King Saud University	College of Sciences		Department of Mathematics
First Examination	Math 132	Semester I (1442)	Time:1H30

Exercice 1: ((2+2)+2+2+2)

1. Decide whether the following propositions are tautology or a contradiction or a contingency?
   (a) $[(p \leftrightarrow q) \vee (\neg p \rightarrow r)] \rightarrow \neg q$.
   (b) $[\neg(p \rightarrow \neg q) \vee \neg(q \rightarrow \neg r)] \wedge \neg(\neg r \rightarrow p)$.
2. Using laws, prove that the following conditional statement is a Tautology:
$$[(p \rightarrow \neg q) \wedge (q \rightarrow \neg r)] \vee (\neg r \rightarrow p).$$
3. Prove that the following conditional statement is a Contradiction:
$$[p \wedge (\neg q \vee r)] \wedge [\neg p \vee (q \wedge \neg r)]$$
4. Without using truth tables, prove the following logical equivalence:
$$(p \leftrightarrow q) \wedge (p \vee q) \equiv (p \wedge q)$$

Exercice 2: ((1+1)+(1+1))

1. Determine the truth value of each of the following statements if the domain consists of all real numbers. (Justify your answer)
   (a) $\exists x \in \mathbb{R};\ (x^2 - 5 = 0)$.
   (b) $\forall x \in \mathbb{R};\ x^2 \geq x$.
2. Determine the truth value of the following statement: if the domain consists of all integers. (Justify your answer)
   (a) $\exists n \in \mathbb{Z};\ (n^2 - 2 = 0)$.
   (b) $\forall n \in \mathbb{Z};\ n^2 \geq n$.

Exercice 3: (3+3+5)

1. Let x and y be two real numbers. Prove that:if $x + y \geq 2$ then, $x \geq 1$ or $y \geq 1$.
2. Let x, y, and z be real numbers. Prove by contradiction that
   if $(2x^3 + 2y + 4z^2 \leq 78)$ then $(x \leq 3$ or $y \leq 4$ or $z \leq 2$.
3. Let n be an integer. Prove that: n is odd if and only if $3n + 3$ is even.

Answer First examination

Exercice 1:

1) a) (2) $\overbrace{[\overbrace{(p \leftrightarrow q) \vee (\neg p \to r)}^{A}]}^{R} \to \neg q$.

p	q	r	$p\leftrightarrow q$	$\neg p$	$\neg p \to r$	A	$\neg q$	R
T	T	T	T	F	T	T	F	F
T	T	F	T	F	T	T	F	F
T	F	T	F	F	T	T	T	T
T	F	F	F	F	T	T	T	T
F	T	T	F	T	T	T	F	F
F	T	F	F	T	F	F	F	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	T	T	T

contingency

b) (2) $\overbrace{[\overbrace{\neg(p \to \neg q) \vee \neg(q \to \neg r)}^{A}] \wedge \neg(\neg r \to p)}^{R}$.

p	q	r	$p\to\neg q$	$\neg(p\to\neg q)$	$q\to\neg r$	$\neg(q\to\neg r)$	A	$(\neg r\to p)$	$\neg(\neg r\to p)$	R
T	T	T	F	T	F	T	T	T	F	F
T	T	F	F	T	T	F	T	T	F	F
T	F	T	T	F	T	F	F	T	F	F
T	F	F	T	F	T	F	F	T	F	F
F	T	T	T	F	F	T	T	T	F	F
F	T	F	T	F	T	F	F	F	T	F
F	F	T	T	F	T	F	F	T	F	F
F	F	F	T	F	T	F	F	F	T	F

contradiction

2) (2) $[(p \to \neg q) \wedge (q \to \neg r)] \vee (\neg r \to p)$

$\equiv [(\neg p \vee \neg q) \wedge (\neg q \vee \neg r)] \vee (r \vee p)$

$\equiv [\neg q \vee (\neg p \wedge \neg r)] \vee (r \vee p)$

$\equiv \neg q \vee \underbrace{\neg(p \vee r) \vee (p \vee r)}_{T}$

$\equiv \neg q \vee T \equiv T$.

3) (2) $[p \wedge (\neg q \vee r)] \wedge [\neg p \vee (q \wedge \neg r)]$

$\equiv [p \wedge (\neg q \vee r)] \wedge \neg[p \wedge (\neg q \vee r)]$

$\equiv F$. $\quad A \wedge \neg A$.

4) (2) $(p \leftrightarrow q) \wedge (p \vee q) \equiv [(p \wedge q) \vee (\neg p \wedge \neg q)] \wedge (p \vee q)$

$\equiv [(p \wedge q) \vee \neg(p \vee q)] \wedge (p \vee q)$

$\equiv [(p \wedge q) \wedge (p \vee q)] \vee [\underbrace{\neg(p \vee q) \wedge (p \vee q)}_{F}]$

$\equiv (p \wedge q) \wedge (p \vee q)$.

$\equiv [p \wedge (p \vee q)] \wedge (q \wedge (p \vee q))$

$\equiv p \wedge q$.

Exercice 2:

1) (1) a) True; $x = \sqrt{5}$ or $x = -\sqrt{5}$.

(1) b) false, $x = \frac{1}{2}$; $\left(\frac{1}{2}\right)^2 \not\geq \frac{1}{2}$.

2) (1) a) false $\sqrt{2} \notin \mathbb{Z}$; $-\sqrt{2} \notin \mathbb{Z}$.

(1) b) True (we showed it in the course)

Exercice 3:

(3) by contraposition, we prove that: if $x < 1$ and $y < 1$ then $x + y < 2$.

$\left.\begin{array}{l} x < 1 \\ y < 1 \end{array}\right\} \Rightarrow x + y < 1 + 1 = 2$

$\Rightarrow$ we have the result

Let $2x^3+2y+4z^2 \leq 78$.

2) By contradiction we assume that

3 $x>3$ and $y>4$ and $z>2$.

so $2x^3 > 2\times3^3 = 54$.

$2y > 2\times4 = 8$.

$4z^2 > 4\times2^2 = 16$

$\Rightarrow 2x^3+2y+4z^2 > 54+8+16 = 78$.

that contradict the fact that

$2x^3+2y+4z^2 \leq 78$

So our assumption is false

then $x \leq 3$ or $y \leq 4$ or $z \leq 2$.

3) . P: n is odd.

5 Q: $3n+3$ is even.

$P \longleftrightarrow Q$.

- $P \longrightarrow Q$: if n is odd then $3n+3$ is even

$n = 2k+1$; $k \in \mathbb{Z}$.

$\Rightarrow 3n+3 = 3(2k+1)+3$

$= 6k+3+3 = 6k+6$

$= 2(3k+3) = 2t$; $t = 3k+3 \in \mathbb{Z}$.

$\Rightarrow P \longrightarrow Q$ (1)

- $Q \longrightarrow P$: if $3n+3$ is even then n is odd.

by contraposition, we prove that,

if n is even then $3n+3$ is odd.

n even $\Rightarrow n = 2k$; $k \in \mathbb{Z}$.

$\Rightarrow 3n+3 = 3(2k)+3$

$= 6k+3 = 2(3k+1)+1$.

$= 2t+1$; $t = 3k+1$.

so $3n+3$ is odd.

$\Rightarrow$ we have $\neg P \longrightarrow \neg Q$.

then $Q \longrightarrow P$ (2)

so from (1) and (2) $P \Longleftrightarrow Q$.