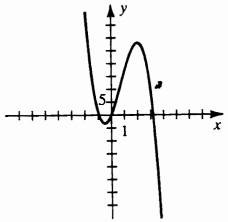
11 (a) 3990 mills **(b)** \$15,420.10

13 The stable point occurs at $\left(\frac{m}{n}, \frac{a}{b}\right)$.

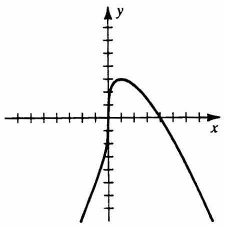
Chapter 3 Review Exercises

1 Max: $f(3) = 1$; min: $f(6) = -8$ **3** $-2, -1, \frac{1}{3}$

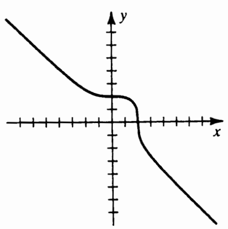
5 Max: $f(2) = 28$; min: $f\left(-\frac{1}{2}\right) = -\frac{13}{4}$; increasing on $\left[-\frac{1}{2}, 2\right]$; decreasing on $\left(-\infty, -\frac{1}{2}\right]$ and $[2, \infty)$



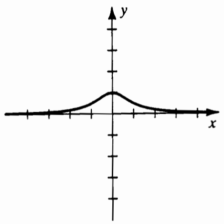
7 Max: $f(1) = 3$; increasing on $(-\infty, 1]$; decreasing on $[1, \infty)$



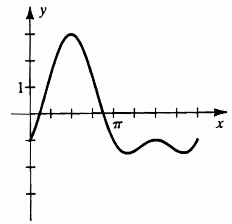
9 Since $f''(0) = 0$ and $f''(2)$ is undefined, use the first derivative test to show that there are no extrema; CU on $(-\infty, 0)$ and $(2, \infty)$; CD on $(0, 2)$; x-coordinates of PI are 0 and 2.



11 Since $f''(0) = -2 < 0$, $f(0) = 1$ is a maximum; CU on $\left(-\infty, -\frac{1}{3}\sqrt{3}\right)$ and $\left(\frac{1}{3}\sqrt{3}, \infty\right)$; CD on $\left(-\frac{1}{3}\sqrt{3}, \frac{1}{3}\sqrt{3}\right)$; x-coordinates of PI are $\pm\frac{1}{3}\sqrt{3}$.



13 Max: $f\left(\frac{\pi}{2}\right) = 3$ and $f\left(\frac{3\pi}{2}\right) = -1$; min: $f\left(\frac{7\pi}{6}\right) = f\left(\frac{11\pi}{6}\right) = -\frac{3}{2}$



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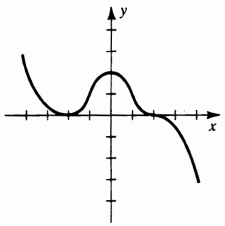
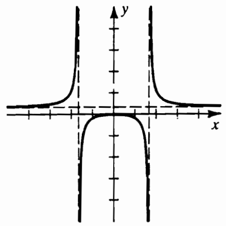
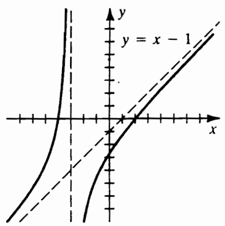


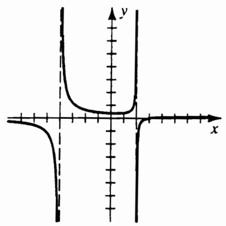
17 Max: $f(0) = 0$



19 No extrema



21 Max: $f(3 + \sqrt{7}) \approx 0.08$; min: $f(3 - \sqrt{7}) \approx 0.37$



23 $\frac{\sqrt{61} - 1}{3}$ **25** 125 yd by 250 yd **27** $\frac{\pi}{2}$

29 Radius of semicircle is $\frac{1}{8\pi}$ mi, length of rectangle is $\frac{1}{8}$ mi.

31 (a) Use all the wire for the circle.
(b) Use length $\frac{5\pi}{4 + \pi} \approx 2.2$ ft for the circle and the remainder for the square.

33 $v(t) = \frac{3(1 - t^2)}{(t^2 + 1)^2}$; $a(t) = \frac{6t(t^2 - 3)}{(t^2 + 1)^3}$; left in $[-2, -1)$; right in $(-1, 1)$; left in $(1, 2]$

35 $C'(100) = 116$; $C(101) - C(100) = 116.11$

37 (a) $18x$ **(b)** $-0.02x^2 + 12x - 500$ **(c)** 300 **(d)** \$1300

39 98 ft/sec² **41** 2.27 **43** ± 0.79

45 Min: $f(1.5345) \approx -10.2624$; PI: none

47 Max: $f(0.3666) \approx 0.3240$; min: $f(0.4780) \approx 0$, $f(0.2527) \approx 0$; PI: $(0.4780, 0)$ and $(0.2527, 0)$

49 Max: $f(1.0810) \approx 2.2948$; min: $f(0.5643) \approx 2.1902$; PI: $(-0.8281, 5.5559)$ and $(0.8281, 2.2434)$

CHAPTER ■ 4

Exercises 4.1

1 $2x^2 + 3x + C$ **3** $3t^3 - 2t^2 + 3t + C$

5 $-\frac{1}{2z^2} + \frac{3}{z} + C$ **7** $2u^{3/2} + 2u^{1/2} + C$

9 $\frac{8}{9}v^{9/4} + \frac{24}{5}v^{5/4} - v^{-3} + C$ **11** $3x^3 - 3x^2 + x + C$

13 $\frac{2}{3}x^3 + \frac{3}{2}x^2 + C$ **15** $\frac{24}{5}x^{5/3} - \frac{15}{2}x^{2/3} + C$

17 $\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$ **19** $-t^{-1} - 2t^{-3} - \frac{9}{5}t^{-5} + C$

21 $\frac{3}{4}\sin u + C$ **23** $-7\cos x + C$

25 $\frac{2}{3}t^{3/2} + \sin t + C$ **27** $\tan t + C$ **29** $-\cot v + C$

31 $\sec w + C$ **33** $-\csc z + C$ **35** $\sqrt{x^2 + 4} + C$

37 $\sin \sqrt[3]{x} + C$ **39** $x^3\sqrt{x - 4}$

43 $a^2x + C$ **45** $\frac{1}{2}at^2 + bt + C$ **47** $(a + b)u + C$

49 $f(x) = 4x^3 - 3x^2 + x + 3$ **51** $y = \frac{8}{3}x^{3/2} - \frac{1}{3}$

53 $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 8x + \frac{65}{6}$

55 $y = -3\sin x + 4\cos x + 5x + 3$ **57** $t^2 - t^3 - 5t + 4$

59 (a) $s(t) = -16t^2 + 1600t$ **(b)** $s(50) = 40{,}000$ ft

61 (a) $s(t) = -16t^2 - 16t + 96$ **(b)** $t = 2$ sec **(c)** -80 ft/sec

63 Solve the differential equation $s''(t) = -g$ for $s(t)$.

65 10 ft/sec² **67** 19.62

69 $C(x) = 20x - 0.0075x^2 + 5.0075$; $C(50) \approx \$986.26$

71 $10x^4 + 4x^3 + 27x^2 - 10x + 4$; $\frac{1}{3}x^6 + \frac{1}{5}x^5 + \frac{9}{4}x^4 - \frac{5}{3}x^3 + 2x^2 + 10x + C$

73 $e^{3x}[(3x^2 + 2x)\cos(4x) - 4x^2\sin(4x)]$; $\frac{1}{15{,}625}e^{3x}[(1875x^2 + 350x - 234)\cos(4x) + 4(625x^2 - 300x + 22)\sin(4x)] + C$

75 $\frac{-(4t^3 - 27t^2 - 30t + 31)}{(2t^3 + 3t^2 - 5t - 6)^2}$; $-\ln(2t - 3)^{1/5} - \ln(t + 2) + \frac{6}{5}\ln(t + 1) + C$

77 (b) Each pair of functions differs only by a constant.

Exercises 4.2

1 $\frac{1}{44}(2x^2 + 3)^{11} + C$ **3** $\frac{1}{12}(3x^3 + 7)^{4/3} + C$

5 $\frac{1}{2}(1 + \sqrt{x})^4 + C$ **7** $\frac{2}{3}\sin\sqrt{x^3} + C$

9 $\frac{2}{9}(3x - 2)^{3/2} + C$ **11** $\frac{3}{32}(8t + 5)^{4/3} + C$

13 $\frac{1}{15}(3z + 1)^5 + C$ **15** $\frac{2}{9}(v^3 - 1)^{3/2} + C$

17 $-\frac{3}{8}(1 - 2x^2)^{2/3} + C$ **19** $\frac{1}{5}s^5 + \frac{2}{3}s^3 + s + C$

21 $\frac{2}{5}(\sqrt{x} + 3)^5 + C$ **23** $-\frac{1}{4(t^2 - 4t + 3)^2} + C$

25 $-\frac{3}{4}\cos 4x + C$ **27** $\frac{1}{4}\sin(4x - 3) + C$

29 $-\frac{1}{2}\cos(v^2) + C$ **31** $\frac{1}{4}(\sin 3x)^{4/3} + C$

33 $x - \frac{1}{2}\cos 2x + C$ **35** $-\cos x - \cos^2 x - \frac{1}{3}\cos^3 x + C$

37 $\frac{1}{3\cos^3 x} + C$ **39** $\frac{1}{1 - \sin t} + C$ **41** $\frac{1}{3}\tan(3x - 4) + C$

43 $\frac{1}{6}\sec^2 3x + C$ **45** $-\frac{1}{5}\cot 5x + C$

47 $-\frac{1}{2}\csc(x^2) + C$ **49** $f(x) = \frac{1}{4}(3x + 2)^{4/3} + 5$

51 $f(x) = 3\sin x - 4\cos 2x + x + 2$

53 (a) $\frac{1}{3}(x + 4)^3 + C_1$
(b) $\frac{1}{3}x^3 + 4x^2 + 16x + C_2$; $C_2 = C_1 + \frac{64}{3}$

55 (a) $\frac{2}{3}(\sqrt{x} + 3)^3 + C_1$
(b) $\frac{2}{3}x^{3/2} + 6x + 18x^{1/2} + C_2$; $C_2 = C_1 + 18$

59 474,592 ft³ **61 (a)** $\frac{dV}{dt} = 0.6 \sin\left(\frac{2\pi}{5}t\right)$ **(b)** $\frac{3}{\pi} \approx 0.95$ L
63 *Hint:* (i) Let $u = \sin x$. (ii) Let $u = \cos x$.
(iii) Use the double angle formula for the sine. The three answers differ by constants.

Exercises 4.3

1 34 **3** 40 **5** 10 **7** 500
9 $\frac{1}{3}n(n^2 + 6n + 20)$ **11** $\frac{1}{12}n(3n^3 + 14n^2 + 9n + 46)$

Exer. 13–18: Answers are not unique.

13 $\sum_{k=1}^{5}(4k-3)$ **15** $\sum_{k=1}^{4}\frac{k}{3k-1}$ **17** $1 + \sum_{k=1}^{n}(-1)^k\frac{x^{2k}}{2k}$
19 111,142.3744 **21** 7.4855
23 0.9441 **25** 21,781,332
27 (a) 10 **(b)** 14
29 (a) $\frac{35}{4}$ **(b)** $\frac{51}{4}$ **31 (a)** 1.04 **(b)** 1.19

Exer. 33–38: Answers for (a) and (b) are the same.

33 28 **35** 18 **37** 6 **39 (a)** 20 **(b)** $\frac{1}{4}(b^4 - a^4)$

Exercises 4.4

1 (a) 1.1, 1.5, 1.1, 0.4, 0.9 **(b)** 1.5
3 (a) 0.3, 1.7, 1.4, 0.5, 0.1 **(b)** 1.7
5 (a) 30 **(b)** 42 **(c)** 36
7 (a) 15.127 **(b)** 15.283 **(c)** 15.3975
9 (a) 141 **(b)** 551 **(c)** 307
11 (a) 292.5 **(b)** 348.5 **(c)** 319.75
13 (a) 0.2668 **(b)** 0.2962 **(c)** 0.2813
15 $\int_{-1}^{2}(3x^2 - 2x + 5)\,dx$ **17** $\int_{0}^{4}2\pi x(1 + x^3)\,dx$
19 $-\frac{14}{3}$ **21** $\frac{14}{3}$ **23** $-\frac{14}{3}$
25 $\int_{0}^{4}\left(-\frac{5}{4}x + 5\right)dx$ **27** $\int_{-1}^{5}\sqrt{9 - (x-2)^2}\,dx$
29 36 **31** 25 **33** 2.5 **35** $\frac{9\pi}{4}$ **37** $12 + 2\pi$

Exercises 4.5

1 30 **3** −12 **5** 2 **7** 78 **9** $-\frac{291}{2}$
11 Use Corollary (4.27). **13** Use Theorem (4.26).
15 Use Theorem (4.26). **17** $\int_{-3}^{1}f(x)\,dx$
19 $\int_{e}^{d}f(x)\,dx$ **21** $\int_{h}^{c+h}f(x)\,dx$ **23 (a)** $\sqrt{3}$ **(b)** 9

25 (a) −1 **(b)** 2 **27 (a)** 3 **(b)** 6
29 (a) $\sqrt[3]{\frac{15}{4}}$ **(b)** 14
31 1.426 **33** Use (4.22) and (4.23)(i).

Exercises 4.6

1 −18 **3** $\frac{265}{2}$ **5** 5 **7** $\frac{31}{32}$ **9** $\frac{20}{3}$ **11** $\frac{352}{5}$
13 $\frac{13}{3}$ **15** $-\frac{7}{2}$ **17** 0 **19** $\frac{10}{3}$ **21** $\frac{53}{2}$ **23** $\frac{14}{3}$
25 0 **27** $\frac{1}{3}$ **29** $\frac{5}{36}$ **31** $\frac{3}{2}(\sqrt{3} - 1) \approx 1.10$
33 $1 - \sqrt{2} \approx -0.41$ **35** 0
37 No, $\sec^2 x$ is not continuous on $[0, \pi]$.
39 Yes, since $\int_{-1}^{0}f(x)\,dx + \int_{0}^{1}f(x)\,dx = \int_{-1}^{1}f(x)\,dx$.
41 (a) $\sqrt{3}$ **(b)** $\frac{1}{2}$ **43 (a)** $\frac{544}{225}$ **(b)** $\frac{38}{15}$
45 0 **47** $\frac{1}{x+1}$ **51 (a)** $\frac{6}{7}cd^{1/6}$
55 *Hint:* Use Part I of the fundamental theorem of calculus (4.30) and the chain rule.
57 $\frac{4x^7}{\sqrt{x^{12} + 2}}$ **59** $3x^2(x^9 + 1)^{10} - 3(27x^3 + 1)^{10}$

Exercises 4.7

1 $L_6 = 10.95$; $R_6 = 11.95$; $M_3 = 11.1$; $T_6 = 11.45$; $S_3 = 11\frac{1}{3}$
3 $L_8 = 12.33375$; $R_8 = 13.60875$; $M_4 = 12.6975$; $T_8 = 12.97125$; $S_4 = 12.88$
5 (a) $L_8 = 1.1501$; $R_8 = 1.2597$ **(b)** 1.2049
7 (a) $L_3 = 0.84$; $L_6 = 0.9$; $L_{12} = 0.93$
(b) 0.96; $E_3 = 0.12$; $E_6 = 0.06$; $E_{12} = 0.03$
(c) The error is reduced by $\frac{1}{2}$ when n doubles.
9 (a) $M_2 = 144$; $M_4 = 153$; $M_8 = 155.25$
(b) 156; $E_2 = 12$; $E_4 = 3$; $E_8 = 0.75$
(c) The error is reduced by $\frac{1}{4}$ when n doubles.
11 (a) $T_2 = 180$; $T_4 = 162$; $T_8 = 157.5$
(b) 156; $E_2 = -24$; $E_4 = -6$; $E_8 = -1.5$
(c) The error is reduced by $\frac{1}{4}$ when n doubles.
13 (a) $S_2 = S_4 = S_8 = 156$
(b) 156; $E_2 = E_4 = E_8 = 0$
(c) Simpson's rule is exact for all n.
15 (a) $T_5 \approx 6.249806$; $T_{10} \approx 6.234926$; $T_{20} \approx 6.231201$; $T_{40} \approx 6.230270$
(b) At least two decimal places
17 (a) $S_2 \approx 2.3987529621$; $S_6 \approx S_{18} \approx S_{54} \approx 2.4039394306$
(b) At least ten decimal places
19 (a) 0.26 **(b)** 4.2×10^{-5}
21 (a) 0.125 **(b)** 6.5×10^{-4}
23 (a) 3,386,880 **(b)** 642 **(c)** 10

25 (a) 25 **(b)** 3 **(c)** 1 **29 (a)** 127.5 **(b)** 131.7
31 0.174 m/sec **33** 0.28 **35** 1.48

Chapter 4 Review Exercises

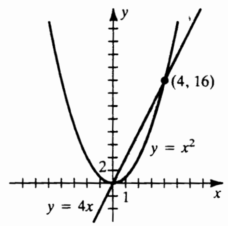
1 $-\frac{8}{x} + \frac{2}{x^2} - \frac{5}{3x^3} + C$ **3** $100x + C$ **5** $\frac{1}{16}(2x+1)^8 + C$
7 $-\frac{1}{16}(1 - 2x^2)^4 + C$ **9** $-\frac{2}{1 + \sqrt{x}} + C$
11 $3x - x^2 - \frac{5}{4}x^4 + C$ **13** $\frac{1}{6}(4x^2 + 2x - 7)^3 + C$
15 $-\frac{1}{x^2} - x^3 + C$ **17** $\frac{3}{5}$ **19** $\frac{1}{6}$ **21** $\sqrt{8} - \sqrt{3} \approx 1.10$
23 $\frac{52}{9}$ **25** $-\frac{37}{6}$ **27** $8\sqrt{3} + 16 \approx 29.86$
29 $\frac{1}{5}\cos(3 - 5x) + C$ **31** $\frac{1}{15}\sin^5 3x + C$
33 $-\frac{1}{6\sin^2 3x} + C$ **35** $\frac{2}{15}(16\sqrt{2} - 3\sqrt{3}) \approx 2.32$ **37** $\frac{1}{6}$
39 $\sqrt[5]{x^4 + 2x^2 + 1} + C$ **41** 0 **43** $y = x^3 - 2x^2 + x + 2$
45 $\frac{135}{4}$ **47** Use Corollary (4.27). **49** $\int_{a}^{e}f(x)\,dx$
51 (a) $-16t^2 - 30t + 900$ **(b)** −190 ft/sec
(c) $\frac{15}{16}(-1 + \sqrt{65}) \approx 6.6$ sec
53 $\int_{-2}^{3}\sqrt{1 + 3x^2}\,dx$ **55** $M_5 \approx 0.824279$; $M_{10} \approx 0.8092539$
57 $S_4 \approx 11.105304$; $S_8 \approx 11.105302$ **59** 81.625 °F

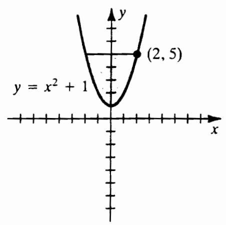
CHAPTER 5

Exercises 5.1

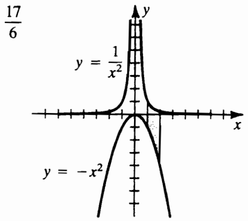
Exer. 1–4: Answers are not unique.

1 $\int_{-2}^{2}[(x^2 + 1) - (x - 2)]\,dx$
3 $\int_{-2}^{1}[(-3y^2 + 4) - y^3]\,dy$
5 $\int_{0}^{4}(4x - x^2)\,dx = \frac{32}{3}$ **7** $2\int_{0}^{2}[5 - (x^2 + 1)]\,dx = \frac{32}{3}$

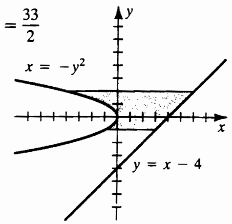




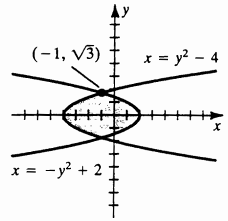
9 $\int_{1}^{2}\left[\frac{1}{x^2} - (-x^2)\right]dx = \frac{17}{6}$



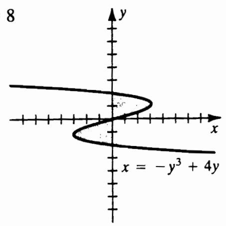
11 $\int_{-1}^{2}[(4 + y) - (-y^2)]\,dy = \frac{33}{2}$



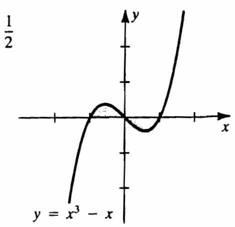
13 $2\int_{0}^{\sqrt{3}}[(2 - y^2) - (y^2 - 4)]\,dy = 8\sqrt{3}$



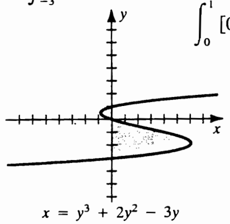
15 $2\int_{0}^{2}[(4y - y^3) - 0]\,dy = 8$



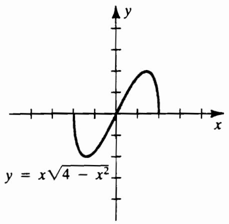
17 $2\int_{0}^{1}[0 - (x^3 - x)]\,dx = \frac{1}{2}$



19 $\int_{-3}^{0} [(y^3 + 2y^2 - 3y) - 0]\, dy + \int_{0}^{1} [0 - (y^3 + 2y^2 - 3y)]\, dy = \frac{71}{6}$



21 $\int_{0}^{2} x\sqrt{4 - x^2}\, dx = \frac{16}{3}$



23 $3 + \frac{3}{2}\sqrt{3} \approx 5.74$

25 (a) $\int_{0}^{1} (3x - x)\, dx + \int_{1}^{2} [(4 - x) - x]\, dx$

(b) $\int_{0}^{2} \left(y - \frac{1}{3}y\right) dy + \int_{2}^{3} \left[(4 - y) - \frac{1}{3}y\right] dy$

27 (a) $\int_{1}^{4} [\sqrt{x} - (-x)]\, dx$

(b) $\int_{-4}^{-1} [4 - (-y)]\, dy + \int_{-1}^{1} (4 - 1)\, dy + \int_{1}^{2} (4 - y^2)\, dy$

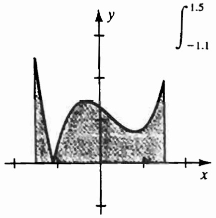
29 (a) $\int_{-6}^{-1} [(x + 3) - (-\sqrt{3 - x})]\, dx + 2\int_{-1}^{3} \sqrt{3 - x}\, dx$

(b) $\int_{-3}^{2} [(3 - y^2) - (y - 3)]\, dy$

31 9 **33** 12 **35** $4\sqrt{2}$

37 $\int_{0}^{1} (x^2 - 6x + 5)\, dx + \int_{1}^{5} -(x^2 - 6x + 5)\, dx + \int_{5}^{7} (x^2 - 6x + 5)\, dx$

41 $\int_{-1.5}^{-1.1} -(x^3 - 0.7x^2 - 0.8x + 1.3)\, dx + \int_{-1.1}^{1.5} (x^3 - 0.7x^2 - 0.8x + 1.3)\, dx$



43 (a) $(a, 0.9052)$, $(b, 5.3623)$, $a \approx 0.0819$, $b \approx 2.8754$

(b) $\int_{a}^{b} [\sqrt{10x} - (x^3 - 2x^2 - x + 1)]\, dx$ **(c)** 10.3259

45 (a) $(\pm a, -8.0061)$, $a \approx 3.4632$

(b) $\int_{-a}^{a} [50 \cos(0.5x) - (x^2 - 20)]\, dx$ **(c)** 308.2566

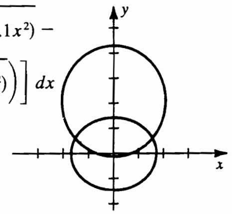
47 (a) $\int_{-5}^{5} [\sqrt{25 - x^2} - (\sqrt{29 - x^2} - 2)]\, dx$ **(b)** 14.7515

49 (a) $\int_{0}^{\pi} [\sin x - \sin(\sin x)]\, dx$ **(b)** 0.2135

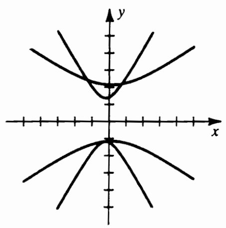
51 (a) $[0, 1]$ **(b)** $\frac{1}{6}$ **53 (a)** $[0, 1]$ **(b)** 2

55 (a) $(\pm 1.540, 0.618)$

(b) $2\int_{0}^{1.54} \left[\sqrt{\frac{1}{2.9}(6.09 - 2.1x^2)} - \left(2.1 - \sqrt{\frac{1}{4.3}(21.07 - 4.9x^2)}\right)\right] dx$



57 (a) $(0.741, 2.206)$

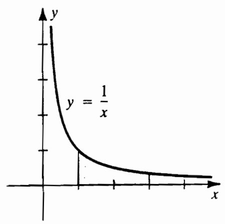


(b) $\int_{0}^{0.74} \left\{0.5 + \sqrt{\frac{1}{5.3}[14.31 + 2.7(x - 0.1)^2]} - [0.1 + \sqrt{1.6 + 3.2(x + 0.2)^2}]\right\} dx$

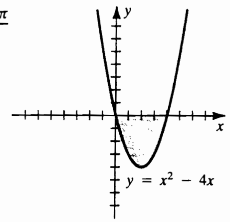
Exercises 5.2

1 $\pi\int_{-1}^{2} \left(\frac{1}{2}x^2 + 2\right)^2 dx$ **3** $2 \cdot \pi\int_{0}^{4} [(\sqrt{25 - y^2})^2 - 3^2]\, dy$

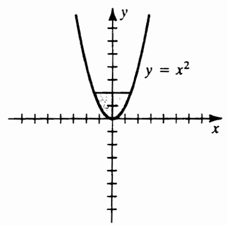
5 $\pi\int_{1}^{3} \left(\frac{1}{x}\right)^2 dx = \frac{2\pi}{3}$

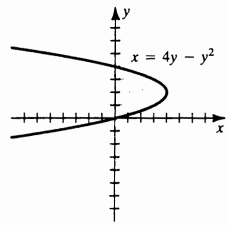


7 $\pi\int_{0}^{4} (x^2 - 4x)^2\, dx = \frac{512\pi}{15}$



9 $\pi\int_{0}^{2} (\sqrt{y})^2\, dy = 2\pi$ **11** $\pi\int_{0}^{4} (4y - y^2)^2\, dy = \frac{512\pi}{15}$

