbb{R}^2 / x^4 + y^2 \le 0\} = \{(0,0)\}$

It is a subspace of $\mathbb{R}^2$ (Trivial)

Question 2 [Marks: 2.5]:

Let $B = \{(1,1,1), (0,1,1), (0,0,1)\}$ and $C = \{(1,0,0), (0,1,0), (0,0,1)\}$ be *bases* of *Euclidean space* $\mathbb{R}^3$ and $[u]_B = (3,2,1)$. **Find** the *transition matrix* ${}_C P_B$ and the *coordinate vector* $[u]_C$.

Answer:

- $${}_C P_B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

- $$[u]_C = {}_C P_B [u]_B = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix}$$

Question 3 [Marks: 2]:

Let A be 4×2 *matrix* with *rank* $(A) = 2$. **Find** *nullity* (A^T).

Answer: A^T is 2×4 matrix

nullity (A^T) + rank $(A^T) = 4$

As Rank (A^T) = Rank (A)

then nullity $(A^T) = 4 - \text{Rank}(A^T)$

$= 4 - 2 = 2$

Question 4 [Marks: 2]:

Explain! why the *function* $<(x_1, y_1, z_1), (x_2, y_2, z_2)> = x_1 y_2 z_1 + x_2 y_1 z_2$ is not an *inner product* on $\mathbb{R}^3$.

Answer:

Take: $u = (1, 1, -1)$

$\|u\|^2 = < u \mid u > = -1 + -1 = -2 < 0$ ↯

So the function $<(x_1, y_1, z_1) \mid (x_2, y_2, z_2)> = x_1 y_2 z_1 + x_2 y_1 z_2$

is not an inner product on $\mathbb{R}$

Question 5 [Marks: 2]:

Let $u_1 = (2, -1, -2)$, $u_2 = (1, 0, 1)$ and $u_3 = (1, x, y)$ in *Euclidean space* $\mathbb{R}^3$.

Find the values of x and y so that the set $\{u_1, u_2, u_3\}$ is orthogonal.

Answer:

As $\{u_1, u_2, u_3\}$ is orthogonal then

$< u_1 \mid u_3 > = 0 \iff 2 - x - 2y = 0$

and $< u_2 \mid u_3 > = 0 \iff 1 + y = 0$

$\iff \begin{cases} x + 2y = 2 \\ y = -1 \end{cases}$

$x = 4, \ y = -1$

$u_3 = (1, 4, -1)$.