Name: Answer ID: Section: **Quiz1- First Semester 1443H – Math 244** **Duration: 30 min**	جامعة الملك سعود King Saud University

Question 1 [Marks: 1.5]:

Find the diagonal matrix $\mathbf{D}^3$ that satisfies $\mathbf{D} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

Answer:

$$D^3 = \begin{pmatrix} (-1)^3 & 0 & 0 \\ 0 & 2^{3/2} & 0 \\ 0 & 0 & 2^3 \end{pmatrix}$$

Question 2 [Marks: 1.5]:

Let $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ be a matrix and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be a unit matrix. Find x and y if

$A^2 + x A = -y I$.

Answer:

$$A^2 = A \cdot A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 9 & 4 \end{pmatrix}$$

$$A^2 + x A = \begin{pmatrix} 1 & 0 \\ 9 & 4 \end{pmatrix} + x \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1+x & 0 \\ 9+3x & 4+2x \end{pmatrix} = -yI = \begin{pmatrix} -y & 0 \\ 0 & -y \end{pmatrix}$$

So $\begin{cases} 1+x = -y \\ 9+3x = 0 \\ 4+2x = -y \end{cases} \Leftrightarrow \begin{cases} x = -3 \\ y = 2 \end{cases}$

Question 3 [Marks: 1.5]:

Let $\mathbf{A} \in \mathbf{M}_{3\times 3}(\mathbb{R})$ with determinant $|A| = 2$. Find $|2(adj(A))^{-1} + A|$.

Answer:

$A^{-1} = \frac{1}{|A|} Adj(A)$ So $(A^{-1})^{-1} = |A| [Adj(A)]^{-1}$

$[Adj(A)]^{-1} = \frac{1}{|A|} A$

$$\left| 2 [Adj(A)]^{-1} + A \right| = \left| \frac{2}{|A|} A + A \right| = |2A| = 2^3 |A| = 16$$

Question 4 [Marks: 1.5]:

Show that the matrix $\mathbf{A}=\begin{bmatrix}1 & 1 & 2\\ 1 & 0 & 1\\ 2 & -1 & 3\end{bmatrix}$ is row equivalent to the matrix $\mathbf{I}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$.

Answer:

$-2 \quad -2 \quad -1$

$$\begin{pmatrix}1 & 1 & 2\\ 1 & 0 & 1\\ 2 & -1 & 3\end{pmatrix} \to \begin{pmatrix}1 & 1 & 2\\ 0 & -1 & -1\\ 0 & -3 & -1\end{pmatrix} \to \begin{pmatrix}1 & 1 & 2\\ 0 & 1 & 1\\ 0 & 0 & +2\end{pmatrix}$$

$$\to \begin{pmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \longrightarrow \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Question 5 [Marks: 2]:

By using the Cramer's rule, find values of x and y in solution of the following linear system:

$$\begin{aligned} x + 2y + 3z &= 1\\ 2x + 5y + 3z &= 2\\ x \quad\quad + 8z &= 0. \end{aligned}$$

Answer:

$$(*)\begin{cases} x + 2y + 3z = 1\\ 2x + 5y + 3z = 2\\ x \quad + 8z = 0 \end{cases} \iff AX = B \text{ with } A = \begin{pmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{pmatrix}$$

$$X = \begin{pmatrix}x\\ y\\ z\end{pmatrix} \quad ; \quad B = \begin{pmatrix}1\\ 2\\ 0\end{pmatrix}$$

$|A| = [40 + 6 + 0] - [15 + 0 + 32] = -1 \neq 0$ So $(*)$ has uniq[ue] solution.

By Cramer's rule:

$$x = \frac{1}{-1}\begin{vmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 0 & 0 & 8\end{vmatrix} = -8$$

$$y = \frac{1}{-1}\begin{vmatrix}1 & 1 & 3\\ 2 & 2 & 3\\ 1 & 0 & 8\end{vmatrix} = -\left[(16 + 3 + 0) - (6 + 0 + 16)\right] = 3$$

Question 6 [Marks: 2]:

Find condition on a, b and c for which the following linear system has infinitely many solutions:

$$\begin{aligned} x - 2y + 5z &= a \\ 4x - 5y + 8z &= b \\ -3x + 3y - 3z &= c \end{aligned}$$

Answer:

Augmented matrix $\left[\begin{array}{ccc|c} 1 & -2 & 5 & a \\ 4 & -5 & 8 & b \\ -3 & 3 & -3 & c \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 5 & a \\ 0 & 3 & -12 & b-4a \\ 0 & -3 & 12 & c+3a \end{array}\right]$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 5 & a \\ 0 & 3 & -12 & b-4a \\ 0 & 0 & 0 & b+c-a \end{array}\right]$$

The System has infinitely many Solutions if and only if

$$\boxed{b+c-a=0}$$

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