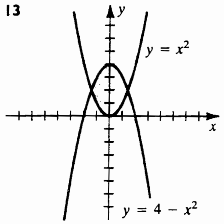




13 $2 \cdot \pi\int_{0}^{\sqrt{2}} [(4 - x^2)^2 - (x^2)^2]\, dx = \frac{64\pi\sqrt{2}}{3}$

15 $\pi\int_{0}^{2} [(4 - x)^2 - (x)^2]\, dx = 16\pi$

13



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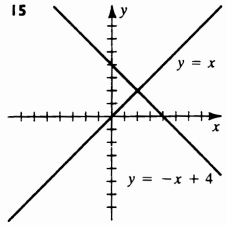
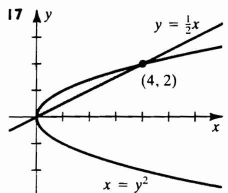


17 $\pi\int_{0}^{2} [(2y)^2 - (y^2)^2]\, dy = \frac{64\pi}{15}$

19 $\pi\int_{-1}^{2} [(y + 2)^2 - (y^2)^2]\, dy = \frac{72\pi}{5}$

17



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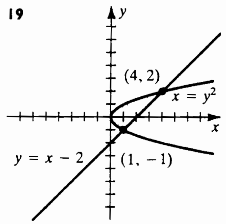
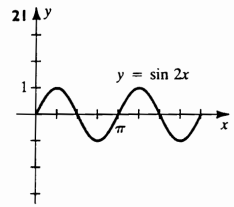


21 $\pi\int_{0}^{\pi} (\sin 2x)^2\, dx = \frac{1}{2}\pi^2$

23 $\pi\int_{0}^{\pi/4} [(\cos x)^2 - (\sin x)^2]\, dx = \frac{\pi}{2}$

21



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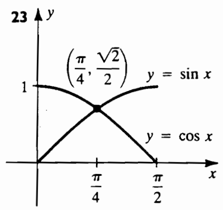

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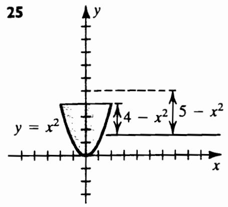
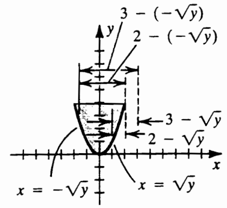




(a) $2 \cdot \pi\int_{0}^{2} (4 - x^2)^2\, dx = \frac{512\pi}{15}$

(b) $2 \cdot \pi\int_{0}^{2} [(5 - x^2)^2 - (5 - 4)^2]\, dx = \frac{832\pi}{15}$

(c) $\pi\int_{0}^{4} \{[2 - (-\sqrt{y})]^2 - [2 - \sqrt{y}]^2\}\, dy = \frac{128\pi}{3}$

(d) $\pi\int_{0}^{4} \{[3 - (-\sqrt{y})]^2 - [3 - \sqrt{y}]^2\}\, dy = 64\pi$

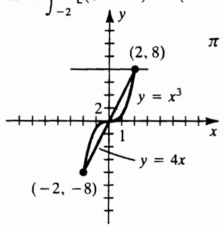
27 (a) $\pi\int_{0}^{4} \left\{\left[\left(-\frac{1}{2}x + 2\right) - (-2)\right]^2 - [0 - (-2)]^2\right\} dx$

(b) $\pi\int_{0}^{4} \left\{(5 - 0)^2 - \left[5 - \left(-\frac{1}{2}x + 2\right)\right]^2\right\} dx$

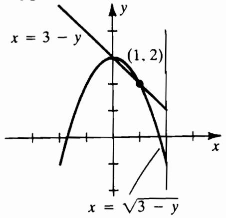
(c) $\pi\int_{0}^{2} \{(7 - 0)^2 - [7 - (-2y + 4)]^2\}\, dy$

(d) $\pi\int_{0}^{2} \{[(-2y + 4) - (-4)]^2 - [0 - (-4)]^2\}\, dy$

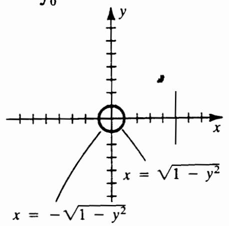
29 $\pi\int_{-2}^{0} [(8 - 4x)^2 - (8 - x^3)^2]\, dx + \pi\int_{0}^{2} [(8 - x^3)^2 - (8 - 4x)^2]\, dx$



31 $\pi \int_2^3 \{[2-(3-y)]^2 - [2-\sqrt{3-y}]^2\}\, dy$



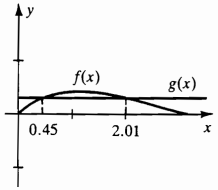
33 $2 \cdot \pi \int_0^1 \{[5-(-\sqrt{1-y^2})]^2 - [5-\sqrt{1-y^2}]^2\}\, dy$



35 $\pi \int_0^h r^2\, dy = \pi r^2 h$ **37** $\pi \int_0^h \left(\frac{r}{h}x\right)^2 dx = \frac{1}{3}\pi r^2 h$

39 $\pi \int_0^h \left(\frac{R-r}{h}x + r\right)^2 dx = \frac{1}{3}\pi h(R^2 + Rr + r^2)$ **41** $\frac{63\pi}{2}$

43



(a) 0.45, 2.01 **(b)** 0.28

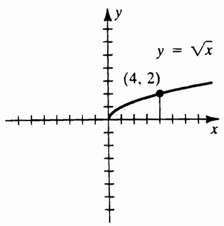
45 $\frac{4}{3}\pi ab^2$ **47 (a)** $p = \frac{r^2}{4h}$

(b) $\frac{1}{2}\pi r^2 h$

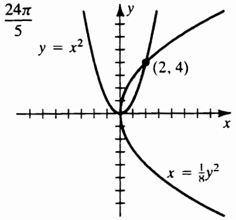
Exercises 5.3

1 $2\pi \int_2^{11} x\sqrt{x-2}\, dx$ **3** $2\pi \int_0^6 y\left(-\frac{1}{2}y + 3\right) dy$

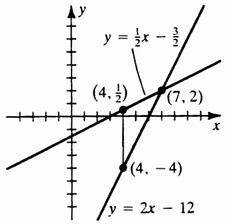
5 $2\pi \int_0^4 x\sqrt{x}\, dx = \frac{128\pi}{5}$



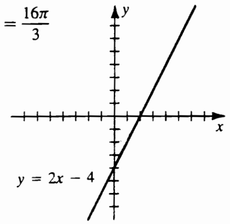
7 $2\pi \int_0^2 x(\sqrt{8x} - x^2)\, dx = \frac{24\pi}{5}$



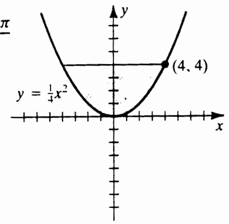
9 $2\pi \int_4^7 x\left[\left(\frac{1}{2}x - \frac{3}{2}\right) - (2x - 12)\right] dx = \frac{135\pi}{2}$



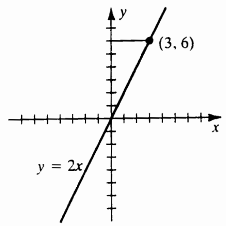
11 $2\pi \int_0^2 x[0 - (2x - 4)]\, dx = \frac{16\pi}{3}$



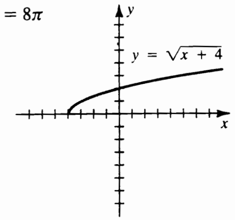
13 $2 \cdot 2\pi \int_0^4 y\sqrt{4y}\, dy = \frac{512\pi}{5}$



15 $2\pi \int_0^6 y\left(\frac{1}{2}y\right) dy = 72\pi$



17 $2\pi \int_0^2 y[0 - (y^2 - 4)]\, dy = 8\pi$



19 (a) $2\pi \int_0^2 (3 - x)(x^2 + 1)\, dx$

(b) $2\pi \int_0^2 [x - (-1)](x^2 + 1)\, dx$

21 (a) $2 \cdot 2\pi \int_0^4 (4 - y)\sqrt{y}\, dy$

(b) $2 \cdot 2\pi \int_0^4 (5 - y)\sqrt{y}\, dy$

(c) $2\pi \int_{-2}^2 (2 - x)(4 - x^2)\, dx$

(d) $2\pi \int_{-2}^2 [x - (-3)](4 - x^2)\, dx$

23 $2\pi \int_0^1 (2 - x)[(3 - x^2) - (3 - x)]\, dx$

25 $2 \cdot 2\pi \int_{-1}^1 (5 - x)\sqrt{1 - x^2}\, dx$

27 (a) $2\pi \int_0^{1/2} y(4 - 1)\, dy + 2\pi \int_{1/2}^1 y[(1/y^2) - 1]\, dy$

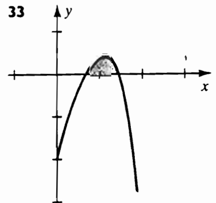
(b) $\pi \int_1^4 \left(\frac{1}{\sqrt{x}}\right)^2 dx$

29 (a) $2\pi \int_0^1 x(x^2 + 2)\, dx$

(b) $\pi \int_0^2 (1)^2\, dy + \pi \int_2^3 [(1)^2 - (\sqrt{y - 2})^2]\, dy$

31 76π

33



(a) 0.68, 1.44

(b) $2\pi \int_{0.68}^{1.44} x(-x^4 + 2.21x^3 - 3.21x^2 + 4.42x - 2)\, dx$

35 (a) $\frac{8}{3}$ **(b)** 2π **(c)** $\frac{16\pi}{5}$

Exercises 5.4

Exer. 1–26: The first integral represents a general formula for the volume. In Exercises 1–8, the vertical distance between the graphs of $y = \sqrt{x}$ and $y = -\sqrt{x}$ is $[\sqrt{x} - (-\sqrt{x})]$, denoted by $2\sqrt{x}$.

1 $\int_c^d s^2\, dx = \int_0^9 (2\sqrt{x})^2\, dx = 162$

3 $\int_c^d \frac{1}{2}\pi r^2\, dx = \int_0^9 \frac{1}{2}\pi(\sqrt{x})^2\, dx = \frac{81\pi}{4}$

5 $\int_c^d \frac{\sqrt{3}}{4}s^2\, dx = \int_0^9 \frac{\sqrt{3}}{4}(2\sqrt{x})^2 dx = \frac{81\sqrt{3}}{2}$

7 $\int_c^d \frac{1}{2}(B + b)h\, dx$

$= \int_0^9 \frac{1}{2}\left[2\sqrt{x} + \frac{1}{2}(2\sqrt{x})\right]\left[\frac{1}{4}(2\sqrt{x})\right] dx = \frac{243}{8}$

9 $\int_c^d s^2\, dx$

$= 2\int_0^a [\sqrt{a^2 - x^2} - (-\sqrt{a^2 - x^2})]^2\, dx = \frac{16}{3}a^3$

11 $\int_c^d \frac{1}{2}bh\, dx$

$= 2\int_0^2 \frac{1}{2}\left[\frac{1}{\sqrt{2}}(4 - x^2)\right]\left[\frac{1}{\sqrt{2}}(4 - x^2)\right] dx = \frac{128}{15}$

13 $\int_c^d lw\, dx = \int_0^h \left(\frac{2ax}{h}\right)\left(\frac{ax}{h}\right) dx = \frac{2}{3}a^2 h$

15 $\int_c^d \frac{1}{2}\pi r^2\, dy = 2\int_0^4 \frac{1}{2}\pi\left[\frac{1}{2}\left(4 - \frac{1}{4}y^2\right)\right]^2 dy = \frac{128\pi}{15}$

17 $\int_c^d lw\, dy = \int_0^a [\sqrt{a^2 - y^2} - (-\sqrt{a^2 - y^2})]y\, dy$

$= \frac{2}{3}a^3$

19 $\int_c^d \frac{1}{2}bh\, dx = \int_{-a}^a \frac{1}{2}[\sqrt{a^2 - x^2} - (-\sqrt{a^2 - x^2})]h\, dx$

$= \frac{1}{2}\pi a^2 h$

21 $\int_c^d \frac{1}{2}bh\, dx = \int_0^4 \frac{1}{2}\left(\frac{2}{4}x\right)\left(\frac{3}{4}x\right) dx = 4\ \text{cm}^3$

23 $\int_c^d \frac{1}{2}\pi r^2\, dy = \int_0^a \frac{1}{2}\pi\left[\frac{1}{2}(a - y)\right]^2 dy = \frac{\pi}{24}a^3$

25 The areas of cross sections of typical disks and washers are $\pi[f(x)]^2$ and $\pi\{[f(x)]^2 - [g(x)]^2\}$, respectively. In each case, the integrand represents $A(x)$ in (5.13).

27 864

Exercises 5.5

1 (a) $\int_1^3 \sqrt{1 + (3x^2)^2}\, dx$

(b) $\int_2^{28} \sqrt{1 + \left[\frac{1}{3}(y - 1)^{-2/3}\right]^2}\, dy$

3 (a) $\int_{-3}^{-1} \sqrt{1 + (-2x)^2}\, dx$

(b) $\int_{-5}^{3} \sqrt{1 + \left[\frac{1}{2}(4 - y)^{-1/2}\right]^2}\, dy$

5 $\int_1^8 \sqrt{1 + \left(\frac{4}{9}x^{-1/3}\right)^2}\, dx = \left(4 + \frac{16}{81}\right)^{3/2} - \left(1 + \frac{16}{81}\right)^{3/2} \approx 7.29$

7 $\int_1^4 \sqrt{1 + \left(-\frac{3}{2}x^{1/2}\right)^2}\, dx = \frac{8}{27}\left[10^{3/2} - \left(\frac{13}{4}\right)^{3/2}\right] \approx 7.63$

9 $\int_1^2 \sqrt{1 + \left(\frac{1}{4}x^2 - \frac{1}{x^2}\right)^2}\, dx = \frac{13}{12}$

11 $\int_1^2 \sqrt{1 + \left(\frac{3}{2}y^{-4} + \frac{1}{6}y^4\right)^2}\, dy = \frac{353}{240}$

13 $\int_0^2 \sqrt{1 + \left(\frac{7}{2} - 3y^2\right)^2}\, dy$

15 $8\int_a^1 \sqrt{1 + [(-x^{-1/3})(1 - x^{2/3})^{1/2}]^2}\, dx = 6,$ where $a = \left(\frac{1}{2}\right)^{3/2}$

17 (a) $\int_1^{1.1} \sqrt{1 + \frac{4}{9}x^{-2/3}}\, dx \approx 0.119599$

(b) $\sqrt{13/30} \approx 0.120185$ **(c)** 0.119598

19 (a) $\int_2^{2.1} \sqrt{1 + 4x^2}\, dx \approx 0.422021$

(b) $\sqrt{17}(0.1) \approx 0.412311$ **(c)** $\sqrt{0.1781} \approx 0.422019$

21 (a) $\int_{\pi/6}^{31\pi/180} \sqrt{1 + \sin^2 x}\, dx \approx 0.0195733$

(b) $\pi\sqrt{5}/360 \approx 0.0195134$ **(c)** 0.0195725

23 9.778303 **25** 1.849432

27 (a) 3.7900; 3.8125; it is smaller

(b) $\int_0^{\pi} \sqrt{1 + \cos^2 x}\, dx$; 3.8199; 3.8202

29 $2\pi \int_0^1 \sqrt{4x}\sqrt{1 + (x^{-1/2})^2}\, dx = \frac{8\pi}{3}(2^{3/2} - 1) \approx 15.32$

31 $2\pi \int_1^2 \left(\frac{1}{4}x^4 + \frac{1}{8}x^{-2}\right)\sqrt{1 + \left(x^3 - \frac{1}{4}x^{-3}\right)^2}\, dx = \frac{16{,}911\pi}{1024} \approx 51.88$

33 $2\pi \int_2^4 \frac{1}{8}y^3\sqrt{1 + \left(\frac{3}{8}y^2\right)^2}\, dy = \frac{\pi}{27}[8(37)^{3/2} - 13^{3/2}] \approx 204.04$

35 $2\pi \int_4^5 \sqrt{25 - y^2}\sqrt{1 + [(-y)(25 - y^2)^{-1/2}]^2}\, dy = 10\pi$

37 $2\pi \int_0^h \left(\frac{r}{h}x\right)\sqrt{1 + \left(\frac{r}{h}\right)^2}\, dx = \pi r\sqrt{h^2 + r^2}$

39 $2 \cdot 2\pi \int_0^r \sqrt{r^2 - x^2}\sqrt{1 + [(-x)(r^2 - x^2)^{-1/2}]^2}\, dx = 4\pi r^2$

41 *Hint*: Regard ds as the slant height of the frustum of a cone that has average radius x.

43 (a) 13.6862; 14.2384; it is smaller

(b) $2\pi \int_0^{\pi} \sin x\sqrt{1 + \cos^2 x}\, dx$; 13.4821; 14.1937

45 201 in^2

47 (a) $x^2 = 500(y - 10)$ **(b)** $\int_{-200}^{200} \sqrt{1 + \left(\frac{1}{250}x\right)^2}\, dx$

(c) 282 ft

49 (a) *Hint*: $S = \int_0^a 2\pi x\sqrt{1 + \left(\frac{1}{2p}x\right)^2}\, dx$ **(b)** 64,968 ft^2

Exercises 5.6

1 (a) and **(b)** 6000 ft-lb **3 (a)** $\frac{128}{3}$ in.-lb **(b)** $\frac{64}{3}$ in.-lb

5 $W_2 = 3W_1$ **7** 27,945 ft-lb **9** 276 ft-lb **11** 2250 ft-lb

13 (a) $\frac{81\pi}{2}(62.5) \approx 7952$ ft-lb **(b)** $\frac{189\pi}{2}(62.5) \approx 18{,}555$ ft-lb

15 500 ft-lb **17** $575\left(\frac{1}{2} - 40^{-1/5}\right) \approx 12.55$ in.-lb

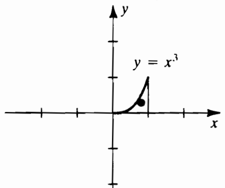
19 $W = \frac{Gm_1m_2h}{(4000)(4000 + h)}$ **21** 36.85 ft-lb

23 (a) $\frac{3}{10}k$ J (k a constant) **(b)** $\frac{9}{40}k$ J

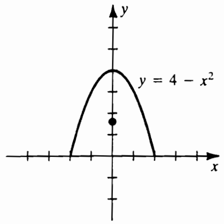
Exercises 5.7

1 250; 140; 0.56 **3** 14; −27; −46; $\left(-\frac{23}{7}, -\frac{27}{14}\right)$

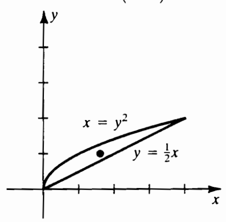
5 $\frac{1}{4}; \frac{1}{14}; \frac{1}{5}; \left(\frac{4}{5}, \frac{2}{7}\right)$



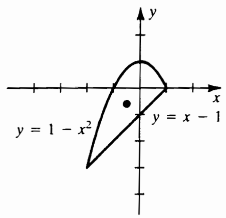
7 $\frac{32}{3}; \frac{256}{15}; 0; \left(0, \frac{8}{5}\right)$



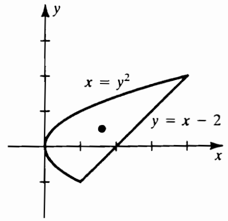
9 $\frac{4}{3}; \frac{4}{3}; \frac{32}{15}; \left(\frac{8}{5}, 1\right)$



11 $\frac{9}{2}; -\frac{27}{10}; -\frac{9}{4}; \left(-\frac{1}{2}, -\frac{3}{5}\right)$



13 $\frac{9}{2}; \frac{9}{4}; \frac{36}{5}; \left(\frac{8}{5}, \frac{1}{2}\right)$



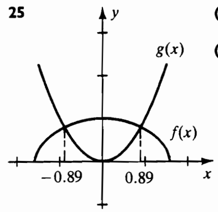
15 $\left(\frac{4a}{3\pi}, \frac{4a}{3\pi}\right)$

17 With the center of the circle at the origin, the centroid is $\left(0, -\frac{20a}{3(8 + \pi)}\right)$.

19 Show that the centroid is $\left(\frac{1}{3}a, \frac{1}{3}(b + c)\right)$.

21 $(2\pi \cdot 3)(\sqrt{2}\sqrt{18}) = 36\pi$ **23** $\left(\frac{4a}{3\pi}, \frac{4a}{3\pi}\right)$

25



(a) $\rho \int_{-0.89}^{0.89} (\sqrt{|\cos x|} - x^2)\, dx$

(b) 1.19ρ

Exercises 5.8

1 (a) $\frac{1}{2}(62.5)$ lb **(b)** $\frac{3}{2}(62.5)$ lb

3 (a) $\frac{\sqrt{3}}{3}(62.5)$ lb **(b)** $\frac{\sqrt{3}}{24}(62.5)$ lb

5 $\frac{16}{3}(60)$ lb **7** $\frac{592}{3}(62.5)$ lb

9 (a) 90(50) lb **(b)** 54(50) lb; 36(50) lb **11** 1.56 L/min

13 In min: **(a)** 20 **(b)** 66 **(c)** 115 **(d)** 197

15 $10\sqrt{11} - 10 \approx 23.17$ min

17 666 **19** 11 **21 (a)** and **(b)** 150 J

23 $9 - \frac{5\sqrt{5}}{3} \approx 5.27$ gal **25** 1.45 coulombs

27 (a) $\int_0^{1/30} 12{,}450\pi \sin(30\pi t)\, dt = 830$ cm^3

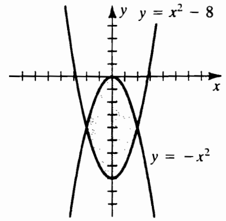
(b) It is not safe, since approximately 0.027 joule is inhaled.

29 32 **31** $x_c = 320$; 2560 **33** $x_c = 800$; 120,000

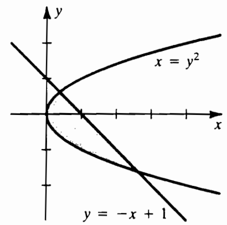
Chapter 5 Review Exercises

1 (a) $2\int_0^2 [(-x^2) - (x^2 - 8)]\, dx = \frac{64}{3}$

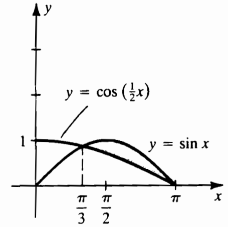
(b) $4\int_{-4}^{0} \sqrt{-y}\, dy = \frac{64}{3}$



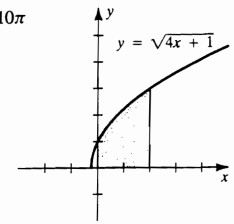
3 $\int_a^b [(1 - y) - y^2]\, dy = \frac{5\sqrt{5}}{6}$, where $a = \frac{1}{2}(-1 - \sqrt{5})$ and $b = \frac{1}{2}(-1 + \sqrt{5})$



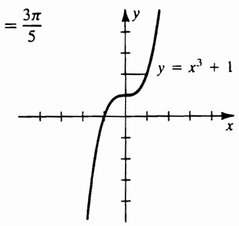
5 $\int_{\pi/3}^{\pi} \left(\sin x - \cos\frac{1}{2}x\right) dx = \frac{1}{2}$



7 $\pi \int_0^2 (\sqrt{4x + 1})^2\, dx = 10\pi$



9 $2\pi \int_0^1 x[2 - (x^3 + 1)]\,dx = \frac{3\pi}{5}$



11 $2\pi \int_0^{\sqrt{\pi/2}} x(\cos x^2)\,dx = \pi$

13 (a) $\pi \int_{-2}^{1} [(-4x + 8)^2 - (4x^2)^2]\,dx = \frac{1152\pi}{5}$

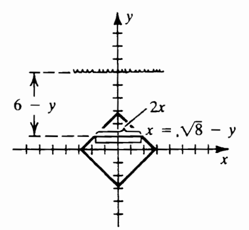
(b) $2\pi \int_{-2}^{1} (1 - x)[(-4x + 8) - 4x^2]\,dx = 54\pi$

(c) $\pi \int_{-2}^{1} \{(16 - 4x^2)^2 - [16 - (-4x + 8)]^2\}\,dx = \frac{1728\pi}{5}$

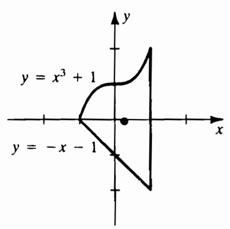
15 $\int_{-2}^{5} \sqrt{1 + \left[\frac{1}{3}(x + 3)^{-1/3}\right]^2}\,dx = \frac{1}{27}(37^{3/2} - 10^{3/2}) \approx 7.16$

17 $\int_0^4 (5 - y)(62.5)\pi(6)^2\,dy = 432\pi(62.5)$ ft-lb

19 $\rho \int_0^{\sqrt{8}} (6 - y)2(\sqrt{8} - y)\,dy + \rho \int_{-\sqrt{8}}^{0} (6 - y)2(y + \sqrt{8})\,dy = 96(62.5)$ lb



21 $4; -\frac{4}{21}; \frac{16}{15}; \left(\frac{4}{15}, -\frac{1}{21}\right)$



23 $2\pi \int_1^2 \left(\frac{1}{3}x^3 + \frac{1}{4}x^{-1}\right)\sqrt{1 + \left(x^2 - \frac{1}{4}x^{-2}\right)^2}\,dx = \frac{515\pi}{64} \approx 25.3$

25 900 ft

27 (a) The area under the graph of $y = 2\pi x^4$

(b) (i) The volume obtained by revolving $y = \sqrt{2}x^2$ about the x-axis

(ii) The volume obtained by revolving $y = x^3$ about the y-axis

(c) The work done by a force of magnitude $y = 2\pi x^4$ as it moves from $x = 0$ to $x = 1$.

