KING SAUD UNIVERSITY **First Semester 37/38**

Math Department **January 15 2017**

Final exam106 **Time: 180mn**

Question 1(2+2+3)

a) Find the constant c that satisfies the equation

$$\sum_{k=1}^{k=9}(3k^2 - k + c) = 900$$

b) Compute the integral $\int \frac{\sin(3\sqrt{x}+1)}{\sqrt{x}}\,dx$

c) Approximate the integral $\int_0^4 \frac{dx}{1+x^3}$ using Simpson's Rule with n=4

Question 2(3+3+3)

a) Evaluate the integral $\int \frac{e^{1+2\cosh^{-1}x}dx}{\sqrt{x^2-1}}$

b) Compute the integral $\int \frac{dx}{x\sqrt{x^4-16}}$

c) Find the limit $\lim_{x\to 0} \frac{\int_0^x (\sin t - t)dt}{x^4}$

Question 3(3+3+2)

a) Compute the following integral $\int x^2 \ln x dx$

b) Find the integral $\int (\sin x)^5 \cos^3 x dx$

c) Evaluate the integral $\int \cos 4x . \cos 2x dx$

Question 4(3+2+3)

a) Evaluate the integral $\int \frac{5x^2+5x+2}{(x+1)^2(x-1)} dx$

b) Compute the integral $\int \frac{dx}{x^{\frac{3}{2}}+x^{\frac{1}{2}}}$

c) Sketch the region bounded by the curves: $y = x^2$, $y = \sqrt{x}$, $x = 0, and\ x = 2$ and find its area.

Question5(3+3+2)

a) Find the volume of the solid generated by revolving the region bounded by the curves $y = x, y = \sqrt{x}$ about the line $y = 2$

b) Sketch the region R that lies inside the curve $r = 1 + cos\theta$

and outside the curve $r = 1 - cos\theta$, and find its area.

c) Find the surface area obtained by revolving the curve $r = 4cos\theta \quad 0 \leq \theta \leq \pi/2$ about the x-axis

1)

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① a) $\sum_{1}^{9} 3b^2 - b + c = 3\sum_{1}^{9} b^2 - \sum_{1}^{9} b + 9c$

$= 810 + 9c$ (1,5)

So $c = 10$ (0,5)

b) $\displaystyle\int \frac{\sin(3\sqrt{x}+1)}{\sqrt{x}}\,dx = \frac{2}{3}\int \sin u\,du$ (2)

$u = 3\sqrt{x}+1$ $\qquad = -\frac{2}{3}\cos(3\sqrt{x}+1) + C$ (1)

c) $S_4 = \frac{4}{12}\left(f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right)$

$= \frac{4}{12}\left(1 + 2 + \frac{2}{9} + \frac{4}{28} + \frac{1}{65}\right)$ (2)

$\simeq 1.12682$ (1)

② a) $\displaystyle\int \frac{e^{1+2\cosh^{-1}x}}{\sqrt{x^2-1}}\,dx = \frac{1}{2}\int e^u\,du$ (2)

$u = 1 + 2\cosh^{-1}x$ $\qquad = \frac{1}{2}e^{1+2\cosh^{-1}x} + C$ (1)

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)

b) $$\int \frac{dx}{x\sqrt{x^4-16}} = \int \frac{x\,dx}{x^2\sqrt{x^4-16}}$$

$$= \frac{1}{2}\int \frac{du}{u\sqrt{u^2-16}} \quad 2$$

$$= \frac{1}{8}\sec^{-1}\left(\frac{x^2}{4}\right) + C \quad 1$$

c) $$\lim_{x\to 0} \frac{\int_0^x \sin t - t \, dt}{x^4}$$

$$= \lim_{x\to 0} \frac{\sin x - x}{4x^3} \quad 1$$

$$= \lim_{x\to 0} \frac{\cos x - 1}{12x^2} \quad 1$$

$$= \lim_{x\to 0} \frac{-\sin x}{24x} = -\frac{1}{24} \quad 1$$

Q3 a) $$\int x^2 \ln x \, dx = \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\cdot\frac{1}{x}\,dx \quad 2$$

$$= \frac{x^3}{3}\ln x - \frac{1}{9}x^3 + C \quad 1$$

3.)

b) $\int \sin^5 x \cdot \cos^3 x \, dx = \int \sin^5 x \cos^2 x \cos x \, dx$

1 $= \int \sin^5 x (1 - \sin^2 x) \cos x \, dx$

1 $= \int u^5 (1 - u^2) \, du$

1 $= \frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + C$

c) $\int \cos 4x \cos 2x \, dx = \frac{1}{2} \int (\cos(6x) + \cos 2x) \, dx$

1

1 $= \frac{\sin 6x}{12} + \frac{\sin 2x}{4} + C$

Q4 a) $\frac{5x^2+5x+2}{(x+1)^2(x-1)} = \frac{2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x-1}$

