Dr Borhen

See 1052

Homework 3 (Math 106)

Ex1 Sketch the bounded region R bounded by the graphs $y = x^2+4$ and $y = -x+10$ and find its area.

Ex2 Sketch the bounded region R by the graphs $x = y^2$ and $x - y = 2$ and find the volume of the solid generated if R is revolved about y-axis.

Ex3: Find the arc length of the curve $y = \ln(\sin x)$ from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{3}$.

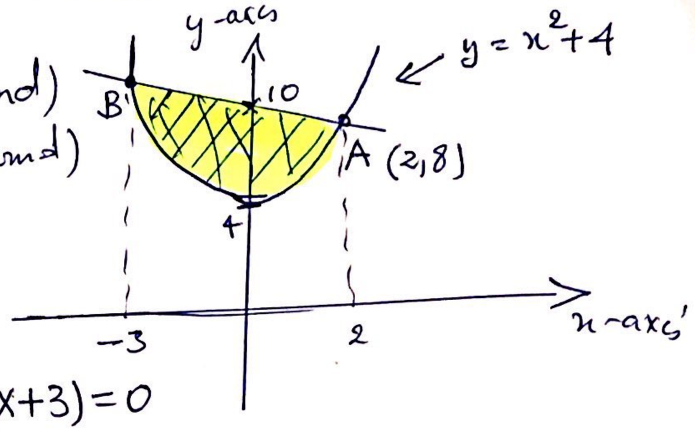
Ex4: Find the area of the surface generated by revolving the curve $y = \sqrt{x}$ on $[1,4]$ about x-axis.

Ex5 Convert the cartesian equation $(x+2)^2 + y^2 = 4$ to polar equation and sketch its graph.

Answer Homework 3 (Dr Borhen)

Ex1 : $f(x) = -x + 10$ (up bound)

$g(x) = x^2 + 4$ (lower bound)



- Intersection points

$x^2 + 4 = -x + 10$

$x^2 + x - 6 = (x-2)(x+3) = 0$

$\begin{cases} x = 2 \\ y = 8 \end{cases}$ and $\begin{cases} x = -3 \\ y = +13 \end{cases}$

Our region $R = \left\{ (x,y) \,/\, \begin{matrix} -3 \le x \le 2 \\ x^2+4 \le y \le -x+10 \end{matrix} \right\}$

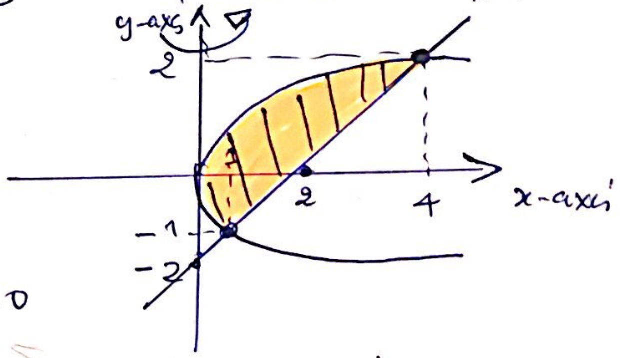
Its area is: $A(R) = \int_{-3}^{2} \left[(-x+10) - (x^2+4)\right] dx$

$= \int_{-3}^{2} (-x^2 + x + 6)\, dx$

$A(R) = \left[-\frac{x^3}{3} - \frac{x^2}{2} + 6x \right]_{-3}^{2} = \dots$

Ex2 : $x = y^2$: eq of Parabola

$x - y = 2$: eq of a line.