29 (a) $(a, 0.67)$, $(b, 1.91)$, $a \approx -0.82$, $b \approx 1.38$

(b) $\int_a^b (\sqrt{1 + x^3} - x^2)\,dx \approx 1.43$

31 (a) $(a, 2.40)$, $(b, 9.53)$, $a \approx 0.29$, $b \approx 4.54$

(b) $\int_a^b [\sqrt{20x} - (x^3 - 4x^2 - x + 3)]\,dx \approx 44.42$

CHAPTER ■ 6

Exercises 6.1

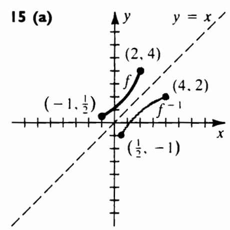
1 $\frac{x - 5}{3}$ **3** $\frac{2x + 1}{3x}$ **5** $\frac{5x + 2}{2x - 3}$ **7** $-\frac{1}{3}\sqrt{6 - 3x}$

9 $3 - x^2, x \geq 0$ **11** $(x - 1)^3$

13 (a) The graph of f is a line of slope $a \neq 0$ and hence is one-to-one. $f^{-1}(x) = \frac{x - b}{a}$

(b) No (not one-to-one)

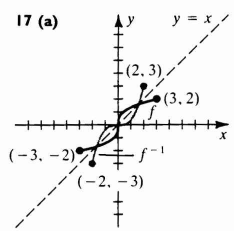
15 (a)



(b) $[-1, 2]; \left[\frac{1}{2}, 4\right]$

(c) $\left[\frac{1}{2}, 4\right]; [-1, 2]$

17 (a)



(b) $[-3, 3]; [-2, 2]$
(c) $[-2, 2]; [-3, 3]$

19 (a) $[-0.27, 1.22]$
(b) $[-0.20, 3.31]; [-0.27, 1.22]$

21 (a) $[-1.43, 1.43]$ **(b)** $[-0.84, 0.84]; [-1.43, 1.43]$

23 (a) $[-2.14, 1]$ **(b)** $[0.5, 2]; [-2.14, 1]$

25 (a) f is increasing on $\left[-\frac{3}{2}, \infty\right)$ and hence is one-to-one. **(b)** $[0, \infty)$ **(c)** x

27 (a) f is decreasing on $[0, \infty)$ and hence is one-to-one.
(b) $(-\infty, 4]$ **(c)** $-\frac{1}{2\sqrt{4 - x}}$

29 (a) f is decreasing on $(-\infty, 0)$ and $(0, \infty)$ and hence is one-to-one.
(b) All real numbers except zero **(c)** $-\frac{1}{x^2}$

31 (a) f is increasing, since $f'(x) > 0$ for every x **(b)** $\frac{1}{16}$

33 (a) f is decreasing, since $f'(x) < 0$ for $x > 0$ **(b)** $-\frac{2}{7}$

35 (a) f is increasing, since $f'(x) > 0$ for every x **(b)** $\frac{1}{16}$

Exercises 6.2

1 $\frac{9}{9x + 4}$ **3** $\frac{2(3x - 1)}{3x^2 - 2x + 1}$ **5** $\frac{2}{2x - 3}$ **7** $\frac{15}{3x - 2}$

9 $\frac{3x^2}{2x^3 - 7}$ **11** $1 + \ln x$ **13** $\frac{1}{2x}\left(1 + \frac{1}{\sqrt{\ln x}}\right)$

15 $-\frac{1}{x}\left[\frac{1}{(\ln x)^2} + 1\right]$ **17** $\frac{20}{5x - 7} + \frac{6}{2x + 3}$

19 $\frac{x}{x^2 + 1} - \frac{18}{9x - 4}$ **21** $\frac{x}{x^2 - 1} - \frac{x}{x^2 + 1}$ **23** $\frac{1}{\sqrt{x^2 - 1}}$

25 $-2\tan 2x$ **27** $9\csc 3x \sec 3x$ **29** $\frac{2\tan 2x}{\ln \sec 2x}$

31 $\tan x$ **33** $\sec x$ **35** $\frac{y(2x^2 - 1)}{x(3y + 1)}$ **37** $\frac{y(y - x\ln y)}{x(x - y\ln x)}$

39 $(5x + 2)^2(6x + 1)(150x + 39)$ **41** $\frac{(14x + 11)(x - 5)^2}{\sqrt{4x + 7}}$

43 $\frac{(19x^2 + 20x - 3)(x^2 + 3)^4}{2(x + 1)^{3/2}}$ **45** $y = 8x - 15$

47 $(10, 5\ln 10 - 5) \approx (10, 6.51)$; $y'' = -(5/x^2) < 0$ implies that the graph is CD for $x > 0$. **49** ± 0.73 yr

51 (a) $s'(0) = 0$ m/sec; $s''(0) = \frac{bc}{m_1 + m_2}$ m/sec²

(b) $s'\left(\frac{m_2}{b}\right) = c\ln\left(\frac{m_1 + m_2}{m_1}\right)$; $s''\left(\frac{m_2}{b}\right) = \frac{bc}{m_1}$

53 The graphs coincide if $x > 0$; however, the graph of $y = \ln(x^2)$ contains points with negative x-coordinates.

55 (a) $-3.18 \leq y \leq 0$
(b) x-int.: $\pi/2 \approx 1.57$; max: $f(\pi/2) = 0$

57 (a) $1.33 \leq y \leq 2.18$
(b) y-int.: 2

59 (a) $-1.97 \leq y \leq 3.79$
(b) x-int.: 0.55; max: $f(2.47) \approx 1.56$, $f(8.14) \approx 2.91$, $f(14.30) \approx 3.49$; min: $f(4.65) \approx 0.34$, $f(10.97) \approx 1.19$, $f(17.26) \approx 1.65$

61 0.5671 **63** −3.2088, 2.0435 **65** 1.7477 **67** 1.8929
69 12.0536 **71** 9.3392

Exercises 6.3

1 $-5e^{-5x}$ **3** $6xe^{3x^2}$ **5** $\frac{e^{2x}}{\sqrt{1 + e^{2x}}}$ **7** $\frac{e^{\sqrt{x+1}}}{2\sqrt{x + 1}}$

9 $2xe^{-2x}(1 - x)$ **11** $\frac{e^x(x - 1)^2}{(x^2 + 1)^2}$ **13** $12e^{4x}(e^{4x} - 5)^2$

15 $-\frac{e^{1/x}}{x^2} - e^{-x}$ **17** $\frac{4}{(e^x + e^{-x})^2}$ **19** $e^{-2x}\left(\frac{1}{x} - 2\ln x\right)$

21 $5e^{5x}\cos e^{5x}$ **23** $e^{-x}\tan e^{-x}$

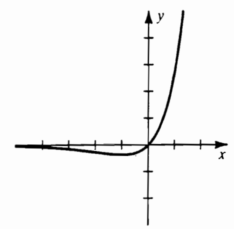
25 $e^{3x}\left(\frac{\sec^2\sqrt{x}}{2\sqrt{x}} + 3\tan\sqrt{x}\right)$

27 $-8e^{-4x}\sec^2(e^{-4x})\tan(e^{-4x})$ **29** $e^{\cot x}(1 - x\csc^2 x)$

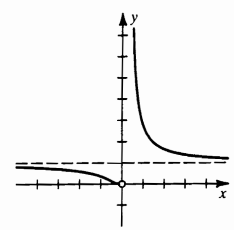
31 $\frac{3x^2 - ye^{xy}}{xe^{xy} + 6y}$ **33** $\frac{e^x\cot y - e^{2y}}{2xe^{2y} + e^x\csc^2 y}$

35 $y = (e + 3)x - (e + 1)$

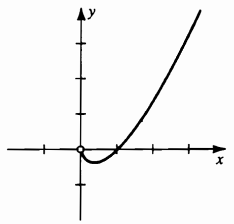
37 Min: $f(-1) = -e^{-1} \approx -0.368$; increasing on $[-1, \infty)$; decreasing on $(-\infty, -1]$; CU on $(-2, \infty)$; CD on $(-\infty, -2)$; PI: $(-2, -2e^{-2}) \approx (-2, -0.271)$



39 Decreasing on $(-\infty, 0)$ and $(0, \infty)$; CU on $\left(-\frac{1}{2}, 0\right)$ and $(0, \infty)$; CD on $\left(-\infty, -\frac{1}{2}\right)$; PI: $\left(-\frac{1}{2}, e^{-2}\right)$



41 Min: $f(e^{-1}) = -e^{-1}$; increasing on $[e^{-1}, \infty)$; decreasing on $(0, e^{-1}]$; CU on $(0, \infty)$; no PI

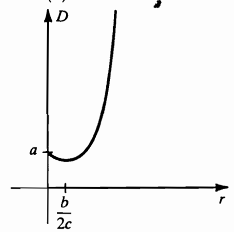


43 $q'(t) = -cq(t)$ **45 (a)** $\dfrac{\ln(a/b)}{a-b}$ **(b)** $\lim_{t\to\infty} C(t) = 0$

47 (a) 75.8 cm; 15.98 cm/yr **(b)** 3 mo; 6 yr

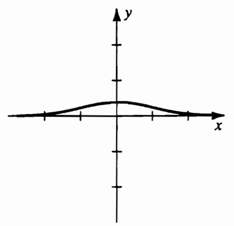
49 (a) $f\left(\dfrac{\pi}{a}\right)$ **(b)** At $x = \dfrac{2}{a}$

51



55 Max: $f(\mu) = \dfrac{1}{\sigma\sqrt{2\pi}}$; increasing on $(-\infty, \mu]$; decreasing on $[\mu, \infty)$; CU on $(-\infty, \mu - \sigma)$ and $(\mu + \sigma, \infty)$; CD on $(\mu - \sigma, \mu + \sigma)$;

PI: $\left(\mu \pm \sigma, \dfrac{1}{\sigma\sqrt{2\pi e}}\right)$; both limits equal 0



57 (a) $[0.054, 1]$ **(b)** y-int.: 1

59 (a) $[-3.18, 6.13]$

(b) x-int.: ± 0.84, 2.52, 4.20, 5.88, 7.56; y-int.: 6; max: $f(-0.11) \approx 6.13$, $f(3.25) \approx 1.65$, $f(6.61) \approx 0.45$; min: $f(1.57) \approx -3.18$, $f(4.93) \approx -0.86$

61 0.5671 **63** 1.2022 **65** $e^{-1/2} \approx 0.607$

Exercises 6.4

1 (a) $\frac{1}{2}\ln|2x+7| + C$ **(b)** $\ln\sqrt{3}$

3 (a) $2\ln|x^2 - 9| + C$ **(b)** $\ln\frac{25}{64}$

5 (a) $-\frac{1}{4}e^{-4x} + C$ **(b)** $-\frac{1}{4}(e^{-12} - e^{-4})$

7 (a) $-\frac{1}{2}\ln|\cos 2x| + C$ **(b)** $\frac{1}{4}\ln 2$

9 (a) $2\ln|\csc\frac{1}{2}x - \cot\frac{1}{2}x| + C$ **(b)** $2\ln(2+\sqrt{3})$

11 $\frac{1}{2}\ln|x^2 - 4x + 9| + C$

13 $\frac{1}{2}x^2 + 4x + 4\ln|x| + C$

15 $\frac{1}{2}(\ln x)^2 + C$ **17** $\frac{1}{2}x^2 + \frac{1}{5}e^{5x} + C$

19 $-\frac{3}{2}\ln|1 + 2\cos x| + C$ **21** $e^x + 2x - e^{-x} + C$

23 $\ln(e^x + e^{-x}) + C$ **25** $3\ln|\sin\sqrt[3]{x}| + C$

27 $\frac{1}{2}\ln|\sec 2x + \tan 2x| + C$ **29** $-\frac{1}{3}\ln|\sec e^{-3x}| + C$

31 $\ln|\csc x - \cot x| + \cos x + C$ **33** $\ln|\csc x| + C$

35 $x + 2\ln|\sec x + \tan x| + \tan x + C$ **37** 4

39 $\pi(1 - e^{-1})$ **41** $y = 2e^{2x} - \frac{3}{2}e^{-2x} + \frac{7}{2}$

43 $y = 3e^{-x} + 4x - 4$ **45** $\dfrac{2}{\ln(13/4)} \approx 1.697$

47 (a) 25 **(b)** 205 **(c)** 12 **49** $\Delta S = c\ln\dfrac{T_2}{T_1}$

51 (a) $\frac{5}{2}(1 - e^{-4t})$ **(b)** $\lim_{t\to\infty} Q(t) = \frac{5}{2}$ coulombs

53 (a) $s(t) = kv_0(1 - e^{-t/k})$ **(b)** $\lim_{t\to\infty} s(t) = kv_0$

55 0.7468 **57** 127.2930 **59** 6.43 **61** 9.34

Exercises 6.5

1 $7^x\ln 7$ **3** $8^{x^2+1}(2x\ln 8)$ **5** $\dfrac{4x^3 + 6x}{(x^4 + 3x^2 + 1)\ln 10}$

7 $5^{3x-4}(3\ln 5)$ **9** $\dfrac{-(x^2+1)10^{1/x}(\ln 10)}{x^2} + (2x)10^{1/x}$

11 $\dfrac{30x}{(3x^2+2)\ln 10}$ **13** $\left(\dfrac{6}{6x+4} - \dfrac{2}{2x-3}\right)\dfrac{1}{\ln 5}$

15 $\dfrac{1}{x\ln x\ln 10}$ **17** $ex^{e-1} + e^x$

19 $(x+1)^x\left[\dfrac{x}{x+1} + \ln(x+1)\right]$ **21** $2^{\sin^2 x}(\sin 2x)\ln 2$

23 (a) 0 **(b)** $5x^4$ **(c)** $\sqrt{5}x^{\sqrt{5}-1}$ **(d)** $(\sqrt{5})^x\ln\sqrt{5}$

(e) $x^{1+x^2}(1 + 2\ln x)$

29 (a) $\dfrac{7^x}{\ln 7} + C$ **(b)** $\dfrac{342}{49\ln 7} \approx 3.59$

31 (a) $\dfrac{-5^{-2x}}{2\ln 5} + C$ **(b)** $\dfrac{12}{625\ln 5} \approx 0.012$ **33** $\dfrac{10^{3x}}{3\ln 10} + C$

35 $\dfrac{-3^{-x^2}}{2\ln 3} + C$ **37** $\dfrac{\ln(2^x+1)}{\ln 2} + C$

39 $(\ln 10)\ln|\log x| + C$ **41** $-\dfrac{3^{\cos x}}{\ln 3} + C$

43 (a) $\pi^\pi x + C$ **(b)** $\frac{1}{5}x^5 + C$ **(c)** $\dfrac{x^{\pi+1}}{\pi+1} + C$

(d) $\dfrac{\pi^x}{\ln\pi} + C$ **45** $\dfrac{1}{\ln 2} - \dfrac{1}{2} \approx 0.94$

47 (a) \$0.05/yr **(b)** \$0.95

49 (a) In trout/yr: 95; 62; 53 **(b)** 9.36

51 pH $\approx$ 2.201; $\pm$0.1%

53 (b) $S = \dfrac{k}{x}$, where $k = \dfrac{a}{\ln 10}$;

$S(x) = 2\,S(2x)$ (twice as sensitive)

55 (a) With $n = r/h$,

$\ln A = \ln[P(1+h)^{rt/h}] = \ln P + rt\ln(1+h)^{1/h}$.

(b) Since $h = r/n$, $n \to \infty$ if and only if $h \to 0^+$. Thus,

$$\ln A = \lim_{h\to 0^+}[\ln P + rt\ln(1+h)^{1/h}] = \ln P + rt\ln e = \ln(Pe^{rt})$$

and $A = Pe^{rt}$.

57 Let $h = x/n$. Then

$$\lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n = \lim_{h\to 0^+}(1+h)^{x/h} = \lim_{h\to 0^+}[(1+h)^{1/h}]^x = e^x.$$

Exercises 6.6

1 $q(t) = 5000(3)^{t/10}$; 45,000; $\dfrac{10\ln 10}{\ln 3} \approx 20.96$ hr

3 $30\left(\dfrac{29}{30}\right)^5 \approx 25.32$ in.

5 $\dfrac{\ln(40/5.5)}{0.02} \approx 99.21$ yr after Jan. 1, 1993 (March 17, 2092)

7 $\dfrac{5\ln(1/6)}{\ln(1/3)} \approx 8.15$ min

9 Proceed as in the solution to Example 1.

11 $P(z) = \left(\dfrac{288 - 0.01z}{288}\right)^{3.42}$ **13** $\dfrac{29\ln(2/5)}{\ln(1/2)} \approx 38.34$ yr

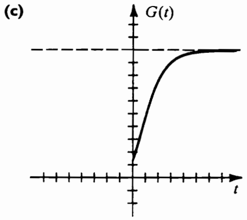
15 $600\left(\dfrac{1}{2}\right)^{-3/16} \approx 683.27$ mg

17 $v(y) = \sqrt{2k\left(\dfrac{1}{y} - \dfrac{1}{y_0}\right) + v_0^2}$

19 $\dfrac{5700\ln(0.2)}{\ln(1/2)} \approx 13{,}235$ yr **21** Use Theorem (4.35).

23 $V(t) = \dfrac{1}{27}(kt + C)^3$

25 (c)



Exercises 6.7

1 (a) $-\frac{\pi}{4}$ **(b)** $\frac{2\pi}{3}$ **(c)** $-\frac{\pi}{3}$

3 (a) Not defined **(b)** Not defined **(c)** $\frac{\pi}{4}$

5 (a) $\frac{\pi}{3}$ **(b)** $\frac{5\pi}{6}$ **(c)** $-\frac{\pi}{6}$ **7 (a)** $\frac{\pi}{3}$ **(b)** $\frac{2\pi}{3}$ **(c)** $\frac{\pi}{6}$

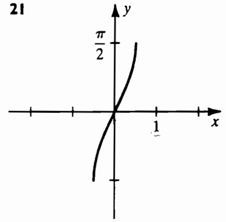
9 (a) $\frac{\sqrt{3}}{2}$ **(b)** 0 **(c)** Not defined

11 (a) $-\frac{\sqrt{21}}{2}$ **(b)** $\frac{\sqrt{65}}{4}$ **(c)** $\frac{5}{\sqrt{24}}$

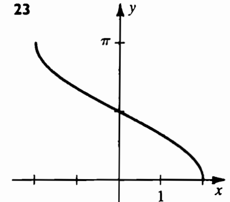
13 (a) -1.1971 **(b)** 0.2712 **15 (a)** 1.0556 **(b)** 0.6183

17 $\dfrac{x}{\sqrt{x^2+1}}$ **19** $\dfrac{3}{\sqrt{9-x^2}}$

21

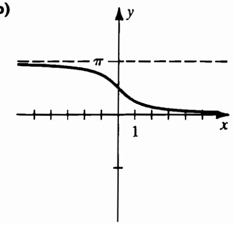


23



27 (a) $y = \cot^{-1}x$ if and only if $x = \cot y$ for $0 < y < \pi$.

(b)



29 (a) $\alpha = \theta - \sin^{-1}\dfrac{d}{k}$ **(b)** 40° **31** $\dfrac{1}{2\sqrt{x}\sqrt{1-x}}$

33 $\dfrac{3}{9x^2 - 30x + 26}$ **35** $\dfrac{-e^{-x}}{\sqrt{e^{-2x} - 1}} - e^{-x}\,\text{arcsec}\, e^{-x}$

37 $\dfrac{2x}{(1 + x^4)\arctan(x^2)}$ **39** $-\dfrac{9(1 + \cos^{-1} 3x)^2}{\sqrt{1 - 9x^2}}$

41 $\left(\dfrac{1}{x^2}\right)\sin\left(\dfrac{1}{x}\right) + \sec x \tan x - \dfrac{1}{\sqrt{1 - x^2}}$

43 $3^{\arcsin(x^3)}\dfrac{(3\ln 3)x^2}{\sqrt{1 - x^6}}$

45 $\dfrac{1 - 2x\arctan x}{(x^2 + 1)^2}$ **47** $\dfrac{1}{2\sqrt{x}}\left(\dfrac{1}{\sqrt{x - 1}} + \sec^{-1}\sqrt{x}\right)$

49 $\dfrac{ye^x - 2x - \sin^{-1} y}{\dfrac{x}{\sqrt{1 - y^2}} - e^x}$

51 (a) $\dfrac{1}{4}\tan^{-1}\left(\dfrac{x}{4}\right) + C$ **(b)** $\dfrac{\pi}{16}$

53 (a) $\dfrac{1}{2}\sin^{-1}(x^2) + C$ **(b)** $\dfrac{\pi}{12}$

55 $2\tan^{-1}\sqrt{x} + C$ **57** $\sin^{-1}\left(\dfrac{e^x}{4}\right) + C$

59 $\dfrac{1}{2}\ln(x^2 + 9) + C$ **61** $\dfrac{1}{5}\sec^{-1}\left(\dfrac{e^x}{5}\right) + C$

63 $\pm\dfrac{7}{3576}$ rad **65** $-\dfrac{25}{1044}$ rad/sec **67** $\sqrt{4800} \approx 69.3$ ft

69 $\dfrac{2\pi}{27} \approx 0.233$ mi/sec **75** $x\sin^{-1}(2x) + \frac{1}{2}\sqrt{1 - 4x^2} + C$

77 $\dfrac{1}{2}x^2\tan^{-1}(x^2) - \dfrac{1}{4}\ln(x^4 + 1) + C$

79 0.7241 **81** 2.0570 **83** 31.9285

Exercises 6.8

1 (a) 27.2899 **(b)** 2.1250 **(c)** −0.9951 **(d)** 1.0000 **(e)** 0.2658 **(f)** −0.8509

3 $5\cosh 5x$ **5** $3x^2\sinh(x^3)$

7 $\dfrac{1}{2\sqrt{x}}(\sqrt{x}\,\text{sech}^2\sqrt{x} + \tanh\sqrt{x})$

9 $\left(\dfrac{1}{x^2}\right)\text{csch}^2\left(\dfrac{1}{x}\right)$

11 $\dfrac{-2x\,\text{sech}(x^2)[(x^2 + 1)\tanh(x^2) + 1]}{(x^2 + 1)^2}$

13 $-12\,\text{csch}^2 6x \coth 6x$

15 (a) $\mathbb{R}$ **(b)** $\dfrac{4x\sinh\sqrt{4x^2 + 3}}{\sqrt{4x^2 + 3}}$

17 (a) $\mathbb{R}$ **(b)** $-\dfrac{\text{sech}^2 x}{(\tanh x + 1)^2}$

19 $\dfrac{1}{3}\sinh(x^3) + C$ **21** $2\cosh\sqrt{x} + C$

23 $\dfrac{1}{3}\tanh 3x + C$ **25** $-2\coth\left(\dfrac{1}{2}x\right) + C$

27 $-\dfrac{1}{3}\,\text{sech}\, 3x + C$ **29** $-\text{csch}\, x + C$

31 $(\ln(2 \pm \sqrt{3}), \pm\sqrt{3})$

33 Show that $A = \dfrac{1}{2}(\cosh t)(\sinh t) - \displaystyle\int_1^{\cosh t}\sqrt{x^2 - 1}\,dx$ and that $\dfrac{dA}{dt} = \dfrac{1}{2}$.

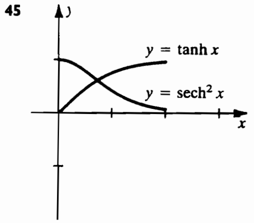
35 (a) 286,574 ft² **(b)** 1494 ft **37** 34.94 ft

39 $10.5\sinh^{-1}\frac{4}{3} \approx 11.54$ ft

41 (b) $y = \dfrac{1}{\alpha}\ln[\cosh(\sqrt{g\alpha}\,t + v_0)] + h_0$

43 (a) $\lim\limits_{h\to\infty} v^2 = \dfrac{gL}{2\pi}$ **(b)** *Hint*: Let $f(h) = v^2$.

45



(a) 0.7 **(b)** 0.722

47 $\cosh x + \sinh x = \dfrac{e^x + e^{-x}}{2} + \dfrac{e^x - e^{-x}}{2} = e^x$

49 $\sinh x \cosh y + \cosh x \sinh y$

$$= \frac{(e^x - e^{-x})(e^y + e^{-y})}{4} + \frac{(e^x + e^{-x})(e^y - e^{-y})}{4}$$

$$= \frac{(e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y}) + (e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y})}{4}$$

$$= \frac{2e^{x+y} - 2e^{-x-y}}{4} = \frac{e^{x+y} - e^{-(x+y)}}{2} = \sinh(x + y)$$

51 $\sinh(x - y)$
$= \sinh(x + (-y))$
$= \sinh x \cosh(-y) + \cosh x \sinh(-y)$ (Exer. 49)
$= \sinh x \cosh y - \cosh x \sinh y$ (Exer. 48)

53 Let $y = x$ in Exercise 49.

55 From Exercise 54,

$$\cosh 2y = \cosh^2 y + \sinh^2 y = (1 + \sinh^2 y) + \sinh^2 y = 1 + 2\sinh^2 y,$$

and hence

$$\sinh^2 y = \frac{\cosh 2y - 1}{2}.$$

Let $y = \dfrac{x}{2}$ to obtain the identity.

57 $\cosh nx + \sinh nx = \dfrac{e^{nx} + e^{-nx}}{2} + \dfrac{e^{nx} - e^{-nx}}{2}$
$= e^{nx} = (e^x)^n = (\cosh x + \sinh x)^n$

59 (a) 0.8814 **(b)** 1.3170 **(c)** −0.5493 **(d)** 1.3170

61 $\dfrac{5}{\sqrt{25x^2 + 1}}$ **63** $\dfrac{1}{2\sqrt{x}\sqrt{x - 1}}$ **65** $\dfrac{4}{16x^2 - 1}$

67 $-\dfrac{2}{x\sqrt{1 - x^4}}$

69 (a) $(\frac{1}{4}, \infty)$ **(b)** $\dfrac{4}{\sqrt{16x^2 - 1}\cosh^{-1}(4x)}$

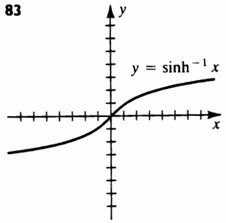
71 (a) $(-2, 0)$ **(b)** $-\dfrac{1}{x(x + 2)}$

73 $\dfrac{1}{4}\sinh^{-1}\left(\dfrac{4}{9}x\right) + C$ **75** $\dfrac{1}{14}\tanh^{-1}\left(\dfrac{2}{7}x\right) + C$

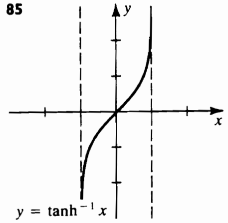
77 $\cosh^{-1}\left(\dfrac{e^x}{4}\right) + C$ **79** $-\dfrac{1}{6}\,\text{sech}^{-1}\left(\dfrac{x^2}{3}\right) + C$

81 $y = \sinh 3t$

83



85



Exer. 87–91: (a) Use a procedure similar to that given in the text for $\sinh^{-1} x$. **(b)** Let $u = x$ in Theorem (6.48) and differentiate $\cosh^{-1} u$. **(c)** Differentiate the right-hand side.

