(SEMESTER 1, 1437 -1438) FIRST MID-TERM EXAM

Question:1 .(a) Let

$$x+y+z=0$$
$$x-2y+2z=4$$
$$x+2y-z=2 \qquad [10]$$

(i) Write the above system of linear equations in the form AX = B.

(ii) Find A^{-1} using elementary matrix method, and use it to solve the system of equations.

Solution (i)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}$$

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(ii)

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -2 & 2 & 0 & 1 & 0 \\ 1 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2/5 & 3/5 & 4/5 \\ 0 & 1 & 0 & 3/5 & -2/5 & -1/5 \\ 0 & 0 & 1 & 4/5 & -1/5 & -3/5 \end{array}\right] \equiv [I \mid A^{-1}]$$

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$$A^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 3 & 4 \\ 3 & -2 & -1 \\ 4 & -1 & -3 \end{bmatrix}$$

(iii)

$$X = A^{-1}B = \frac{1}{5}\begin{bmatrix} -2 & 3 & 4 \\ 3 & -2 & -1 \\ 4 & -1 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 20 \\ -10 \\ -10 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix}$$

$x=+4$, $y=-2$, $z=-2$

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(b) Find the relationship between a, b and c for which the system of linear equation will be consistent

$$3x-9y+3z=a$$
$$x-2y-z=b \qquad [8]$$
$$5x-13y+z=c$$

Solution. Augmented matrix

$$\left[\begin{array}{ccc|c} 3 & -9 & 3 & a \\ 1 & -2 & -1 & b \\ 5 & -13 & 1 & c \end{array}\right] \equiv \left[\begin{array}{ccc|c} 1 & -2 & -1 & b \\ 3 & -9 & 3 & a \\ 5 & -13 & 1 & c \end{array}\right] \quad R1 \leftrightarrow R2$$

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$$\equiv \left[\begin{array}{ccc|c} 1 & -2 & -1 & b \\ 0 & -3 & 6 & a-3b \\ 0 & -3 & 6 & c-5b \end{array}\right] \begin{array}{l} \\ -3R_1+R_2 \\ -5R_1+R_3 \end{array}$$

$$\equiv \left[\begin{array}{ccc|c} 1 & -2 & -1 & b \\ 0 & -3 & 6 & a-3b \\ 0 & 0 & 0 & c-a-2b \end{array}\right] \begin{array}{l} \\ \\ -R_2+R_3 \end{array}$$

system is consistent if $c-a-2b=0 \Rightarrow c=a+2b$

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Question: 2 . (a) Find A if inverse of [3A+I] is equal to $\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$ [6]

(b) Let A and B be 3x3 matrices with
$\det A = -10$ **and** $\det B = 5$.

Find (a) det (6A), (b) det($A^T B^{-1}$ **)** **[6]**

Solution. 2 (a)

$$[3A + I]^{-1} = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$$

taking inverse of both sides

$$3A + I = \frac{1}{1}\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

$$3A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ -2 & 6 \end{bmatrix}$$

$$A = \frac{1}{3}\begin{bmatrix} 0 & -3 \\ -2 & 6 \end{bmatrix}$$ 6

(b) (i) $\det 6A = 6^3 \det A = 6^3(-10) = (216)(-10)$
$= -2160$ 3

(ii) $\det(A^T B^{-1}) = \det A^T \det B^{-1}$
$= \det A \cdot \frac{1}{\det B} = \frac{-10}{5} = -2$ 3

Question: 3 . Solve the linear system by using Cramer's Rule

$$\begin{aligned} 2x_1 + 2x_2 \quad &= 1 \\ -2x_1 + x_2 + x_3 &= 0 \\ 3x_1 \quad + x_3 &= 1 \end{aligned}$$

[10]

Solution

$$\begin{bmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\rightarrow$ $\det A = 12$, 2 $\det A_1 = 3$, 2 $\det A_2 = 3$, 2 $\det A_3 = 3$ 2

2 $x = \frac{1}{4}$, $y = \frac{1}{4}$, $z = \frac{1}{4}$.

Question: 4. **Suppose the points** $(2,5)$, $(3,2)$ and $(4,5)$ **lie on the curve**

$$y = a + bx + cx^2$$

i. **Find the system of linear equations in a, b and c.**
ii. **Solve the system by Gauss – Jordan method.**
iii. **Write the equation of the curve.** **[10]**

Solution

(i)

at point $(2,5)$

$$a + 2b + 4c = 5$$

at point $(3,2)$

$$a + 3b + 9c = 2$$

at point $(4,5)$

$$a + 4b + 16c = 5$$

Linear system is

$$a + 2b + 4c = 5$$
$$a + 3b + 9c = 2$$
$$a + 4b + 16c = 5$$

3

(ii) The augmented Mx is

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 1 & 3 & 9 & 2 \\ 1 & 4 & 16 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 0 & 2 & 12 & 0 \end{array}\right] \quad -R1+R2, \; -R1+R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -6 & 11 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 2 & 6 \end{array}\right] \quad -2R2+R3, \; -2R2+R1$$

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$$\left[\begin{array}{ccc|c} 1 & 0 & -6 & 11 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \tfrac{1}{2}R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & -18 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad 6R3+R1, \; -5R3+R2$$

$\therefore$ $a = 29$, $b = -18$ and $c = 3$

$\Rightarrow$ the eqn of the curve is

$$y = 29 - 18x + 3x^2$$

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