Arbitrage opportunities in forward contracts

Prepaid forward contract

Continuous Dividends

$F_{0,T}^{P} < S_0 e^{-\delta T}$

$t=0$		$t=T$	
Activity	*Payoff*	*Activity*	*Payoff*
$-$*Enter long prepaid forward contract .* $-$*Sell short* $e^{-\delta T}$*of a stock .*	$-F_{0,T}^{P}$ $S_0 e^{-\delta T}$	$-$*Execute prepaid forward contract .* $-$*Cover short sell .*	S_T $-S_T$
Total Payoff $= S_0 e^{-\delta T} - F_{0,T}^{P} > 0$		*Total Payoff* $= 0$	

$F_{0,T}^{P} > S_0 e^{-\delta T}$

$t=0$		$t=T$	
Activity	*Payoff*	*Activity*	*Payoff*
$-$*Enter Short prepaid forward contract .* $-$*Buy* $e^{-\delta T}$*of a stock .*	$F_{0,T}^{P}$ $-S_0 e^{-\delta T}$	$-$*Execute prepaid forward contract .* $-$*Own a whole stock .*	$-S_T$ S_T
Total Payoff $= F_{0,T}^{P} - S_0 e^{-\delta T} > 0$		*Total Payoff* $= 0$	

Forward contract

No Dividend

$F_{0,T} < S_0 e^{\delta T}$

time $=0$		*time* $=T$	
Activity	*Payoff*	*Activity*	*Payoff*
$-$*Enter long forward contract .* $-$*Sell short one stock .* $-$*Invest* S_0 *in risk free account.*	0 S_0 $-S_0$	$-$*Execute forward contract .* $-$*Cover short sell .* $-$*Collect risk free invesment .*	$S_T - F_{0,T}$ $-S_T$ $S_0 e^{\delta T}$
Total Payoff $= 0$		*Total Payoff* $= S_0 e^{\delta T} - F_{0,T} > 0$	

$F_{0,T} > S_0 e^{\delta T}$

time $=0$		*time* $=T$	
Activity	*Payoff*	*Activity*	*Payoff*
$-$*Enter short forward contract .* $-$*Borrow* S_0 *.* $-$*Buy one stock.*	0 S_0 $-S_0$	$-$*Execute forward contract .* $-$*Clear loan.* $-$*Collect risk free invesment .*	$F_{0,T} - S_T$ $-S_0 e^{\delta T}$ S_T
Total Payoff $= 0$		*Total Payoff* $= F_{0,T} - S_0 e^{\delta T} > 0$	

Discrete Dividends

(*for simplicity assume only one dividend*)

$F_{0,T} > S_0 e^{\delta T} - De^{\delta(T-t)}$

time $=0$		*time* $=t$		*time* $=T$	
Activity	*Payoff*	*Activity*	*Payoff*	*Activity*	*Payoff*
$-$*Enter short forward contract .* $-$*Borrow* S_0 *.* $-$*Buy one stock.*	0 S_0 $-S_0$	$-$*collect D and put it in risk free investment*	$De^{-\delta t}$	$-$*Execute forward contract .* $-$*Clear loan.* $-$*Collect risk free invesment .* $-$*own a stock.*	$F_{0,T} - S_T$ $-S_0 e^{\delta T}$ $De^{\delta(T-t)}$ S_T
Total Payoff $= 0$				*Total Payoff* $= F_{0,T} - S_0 e^{\delta T} + De^{\delta(T-t)} > 0$	

$F_{0,T} < S_0 e^{\delta T} - De^{\delta(T-t)}$

time $=0$		*time* $=t$		*time* $=T$	
Activity	*Payoff*	*Activity*	*Payoff*	*Activity*	*Payoff*
$-$*Enter long forward contract .* $-$*Sell short one stock .* $-$*Invest* S_0 *in risk free account.*	0 S_0 $-S_0$	$-$*Stock pays D .*	$-De^{-\delta t}$	$-$*Execute forward contract .* $-$*Cover short sell .* $-$*Collect risk free invesment .*	$S_T - F_{0,T}$ $-S_T - De^{\delta(T-t)}$ $S_0 e^{\delta T}$
Total Payoff $= 0$				*Total Payoff* $= S_0 e^{\delta T} - F_{0,T} - De^{\delta(T-t)} > 0$	

Continuous Dividends

$F_{0,T} < S_0 e^{(r-\delta)T}$

time $=0$		*time* $=T$	
Activity	*Payoff*	*Activity*	*Payoff*
$-$*Enter long forward contract .* $-$*Sell short* $e^{-\delta T}$*of a stock .* $-$*Invest proceeds in risk free account.*	0 $S_0 e^{-\delta T}$ $-S_0 e^{-\delta T}$	$-$*Execute forward contract .* $-$*Cover short sell.* $-$*Collect risk free invesment .*	$S_T - F_{0,T}$ $-S_T$ $S_0 e^{(r-\delta)T}$
Total Payoff $= 0$		*Total Payoff* $= S_0 e^{(r-\delta)T} - F_{0,T} > 0$	

$F_{0,T} > S_0 e^{(r-\delta)T}$

time $=0$		*time* $=T$	
Activity	*Payoff*	*Activity*	*Payoff*
$-$*Enter short forward contract .* $-$*Borrow* $S_0 e^{\delta T}$ *.* $-$*Buy* $e^{\delta T}$ *of a stock.*	0 $S_0 e^{\delta T}$ $-S_0 e^{\delta T}$	$-$*Execute forward contract .* $-$*Clear loan.* $-$*own a stock.*	$F_{0,T} - S_T$ $-S_0 e^{(r-\delta)T}$ S_T
Total Payoff $= 0$		*Total Payoff* $= F_{0,T} - S_0 e^{(r-\delta)T} > 0$	