Exercises 6.9

1 $\frac{1}{2}$ **3** $\frac{1}{40}$ **5** $\frac{3}{13}$ **7** $-\frac{1}{2}$ **9** $\frac{1}{6}$ **11** ∞ **13** $\frac{1}{3}$ **15** 1

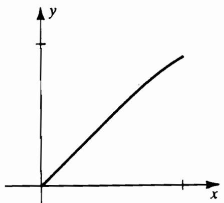
17 ∞ **19** $\frac{2}{5}$ **21** 0 **23** ∞ **25** 2 **27** $\frac{3}{5}$ **29** −3

31 0 **33** ∞ **35** 1

37 0.9129, 0.9901, 0.9990, 0.9999; predict limit of 1

39 gt **41** $\frac{1}{2}A\omega_0 t \sin\omega_0 t$ **43 (a)** 1 **(b)** $-\frac{1}{18}$

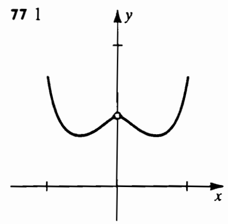
45 (a) 0.2499, 0.4969, 0.7266, 0.9045
(b)



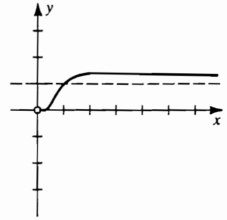
49 0 **51** 0 **53** 0 **55** 1 **57** e^5 **59** 1 **61** ∞

63 e^2 **65** 0 **67** $-\infty$ **69** $\frac{1}{2}$ **71** e **73** $e^{1/3}$ **75** ∞

77 1



79 (a) Max: $f(e) = e^{1/e} \approx 1.44$; $\lim\limits_{x\to 0^+} x^{1/x} = 0$
(b) $y = 1$



83 gt

Chapter 6 Review Exercises

1 $\dfrac{10 - x}{15}$

3 f is decreasing, since $f'(x) < 0$ for $-1 \le x \le 1$; $-\dfrac{1}{8}$

5 $\dfrac{75x^2}{5x^3 - 4}$ **7** $\dfrac{12}{3x + 2} + \dfrac{3}{6x - 5} - \dfrac{8}{8x - 7}$

9 $\dfrac{-4x}{(2x^2 + 3)[\ln(2x^2 + 3)]^2}$ **11** $2x$

13 $\dfrac{10^x}{x\ln 10} + 10^x(\ln 10)\log x$ **15** $\dfrac{1}{4x\sqrt{\ln\sqrt{x}}}$

17 $2xe^{-x^2}(1 - x^2)$ **19** $\dfrac{10^{\ln x}\ln 10}{x}$ **21** $\dfrac{2\ln x\,(x^{\ln x})}{x}$

23 $2e^{-2x}\csc e^{-2x}(\csc^2 e^{-2x} + \cot^2 e^{-2x})$

25 $-16\tan 4x$ **27** $-\dfrac{y}{x}$

29 $\left[\dfrac{4}{3(x + 2)} + \dfrac{3}{2(x - 3)}\right](x + 2)^{4/3}(x - 3)^{3/2}$

31 (a) $-2e^{-\sqrt{x}} + C$ **(b)** $2(e^{-1} - e^{-2}) \approx 0.465$

33 $-\dfrac{1}{2}\ln|\cos x^2| + C$ **35** $\dfrac{x^{e+1}}{e + 1} + C$

37 $-\dfrac{1}{2}e^{-2x} - 2e^{-x} + x + C$

39 $\dfrac{1}{2}x^2 - 2x + 4\ln|x + 2| + C$ **41** $-\dfrac{1}{8}e^{4/x^2} + C$

43 $\ln(1 + e^x) + C$ **45** $\dfrac{(5e)^x}{\ln(5e)} + C$

47 $\cos e^{-x} + C$ **49** $-\ln|1 + \cot x| + C$

51 $-\ln|\cos e^x| + C$

53 $-\dfrac{1}{3}\cot 3x + \dfrac{2}{3}\ln|\csc 3x - \cot 3x| + x + C$

55 $y = -\dfrac{1}{9}e^{-3x} + \dfrac{5}{3}x - \dfrac{8}{9}$ **57** $4e^2 + 12 \approx 41.56$ cm

59 $y - e = -2(1 + e)(x - 1)$

61 $\dfrac{\pi}{8}(e^{-16} - e^{-24}) \approx 4.42 \times 10^{-8}$

63 $\frac{5\ln(1/100)}{\ln(1/2)} \approx 33.2$ days

65 (a) $\frac{3\ln(3/10)}{\ln(1/2)} \approx 5.2$ hr or 2.2 additional hr

(b) $10\left[1 - \left(\frac{1}{2}\right)^{7/3}\right] \approx 8.016$ lb

67 $100{,}000(2)^6 = 6{,}400{,}000$

69 $\frac{1}{2x\sqrt{x-1}}$ **71** $\frac{2x}{\sqrt{x^4-1}} + 2x \operatorname{arcsec}(x^2)$

73 $\frac{2x}{(1+x^4)\tan^{-1}(x^2)}$ **75** $\frac{-x}{\sqrt{x^2(1-x^2)}}$

77 $\frac{1}{(1+x^2)[1+(\tan^{-1}x)^2]}$ **79** $-5e^{-5x}\sinh e^{-5x}$

81 $(\cosh x - \sinh x)^{-2}$, or e^{2x} **83** $\frac{2x}{\sqrt{x^4+1}}$

85 $\frac{1}{6}\tan^{-1}\left(\frac{3}{2}x\right) + C$ **87** $-\sqrt{1-e^{2x}} + C$

89 $\frac{1}{2}\sinh(x^2) + C$ **91** $\frac{\pi}{3}$

93 $\frac{1}{2}\sin^{-1}\left(\frac{2}{3}x\right) + C$ **95** $-\frac{1}{3}\operatorname{sech}^{-1}\left(\frac{2}{3}|x|\right) + C$

97 $\frac{1}{25}\sqrt{25x^2+36} + C$ **99** $\left(\pm\frac{4}{15}, \sin^{-1}\left(\pm\frac{4}{5}\right)\right)$

101 Let $c = \tan^{-1}\frac{1}{2}$. Min: $f(c) = 5\sqrt{5}$; increasing on $\left[c, \frac{\pi}{2}\right)$; decreasing on $(0, c]$.

103 (a) $\frac{1}{2}\tan^{-1} 4 + \frac{\pi}{2}n$ for $n = 0, 1, 2, 3$

(b) 0.66, 2.23, 3.80, 5.38

105 $\frac{1}{260}$ rad/sec $\approx 0.22°$/sec

107 $-\frac{800}{2581} \approx -0.31$ rad/sec **109** $\frac{1}{2}\ln 2$ **111** ∞

113 0 **115** $-\infty$ **117** e **119** 1 **121** 0

CHAPTER ▪ 7

Exercises 7.1

1 $-(x+1)e^{-x} + C$ **3** $\frac{1}{27}e^{3x}(9x^2 - 6x + 2) + C$

5 $\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$

7 $x\sec x - \ln|\sec x + \tan x| + C$

9 $x^2\sin x + 2x\cos x - 2\sin x + C$

11 $x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$

13 $\frac{2}{9}x^{3/2}(3\ln x - 2) + C$ **15** $-x\cot x + \ln|\sin x| + C$

17 $-\frac{1}{2}e^{-x}(\sin x + \cos x) + C$

19 $\cos x(1 - \ln\cos x) + C$

21 $-\frac{1}{2}\csc x\cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$

23 $\frac{1}{3}(2 - \sqrt{2}) \approx 0.20$ **25** $\frac{\pi}{4}$

27 $\frac{1}{40{,}400}(2x+3)^{100}(200x - 3) + C$

29 $\frac{1}{41}e^{4x}(4\sin 5x - 5\cos 5x) + C$

31 $x(\ln x)^2 - 2x\ln x + 2x + C$

33 $x^3\cosh x - 3x^2\sinh x + 6x\cosh x - 6\sinh x + C$

35 $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

37 $x\cos^{-1} x - \sqrt{1-x^2} + C$ **39** Let $u = x^m$.

41 Let $u = (\ln x)^m$.

43 $e^x(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120) + C$

45 2π **47** $\frac{\pi}{2}(e^2 + 1) \approx 13.18$ **49** $\left(\frac{3\ln 3 - 2}{2}, 1\right)$

Exercises 7.2

1 $\sin x - \frac{1}{3}\sin^3 x + C$ **3** $\frac{1}{8}x - \frac{1}{32}\sin 4x + C$

5 $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

7 $\frac{1}{8}\left(\frac{5}{2}x - 2\sin 2x + \frac{3}{8}\sin 4x + \frac{1}{6}\sin^3 2x\right) + C$

9 $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$ **11** $\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

13 $\frac{1}{5}\tan^5 x - \frac{1}{3}\tan^3 x + \tan x - x + C$

15 $\frac{2}{3}\sin^{3/2} x - \frac{2}{7}\sin^{7/2} x + C$ **17** $\tan x - \cot x + C$

19 $\frac{2}{3} - \frac{5}{6\sqrt{2}} \approx 0.08$ **21** $\frac{1}{2}\left(\frac{1}{2}\sin 2x - \frac{1}{8}\sin 8x\right) + C$

23 $\frac{3}{5}$ **25** $-\frac{1}{5}\cot^5 x - \frac{1}{7}\cot^7 x + C$

27 $-\ln(2 - \sin x) + C$ **29** $-\frac{1}{1+\tan x} + C$

31 $\frac{3}{4}\pi^2 \approx 7.40$ **33** $\frac{5}{2}$

35 (a) Use the trigonometric product-to-sum formulas.

(b) $\int \sin mx\cos nx\,dx = \begin{cases} -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C & \text{if } m \ne n \\ -\frac{\cos 2mx}{4m} + C & \text{if } m = n \end{cases}$

$\int \cos mx\cos nx\,dx = \begin{cases} \frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + C & \text{if } m \ne n \\ \frac{x}{2} + \frac{\sin 2mx}{4m} + C & \text{if } m = n \end{cases}$

Exercises 7.3

1 $\frac{1}{2}\ln\left|\frac{2}{x} - \frac{\sqrt{4-x^2}}{x}\right| + C$ **3** $\frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}}{x} - \frac{3}{x}\right| + C$

5 $\frac{\sqrt{x^2-25}}{25x} + C$ **7** $-\sqrt{4-x^2} + C$

9 $-\frac{x}{\sqrt{x^2-1}} + C$ **11** $\frac{1}{432}\left[\tan^{-1}\left(\frac{x}{6}\right) + \frac{6x}{x^2+36}\right] + C$

13 $\sin^{-1}\left(\frac{x}{3}\right) + C$ **15** $\frac{1}{2(16-x^2)} + C$

17 $\frac{1}{243}(9x^2+49)^{3/2} - \frac{49}{81}\sqrt{9x^2+49} + C$

19 $\frac{(3+2x^2)\sqrt{x^2-3}}{27x^3} + C$ **21** $-\frac{8}{x^2} + 8\ln|x| + \frac{1}{2}x^2 + C$

23 $25\pi[\sqrt{2} - \ln(\sqrt{2}+1)] \approx 41.85$ **25** 509×10^6 km^2

27 $y = \sqrt{x^2-16} - 4\sec^{-1}\frac{x}{4}$

29 Let $u = a\tan\theta$. **31** Let $u = a\sin\theta$.

33 Let $u = a\sec\theta$.

Exercises 7.4

Answers are expressed as sums that correspond to partial fraction decompositions. Logarithms can be combined. Thus, an equivalent answer for Exercise 1 is $\ln|x|^3(x-4)^2 + C$.

1 $3\ln|x| + 2\ln|x-4| + C$

3 $4\ln|x+1| - 5\ln|x-2| + \ln|x-3| + C$

5 $6\ln|x-1| + \frac{5}{x-1} + C$

7 $3\ln|x-2| - 2\ln|x+4| + C$

9 $2\ln|x| - \ln|x-2| + 4\ln|x+2| + C$

11 $5\ln|x+1| - \frac{1}{x+1} - 3\ln|x-5| + C$

13 $5\ln|x| - \frac{2}{x} + \frac{3}{2x^2} - \frac{1}{3x^3} + 4\ln|x+3| + C$

15 $x + 4\ln|x| + \ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

17 $\ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + 3\ln|x+5| + C$

19 $-\frac{1}{2}\ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{2}\ln(x^2+1) + C$

21 $\ln(x^2+1) - \frac{4}{x^2+1} + C$

23 $\frac{1}{2}x^2 + x + 2\ln|x| + 2\ln|x-1| + C$

25 $\frac{1}{3}x^3 - 9x - \frac{1}{9x} - \frac{1}{2}\ln(x^2+9) + \frac{728}{27}\tan^{-1}\left(\frac{x}{3}\right) + C$

27 $2\ln|x+4| + \frac{6}{x+4} - \frac{5}{x-3} + C$

29 $\frac{13}{6}\ln(6x+5) + \frac{8}{3}\ln(3x-2) - \ln(2x+7) + 4\ln(x-1) + C$

31 $-\frac{34}{5}\ln(5x+2) - \frac{17}{3}\ln(3x+25) + \frac{3}{2}\ln(2x-5) + C$

33 $\frac{1}{2a}(\ln|a+u| - \ln|a-u|) + C = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$

35 $-\frac{b}{a^2}\ln|u| - \frac{1}{au} + \frac{b}{a^2}\ln|a+bu| + C = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

37 $\frac{1}{2}\ln 3 \approx 0.55$ **39** $\frac{\pi}{27}(4\ln 2 + 3) \approx 0.67$

41 $\frac{\frac{7}{4}}{x} + \frac{-\frac{5}{3}}{x-1} + \frac{-\frac{1}{3}}{x+1} + \frac{\frac{29}{24}}{x-2} + \frac{\frac{25}{24}}{x+2}$

Exercises 7.5

1 $\frac{1}{2}\tan^{-1}\frac{x+1}{2} + C$ **3** $\frac{1}{2}\tan^{-1}\frac{x-2}{2} + C$

5 $\sin^{-1}\frac{x-2}{2} + C$

7 $-2\sqrt{9-8x-x^2} - 5\sin^{-1}\frac{x+4}{5} + C$

9 $\frac{1}{2}\left[\tan^{-1}(x+2) + \frac{x+2}{x^2+4x+5}\right] + C$

11 $\frac{x+3}{4\sqrt{x^2+6x+13}} + C$ **13** $\frac{2}{3\sqrt{7}}\tan^{-1}\frac{4x-3}{3\sqrt{7}} + C$

15 $\ln\left(\frac{e^x+1}{e^x+2}\right) + C$ **17** $1 + \frac{\pi}{4} \approx 1.79$ **19** $\frac{\pi}{20} \approx 0.16$

21 $\frac{3}{7}(x+9)^{7/3} - \frac{27}{4}(x+9)^{4/3} + C$

23 $\frac{5}{81}(3x+2)^{9/5} - \frac{5}{18}(3x+2)^{4/5} + C$

25 $2 + 8\ln\frac{6}{7} \approx 0.767$

27 $\frac{6}{7}x^{7/6} - \frac{6}{5}x^{5/6} + 2x^{1/2} - 6x^{1/6} + 6\tan^{-1}(x^{1/6}) + C$

29 $\frac{2}{\sqrt{3}}\tan^{-1}\sqrt{\frac{x-2}{3}} + C$

31 $\frac{3}{5}(x+4)^{5/3} - \frac{9}{2}(x+4)^{2/3} + C$

33 $\frac{2}{7}(1+e^x)^{7/2} - \frac{4}{5}(1+e^x)^{5/2} + \frac{2}{3}(1+e^x)^{3/2} + C$

35 $e^x - 4\ln(e^x+4) + C$

37 $2\sin\sqrt{x+4} - 2\sqrt{x+4}\cos\sqrt{x+4} + C$ **39** $\frac{137}{320}$

41 $\ln|\cos x| - \ln(1 - \cos x) + C$

43 $\frac{1}{2}\ln|e^x - 1| - \frac{1}{2}\ln(e^x+1) + C$

45 $\frac{4}{3}\ln(4 - \sin x) + \frac{2}{3}\ln(\sin x + 2) + C$

47 $\frac{2}{\sqrt{3}}\tan^{-1}\frac{2\tan(x/2)+1}{\sqrt{3}} + C$ **49** $\ln\left|\tan\frac{x}{2} + 1\right| + C$

51 $-\frac{1}{5}\ln\left|2\tan\frac{x}{2} - 1\right| + \frac{1}{5}\ln\left|\tan\frac{x}{2} + 2\right| + C$

Exercises 7.6

1 $\sqrt{4+9x^2} - 2\ln\left|\dfrac{2+\sqrt{4+9x^2}}{3x}\right| + C$

3 $-\frac{x}{8}(2x^2-80)\sqrt{16-x^2} + 96\sin^{-1}\frac{x}{4} + C$

5 $-\frac{2}{135}(9x+4)(2-3x)^{3/2} + C$

7 $-\frac{1}{18}\sin^5 3x\cos 3x - \frac{5}{72}\sin^3 3x\cos 3x - \frac{5}{48}\sin 3x\cos 3x + \frac{5}{16}x + C$

9 $-\frac{1}{3}\cot x\csc^2 x - \frac{2}{3}\cot x + C$

11 $\frac{2x^2-1}{4}\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + C$

13 $\frac{1}{13}e^{-3x}(-3\sin 2x - 2\cos 2x) + C$

15 $\sqrt{5x-9x^2} + \frac{5}{6}\cos^{-1}\frac{5-18x}{5} + C$

17 $\frac{1}{4\sqrt{15}}\ln\left|\dfrac{\sqrt{5x^2}-\sqrt{3}}{\sqrt{5x^2}+\sqrt{3}}\right| + C$

19 $\frac{1}{4}(2e^{2x}-1)\cos^{-1}e^x - \frac{1}{4}e^x\sqrt{1-e^{2x}} + C$

21 $\frac{2}{315}(35x^3-60x^2+96x-128)(2+x)^{3/2} + C$

23 $\frac{2}{81}(4+9\sin x - 4\ln|4+9\sin x|) + C$

25 $2\sqrt{9+2x} + 3\ln\left|\dfrac{\sqrt{9+2x}-3}{\sqrt{9+2x}+3}\right| + C$

27 $\frac{3}{4}\ln\left|\dfrac{\sqrt[3]{x}}{4+\sqrt[3]{x}}\right| + C$

29 $\sqrt{16-\sec^2 x} - 4\ln\left|\dfrac{4+\sqrt{16-\sec^2 x}}{\sec x}\right| + C$

31 $\frac{1}{2}\ln(\cos x + \sin x + 1) - \frac{1}{2}\ln(5\cos x + \sin x + 5) + C$

33 $e^{4x}\left[\frac{1}{5000}(1000x^3 - 450x^2 + 60x + 21)\sin 2x - \frac{1}{2500}(250x^3 - 300x^2 + 165x - 36)\cos 2x\right] + C$

35 $2\sqrt{x} - \ln(x+\sqrt{x}+3) - \frac{10}{11}\sqrt{11}\tan^{-1}\left(\frac{\sqrt{11}(2\sqrt{x}+1)}{11}\right) + C$

37 $\ln\left(\frac{1-\cos x}{\sin x}\right) + C$ **39** $2\sqrt{x} - 2\ln(\sqrt{x}+1) + C$

Exercises 7.7

C denotes that the integral converges; D denotes that it diverges.

1 C; 3 **3** D **5** D **7** C; $\frac{1}{2}$ **9** C; $-\frac{1}{2}$
11 D **13** D **15** C; 0 **17** D **19** D **21** C **23** D

25 (a) Not possible **(b)** π **27** π

29 (b) No **31** If $F(x) = \frac{k}{x^2}$, then $W = k$.

33 (a) $\frac{1}{k}$ **(b)** No, the improper integral diverges.

35 (b) $c = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2}$ **37** $\frac{1}{s}$, $s>0$ **39** $\frac{s}{s^2+1}$, $s>0$

41 $\frac{1}{s-a}$, $s>a$

43 (a) 1; 1; 2 **(b)** *Hint:* Let $u = x^n$ and integrate by parts.
45 0.49
47 C; 6 **49** D **51** D **53** D **55** C; $3\sqrt[3]{4}$ **57** D

59 C; $\frac{\pi}{2}$ **61** D **63** C; $-\frac{1}{4}$ **65** D **67** D

69 D **71** C **73** D
75 $n > -1$ **77 (a)** 2 **(b)** Not possible **79** 1.79

81 (b) $T = 2\pi\sqrt{\frac{m}{k}}$ **83 (a)** t is undefined at $y = 0$.

Chapter 7 Review Exercises

1 $\frac{1}{2}x^2\sin^{-1}x - \frac{1}{4}\sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + C$

3 $2\ln 2 - 1 \approx 0.39$ **5** $\frac{1}{6}\sin^3 2x - \frac{1}{10}\sin^5 2x + C$

7 $\frac{1}{5}\sec^5 x + C$ **9** $\frac{x}{25\sqrt{x^2+25}} + C$

11 $2\ln\left|\dfrac{2-\sqrt{4-x^2}}{x}\right| + \sqrt{4-x^2} + C$

13 $2\ln|x-1| - \ln|x| - \frac{x}{(x-1)^2} + C$

15 $-5\ln|x-3| + 2\ln|x+3| + 2\ln(x^2+9) + \frac{1}{3}\tan^{-1}\frac{x}{3} + C$

17 $-\sqrt{4+4x-x^2} + 2\sin^{-1}\frac{x-2}{\sqrt{8}} + C$

19 $3(x+8)^{1/3} + \ln[(x+8)^{1/3} - 2]^2 - \ln|(x+8)^{2/3} + 2(x+8)^{1/3} + 4| - \frac{6}{\sqrt{3}}\tan^{-1}\frac{(x+8)^{1/3}+1}{\sqrt{3}} + C$

21 $\frac{1}{13}e^{2x}(2\sin 3x - 3\cos 3x) + C$

23 $\frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C$ **25** $-\sqrt{4-x^2} + C$

27 $\frac{1}{3}x^3 - x^2 + 3x - \frac{1}{4}\ln|x| - \frac{1}{2x} - \frac{23}{4}\ln|x+2| + C$

29 $2\tan^{-1}\sqrt{x} + C$ **31** $\ln|\sec e^x + \tan e^x| + C$

33 $\frac{1}{125}[10x\sin 5x - (25x^2-2)\cos 5x] + C$

35 $\frac{2}{7}\cos^{7/2}x - \frac{2}{3}\cos^{3/2}x + C$ **37** $\frac{2}{3}(1+e^x)^{3/2} + C$

39 $\frac{1}{16}[2x\sqrt{4x^2+25} - 25\ln(\sqrt{4x^2+25}+2x)] + C$

41 $\frac{1}{3}\tan^3 x + C$ **43** $-x\csc x + \ln|\csc x - \cot x| + C$

45 $-\frac{1}{4}(8-x^3)^{4/3} + C$

47 $-2x\cos\sqrt{x} + 4\sqrt{x}\sin\sqrt{x} + 4\cos\sqrt{x} + C$

49 $\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + C$

51 $\frac{2}{5}x^{5/2} - \frac{8}{3}x^{3/2} + 6x^{1/2} + C$

53 $\frac{1}{3}(16-x^2)^{3/2} - 16(16-x^2)^{1/2} + C$

55 $\frac{11}{2}\ln|x+5| - \frac{15}{2}\ln|x+7| + C$

57 $x\tan^{-1}5x - \frac{1}{10}\ln(1+25x^2) + C$ **59** $e^{\tan x} + C$

61 $\frac{1}{\sqrt{5}}\ln|\sqrt{7+5x^2} + \sqrt{5}x| + C$

63 $-\frac{1}{5}\cot^5 x + \frac{1}{3}\cot^3 x - \cot x - x + C$

65 $\frac{1}{5}(x^2-25)^{5/2} + \frac{25}{3}(x^2-25)^{3/2} + C$

67 $\frac{1}{3}x^3 - \frac{1}{4}\tanh 4x + C$

69 $-\frac{1}{4}x^2e^{-4x} - \frac{1}{8}xe^{-4x} - \frac{1}{32}e^{-4x} + C$

71 $3\sin^{-1}\frac{x+5}{6} + C$ **73** $-\frac{1}{7}\cos 7x + C$

75 $-9\ln|x-1| + 18\ln|x-2| - 5\ln|x-3| + C$
77 $x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + \sin x + C$

79 $-\frac{\sqrt{9-4x^2}}{x} - 2\sin^{-1}\left(\frac{2}{3}x\right) + C$

81 $24x - \frac{10}{3}\ln|\sin 3x| - \frac{1}{3}\cot 3x + C$

83 $-\ln x - \frac{4}{\sqrt[4]{x}} + 4\ln(\sqrt[4]{x}+1) + C$

85 $-2\sqrt{1+\cos x} + C$

87 $-\frac{x}{2(25+x^2)} + \frac{1}{10}\tan^{-1}\frac{x}{5} + C$

89 $\frac{1}{3}\sec^3 x - \sec x + C$

91 $\frac{7}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) - \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + \ln(x^2+4) + C$

93 $\frac{1}{4}x^4 - 2x^2 + 4\ln|x| + C$

95 $\frac{2}{5}x^{5/2}\ln x - \frac{4}{25}x^{5/2} + C$

97 $\frac{3}{64}(2x+3)^{8/3} - \frac{9}{20}(2x+3)^{5/3} + \frac{27}{16}(2x+3)^{2/3} + C$

99 $\frac{1}{2}e^{(x^2)}(x^2-1) + C$

101 D **103** D **105** C; $-\frac{9}{2}$ **107** D

109 C; $\frac{\pi}{2}$ **111** D **113** 0.14

115 (a) 1 **(b)** $\frac{\pi}{32}$

117 (a) Not possible **(b)** Not possible

CHAPTER ■ 8

Exercises 8.1

1 $\frac{1}{5}, \frac{1}{4}, \frac{3}{11}, \frac{2}{7}; \frac{1}{3}$ **3** $\frac{3}{5}, -\frac{9}{11}, -\frac{29}{21}, -\frac{57}{35}; -2$

5 $-5, -5, -5, -5; -5$ **7** $2, \frac{7}{3}, \frac{25}{14}, \frac{7}{5}; 0$

9 $\frac{2}{\sqrt{10}}, \frac{2}{\sqrt{13}}, \frac{2}{\sqrt{18}}, \frac{2}{5}; 0$ **11** $\frac{3}{10}, -\frac{6}{17}, \frac{9}{26}, -\frac{12}{37}; 0$
13 1.1, 1.01, 1.001, 1.0001; 1 **15** 2, 0, 2, 0; DNE

17 C; 0 **19** C; $\frac{\pi}{2}$ **21** D **23** C; 0 **25** D **27** D

29 C; e **31** C; 0 **33** C; $\frac{1}{2}$ **35** D **37** C; 1

39 C; 0 **41** C; 0
43 (b) 10,000 on A; 5000 on B; 20,000 on C
45 (a) The sequence appears to converge to 1.
(b) Use mathematical induction; 1
47 (a) The sequence appears to converge to approximately 0.739.
49 (a) $x_2 = 3.5$, $x_3 = 3.178571429$, $x_4 = 3.162319422$, $x_5 = 3.162277660$, $x_6 = 3.162277660$

51 (a) $B = \frac{1}{4}$ **(b)** 1.10

Exercises 8.2

1 (a) $-\frac{2}{35}, -\frac{4}{45}, -\frac{6}{55}$ **(b)** $-\frac{2n}{5(2n+5)}$ **(c)** C; $-\frac{1}{5}$

3 (a) $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}$ **(b)** $\frac{n}{2n+1}$ **(c)** C; $\frac{1}{2}$

5 (a) $-\ln 2, -\ln 3, -\ln 4$ **(b)** $-\ln(n+1)$ **(c)** D

7 C; 4 **9** C; $\frac{\sqrt{5}}{\sqrt{5}+1}$ **11** C; $\frac{37}{99}$ **13** D **15** D

17 $-1 < x < 1$; $\frac{1}{1+x}$ **19** $1 < x < 5$; $\frac{1}{5-x}$ **21** $\frac{23}{99}$

23 $\frac{16{,}181}{4995}$ **25** C **27** C **29** D **31** D **33** D
35 Needs further investigation **37** D **39** D

41 C; $\frac{41}{24}$ **43** C; $\frac{6}{7}$ **45** C; $\frac{8}{7}$ **47** C; $\frac{5}{3}$

49 (a) 0.21037; 0.26720; 0.26940 **(b)** 0.265
51 $S_{32} \approx 4.06$

53 1.423611; 1.527422; 1.564977; 1.584347; 1.596163
55 1.040293; 1.573514; 1.921645; 2.179883; 2.385110
57 Disprove; let $a_n = 1$ and $b_n = -1$ **59** 30 m
61 (b) $\dfrac{Q}{1-e^{-cT}}$ **(c)** $-\dfrac{1}{c}\ln\dfrac{M-Q}{M}$ **63 (b)** 2000
65 (a) $a_{k+1} = \frac{1}{4}\sqrt{10}\,a_k$
(b) $a_n = \left(\frac{1}{4}\sqrt{10}\right)^{n-1} a_1$; $A_n = \left(\frac{5}{8}\right)^{n-1} A_1$;
$P_n = \left(\frac{1}{4}\sqrt{10}\right)^{n-1} P_1$
(c) $\dfrac{16}{4-\sqrt{10}}a_1$; $\frac{8}{3}a_1^2$

Exercises 8.3

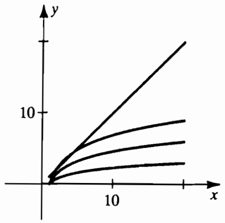
Exer. 1–12: (a) Each function f is positive-valued and continuous on the interval of integration. Since $f'(x)$ is negative, f is decreasing. (b) The value of the improper integral is given, if it exists.

