# KING SAUD UNIVERSITY

## Department of Mathematics

## (Summer Term 1437/1438)

## Second Mid Term

Max. Marks: 25 (M-106) Time: 90 mins

**Question 1(3+3)**

(a) Find the $\lim_{x\to\frac{\pi}{2}} \frac{1-\sin(x)}{1+\cos(2x)}$ if it exists. ( b) Evaluate $\int x^2 \sinh x \, dx$.

**Question 2(3+3)**

Evaluate the integrals a) $\int \sin^3 x \sqrt{\cos x} \, dx$. b) $\int \frac{x^2}{(4-x^2)^{3/2}} \, dx$.

**Question 3(3+4)**

Evaluate the integral (a) $\int \frac{1}{\sqrt{-x^2-6x-5}} \, dx$. (b) $\int \frac{x+2}{(x-1)(x^2-3x+2)} \, dx$.

**Question 4(3+3)**

a) Determine whether the integral $\int_0^9 \frac{1}{\sqrt[3]{x-1}} \, dx$ is convergent or divergent and if it is convergent, find its value.

b) Evaluate the integral $\int \frac{x^{1/3}}{x^{2/3}+1} \, dx$.

**Q:1** (6 points)

(a) $\lim_{x \to \pi/2} \frac{1-\sin x}{1+\cos 2x}$ $\left(\frac{0}{0}\right)$ - form

Hospital's rule

$\Rightarrow \lim_{x \to \pi/2} \frac{-\cos x}{-2\sin 2x} = \left(\frac{0}{0}\right)$ - form (2)

$\lim_{x \to \pi/2} \frac{-\cos x}{-4\sin x \cos x} = +\frac{1}{4}$ (1)

(b) $\int x^2 \sinh x \, dx = x^2 \cosh x - \int 2x \cosh x \, dx$ (1)

Integration by parts

$= x^2 \cosh x - 2\left[x \sinh x - \int \sinh x \, dx\right]$ (1)

$= x^2 \cosh x - 2x \sinh x + 2\cosh x + c$ (1)

**Q. 2** (6 points)

(a) $\int \sin^3 x \sqrt{\cos x} \, dx = \int \sin^2 x \sqrt{\cos x} \cdot \sin x \, dx$ (2)

Put $u = \cos x \Rightarrow du = -\sin x \, dx$

$= -\int (1-u^2)\sqrt{u} \, du = -\int (\sqrt{u} - u^{5/2}) \, du$ (1)

$= -\frac{u^{3/2}}{3/2} + \frac{u^{7/2}}{7/2} + c = -\frac{2}{3}(\cos x)^{3/2} + \frac{2}{7}(\cos x)^{7/2} + c$

(b) $\int \frac{x^2}{(4-x^2)^{3/2}} dx$

Put $x = 2\sin\theta$ (1)

$dx = 2\cos\theta \, d\theta$

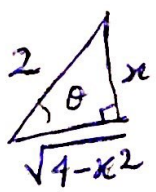
$\sqrt{4-x^2} = 2\cos\theta$

$= \int \frac{4\sin^2\theta}{(2\cos\theta)^3} 2\cos\theta \, d\theta$ (1)

$= \int \tan^2\theta \, d\theta = \int (\sec^2\theta - 1) \, d\theta = \tan\theta - \theta + c$

$= \frac{x}{\sqrt{4-x^2}} - \sin^{-1}(x/2) + c$

$\sin\theta = \frac{x}{2}$ (1)



(4 parts)

**Q: 3**

(i) $\int \frac{1}{\sqrt{-x^2-6x+5}}\,dx$

$-(x^2+6x+9-9+5)$

$-(x+3)^2+4$

$= \int \frac{1}{\sqrt{4-(x+3)^2}}\,dx$ (2)

$= \sin^{-1}\left(\frac{x+3}{2}\right)+c$ (1)

(b) $I=\int \frac{x+2}{(x-1)(x^2-3x+2)}\,dx = \int \frac{x+2}{(x-1)(x-1)^2(x-2)}\,dx$

$x \in \mathbb{R}\setminus\{1,2\}$, $\frac{x+2}{(x-1)^2(x-2)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$ (1)

$x+2 = A(x-1)(x-2)+B(x-2)+C(x-1)^2$

$x=1 \Rightarrow +3 = -B \Rightarrow B=-3$

$x=2 \Rightarrow 4=C$ (2)

$x=0 \Rightarrow 2 = 2A-2B+C$

$2 = 2A+6+4 \Rightarrow 2A=-8$

$A=-4$

$I=\int \frac{-4}{x-1}\,dx - \int \frac{3}{(x-1)^2}\,dx + \int \frac{4}{x-2}\,dx$

$= -4\ln|x-1| + \frac{3}{x-1} + 4\ln|x-2| + c$ (3)

Q:4 (6 points)

(a) $\int_0^1 \frac{1}{(x-1)^{1/3}}\,dx + \int_1^9 \frac{1}{(x-1)^{1/3}}\,dx$ (1)

$\lim_{t\to 1^-} \left[\frac{(x-1)^{2/3}}{2/3}\right]_0^t + \lim_{t\to 1^+} \left[\frac{(x-1)^{1/3}}{1/3}\right]_t^9$ (1)

$-\frac{3}{2}[0-1] + \frac{3}{2}[4-0] = -\frac{3}{2} + 6$ (1)

$= 9/2$ (1)

(b) $\int \frac{x^{1/3}}{1+x^{2/3}}\,dx$

$u = x^{1/3}$

$u^2 = x^{2/3}$

$u^3 = x$

$3u^2\,du = dx$ (1)

$= \int \frac{u}{u^2+1} 3u^2\,du$

$= 3\int \frac{u^3}{u^2+1}\,du$ (1)

$u^2+1 \;\overline{)\; u^3}\;\; u$

$u^3 + u$

$-u$

$= 3\int \left[u - \frac{u}{u^2+1}\right] du$

$= 3\left\{\frac{u^2}{2} - \frac{1}{2}\ln[u^2+1]\right\} + c$ (1)

$= 3\left\{\frac{1}{2}x^{2/3} - \frac{1}{2}\ln\{x^{2/3}+1\}\right\} + c$