1.5

$\int \frac{5x^2+5x+2}{(x+1)^2(x-1)} dx = 2\ln|x+1| + \frac{1}{x+1} + 3\ln|x-1| + C$

1.5

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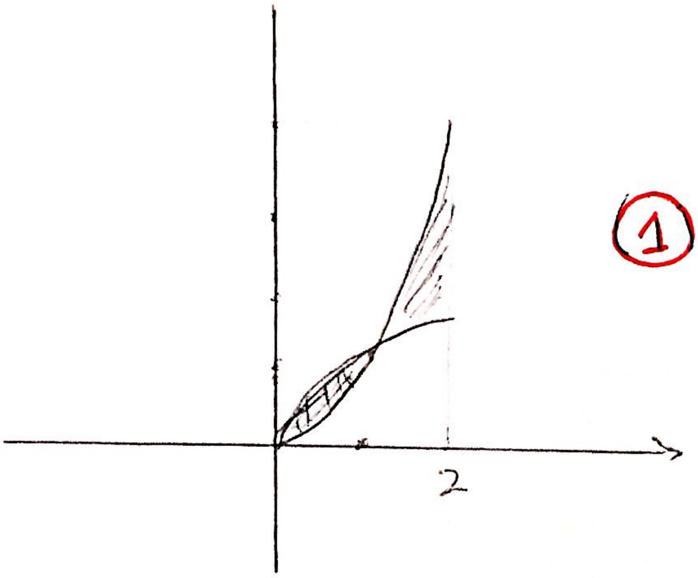
4)

b) $$\int \frac{dx}{x^{3/2}+x^{1/2}} = \int \frac{2u\,du}{u^3+u}$$ 1

$$= 2\int \frac{du}{u^2+1} = 2\tan^{-1}(\sqrt{x})+C$$ 1

c)



1

$x^2 = \sqrt{x} \Rightarrow x = 0$ or $x = 1$ 0.5

$$A = \int_0^1 \sqrt{x} - x^2\,dx + \int_1^2 x^2 - \sqrt{x}\,dx$$ 1

$$= \left[\tfrac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3} - \frac{2}{3}x^{3/2}\right]_1^2$$

$$= \frac{10}{3} - \frac{4}{3}\sqrt{2} \simeq 1.4477$$ 0.5

Q5) a)

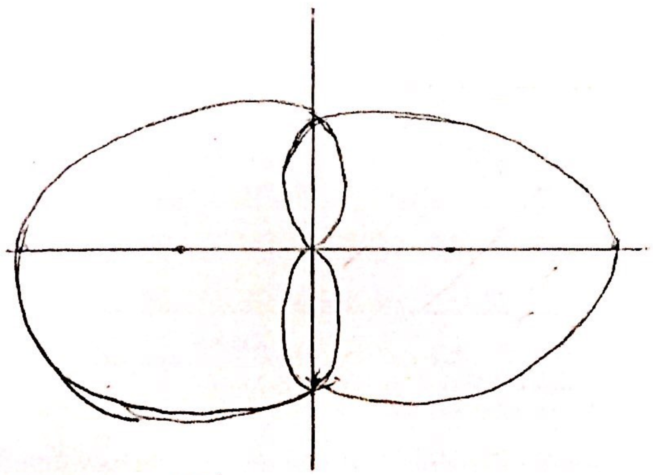
$$V = \int_0^1 \pi\left((2-x)^2 - (2-\sqrt{x})^2\right) dx \quad (1,5)$$

$$= \pi \int_0^1 (4-4x+x^2) - (4-4\sqrt{x}+x)\, dx$$

$$= \pi \int_0^1 x^2 - 5x + 4\sqrt{x}\, dx \quad (1)$$

$$= \pi \left[\frac{x^3}{3} - \frac{5}{2}x^2 + \frac{8}{3}x^{3/2}\right]_0^1 = \frac{\pi}{2} \quad (0.5)$$

b)



(1)

$1 - \cos\theta = 1 + \cos\theta$

$\Rightarrow \theta = \frac{\pi}{2}$ or $\theta = -\pi/2$

$\left(\frac{1}{2}\right) = (0.5)$

$$A = \int_{-\pi/2}^{\pi/2} \frac{1}{2}\left((1+\cos\theta)^2 - (1-\cos\theta)^2\right) d\theta \quad (1)$$

$$= \int_{-\pi/2}^{\pi/2} 2\cos\theta\, d\theta = 4 \quad (0.5)$$

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c)

c) $S = \int_0^{\pi/2} 2\pi |y| ds$

(1) $= \int_0^{\pi/2} 2\pi . 4\cos\theta\sin\theta \sqrt{16\cos^2\theta + 16\sin^2\theta}\, d\theta$

$= 32\pi \int_0^{\pi/2} \cos\theta\sin\theta\, d\theta$

$= 16\pi$ (1)