1 (a) $f'(x) = \dfrac{-4}{(2x+3)^3} < 0$ if $x \geq 1$
(b) $\int_1^\infty f(x)\,dx = \frac{1}{10}$; C
3 (a) $f'(x) = \dfrac{-4}{(4x+7)^2} < 0$ if $x \geq 1$
(b) $\int_1^\infty f(x)\,dx = \infty$; D
5 (a) $f'(x) = x(2-3x^3)e^{-x^3} < 0$ if $x \geq 1$
(b) $\int_1^\infty f(x)\,dx = \frac{1}{3e}$; C
7 (a) $f'(x) = \dfrac{1-\ln x}{x^2} < 0$ if $x \geq 3$
(b) $\int_3^\infty f(x)\,dx = \infty$; D
9 (a) $f'(x) = \dfrac{1-2x^2}{x^2(x^2-1)^{3/2}} < 0$ if $x \geq 2$
(b) $\int_2^\infty f(x)\,dx = \frac{\pi}{6}$; C
11 (a) $f'(x) = \dfrac{1-2x\arctan x}{(1+x^2)^2} < 0$ if $x \geq 1$
(b) $\int_1^\infty f(x)\,dx = \frac{3\pi^2}{32}$; C

Exer. 13–28: A typical b_n is listed; however, there are many other possible choices.

13 $b_n = \frac{1}{n^4}$; C **15** $b_n = \frac{1}{3^n}$; C **17** $b_n = \frac{\pi/4}{n}$; D
19 $b_n = \frac{1}{n^2}$; C **21** $b_n = \frac{1}{\sqrt{n}}$; D **23** $b_n = \frac{1}{n^{3/2}}$; C

25 $b_n = \frac{1}{e^n}$; C **27** $b_n = \frac{1}{\sqrt{n}}$; D **29** D **31** C
33 D **35** D **37** C **39** C **41** C **43** C
45 C **47** $k > 1$ **49 (b)** $n > e^{100} - 1 \approx 2.688 \times 10^{43}$
51 Since $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, there is an M such that if $K > M$, then $\frac{a_k}{b_k} < 1$, or $a_k < b_k$. Since Σb_n converges and $a_n < b_n$ for all but at most a finite number of terms, Σa_n must also converge.
53 $\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{n} a_k + \sum_{k=n+1}^{\infty} a_k$, where the error $E = \sum_{k=n+1}^{\infty} a_k < \int_n^\infty f(x)\,dx$. (See Figure 8.8.)
55 4
57 Since Σa_n converges, $\lim_{n\to\infty} a_n = 0$ and $\lim_{n\to\infty} \frac{1}{a_n} = \infty$. By (8.17), $\Sigma\frac{1}{a_n}$ diverges.
59 $S_3 \approx 0.40488$ **61** $S_9 \approx 1.08194$ **63** $S_{21,998} \approx 0.93705$
65 The series diverges for $k = 1$, 2, and 3.



Exercises 8.4

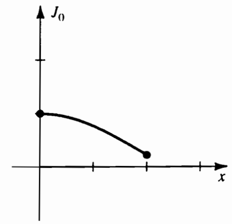
1 $\frac{1}{2}$; C **3** $\frac{5}{3}$; D **5** 0; C **7** 1; inconclusive
9 ∞; D **11** 0; C **13** 2; D **15** $\frac{1}{3}$; C **17** $\frac{1}{2}$; C
19 C **21** C **23** C **25** C **27** D **29** C
31 D **33** C **35** D **37** D **39** D

Exercises 8.5

1 (a) Conditions (i) and (ii) are satisfied.
(b) Converges, by (8.30)
3 (a) Condition (i) is satisfied, but (ii) is not.
(b) Diverges, by (8.17)
5 CC **7** CC **9** D **11** AC **13** AC **15** D **17** CC
19 AC **21** D **23** CC **25** D **27** D **29** D **31** AC
33 0.368 **35** 0.901 **37** 0.306 **39** 141 **41** 5
45 No. If $a_n = b_n = \frac{(-1)^n}{\sqrt{n}}$, then both Σa_n and Σb_n converge by the alternating series test. However, $\Sigma a_n b_n = \Sigma\frac{1}{n}$, which diverges.

Exercises 8.6

1 $[-1, 1)$ **3** $(-2, 2)$ **5** $(-1, 1]$ **7** $[-1, 1)$
9 $[-1, 1]$ **11** $(-6, 14)$ **13** Converges only for $x = 0$
15 $(-2, 2)$ **7** $(-\infty, \infty)$ **19** $\left[\frac{17}{9}, \frac{19}{9}\right)$ **21** $(-12, 4)$
23 Converges only for $x = 3$ **25** $(0, 2e)$ **27** $\left(-\frac{5}{2}, \frac{7}{2}\right]$
29 $(-\infty, \infty)$
31 (a) $\frac{3}{2}$ **33 (a)** $\frac{1}{e}$ **35** ∞ **37** Use (8.35).
39 $J_0(x) \approx 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304}$ **41** Use (8.35).
43 Use (8.37).



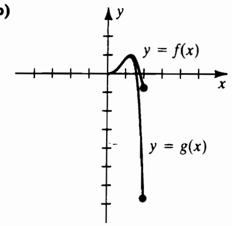
45 Assume that $\Sigma a_n x^n$ is absolutely convergent at $x = r$. Let $x = -r$. Then $\Sigma|a_n(-r)^n| = \Sigma|a_n r^n|$ is absolutely convergent, which implies that $\Sigma a_n(-r)^n$ is convergent. This is a contradiction.

Exercises 8.7

1 (a) $\sum_{n=0}^{\infty} 3^n x^n$ **(b)** $\sum_{n=1}^{\infty} n3^n x^{n-1}$; $\sum_{n=0}^{\infty} \frac{3^n}{n+1}x^{n+1}$
3 (a) $\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{7}{2}\right)^n x^n$
(b) $\frac{1}{2}\sum_{n=1}^{\infty}(-1)^n \frac{n7^n}{2^n}x^{n-1}$; $\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n \frac{7^n}{(n+1)2^n}x^{n+1}$
5 $\sum_{n=0}^{\infty} x^{2n+2}$; $r = 1$ **7** $\sum_{n=0}^{\infty} \frac{3^n}{2^{n+1}}x^{n+1}$; $r = \frac{2}{3}$
9 $-1 - x - 2\sum_{n=2}^{\infty} x^n$; $r = 1$ **11 (b)** 0.183; 0.182321557
15 $\sum_{n=0}^{\infty} \frac{3^n}{n!}x^{n+1}$ **17** $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n!}x^{n+3}$
19 $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n+1}x^{2n+4}$ **21** $\sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1}x^{(2n+1)/2}$
23 $\sum_{n=0}^{\infty} \frac{-5^{2n+1}}{(2n+1)!}x^{2n+1}$ **25** $\sum_{n=0}^{\infty} \frac{1}{(2n)!}x^{6n+2}$ **27** 0.3333
29 0.0992 **31** 0.9677 **33** $\sum_{n=1}^{\infty}(2n)x^{2n-1}$ **37** $-\sum_{n=1}^{\infty}\frac{1}{n^2}x^n$
39 (a) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n+1}}{4n+1}$ **41 (a)** $\sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{(n+1)(n+1)!}$

Exercises 8.8

1 $\frac{3^n}{n!}$
3 $a_n = 0$ if $n = 2k$, and $a_n = (-1)^k \frac{2^{2k+1}}{(2k+1)!}$ if $n = 2k+1$
5 $(-1)^n 3^n$ **7 (b)** $\sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n)!}x^{2n}$
9 $\sum_{n=0}^{\infty}(-1)^n \frac{3^{2n+1}}{(2n+1)!}x^{2n+2}$ **11** $\sum_{n=0}^{\infty}(-1)^n \frac{2^{2n}}{(2n)!}x^{2n}$
13 $1 + \sum_{n=1}^{\infty}(-1)^n \frac{2^{2n-1}}{(2n)!}x^{2n}$ **15** $\sum_{n=0}^{\infty} \frac{(\ln 10)^n}{n!}x^n$
17 $\sum_{n=0}^{\infty}(-1)^n \frac{1}{\sqrt{2}(2n+1)!}\left(x - \frac{\pi}{4}\right)^{2n+1} + \sum_{n=0}^{\infty}(-1)^n \frac{1}{\sqrt{2}(2n)!}\left(x - \frac{\pi}{4}\right)^{2n}$
19 $\sum_{n=0}^{\infty}(-1)^n \frac{1}{2^{n+1}}(x-2)^n$ **21** $\sum_{n=0}^{\infty} \frac{2^n}{e^2 n!}(x+1)^n$
23 $2 + 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 7\left(x - \frac{\pi}{3}\right)^2$
25 $\frac{\pi}{6} + \frac{2}{\sqrt{3}}\left(x - \frac{1}{2}\right) + \frac{2}{3\sqrt{3}}\left(x - \frac{1}{2}\right)^2$
27 $-\frac{1}{e} + \frac{1}{2e}(x+1)^2 + \frac{1}{3e}(x+1)^3$ **29** 0.5; 0.125
31 0.9986; 3.13×10^{-7} **33** 0.0997; 2×10^{-6}
35 0.6667; 0.1 **37** 0.4969; 9.04×10^{-6} **39** 0.4864
41 0.4484
43 (a) 0.309524, −0.690476
(b)



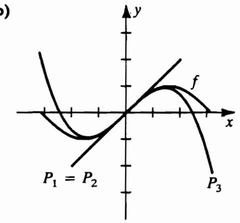
The first approximation is more accurate.

45 $2\sum_{n=0}^{\infty} \frac{1}{2n+1}x^{2n+1}$
47 (a) $\pi = 4\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + (-1)^n \frac{1}{2n+1} + \cdots\right]$
(b) 3.34 with an error of less than $\frac{4}{11}$ **(c)** 40,000
49 (c) 16.7 ft
53 (a) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{(3x/5)^{2n-1}}{(2n-1)!}$ **55 (a)** $\sum_{n=0}^{\infty}(-1)^n \frac{x^{4n}}{(2n)!}$
57 (a) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)(2n+1)!}$
59 (a) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{x^{6n-2}}{(6n-2)(2n-1)!}$

Exercises 8.9

1 (a) x; x; $x - \frac{1}{6}x^3$

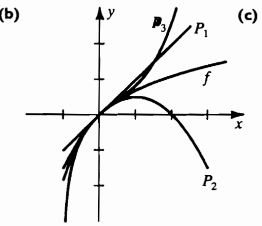
(b)



(c) 0.0500; 2.6×10^{-7}

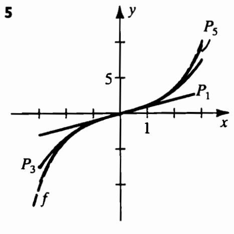
3 (a) x; $x - \frac{1}{2}x^2$; $x - \frac{1}{2}x^2 + \frac{1}{3}x^3$

(b)



(c) 0.7380; 0.164

5



$P_1(x) = x$;
$P_3(x) = x + \frac{1}{6}x^3$;
$P_5(x) = x + \frac{1}{6}x^3 + \frac{1}{120}x^5$

7 $\sin x = 1 - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 + \frac{1}{24}\sin z\left(x - \frac{\pi}{2}\right)^4$, z is between x and $\frac{\pi}{2}$.

9 $\sqrt{x} = 2 + \frac{1}{4}(x - 4) - \frac{1}{64}(x - 4)^2 + \frac{1}{512}(x - 4)^3 - \frac{5}{128}z^{-7/2}(x - 4)^4$, z is between x and 4.

11 $\tan x = 1 + 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 + \frac{1}{3}(3\tan^4 z + 4\tan^2 z + 1)\left(x - \frac{\pi}{4}\right)^3$, z is between x and $\frac{\pi}{4}$.

13 $\frac{1}{x} = -\frac{1}{2} - \frac{1}{4}(x + 2) - \frac{1}{8}(x + 2)^2 - \frac{1}{16}(x + 2)^3 - \frac{1}{32}(x + 2)^4 - \frac{1}{64}(x + 2)^5 + z^{-7}(x + 2)^6$, z is between x and -2.

15 $\tan^{-1} x = \frac{\pi}{4} + \frac{1}{2}(x - 1) - \frac{1}{4}(x - 1)^2 + \frac{3z^2 - 1}{3(1 + z^2)^3}(x - 1)^3$, z is between x and 1.

17 $xe^x = -\frac{1}{e} + \frac{1}{2e}(x + 1)^2 + \frac{1}{3e}(x + 1)^3 + \frac{1}{8e}(x + 1)^4 + \frac{ze^z + 5e^z}{120}(x + 1)^5$, z is between x and -1.

Exer. 19–30: Since $c = 0$, z is between x and 0.

19 $\ln(x + 1) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{x^5}{5(z + 1)^5}$

21 $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{\sin z}{9!}x^9$

23 $e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5 + \frac{4}{45}e^{2z}x^6$

25 $\frac{1}{(x - 1)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6(z - 1)^{-8}$

27 $\arcsin x = x + \frac{1 + 2z^2}{6(1 - z^2)^{5/2}}x^3$ **29** $f(x) = -5x^3 + 2x^4$

31 0.9998; $|R_3(x)| < 4 \times 10^{-9}$
33 2.0075; $|R_3(x)| < 3 \times 10^{-10}$
35 -0.454545; $|R_5(x)| \le 5 \times 10^{-7}$
37 0.223; $|R_4(x)| < 2 \times 10^{-4}$
39 0.8660254; $|R_8(x)| < 8.2 \times 10^{-9}$
41 Five decimal places, since $|R_3(x)| \le 4.2 \times 10^{-6} < 0.5 \times 10^{-5}$
43 Three decimal places, since $|R_2(x)| \le 1.85 \times 10^{-4} < 0.5 \times 10^{-3}$
45 Four decimal places, since $|R_3(x)| \le 3.82 \times 10^{-5} < 0.5 \times 10^{-4}$
47 If f is a polynomial of degree n, then the Taylor remainder $R_n(x) = 0$, since $f^{(n+1)}(x) = 0$. By (8.45), we have $f(x) = P_n(x)$.

Chapter 8 Review Exercises

1 C; 0 **3** D **5** C; 5 **7** The terms approach 0.589388.
9 D **11** AC **13** D **15** D **17** AC **19** D
21 D **23** AC **25** CC **27** C **29** C **31** C
33 CC **35** C **37** C **39** D **41** 0.158
43 $S_4 \approx 0.63092$ **45** $(-3, 3)$ **47** $[-12, -8)$ **49** $\frac{1}{4}$

51 $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n)!}x^{2n-1}$; ∞

53 $\sum_{n=0}^{\infty}(-1)^n\frac{2^{2n}}{(2n + 1)!}x^{2n+1}$; ∞ **55 (a)** $\sum_{n=1}^{\infty}\frac{(3x/5)^{2n-1}}{(2n - 1)!}$

57 (a) $\sum_{n=0}^{\infty}\frac{x^{4n+1}}{(4n + 1)(2n)!}$ **59** $e^{-x} = e^2\sum_{n=0}^{\infty}(-1)^n\frac{1}{n!}(x + 2)^n$

61 0.189 **63** 0.621

65 $\ln\cos x = \ln\left(\frac{1}{2}\sqrt{3}\right) - \frac{1}{3}\sqrt{3}\left(x - \frac{\pi}{6}\right) - \frac{2}{3}\left(x - \frac{\pi}{6}\right)^2 - \frac{4}{27}\sqrt{3}\left(x - \frac{\pi}{6}\right)^3 - \frac{1}{12}(\sec^4 z + 2\sec^2 z\tan^2 z)\left(x - \frac{\pi}{6}\right)^4$, z is between x and $\frac{\pi}{6}$.

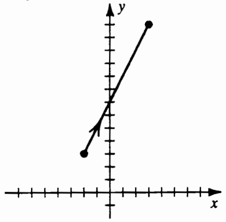
67 $e^{-x^2} = 1 - x^2 + \frac{1}{6}(4z^4 - 12z^2 + 3)e^{-z^2}x^4$, z is between x and 0.

69 0.7314

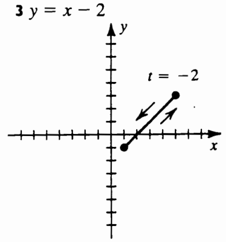
CHAPTER • 9

Exercises 9.1

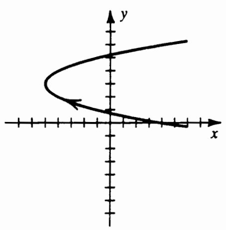
1 $y = 2x + 7$



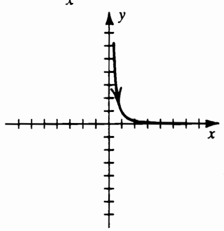
3 $y = x - 2$



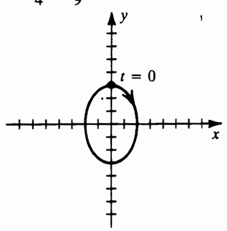
5 $(y - 3)^2 = x + 5$



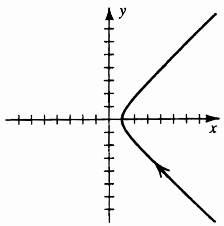
7 $y = \frac{1}{x^2}$



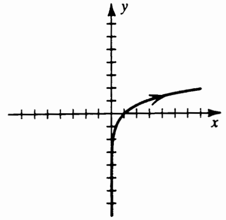
9 $\frac{x^2}{4} + \frac{y^2}{9} = 1$



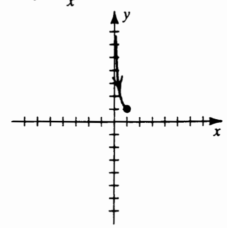
11 $x^2 - y^2 = 1$



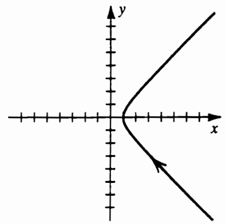
13 $y = \ln x$



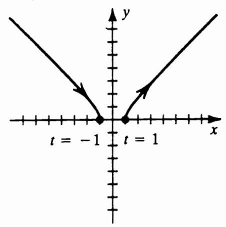
15 $y = \frac{1}{x}$



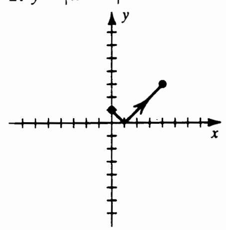
17 $x^2 - y^2 = 1$



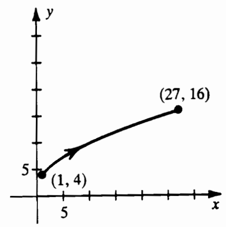
19 $y = \sqrt{x^2 - 1}$



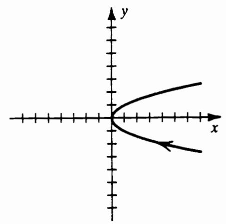
21 $y = |x - 1|$



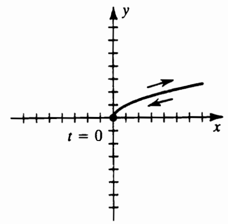
23 $y = (x^{1/3} + 1)^2$



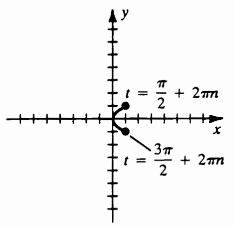
23 C_1



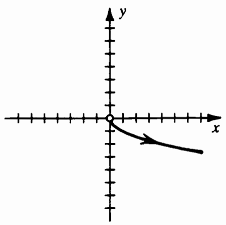
C_2



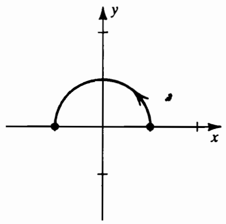
23 C_3



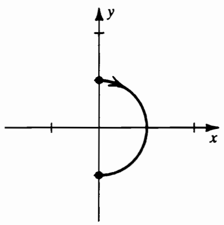
C_4



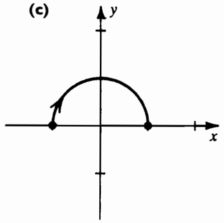
27 (a)



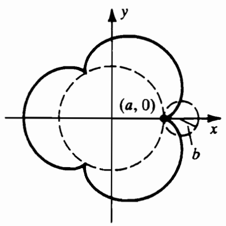
(b)



(c)

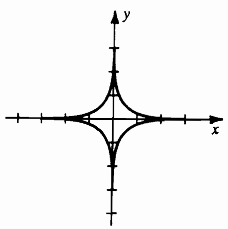


35 $x = 4b \cos t - b \cos 4t$, $y = 4b \sin t - b \sin 4t$

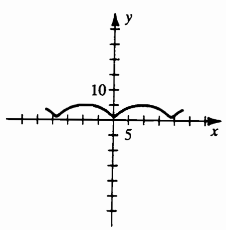


39 (a) The figure is an ellipse with center (0, 0) and axes of lengths $2a$ and $2b$.

