KING SAUD UNIVERSITY. DEPARTMENT OF PHYSICS

QUANTUM MECHANICS H.W №2

Salwa Al Saleh

PROBLEM (1)

For $|\psi\rangle = \begin{pmatrix} i \\ -2 \\ 1 \end{pmatrix}$ and $|\phi\rangle = \begin{pmatrix} -1 \\ 3i \\ \sqrt{2} \end{pmatrix}$.

1. Calculate $4|\psi\rangle - i|\phi\rangle$

2. Find $\langle\phi|\psi\rangle$ and $\langle\psi|\phi\rangle$, what do you observe ?

3. Express the vector $|\psi\rangle$ in terms of the basis:
   $|\varepsilon_1\rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $|\varepsilon_2\rangle = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $|\varepsilon_3\rangle = \begin{pmatrix} -i \\ 1 \\ -i \end{pmatrix}$.

4. Normalise the vector $|\phi\rangle$.

PROBLEM (2)

Given the (canonical) basis for a 3-D vector space. :

$$|e_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad |e_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad |e_1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Show that they truely form a basis for the vector space.

PROBLEM (3)

Given the following polynomials $p_1(x) = x$ and $p_2(x) = x^2 - \frac{1}{3}$, defined over the interval $[-1, 1]$, Are they orthonormal ?

PROBLEM (4)

Given the function defined on the interval $]0, 1[$:

$$f(x) = \begin{cases} 1, & 0 < x < 1/2 \\ 0, & 1/2 < x < 1. \end{cases}$$

Express the function $f(x)$ as a Fourier series.

PROBLEM (5)

Use Schwartz inequality to show that the following is true :

$$\int_{-\infty}^{+\infty} \left(t^{10} - t^6 + 5t^4 - 5\right) e^{-x^2} dt \leq \sqrt{\int_{-\infty}^{+\infty} (t^4 - 1)^2 e^{-x^2} dt} \sqrt{\int_{-\infty}^{+\infty} (t^6 + 5)^2 e^{-x^2} dt}$$