- Intersection points:

$x = 2 + y = y^2$

$y^2 - y - 2 = (y+1)(y-2) = 0$

$\begin{cases} x = 1 \\ y = -1 \end{cases}$ or $\begin{cases} x = 4 \\ y = 2 \end{cases}$

$R = \left\{ (x,y) \,/\, \begin{matrix} -1 \le y \le 2 \\ y^2 \le x \le y+2 \end{matrix} \right\}$

Using Disk method the volume of the solids S generated by revolving R about y-axis is given by

$V(S) = \pi \int_{-1}^{2} \left[(2+y)^2 - (y^2)^2 \right] dy$

$= \pi \int_{-1}^{2} \left[y^2 + 4y + 4 - y^4 \right] dy$

$= \pi \left[y^3/3 + 2y^2 + 4y - \frac{y^5}{5} \right]_{-1}^{2} = \frac{72\pi}{5}$

Ex3: $\mathcal{L}(\mathcal{C}) = \int_{\pi/6}^{\pi/3} \sqrt{1+(f'(x))^2}\, dx$

with $f(x) = \ln(\sin x)$

$$f'(x) = \frac{\cos x}{\sin x} = \cot x$$

$$1 + (f'(x))^2 = 1 + \cot^2 x = \csc^2 x$$

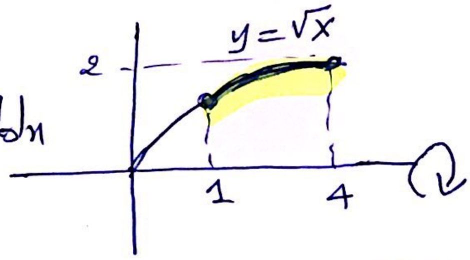
$$\mathcal{L}(\mathcal{C}) = \int_{\pi/6}^{\pi/3} \csc x \, dx = \Big[\ln|\csc x - \cot x|\Big]_{\pi/6}^{\pi/3}$$

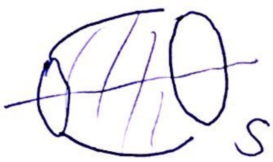
$$\begin{cases} \csc\frac{\pi}{3} = \frac{2}{\sqrt{3}} \ ; \ \cot\frac{\pi}{3} = 1/\sqrt{3} \\ \csc\frac{\pi}{6} = 2 \ ; \ \cot\frac{\pi}{6} = \sqrt{3} \end{cases}$$

$$\mathcal{L}(\mathcal{C}) = \ln\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right) - \ln(2-\sqrt{3})$$
$$= -\frac{1}{2}\ln 3 - \ln(2-\sqrt{3}) > 0$$

Ex 4:

$$A.S = 2\pi \int_1^4 |f(x)| \sqrt{1+(f'(x))^2}\, dx$$





$$f(x) = \sqrt{x}$$
$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$1 + (f'(x))^2 = 1 + \frac{1}{4x} = \frac{4x+1}{4x}$$

$$A.S = 2\pi \int_1^4 \cancel{\sqrt{x}}\, \frac{\sqrt{4x+1}}{2\cancel{\sqrt{x}}}\, dx = \frac{\pi}{4}\int_1^4 4\sqrt{4x+1}\, dx$$

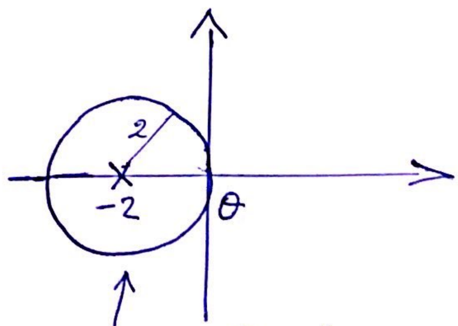
$$A.S = \frac{\pi}{4}\cdot\frac{2}{3}\Big[(4x+1)^{3/2}\Big]_1^4 = \frac{\pi}{6}\left[17^{3/2} - 5^{3/2}\right].$$

Ex5: replace x by $r\cos\theta$ and y by $r\sin\theta$, we get

$$x^2 + 4x + \cancel{4} + y^2 = \cancel{4}$$

$$\underbrace{x^2+y^2}_{} = -4x$$

$$r^2 = -4r\cos\theta$$



$(x+2)^2 + y^2 = 4$

$\boxed{r = -4\cos\theta}$ eq of a circle centered at $I(2,\pi)$ and radius $R = 2$.