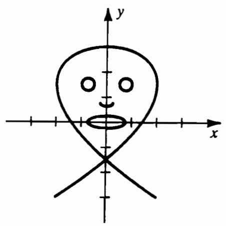
41



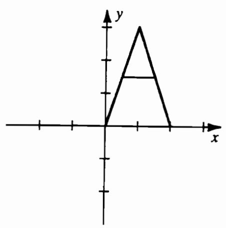
43



45 A mask with a mouth, nose, and eyes



47 The letter A



49 270 ft; yes

59 Try using: $P_0(50, 35)$, $P_1(55, 49)$, $P_2(15, 49)$, $P_3(35, 25)$ (repeated), $P_4(60, -5)$, $P_5(15, -5)$, $P_6(20, 15)$

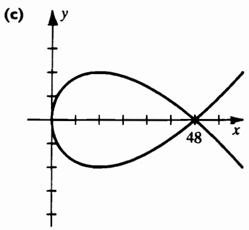
Exercises 9.2

1 1; −1 **3** $\frac{1}{4}$; −4 **5** $-\frac{2}{e^3}$; $\frac{1}{2}e^3$

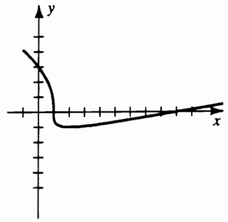
7 $-\frac{3}{2}\tan 1 \approx -2.34$; $\frac{2}{3}\cot 1 \approx 0.43$

9 (−27, −108), (1, 12)

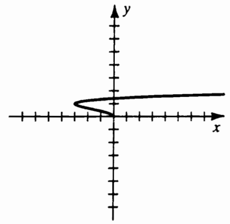
11 (a) Horizontal: (16, ±16); vertical: (0, 0)
(b) $\frac{3t^2 + 12}{64t^3}$
(c)



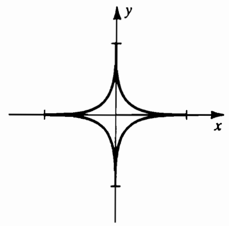
13 (a) Horizontal; (2, −1); vertical: (1, 0) **(b)** $\frac{-2t + 4}{9t^5}$
(c)



15 (a) Horizontal: none; vertical: (0, 0), (−3, 1)
(b) $\frac{1 - 3t}{144t^{3/2}(t - 1)^3}$
(c)



17 (a) Horizontal: (±1, 0); vertical: (0, ±1)
(b) $\frac{1}{3}\sec^4 t \csc t$
(c)



19 Horizontal: (0, ±2), $(2\sqrt{3}, \pm 2)$, $(-2\sqrt{3}, \pm 2)$; vertical: $(4, \pm\sqrt{2})$, $(-4, \pm\sqrt{2})$

21 $\frac{2}{27}(34^{3/2} - 125) \approx 5.43$ **23** $\sqrt{2}(e^{\pi/2} - 1) \approx 5.39$

25 $\frac{1}{8}\pi^2 \approx 1.23$ **27** 15.9 **29** $\frac{8\pi}{3}(17^{3/2} - 1) \approx 578.83$

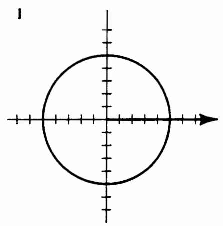
31 $\frac{11\pi}{9} \approx 3.84$ **33** $\frac{64\pi}{3} \approx 67.02$ **35** $\frac{536\pi}{5} \approx 336.78$

37 $\frac{2}{5}\sqrt{2}\pi(2e^{\pi} + 1) \approx 84.03$ **39** 2.2

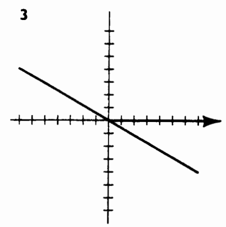
43 Arc length: 142.29; segments: 203.7

Exercises 9.3

1



3



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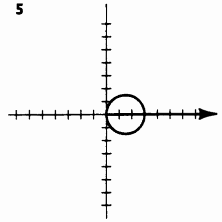

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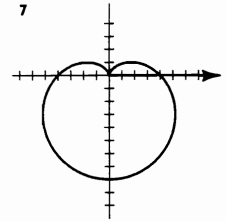

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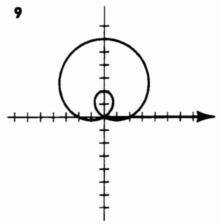
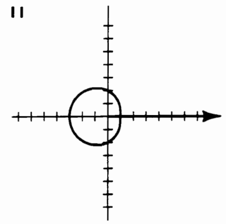


11



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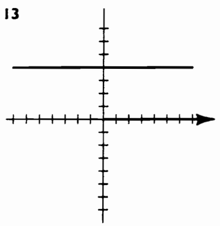

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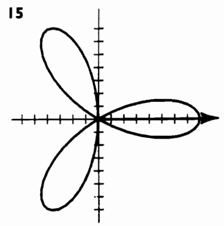

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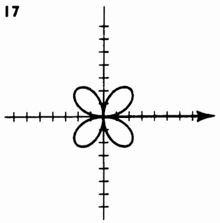

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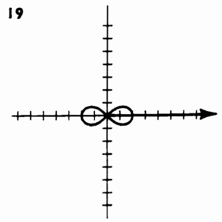

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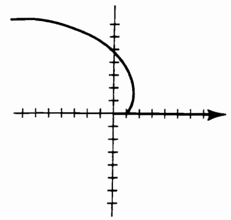

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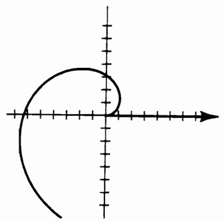

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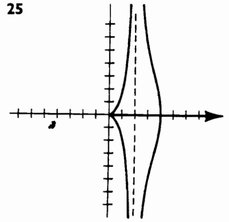


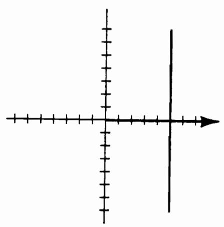
27 $r = -3 \sec \theta$ 29 $r = 4$

31 $\theta = \tan^{-1}\left(-\frac{1}{2}\right)$

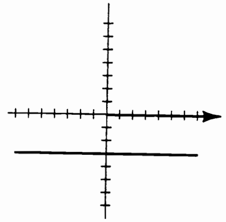
33 $r^2 = -4 \sec 2\theta$

35 $r\theta = a \sin \theta$

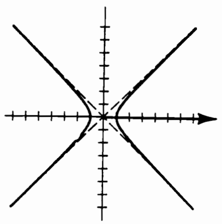
37 $x = 5$



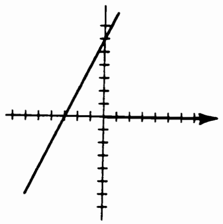
39 $y = -3$



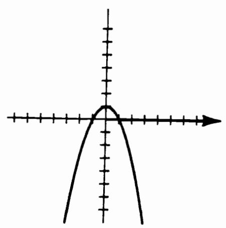
41 $x^2 - y^2 = 1$



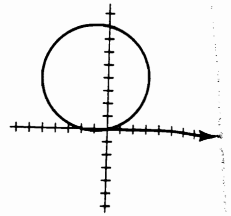
43 $y - 2x = 6$



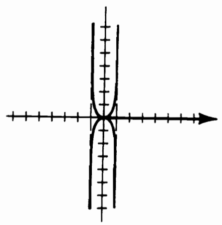
45 $y = -x^2 + 1$



47 $(x + 1)^2 + (y - 4)^2 = 1$



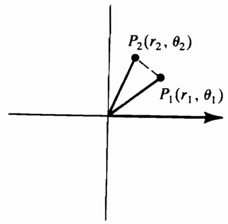
49 $y^2 = \frac{x^4}{1 - x^2}$



51 $\sqrt{3}/3$ 53 -1

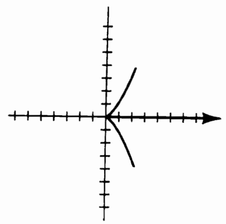
55 2 57 0 59 $\frac{1}{\ln 2}$

61 Let $P_1(r_1, \theta_1)$ and $P_2(r_2, \theta_2)$ be points in an $r\theta$-plane. Let $a = r_1$, $b = r_2$, $c = d(P_1, P_2)$, and $\gamma = \theta_2 - \theta_1$. Substituting into the law of cosines, $c^2 = a^2 + b^2 - 2ab \cos \gamma$, gives us the formula.

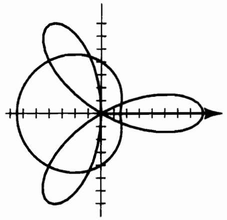


65 Use (9.11).

67 Symmetric with respect to the polar axis



69 The approximate polar coordinates are $(1.75, \pm 0.45)$, $(4.49, \pm 1.77)$, and $(5.76, \pm 2.35)$.



Exercises 9.4

1 π 3 $\frac{3\pi}{2}$ 5 $\frac{\pi}{2}$ 7 $\frac{1}{4}(e^{\pi} - 1) \approx 5.54$ 9 2

11 $\frac{9\pi}{20}$ 13 $\int_0^{\arctan(3/4)} \frac{1}{2}(4 \sec \theta)^2\, d\theta + \int_{\arctan(3/4)}^{\pi/2} \frac{1}{2}(5)^2\, d\theta$

15 $\int_{\pi/4}^{\arctan 3} \frac{1}{2}[(4 \csc \theta)^2 - (2)^2]\, d\theta$

17 (a) $8\int_0^{\pi/6} \frac{1}{2}[(4 \cos 2\theta)^2 - (2)^2]\, d\theta$

(b) $8\left[\int_0^{\pi/6} \frac{1}{2}(2)^2\, d\theta + \int_{\pi/6}^{\pi/4} \frac{1}{2}(4 \cos 2\theta)^2\, d\theta\right]$

19 $2\pi + \frac{9}{2}\sqrt{3} \approx 14.08$ 21 $4\sqrt{3} - \frac{4\pi}{3} \approx 2.74$

23 $\frac{5\pi}{24} - \frac{1}{4}\sqrt{3} \approx 0.22$

25 $\frac{3\pi}{4} + 11 \arcsin\frac{1}{4} - \frac{1}{4}\sqrt{15} \approx 4.17$

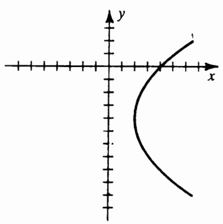
27 $\sqrt{2}(1 - e^{-2\pi}) \approx 1.41$ 29 2 31 $\frac{3\pi}{2}$ 33 2.4

35 $\frac{128\pi}{5} \approx 80.42$ 37 $4\pi^2 a^2$ 39 4.2 41 $4\pi^2 ab$

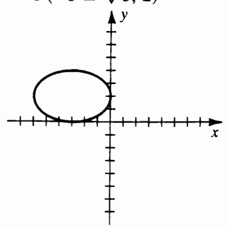
43 $\frac{2}{5}\pi\sqrt{2}(2 + e^{-\pi}) \approx 3.63$

Exercises 9.5

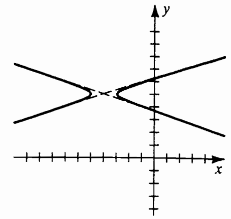
1 $V(2, -4)$; $F(4, -4)$



3 $V(-3 \pm 3, 2)$; $F(-3 \pm \sqrt{5}, 2)$

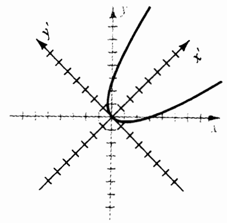


5 $V(-4 \pm 1, 5)$; $F\left(-4 \pm \frac{1}{3}\sqrt{10}, 5\right)$

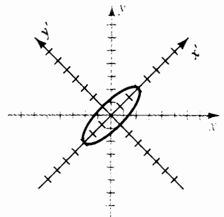


Exer. 7–19: The answer in part (a) gives the value of $B^2 - 4AC$ in the identification theorem.

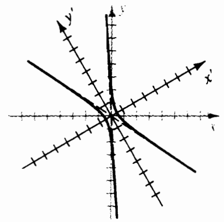
7 (a) 0, parabola
(b) $(y')^2 = 2(x')$



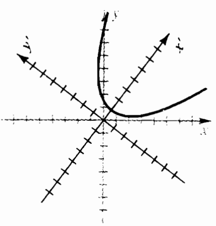
9 (a) -36, ellipse
(b) $\frac{(x')^2}{9} + (y')^2 = 1$



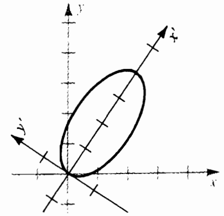
11 (a) 256, hyperbola
(b) $\frac{(x')^2}{1/4} - (y')^2 = 1$



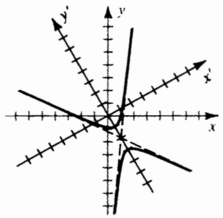
13 (a) 0, parabola
(b) $(y')^2 = 4(x' - 1)$



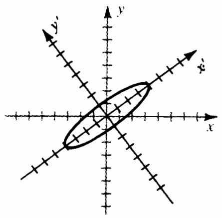
15 (a) -2704, ellipse
(b) $\frac{(x' - 2)^2}{4} + (y')^2 = 1$



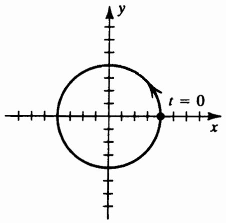
17 (a) 128, hyperbola
(b) $(y' + 2)^2 - \frac{(x')^2}{1/2} = 1$



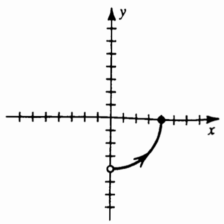
19 (a) -1600, ellipse
(b) $\frac{(x')^2}{16} + (y')^2 = 1$



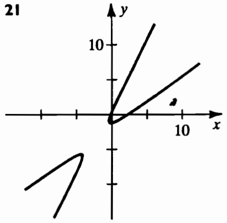
C_3



C_4

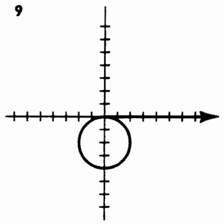


21

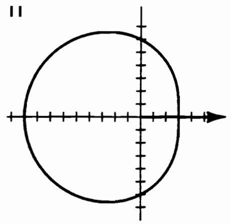


7 (a) $\frac{3t^2 + 2}{t}$ **(b)** Horizontal: none; vertical: 0
(c) $\frac{3t^2 - 2}{2t^3}$

9

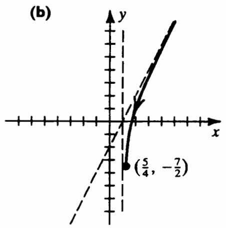


11

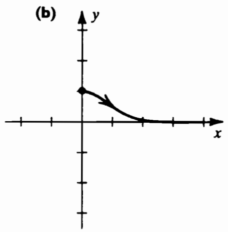


Chapter 9 Review Exercises

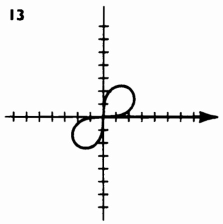
1 (a) $y = \frac{2x^2 - 4x + 1}{x - 1}$
(b)



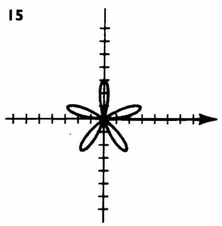
3 (a) $y = 2^{-x^2}$
(b)



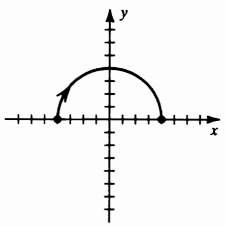
13



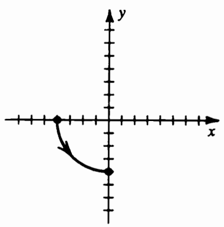
15



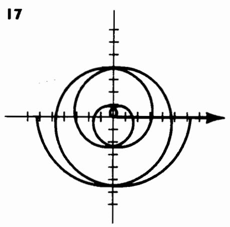
5 C_1



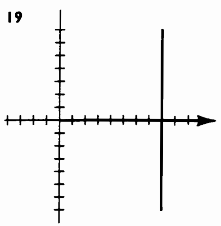
C_2



17



19



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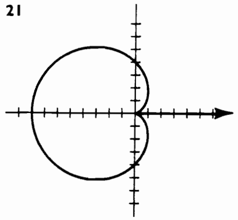
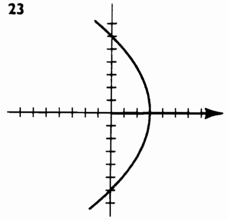
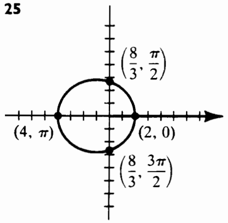


23

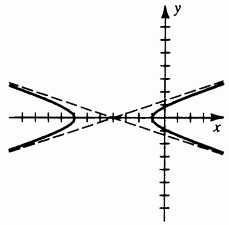


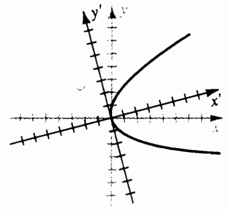
25



27 $r = 4 \cot \theta \csc \theta$
29 $r(2 \cos \theta - 3 \sin \theta) = 8$
31 $r = 2 \cos \theta \sec 2\theta$
33 $x^3 + xy^2 = y$
35 $(x^2 + y^2)^2 = 8xy$
37 $y = (\tan \sqrt{3})x$
39 -1
41 2

43 $\sqrt{2} + \ln(1 + \sqrt{2}) \approx 2.30$
45 $2\pi[5\sqrt{2} + \ln(1 + \sqrt{2})] \approx 49.97$
47 $2\pi a^2(2 - \sqrt{2}) \approx 3.68a^2$ **49** $V(2, 1)$; $F(1, 1)$
51 $V(-4 \pm 3, 0)$; $F(-4 \pm \sqrt{10}, 0)$ **53** $(y')^2 = 3x'$

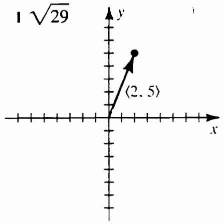




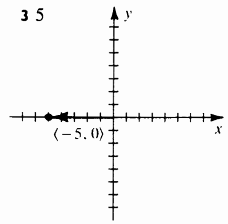
CHAPTER ■ 10

Exercises 10.1

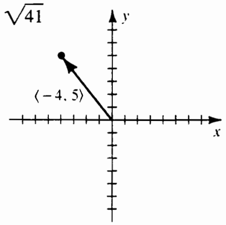
1 $\sqrt{29}$



3 5

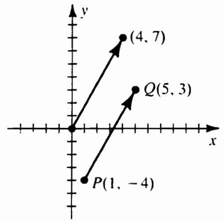


5 $\sqrt{41}$

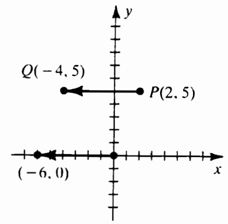


7 $\langle 3, 1\rangle$, $\langle 1, -7\rangle$, $\langle 4, -6\rangle$, $\langle -3, -12\rangle$, $\langle 3, -32\rangle$
9 $\langle -15, 6\rangle$, $\langle 1, -2\rangle$, $\langle -14, 4\rangle$, $\langle 24, -12\rangle$, $\langle 12, -12\rangle$
11 $2\mathbf{i} + 7\mathbf{j}$, $4\mathbf{i} - 3\mathbf{j}$, $6\mathbf{i} + 4\mathbf{j}$, $3\mathbf{i} - 15\mathbf{j}$, $17\mathbf{i} - 17\mathbf{j}$
13 $-\mathbf{b}$ **15** $\mathbf{f}$ **17** $-\frac{1}{2}\mathbf{e}$

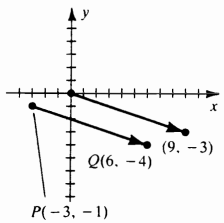
19 $\langle 4, 7\rangle$



21 $\langle -6, 0\rangle$



23 $\langle 9, -3\rangle$



25 (a) $-\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}$
(b) $\frac{8}{17}\mathbf{i} - \frac{15}{17}\mathbf{j}$

27 (a) $\left\langle \frac{2}{\sqrt{29}}, -\frac{5}{\sqrt{29}} \right\rangle$
(b) $\left\langle -\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle$

29 (a) $\langle -12, 6\rangle$ **(b)** $\left\langle -3, \frac{3}{2} \right\rangle$ **31** $\frac{24}{\sqrt{65}}\mathbf{i} - \frac{42}{\sqrt{65}}\mathbf{j}$

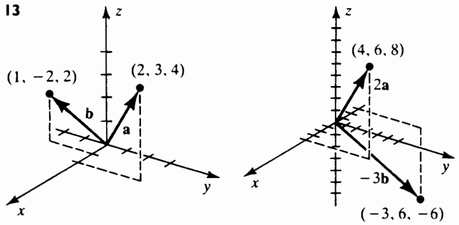
33 (a) $\pm\frac{3}{5}$ **(b)** None **(c)** 0 **35** 40.96; 28.68

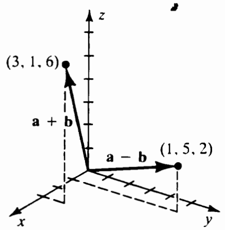
37 -6.180; 19.021 **39** 301.5 mi/hr; 144.3°
41 $\sin^{-1}(0.4) \approx 23.6°$ **43** $p = 2$; $q = -3$
45 If **a** and **b** have the same direction
47 The circle with center (x_0, y_0) and radius c

Exercises 10.2

1 (a) $\sqrt{104}$ **(b)** $(3, 1, -1)$ **(c)** $\langle 2, -6, 8\rangle$
3 (a) $\sqrt{53}$ **(b)** $\left(-\frac{1}{2}, -1, 1\right)$ **(c)** $\langle 7, -2, 0\rangle$

5 (a) $\sqrt{3}$ **(b)** $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ **(c)** $\langle -1, 1, 1\rangle$
7 (a) $\langle 1, 3, 0\rangle$ **(b)** $\langle -5, 9, 2\rangle$ **(c)** $\langle -22, 42, 9\rangle$
(d) $\sqrt{41}$ **(e)** $3\sqrt{41}$
9 (a) $4\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ **(b)** $2\mathbf{i} - 6\mathbf{j} + 7\mathbf{k}$
(c) $11\mathbf{i} - 28\mathbf{j} + 30\mathbf{k}$ **(d)** $\sqrt{29}$ **(e)** $3\sqrt{29}$
11 (a) $\mathbf{i} + \mathbf{k}$ **(b)** $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ **(c)** $5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$ **(d)** $\sqrt{2}$
(e) $3\sqrt{2}$
13





15 $\frac{1}{\sqrt{30}}\langle -2, 5, -1\rangle$
17 (a) $28\mathbf{i} - 30\mathbf{j} + 12\mathbf{k}$ **(b)** $-\frac{14}{3}\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$
(c) $\frac{2}{\sqrt{457}}(14\mathbf{i} - 15\mathbf{j} + 6\mathbf{k})$
19 $(x - 3)^2 + (y + 1)^2 + (z - 2)^2 = 9$
21 $(x + 5)^2 + y^2 + (z - 1)^2 = \frac{1}{4}$
23 (a) $(x + 2)^2 + (y - 4)^2 + (z + 6)^2 = 36$
(b) $(x + 2)^2 + (y - 4)^2 + (z + 6)^2 = 16$
(c) $(x + 2)^2 + (y - 4)^2 + (z + 6)^2 = 4$
25 $(x + 3)^2 + \left(y - \frac{5}{2}\right)^2 + z^2 = \frac{89}{4}$
27 $(x - 1)^2 + (y - 1)^2 + (z - 1)^2 = 1$
29 $(-2, 1, -1)$; 2 **31** $(4, 0, -4)$; 4 **33** $(0, -2, 0)$; 2
35 All points inside or on the sphere of radius 1 with center at the origin
37 All points inside or on a rectangular box with center at the origin and having edges of lengths 2, 4, and 6 in the x, y, and z directions, respectively
39 All points inside or on a cylindrical region of radius 5 and altitude 6 with center at the origin and axis along the z-axis
41 All points not on a coordinate plane
43 *Hint*: P is $\left(\frac{x_1 + x_2 + x_3}{4}, \frac{y_1 + y_2 + y_3}{4}, \frac{z_4}{4}\right)$.

Exercises 10.3

1 3 **3 (a)** -12 **(b)** -12 **5** -99 **7** $-\frac{3}{\sqrt{30}}$
9 0 **11** $\arccos\frac{-3}{\sqrt{534}} \approx 97.5°$ **13** $\arccos\frac{6}{13} \approx 62.5°$
15 *Hint*: Use Theorem (10.21). **17** $-3, 5$ **19** 74
21 $\arccos\frac{37}{\sqrt{3081}} \approx 48.2°$ **23** $\frac{-82}{\sqrt{126}}$ **25** 1
27 $-4\sqrt{3} \approx 6.93$ ft-lb **29** $1000\sqrt{3} \approx 1732$ ft-lb
35 (a) *Hint*: $a_1 = \mathbf{a} \cdot \mathbf{i} = \|\mathbf{a}\|\,\|\mathbf{i}\| \cos \alpha$.
47 When **a** and **b** have the same or opposite direction

Exercises 10.4

1 $\langle 5, 10, 5\rangle$ **3** $\langle -4, 2, -1\rangle$ **5** $-6\mathbf{i} - 8\mathbf{j} + 18\mathbf{k}$
7 $0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k} = \mathbf{0}$ **9** $0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k} = \mathbf{0}$
11 *Hint*: Use Corollary (10.31).
13 $\langle 12, -14, 24\rangle$; $\langle 16, -2, -5\rangle$

Exer. 15–18: c is a nonzero scalar.

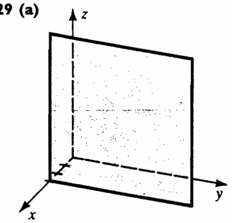
15 (a) $c\langle 13, 7, 5\rangle$ **(b)** $\frac{9}{2}\sqrt{3}$
17 (a) $c\langle -10, -8, -20\rangle$ **(b)** $\sqrt{141}$
19 $\sqrt{\frac{282}{11}} \approx 5.06$
23 4

Exercises 10.5

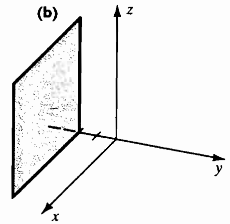
In answers it is assumed that the domain of each parameter is $\mathbb{R}$.

1 $x = 4 + \frac{1}{3}t$, $y = 2 + 2t$, $z = -3 + \frac{1}{2}t$
3 $x = 0$, $y = t$, $z = 0$
5 $x = 5 - 3t$, $y = -2 + 8t$, $z = 4 - 3t$;
$\left(1, \frac{26}{3}, 0\right), \left(\frac{17}{4}, 0, \frac{13}{4}\right), \left(0, \frac{34}{3}, -1\right)$
7 $x = 2 - 8t$, $y = 0$, $z = 5 - 2t$;
$(-18, 0, 0)$, lies in xz-plane, $\left(0, 0, \frac{9}{2}\right)$
9 $x = -6 - 3s$, $y = 4 + s$, $z = -3 + 9s$ **11** $(5, -7, 3)$
13 Do not intersect
15 $\theta = \cos^{-1}\left(\frac{9}{\sqrt{38}\sqrt{33}}\right) \approx 75°$ and $180° - \theta$
17 $\theta = \cos^{-1}\left(\frac{71}{\sqrt{82}\sqrt{249}}\right) \approx 60°$ and $180° - \theta$
19 (a) $z = 4$ **(b)** $x = 6$ **(c)** $y = -7$

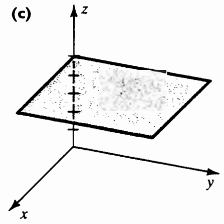
21 $6x - 5y - z = -84$ **23** $3x - y + 2z = -11$
25 $x + 42y - 5z = 8$ **27** $2x + y - 2z = -3$
29 (a)



(b)



(c)



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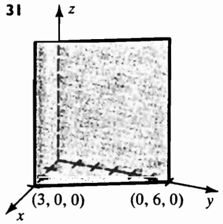

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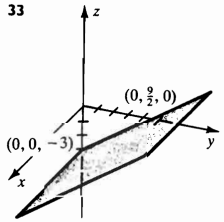

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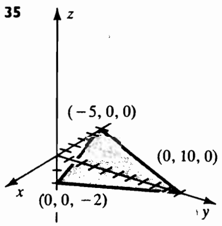
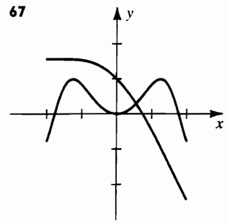


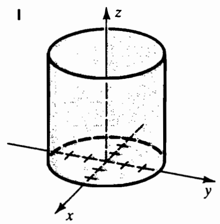
37 $x + z = 5$
39 $3x + 2y = 6$
41 $4x - y + 3z + 7 = 0$
43 $\frac{x - 5}{-3} = \frac{y + 2}{8} = \frac{z - 4}{-3}$
45 $\frac{x - 4}{-7} = \frac{z + 3}{8}$; $y = 2$
47 $x = 3 - t$, $y = 2 + 5t$, $z = t$
49 $x = 3t$, $y = 4 - t$, $z = t$
51 $\frac{7}{\sqrt{59}} \approx 0.91$ **53** $\frac{17}{6\sqrt{14}} \approx 0.76$ **55** $\frac{89}{\sqrt{521}} \approx 3.90$
57 $6x + 11y + 4z = 38$ **59** $\sqrt{\frac{5411}{90}} \approx 7.75$
61 $\sqrt{\frac{474}{17}} \approx 5.28$ **63** $\frac{x}{3} + \frac{y}{-2} + \frac{z}{5} = 1$
65 $6x + 4y + 3z = 12$
67



(a) $(0.55, 0.30)$
(b) $103°$; $77°$

Exercises 10.6

1



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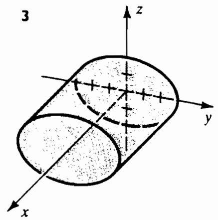

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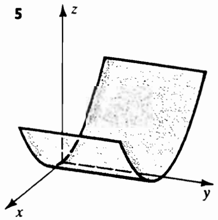

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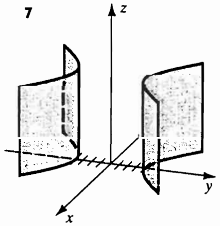
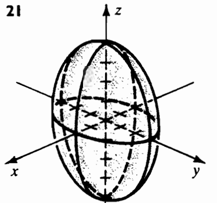
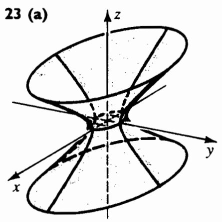


9 K **11** C **13** Q
15 P **17** A **19** E

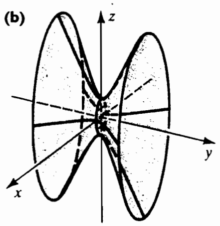
21



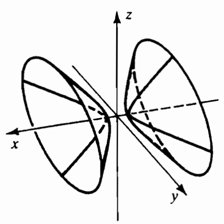
23 (a)



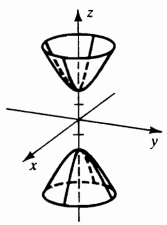
(b)



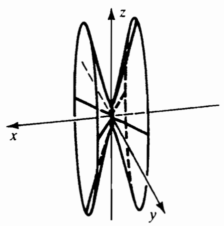
25 (a)



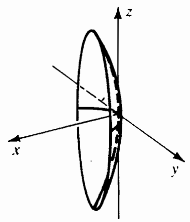
(b)



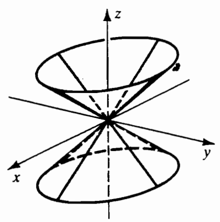
33 Hyperboloid of one sheet



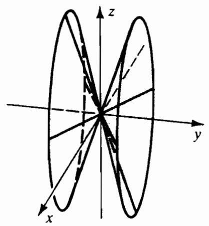
35 Paraboloid



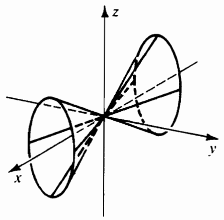
27 (a)



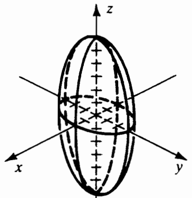
(b)



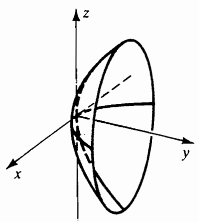
37 Cone



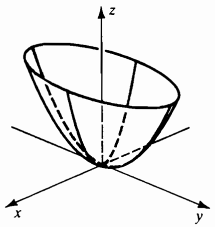
39 Ellipsoid



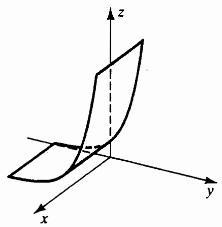
29 (a)



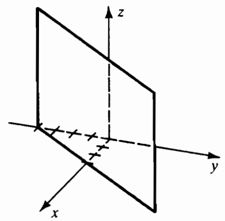
(b)



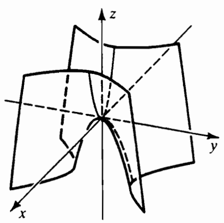
41 Exponential cylinder



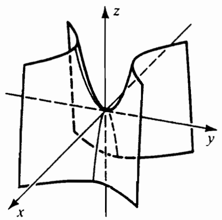
43 Plane



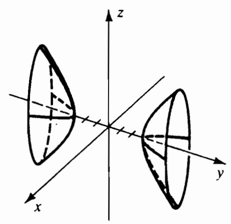
31 (a)



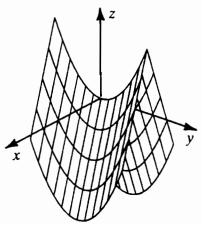
(b)



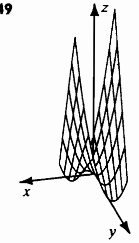
45 Hyperboloid of two sheets



47



49

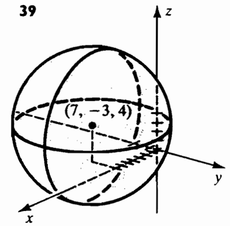


51 $x^2 + z^2 + 4y^2 = 16$ **53** $z = 4 - x^2 - y^2$
55 $y^2 + z^2 - x^2 = 1$
57 (a) The Clarke ellipsoid is flatter at the north and south poles.
(b) Ellipses **(c)** Ellipses

Chapter 10 Review Exercises

1 $5\mathbf{i} - 13\mathbf{j} - 8\mathbf{k}$ **3** $3\sqrt{33}$ **5** 26
7 $\arccos \dfrac{-27}{\sqrt{962}} \approx 150.52°$ **9** $\dfrac{1}{\sqrt{26}}(3\mathbf{i} - \mathbf{j} - 4\mathbf{k})$
11 $22\mathbf{i} - 2\mathbf{j} + 17\mathbf{k}$ **13** $\dfrac{9}{\sqrt{33}} \approx 1.57$ **15** 156 **17** 0
19 80 **21** *Hint*: Use Theorem (10.21).
23 (a) $\sqrt{38}$
(b) $\left(2, -\frac{7}{2}, \frac{5}{2}\right)$
(c) $(x + 1)^2 + (y + 4)^2 + (z - 3)^2 = 16$
(d) $y = -4$
(e) $x = 5 + 6t,\ y = -3 + t,\ z = 2 - t$
(f) $6x + y - z = 25$
25 $6x - 15y + 5z = 30$
27 $x = -13t + 5,\ y = 6t - 2,\ z = 5t$
29 $4x + 3y - 4z = 11$ **31** $\dfrac{x^2}{64} + \dfrac{y^2}{9} + z^2 = 1$
33 (a) $\dfrac{1}{\sqrt{66}}\langle 1, 4, 7\rangle$ **(b)** $x + 4y + 7z = 5$
(c) $x = 2 + 7t,\ y = -1 - 7t,\ z = 1 + 3t$ **(d)** 59
(e) $\arccos \dfrac{59}{\sqrt{3745}} \approx 15.40°$
(f) $\sqrt{66} \approx 8.12$ **(g)** $\sqrt{\dfrac{264}{35}} \approx 2.75$
35 $x = 3 + 2t,\ y = -1 - 4t,\ z = 5 + 8t;$
$x = -1 + 7t,\ y = 6 - 2t,\ z = -\dfrac{7}{2} - 2t$
37 $\theta = \arccos \dfrac{-25}{\sqrt{2295}} \approx 121.46°$ and $180° - \theta$

39



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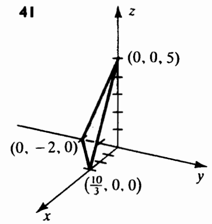

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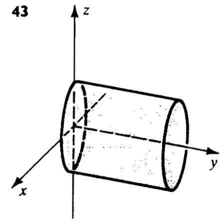

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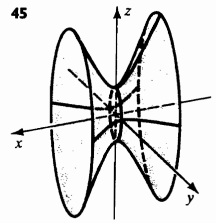

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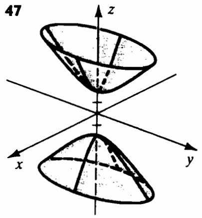

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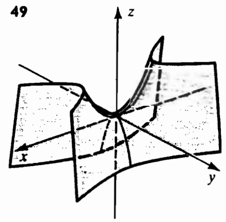
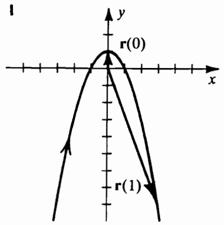


CHAPTER ■ 11

Exercises 11.1

1



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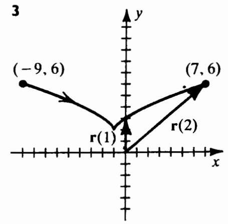

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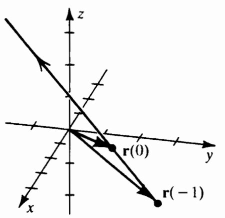

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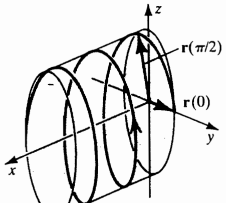

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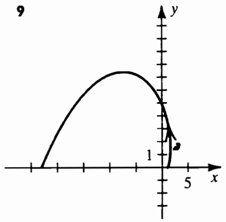

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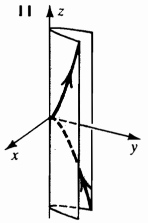

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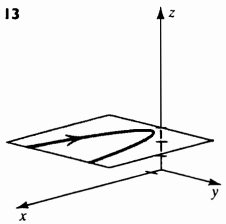

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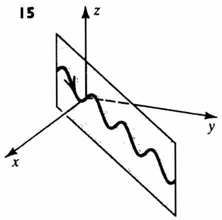

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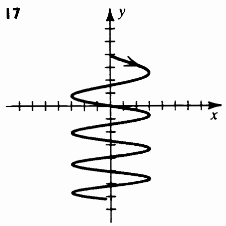
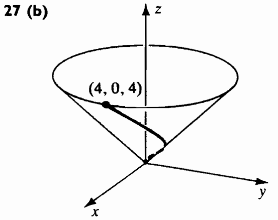


21 $\frac{5}{4}[4\sqrt{17} + \ln(4 + \sqrt{17})]$
≈ 23.2339
23 $\sqrt{3}(e^{2\pi} - 1) \approx 925.7667$
25 8.2182

27 (b)


(c) $4\sqrt{3}$

29 (a) *Hint*: Let $Ax + By + Cz = D$ be the equation of an arbitrary plane. **(b)** 7

Exercises 11.2

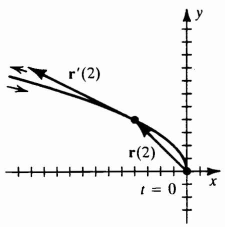
1 (a) $[1, 2]$
(b) $\frac{1}{2}(t-1)^{-1/2}\mathbf{i} - \frac{1}{2}(2-t)^{-1/2}\mathbf{j}$;
$-\frac{1}{4}(t-1)^{-3/2}\mathbf{i} - \frac{1}{4}(2-t)^{-3/2}\mathbf{j}$

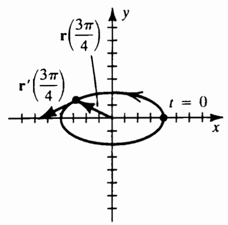
3 (a) $\left\{t: t \neq \frac{\pi}{2} + \pi n\right\}$
(b) $\sec^2 t\,\mathbf{i} + (2t+8)\mathbf{j}$; $2\sec^2 t \tan t\,\mathbf{i} + 2\mathbf{j}$

5 (a) $\left\{t: t \neq \frac{\pi}{2} + \pi n\right\}$
(b) $2t\mathbf{i} + \sec^2 t\,\mathbf{j}$; $2\mathbf{i} + 2\sec^2 t \tan t\,\mathbf{j}$

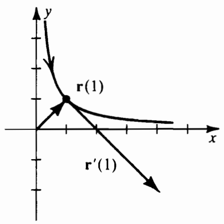
7 (a) $\{t: t \geq 0\}$ **(b)** $\frac{1}{2\sqrt{t}}\mathbf{i} + 2e^{2t}\mathbf{j} + \mathbf{k}$; $-\frac{1}{4t\sqrt{t}}\mathbf{i} + 4e^{2t}\mathbf{j}$

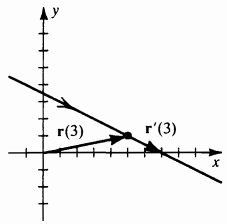
9 $-t^3\mathbf{i} + 2t\mathbf{j}$ **11** $-4\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}$





13 $3t^2\mathbf{i} - 3t^{-4}\mathbf{j}$ **15** $2\mathbf{i} - \mathbf{j}$

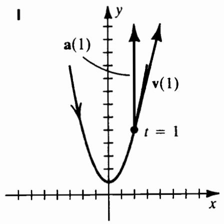




17 $x = 1 + 6t$, $y = -2 - 10t$, $z = 10 + 8t$
19 $x = 1 + t$, $y = t$, $z = 4$ **21** $\pm\frac{1}{\sqrt{5}}(2\mathbf{i} - \mathbf{j})$
25 $x = ae^t$, $y = be^t$, and $z = ce^t$ for constants a, b, and c; the graph is a half-line with endpoint O deleted.
27 $16\mathbf{i} - 8\mathbf{j} + 6\mathbf{k}$ **29** $\left(1 - \frac{1}{\sqrt{2}}\right)\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j} + (\ln\sqrt{2})\mathbf{k}$
31 $\left(\frac{1}{3}t^3 + 2\right)\mathbf{i} + (3t^2 + t - 3)\mathbf{j} + (2t^4 + 1)\mathbf{k}$
33 $(t^3 + t + 7)\mathbf{i} + (2t - t^4)\mathbf{j} + \left(\frac{1}{2}t^2 - 3t + 1\right)\mathbf{k}$

35 $x + y = 1$
37 $(1 + 5t^2)\sin t + (2t^3 + 3t)\cos t$;
$[(t^3 + 4t)\sin t - t^2\cos t]\mathbf{i} + [(3t^2 - 2)\sin t + (t^3 - 2t)\cos t]\mathbf{j} + [-3t\sin t + (1 - t^2)\cos t]\mathbf{k}$

Exercises 11.3

1


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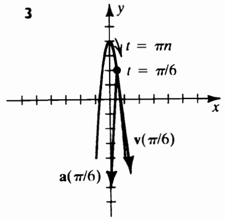

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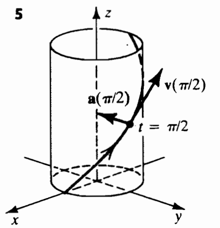

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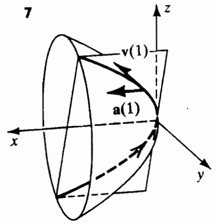
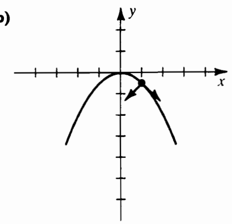
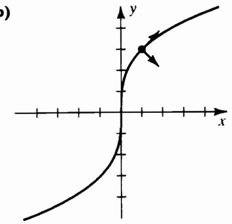
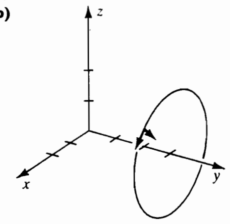


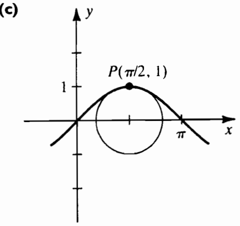
9 $-\frac{1}{2}\mathbf{i} - \frac{1}{3}\mathbf{j}$; $\frac{1}{2}\mathbf{i} + \frac{2}{9}\mathbf{j}$; $\frac{1}{6}\sqrt{13}$ **11** $2\mathbf{i} - \mathbf{j}$; $4\mathbf{i} + \mathbf{j}$; $\sqrt{5}$
13 $e^{\pi/2}(-\mathbf{i} + \mathbf{j} + \mathbf{k})$; $e^{\pi/2}(-2\mathbf{i} + \mathbf{k})$; $\sqrt{3}e^{\pi/2}$
15 $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$; **0**; $\sqrt{14}$ **19** 4.7
21 (a) 18,054 mi/hr **(b)** 86.7 min
23 (a) $750\sqrt{3}\mathbf{i} + (-gt + 750)\mathbf{j}$ **(b)** $\frac{(1500)^2}{8g} \approx 8789$ ft
(c) $\frac{(1500)^2\sqrt{3}}{2g} \approx 60{,}892$ ft **(d)** 1500 ft/sec
25 $\sqrt{250g} \approx 89.4$ ft/sec **27** 0.46 rev/sec
31 2.51 ft **33** 0.14 (mi/hr)/ft

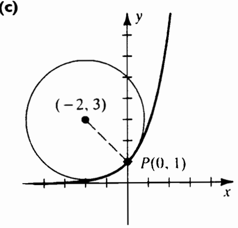
Exercises 11.4

1 (a) $\frac{1}{(1+t^2)^{1/2}}\mathbf{i} - \frac{t}{(1+t^2)^{1/2}}\mathbf{j}$; $-\frac{t}{(1+t^2)^{1/2}}\mathbf{i} - \frac{1}{(1+t^2)^{1/2}}\mathbf{j}$
(b)


3 (a) $\frac{t^2}{(t^4+1)^{1/2}}\mathbf{i} + \frac{1}{(t^4+1)^{1/2}}\mathbf{j}$; $\frac{1}{(t^4+1)^{1/2}}\mathbf{i} - \frac{t^2}{(t^4+1)^{1/2}}\mathbf{j}$
(b)


5 (a) $\cos t\,\mathbf{i} - \sin t\,\mathbf{k}$; $-\sin t\,\mathbf{i} - \cos t\,\mathbf{k}$
(b)


7 $\frac{6}{10^{3/2}} \approx 0.19$ **9** 2 **11** 4 **13** $\frac{2}{17^{3/2}} \approx 0.03$
15 0 **17** $\frac{48}{21^{3/2}} \approx 0.50$
19 (a) 1 **(b)** $\left(\frac{\pi}{2}, 0\right)$
(c)


21 (a) $2\sqrt{2}$ **(b)** $(-2, 3)$
(c)


23 0.6439 **25** 0.6034

27 $\left(\ln\sqrt{2}, \frac{1}{\sqrt{2}}\right)$ **29** $(0, \pm 3)$ **31** $\left(\frac{1}{\sqrt{2}}, -\frac{1}{2}\ln 2\right)$

33 $(\pm\sqrt{2}, -20)$ **35** $(0, 0)$ **39** $\frac{8 - 3\sin^2 2\theta}{(1 + 3\cos^2 2\theta)^{3/2}}$

43 $\left(-\frac{14}{3}, \frac{29}{12}\right)$ **45** $(4, -2)$ **47** $(0, -4)$

53 $x = \frac{4}{5}s - 3,\ y = \frac{3}{5}s + 5;\ s \geq 0$

55 $x = 4\cos\frac{1}{4}s,\ y = 4\sin\frac{1}{4}s;\ 0 \leq s \leq 8\pi$

Exercises 11.5

1 $\frac{4t}{(4t^2+9)^{1/2}}; \frac{6}{(4t^2+9)^{1/2}}; \frac{6}{(4t^2+9)^{3/2}}$

3 $\frac{6t(t^2+2)}{(t^4+4t^2+1)^{1/2}}; \frac{6(t^4+t^2+1)^{1/2}}{(t^4+4t^2+1)^{1/2}}; \frac{2(t^4+t^2+1)^{1/2}}{3(t^4+4t^2+1)^{3/2}}$

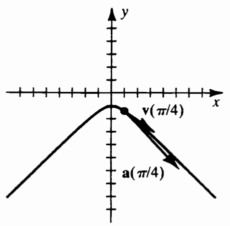
5 $\frac{t}{(1+t^2)^{1/2}}; \frac{2+t^2}{(1+t^2)^{1/2}}; \frac{2+t^2}{(1+t^2)^{3/2}}$

7 $\frac{-65\sin t\cos t}{(16\sin^2 t + 81\cos^2 t + 1)^{1/2}};$
$\frac{(81\sin^2 t + 16\cos^2 t + 1296)^{1/2}}{(16\sin^2 t + 81\cos^2 t + 1)^{1/2}};$
$\frac{(81\sin^2 t + 16\cos^2 t + 1296)^{1/2}}{(16\sin^2 t + 81\cos^2 t + 1)^{3/2}}$

9 $\frac{36}{\sqrt{5}} \approx 16.10; \frac{18}{\sqrt{5}} \approx 8.05$

Chapter 11 Review Exercises

1



3 $2t\mathbf{i} + (8t - 4t^3)\mathbf{j}; 2\mathbf{i} + (8 - 12t^2)\mathbf{j};$
$2|t|\sqrt{17 - 16t^2 + 4t^4}$

5 **(a)** $\frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$ **(b)** $x = t, y = 1 + t, z = 1 + t$

7 $0, \frac{10}{3}$

9 $3\mathbf{i} + 3t^2\mathbf{j} + 4t^3\mathbf{k}, 6t\mathbf{j} + 12t^2\mathbf{k}, \sqrt{9 + 9t^4 + 16t^6};$
$3\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}, 6\mathbf{j} + 12\mathbf{k}, \sqrt{34}$

11 $2\mathbf{i} + \frac{1}{4}\mathbf{j} - \mathbf{k}$

13 $\mathbf{r}(t) = (3t - 4)\mathbf{i} + (2t^3 - 8t + 9)\mathbf{j} + \left(\frac{5}{2}t^2 - t\right)\mathbf{k}$

17 $\frac{108}{82^{3/2}} \approx 0.15$

19 $\pm\sqrt{\frac{36 + \sqrt{3096}}{90}} \approx \pm 1.009$ **21** $\frac{5}{6}\sqrt{5} \approx 1.86$

23 $\frac{-8\cos 2t \sin 2t + \sin t \cos t}{(4\cos^2 2t + \sin^2 t)^{1/2}}; \frac{2|\cos 2t \cos t + 2\sin 2t \sin t|}{(4\cos^2 2t + \sin^2 t)^{1/2}}$

25 2π

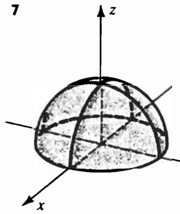
CHAPTER ■ 12

Exercises 12.1

1 $\mathbb{R}^2$; $-29, 6, -4$ **3** $\{(u, v): u \neq 2v\}$; $-\frac{3}{2}, \frac{4}{9}, 0$

5 $\{(x, y, z): x^2 + y^2 + z^2 \leq 25\}$; $4, 2\sqrt{3}$

7



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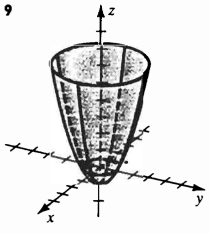

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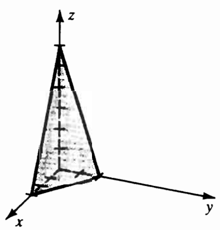

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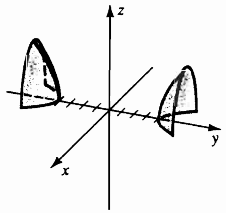

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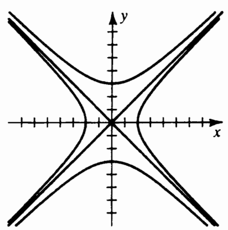
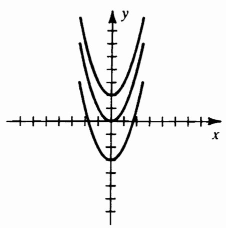
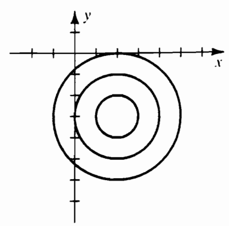


23



25



27 $y \arctan x = \pi$ **29** $x^2 + 4y^2 - z^2 = -1$

31 **(a)** Ellipses **(b)** None **(c)** None

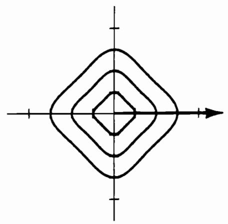
33 **(a)** $k > 8$: none; $k = 8$: the point $(0, 0, 8)$; $0 < k < 8$: circles
(b) None **(c)** None

35 $k = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$

37 $k = 1.1, 1.2, 1.4, 1.6, 1.8$

39 **(c)** **41** **(b)** **43** **(c)**

45



47 None; the origin; the sphere with center $(0, 0, 0)$ and radius 2

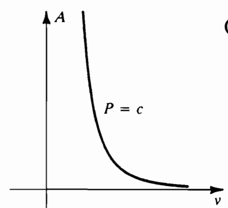
49 Planes with x-intercept k, y-intercept $\frac{k}{2}$, and z-intercept $\frac{k}{3}$

51 None; the z-axis; the right circular cylinder with the z-axis as its axis and radius 2

53 **(a)** Circles with center at the origin **(b)** $x^2 + y^2 = 100$

55 Five; spheres with centers at the origin; the force F is constant if (x, y, z) moves along a level surface.

57 **(a)** $P = kAv^3$ for $k > 0$
(b) A typical level curve (see figure) shows the combinations of areas and wind velocities that result in a fixed power $P = c$.



(c) $Av^3 = \frac{8}{3}\pi \times 10^5$

59 *Example*: 5′11″ and 175 lb are approximately 180 cm and 80 kg. From the graph, we have a surface area of approximately 2.0 m². Using the formula, we obtain $S \approx 1.996$ m².

Exercises 12.2

1 $-\frac{2}{3}$ **3** 1 **5** 0 **7** 0 **9** 4

Exer. 11–20: The answer gives equations of possible paths, and their resulting values, to use in (12.4).

11 $x = 0, -\frac{1}{2}; y = 0, 2$ **13** $y - 2 = m(x - 1), \frac{m}{1 + m^2}$

15 $y = mx, \frac{4m}{2 + 3m^4}$ **17** $x = y = 0, 0; x = y = z, 1$

19 $x = -3 + at, y = bt, z = ct, \frac{(a - c)^2}{a^2 + b^2 + c^2}$ **21** 0 **23** 1

25 $\{(x, y): x + y > 1\}$ **27** $\{(x, y): x \geq 0 \text{ and } |y| \leq 1\}$

29 $\{(x, y, z): z^2 \neq x^2 + y^2\}$ **31** $\{(x, y, z): x \geq 2, yz > 0\}$

33 0 **35** $\frac{x^4 - 2x^2y^2 + y^4 - 4}{x^2 - y^2}; \{(x, y): x^2 \neq y^2\}$

37 $x^2 + 2x\tan y + \tan^2 y + 1; \left\{(x, y): y \neq \frac{\pi}{2} + \pi n\right\}$

39 $e^{x^2+2y}; (x^2 + 2y)(x^2 + 2y - 3); e^{2t} + 2(t^2 - 3t)$

41 $2x^2 - 3xy - 2y^2 - x + 7y$

43 The statement $\lim_{(x,y,z,w)\to(a,b,c,d)} f(x, y, z, w) = L$ means that for every $\epsilon > 0$ there is a $\delta > 0$ such that if $0 < \sqrt{(x - a)^2 + (y - b)^2 + (z - c)^2 + (w - d)^2} < \delta$, then $|f(x, y, z, w) - L| < \epsilon$.

Exercises 12.3

1 $f_x(x, y) = 8x^3y^3 - y^2; f_y(x, y) = 6x^4y^2 - 2xy + 3$

3 $f_r(r, s) = \frac{r}{(r^2 + s^2)^{1/2}}; f_s(r, s) = \frac{s}{(r^2 + s^2)^{1/2}}$

5 $f_x(x, y) = e^y + y\cos x; f_y(x, y) = xe^y + \sin x$

7 $f_t(t, v) = -\frac{v}{t^2 - v^2}; f_v(t, v) = \frac{t}{t^2 - v^2}$

9 $f_x(x, y) = \cos\frac{x}{y} - \frac{x}{y}\sin\frac{x}{y}; f_y(x, y) = \left(\frac{x}{y}\right)^2 \sin\frac{x}{y}$

11 $f_x(x, y, z) = 6xz + y^2; f_y(x, y, z) = 2xy;$
$f_z(x, y, z) = 3x^2$

13 $f_r(r, s, t) = 2re^{2s}\cos t; f_s(r, s, t) = 2r^2e^{2s}\cos t;$
$f_t(r, s, t) = -r^2e^{2s}\sin t$

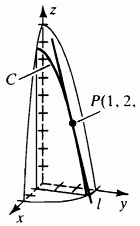
15 $f_x(x, y, z) = e^z - ye^x; f_y(x, y, z) = -e^x - ze^{-y};$
$f_z(x, y, z) = xe^z + e^{-y}$

17 $f_q(q, v, w) = \frac{v}{2\sqrt{qv}\sqrt{1 - qv}};$
$f_v(q, v, w) = \frac{q}{2\sqrt{qv}\sqrt{1 - qv}} + w\cos vw;$
$f_w(q, v, w) = v\cos vw$

19 $w_{xy} = w_{yx} = 4y^3 - 12xy^2$

21 $w_{xy} = w_{yx} = -6x^2e^{-2y} + 2y^{-3}\sin x$

23 $w_{xy} = w_{yx} = -\dfrac{2xz}{y^2}\sinh\dfrac{z}{y}$ **25** $18xy^2 + 16y^3z$
27 $t^2(\sec rt)(\sec^2 rt + \tan^2 rt)$
29 $(1 - x^2y^2z^2)\cos xyz - 3xyz \sin xyz$
31 $w_{rrs} = w_{rsr} = w_{srr} = 36r^2s^2t - 6st^2e^{rt}$
33 Show that $\dfrac{\partial^2 f}{\partial x^2} = \dfrac{y^2 - x^2}{(x^2+y^2)^2} = -\dfrac{\partial^2 f}{\partial y^2}$.
35 Show that $\dfrac{\partial^2 f}{\partial x^2} = -\cos x \sinh y - \sin x \cosh y = -\dfrac{\partial^2 f}{\partial y^2}$.
37 Show that $\dfrac{\partial^2 w}{\partial x^2} = -\cos(x - y) - \dfrac{1}{(x+y)^2} = \dfrac{\partial^2 w}{\partial y^2}$.
39 Show that $w_{xx} = -c^2e^{-c^2t}\sin cx = w_t$.
41 Show that $\dfrac{\partial^2 v}{\partial t^2} = a^2[-k^2(\sin akt)(\sin kx)] = a^2\dfrac{\partial^2 v}{\partial x^2}$.
43 Show that $u_x = 2x = v_y$ and $u_y = -2y = -v_x$.
45 Show that $u_x = e^x\cos y = v_y$ and $u_y = -e^x \sin y =$
47 $w_{xx}, w_{xy}, w_{xz}, w_{yx}, w_{yy}, w_{yz}, w_{zx}, w_{zy}, w_{zz}$
49 In deg/cm: **(a)** 200 **(b)** 400
51 In volts/in.: **(a)** $-\dfrac{100}{9}$ **(b)** $\dfrac{50}{9}$ **(c)** $-\dfrac{50}{9}$
53 $\dfrac{\partial C}{\partial x} \approx -36.58$ (μg/m³)/m is the rate at which the concentration changes in the horizontal direction at (2, 5). $\dfrac{\partial C}{\partial y} \approx -0.229$ (μg/m³)/m is the rate at which the concentration changes in the vertical direction at (2, 5).
55 **(a)** 3.57; 4.81; 4.98; $\lim_{p\to\infty}\dfrac{p}{0.8 + 0.2p} = 5$
(b) $\dfrac{p(p-1)}{[q + p(1-q)]^2}$
(c) $p(p - 1)$; as the number of processors increases, the rate of change of the speedup increases.
57 **(a)** $\partial T/\partial t = T_0\omega e^{-\lambda x}\cos(\omega t - \lambda x)$ is the rate of change of temperature with respect to time at depth x. $\partial T/\partial x = -T_0\lambda e^{-\lambda x}[\cos(\omega t - \lambda x) + \sin(\omega t - \lambda x)]$ is the rate of change of temperature with respect to the depth at time t. **(b)** Show that $k = \omega/(2\lambda^2)$
59 **(a)** $\partial V/\partial x = -0.112y$ mL/yr is the rate at which lung capacity decreases with age for an adult male.
(b) $\partial V/\partial y = 27.63 - 0.112x$ mL/cm is difficult to interpret because we usually think of adult height y as fixed instead of a function of age x.
61 $e_k = -a_{kk}$ for every k
63 $x = 1, y = t, z = -4t + 12$



65 0.007960028, 0.059882618; 0.007960033, 0.05983345
67 0.00256544, 0.12014571; 0.00256369, 0.12017305
69 1.8369; 4.1743

Exercises 12.4

1 **(a)** $10y\Delta y - x\Delta y - y\Delta x + 5(\Delta y)^2 - \Delta x\,\Delta y$
(b) $-y\,dx + (10y - x)\,dy$ **(c)** $\Delta x\,\Delta y - 5(\Delta y)^2$

Exer. 3–6: The expressions for ϵ_1 and ϵ_2 are not unique.
3 $\epsilon_1 = -3\,\Delta y$; $\epsilon_2 = 4\,\Delta y$
5 $\epsilon_1 = 3x\,\Delta x + (\Delta x)^2$; $\epsilon_2 = 3y\Delta y + (\Delta y)^2$
7 $(3x^2 - 2xy)\,dx + (-x^2 + 6y)\,dy$
9 $(2x \sin y)\,dx + (x^2\cos y + 3y^{1/2})\,dy$
11 $xe^{xy}(xy + 2)\,dx + (x^3e^{xy} - 2y^{-3})\,dy$
13 $[2x\ln(y^2 + z^2)]\,dx + \left(\dfrac{2x^2y}{y^2+z^2}\right)dy + \left(\dfrac{2x^2z}{y^2+z^2}\right)dz$
15 $\left[\dfrac{yz(y+z)}{(x+y+z)^2}\right]dx + \left[\dfrac{xz(x+z)}{(x+y+z)^2}\right]dy + \left[\dfrac{xy(x+y)}{(x+y+z)^2}\right]dz$
17 $(2xz - z^2t)\,dx + (4t^3)\,dy + (x^2 - 2xzt)\,dz + (12yt^2 - xz^2)\,dt$
19 7.38 **21** 1.87 **23** **(a)** $\pm\frac{1}{4}$ft² **(b)** $\pm\dfrac{47}{192}$ft³
25 **(a)** 380 lb **(b)** ±11.5% **27** ±0.0185
29 $\pm\dfrac{6W}{A - W}\%$ **31** ±7% **33** 2.96
35 Maximum error in x must not exceed ±2.9 ft.
37 $\pm1.7\pi$ in² **39** Use Theorem (12.17).
43 $(x_4, y_4) \approx (1.8460, 1.1546)$
45 $\begin{bmatrix} f_x & f_y & f_z \\ g_x & g_y & g_z \\ h_x & h_y & h_z \end{bmatrix}\begin{bmatrix}\Delta x \\ \Delta y \\ \Delta z\end{bmatrix} = \begin{bmatrix} -f \\ -g \\ -h\end{bmatrix}$

Exercises 12.5

1 $2x\sin(xy) + y(x^2 + y^2)\cos(xy)$;
$2y\sin(xy) + x(x^2+y^2)\cos(xy)$
3 $2r(\ln s)^2 + 8r\ln s + 2s\ln s$;
$\dfrac{2r^2\ln s}{s} + \dfrac{4r^2}{s} + 2r + 2r\ln s$
5 $3x^2e^{3y} + ye^x + 4x^3y^2$; $3x^3e^{3y} + e^x + 2x^4y$
7 $3\ln(uvt) + 3 + \dfrac{vt}{u}$; $t\ln(uvt) + \dfrac{3u}{v} + t$;
$v\ln(uvt) + \dfrac{3u}{t} + v$
9 $-34y + 6r - 24s$ **11** $\dfrac{-3(1 + t^2)}{(t+1)^4}$
13 $4\sin^3 t\cos t + \tan 4t \sin t - 4\cos t \sec^2 4t$
15 $-\dfrac{6x^2 + 2xy}{x^2 + 3y^2}$ **17** $\dfrac{12\sqrt{xy} + y}{6\sqrt{xy} - x}$
19 $-\dfrac{2z^3 + 2xy^2}{6xz^2 - 6yz + 4}$; $-\dfrac{2x^2y - 3z^2}{6xz^2 - 6yz + 4}$
21 $-\dfrac{e^{yz} - 2yze^{xz} + 3yze^{xy}}{xye^{yz} - 2xye^{xz} + 3e^{xy}}$; $-\dfrac{xze^{yz} - 2e^{xz} + 3xze^{xy}}{xye^{yz} - 2xye^{xz} + 3e^{xy}}$

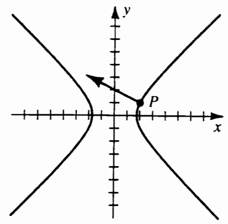
23 **(a)** $0.88\pi \approx 2.76$ in³/min **(b)** $0.3\pi \approx 0.94$ in²/min
25 $\dfrac{dT}{dt} = \dfrac{V}{k}\dfrac{dP}{dt} + \dfrac{P}{k}\dfrac{dV}{dt}$ **27** −6.4 in³/min
29 762.6 cm²/yr **33** 3 **35** 0

Exercises 12.6

1 $-\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$ **3** $3\mathbf{i} + 2\mathbf{j}$ **5** $-8\mathbf{i} + \mathbf{j} - 9\mathbf{k}$
7 0.154841, 0.154669, 0.154583
9 −0.229378, −0.114097, −0.056901
11 $-\dfrac{10}{\sqrt{2}} \approx -7.07$ **13** $-\dfrac{1}{8\sqrt{13}} \approx -0.03$
15 $\dfrac{67}{8\sqrt{26}} \approx 1.64$ **17** $\dfrac{1}{2\sqrt{26}} \approx 0.098$ **19** $16\sqrt{14} \approx 59.87$
21 $\dfrac{15e^{-2}}{\sqrt{35}} \approx 0.34$ **23** $-\dfrac{12}{\sqrt{10}} \approx -3.79$
25 **(a)** $-\dfrac{28}{\sqrt{26}}$ **(b)** $\dfrac{1}{\sqrt{5}}\mathbf{i} - \dfrac{2}{\sqrt{5}}\mathbf{j}$; $\sqrt{80}$
(c) $-\dfrac{1}{\sqrt{5}}\mathbf{i} + \dfrac{2}{\sqrt{5}}\mathbf{j}$; $-\sqrt{80}$
27 **(a)** $-\dfrac{25}{7\sqrt{22}}$ **(b)** $-\dfrac{2}{\sqrt{14}}\mathbf{i} + \dfrac{3}{\sqrt{14}}\mathbf{j} + \dfrac{1}{\sqrt{14}}\mathbf{k}$; 1
(c) $\dfrac{2}{\sqrt{14}}\mathbf{i} - \dfrac{3}{\sqrt{14}}\mathbf{j} - \dfrac{1}{\sqrt{14}}\mathbf{k}$; −1
29 **(a)** $-\dfrac{28}{\sqrt{2}}$ **(b)** The direction of $-12\mathbf{i} - 16\mathbf{j}$
(c) The direction of $12\mathbf{i} + 16\mathbf{j}$
(d) The direction of $4\mathbf{i} - 3\mathbf{j}$
31 **(a)** $-\dfrac{178}{\sqrt{14}}$ **(b)** The direction of $4\mathbf{i} - 8\mathbf{j} + 54\mathbf{k}$
(c) $\sqrt{2996} \approx 54.7$
33 **(b)** $\partial T/\partial r$ is the rate of change of temperature in the direction normal to the circular boundary.
35 **(b)** $\nabla f(1, 2) \approx 1.00003333\mathbf{i} - 0.11111235\mathbf{j}$;
$\nabla f(1, 2) = \mathbf{i} - \frac{1}{9}\mathbf{j}$
37 0.1294 **45** **(b)** $5 + \sqrt{3}$

Exercises 12.7

1 $16(x - 2) + 6(y + 3) + 6(z - 1) = 0$;
$x = 2 + 16t, y = -3 + 6t, z = 1 + 6t$
3 $16(x + 2) + 18(y + 1) + (z - 25) = 0$;
$x = -2 + 16t, y = -1 + 18t, z = 25 + t$
5 $4(x + 5) - 3(y - 5) + 20(z - 1) = 0$;
$x = -5 + 4t, y = 5 - 3t, z = 1 + 20t$
7 $x + \sqrt{3}\left(y - \dfrac{\pi}{3}\right) + (z - 1) = 0$; $x = t, y = \dfrac{\pi}{3} + \sqrt{3}t$,
$z = 1 + t$
9 $-x + \frac{1}{2}(y - 2) - (z - 1) = 0$; $x = -t, y = 2 + \frac{1}{2}t$,
$z = 1 - t$

11


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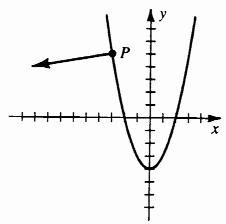

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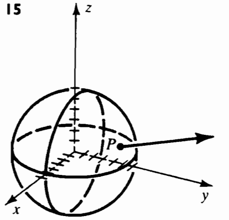

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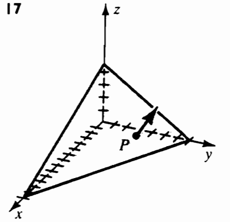

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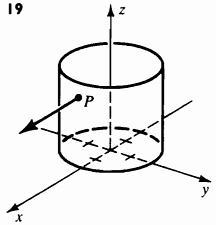
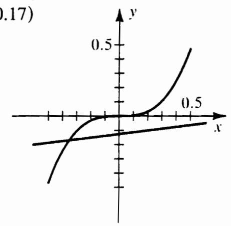


25 $\left(\dfrac{8\sqrt{2}}{\sqrt{5}}, \dfrac{2\sqrt{2}}{\sqrt{5}}, -\dfrac{2\sqrt{2}}{\sqrt{5}}\right)$,
$\left(-\dfrac{8\sqrt{2}}{\sqrt{5}}, -\dfrac{2\sqrt{2}}{\sqrt{5}}, \dfrac{2\sqrt{2}}{\sqrt{5}}\right)$
31 (1.2718, 1.1787),
(1.2729, 1.1746)
33 (0.8239, 0.1372),
(0.8210, 0.1384)

Exercises 12.8

1 Max: $f(-2, 1) = 4$ **3** Min: $f\left(\frac{1}{2}, -\frac{1}{4}\right) = -\frac{1}{2}$
5 Min: $f(0, 0) = 0$
7 SP: $(0, 0, f(0, 0))$; min: $f(1, -1) = -1$
9 SP: $(3, -2, f(3, -2))$
11 SP: $(2, 4, f(2, 4))$, $(-3, -4, f(-3, -4))$;
min: $f(2, -4) = -\dfrac{266}{3}$; max: $f(-3, 4) = \dfrac{617}{6}$
13 SP: $(0, 0, f(0, 0))$; min: $f(4, -8) = -64$,
$f(-1, 2) = -\dfrac{3}{2}$
15 SP: $(-2, -\sqrt{3}, f(-2, -\sqrt{3}))$;
min: $f(-2, \sqrt{3}) = -48 - 6\sqrt{3}$
17 No extrema or saddle points
19 Min: $f(\sqrt[3]{2}, 2\sqrt[3]{2}) = \dfrac{12}{\sqrt[3]{2}}$
21 $(0, 0)$, $(\pm 1, 0)$, $(0, \pm 1)$; min: $f(0, 0) = 0$;
max: $f(0, \pm 1) = \dfrac{3}{e}$

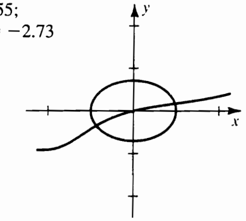
23 Min: $f\left(\frac{1}{2}, -\frac{1}{4}\right) = -\frac{1}{2}$; max: $f\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right) = 1 + \sqrt{2}$
25 Min: $f(0, 0) = 0$; max: $f(4, 3) = 67$
27 Min: $f(1, 2) = f(1, -1) = -1$; max: $f(-1, -2) = 13$
29 $\frac{1}{\sqrt{26}}$
31 $\left(\frac{2}{\sqrt[4]{12}}, \sqrt[4]{12}, \pm\frac{2\sqrt{2}}{\sqrt[4]{12}}\right), \left(-\frac{2}{\sqrt[4]{12}}, -\sqrt[4]{12}, \pm\frac{2\sqrt{2}}{\sqrt[4]{12}}\right)$
33 Square base, altitude $\frac{1}{2}$ the length of the side of the base
35 $\frac{8}{\sqrt{3}}, \frac{6}{\sqrt{3}}, \frac{12}{\sqrt{3}}$ **37** $1, \frac{4}{3}, 4$
39 Square base of side $\sqrt[3]{4}$ ft, height $2\sqrt[3]{4}$ ft
41 (18 in.) × (18 in.) × (36 in.)
43 $\left(2 - \frac{2}{3}\sqrt{3}, 2 - \frac{2}{3}\sqrt{3}\right)$ **47** $y = \frac{1}{2}x + \frac{8}{3}$
49 $y = mx + b$ with $m \approx 1.23$, $b \approx -18.09$; grade of 68
51 $\left(\frac{14}{3}, \frac{11}{3}\right)$
53 (b) $4x - y - 2z + 1 = 0$
55 $(-0.35, -0.17)$



Exercises 12.9

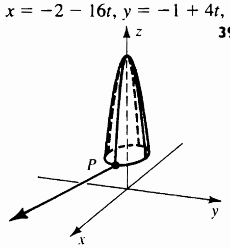
1 Min: $f\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right) = f\left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right) = 0$;
max: $f\left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right) = f\left(-\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) = 5$
3 Min: $f\left(-\frac{5}{\sqrt{3}}, -\frac{5}{\sqrt{3}}, -\frac{5}{\sqrt{3}}\right) = -5\sqrt{3}$;
max: $f\left(\frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}\right) = 5\sqrt{3}$
5 Min: $f\left(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right) = \frac{1}{3}$
7 Min: $f(0, -1, 0) = 1$ and $f(2, 1, 0) = 5$
9 Min: $f\left(1, \frac{2}{\sqrt{3}}, -1, \frac{8}{3}\right) = -\frac{16}{3\sqrt{3}}$;
max: $f\left(1, -\frac{2}{\sqrt{3}}, -1, \frac{8}{3}\right) = \frac{16}{3\sqrt{3}}$
11 $\left(\frac{6}{\sqrt{29}}, \frac{9}{\sqrt{29}}, \frac{12}{\sqrt{29}}\right)$

13 Square base of side $\frac{2}{\sqrt[3]{7}}$, height $\frac{7}{2\sqrt[3]{7}}$
15 $\frac{8}{3}$ **17** Height is twice the radius.
21 Width $= 8\sqrt{3}$ in., depth $= \frac{16}{3}\sqrt{6}$ in.
23 Max: $f(0.97, 0.17) \approx 1.55$;
min: $f(-0.87, -0.35) \approx -2.73$



Chapter 12 Review Exercises

1 $\{(x, y): 4x^2 - 9y^2 \le 36\}$;
the hyperbola $9y^2 - 4x^2 = 108$
3 $\{(x, y, z): z^2 > x^2 + y^2\}$;
the hyperboloid of two sheets $z^2 - x^2 - y^2 = 1$
5 $\frac{5}{4}$ **7** DNE **9** Halves of four-leafed roses; DNE
11 $f_x(x, y) = 3x^2 \cos y + 4$; $f_y(x, y) = -x^3 \sin y - 2y$
13 $f_x(x, y, z) = -\frac{2x}{y^2 + z^2}$; $f_y(x, y, z) = \frac{2y(z^2 - x^2)}{(y^2 + z^2)^2}$;
$f_z(x, y, z) = -\frac{2z(x^2 + y^2)}{(y^2 + z^2)^2}$
15 $f_x(x, y, z, t) = 2xz\sqrt{2y + t}$; $f_y(x, y, z, t) = \frac{x^2 z}{\sqrt{2y + t}}$;
$f_z(x, y, z, t) = x^2\sqrt{2y + t}$; $f_t(x, y, z, t) = \frac{x^2 z}{2\sqrt{2y + t}}$
17 $f_{xx}(x, y) = 6xy^2 + 12x^2$;
$f_{xy}(x, y) = f_{yx}(x, y) = 6x^2y - 9y^2$; $f_{yy}(x, y) = 2x^3 - 18xy$
21 (a) $(2x + 3y)\Delta x + (3x - 2y)\Delta y + (\Delta x)^2 + 3\Delta x\Delta y - (\Delta y)^2$;
$(2x + 3y)\,dx + (3x - 2y)\,dy$ **(b)** -1.13; -1.1
25 $12x + 18y$; $18x - 22y$ **27** $3e^{-t}(9t^2 \cos 3t^3 - \sin 3t^3)$
29 $\frac{z^3 - 2xy}{\cos y - 3xz^2}$; $\frac{z \sin y - x^2}{\cos y - 3xz^2}$ **31 (a)** $-\frac{14}{\sqrt{41}}$ **(b)** 14
33 $-16(x + 2) + 4(y + 1) - 7(z - 2) = 0$;
$x = -2 - 16t$, $y = -1 + 4t$, $z = 2 - 7t$
37



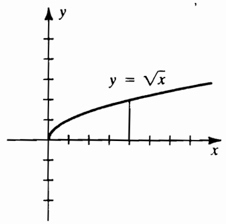
39 Min: $f(0, -1) = -2$

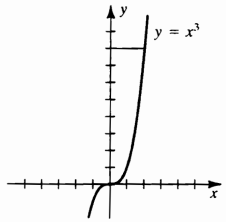
41 Min: $f\left(-\sqrt{\frac{8}{3}}, -\sqrt{\frac{2}{3}}, -\sqrt{\frac{4}{3}}\right)$
$= f\left(-\sqrt{\frac{8}{3}}, \sqrt{\frac{2}{3}}, \sqrt{\frac{4}{3}}\right)$
$= f\left(\sqrt{\frac{8}{3}}, -\sqrt{\frac{2}{3}}, \sqrt{\frac{4}{3}}\right)$
$= f\left(\sqrt{\frac{8}{3}}, \sqrt{\frac{2}{3}}, -\sqrt{\frac{4}{3}}\right) = -\frac{8}{9}\sqrt{3}$;
max: $f\left(\sqrt{\frac{8}{3}}, \sqrt{\frac{2}{3}}, \sqrt{\frac{4}{3}}\right) = f\left(-\sqrt{\frac{8}{3}}, -\sqrt{\frac{2}{3}}, \sqrt{\frac{4}{3}}\right)$
$= f\left(-\sqrt{\frac{8}{3}}, \sqrt{\frac{2}{3}}, -\sqrt{\frac{4}{3}}\right)$
$= f\left(\sqrt{\frac{8}{3}}, -\sqrt{\frac{2}{3}}, -\sqrt{\frac{4}{3}}\right)$
$= \frac{8}{9}\sqrt{3}$
43 $(0, 4, 4)$ **45** $\sqrt{\frac{TC}{k_0 k_1}}$
51 $k = \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}$
53 (a) -2.351176, -2.314361, -2.296035
(b) -2.277764
55 $(4.3418, 3.0227)$, $(4.8003, 2.5887)$
57 $F(\psi, \theta, \varphi) \approx \theta$, $G(\psi, \theta, \varphi) \approx \varphi$, $H(\psi, \theta, \varphi) \approx \psi$,
$K(\psi, \theta, \varphi) \approx 1$

CHAPTER ▪ 13

Exercises 13.1

1 R_x **3** R_y **5** Neither **7** R_x and R_y **9** Neither
11 (a) 39 **(b)** 81 **(c)** 60 **13** -36 **15** $\frac{163}{120}$ **17** $\frac{36}{5}$
19 $\frac{1}{2}(4e - e^4) \approx -21.86$
21 (a) $\int_0^4 \int_0^{\sqrt{x}} f(x, y)\,dy\,dx$ **23 (a)** $\int_0^2 \int_{x^3}^8 f(x, y)\,dy\,dx$
(b) $\int_0^2 \int_{y^2}^4 f(x, y)\,dx\,dy$ **(b)** $\int_0^8 \int_0^{y^{1/3}} f(x, y)\,dx\,dy$





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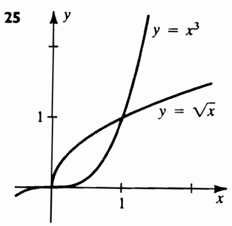
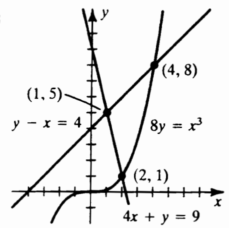


(a) $\int_0^1 \int_{x^3}^{\sqrt{x}} f(x, y)\,dy\,dx$
(b) $\int_0^1 \int_{y^2}^{y^{1/3}} f(x, y)\,dx\,dy$

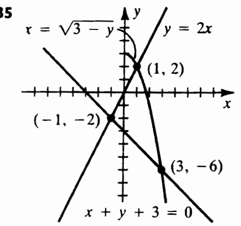
Exer. 27–32: Answers are not unique.

27 $\int_{-1}^{4} \int_{-1}^{2} (y + 2x)\,dx\,dy = \frac{75}{2}$
29 $\int_0^1 \int_{-2y}^{3y} xy^2 dx\,dy = \frac{1}{2}$
31 $\int_0^2 \int_0^{x^2} x^3 \cos xy\,dy\,dx = \frac{1}{3}(1 - \cos 8) \approx 0.38$
33

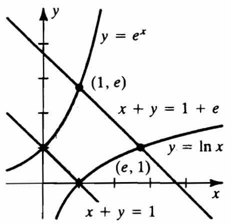


(a) $\int_1^2 \int_{9-4x}^{x+4} f(x, y)\,dy\,dx + \int_2^4 \int_{x^3/8}^{x+4} f(x, y)\,dy\,dx$
(b) $\int_1^5 \int_{(9-y)/4}^{2y^{1/3}} f(x, y)\,dx\,dy + \int_5^8 \int_{y-4}^{2y^{1/3}} f(x, y)\,dx\,dy$

35

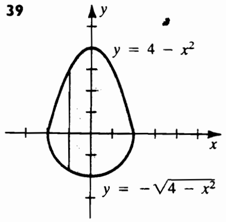


(a) $\int_{-1}^{1} \int_{-x-3}^{2x} f(x, y)\,dy\,dx + \int_1^3 \int_{-x-3}^{3-x^2} f(x, y)\,dy\,dx$
(b) $\int_{-6}^{-2} \int_{-y-3}^{\sqrt{3-y}} f(x, y)\,dx\,dy + \int_{-2}^{2} \int_{y/2}^{\sqrt{3-y}} f(x, y)\,dx\,dy$

37


(a) $\int_0^1 \int_{1-x}^{e^x} f(x, y)\, dy\, dx + \int_1^e \int_{\ln x}^{1+e-x} f(x, y)\, dy\, dx$

(b) $\int_0^1 \int_{1-y}^{e^y} f(x, y)\, dx\, dy + \int_1^e \int_{\ln y}^{1+e-y} f(x, y)\, dx\, dy$

39


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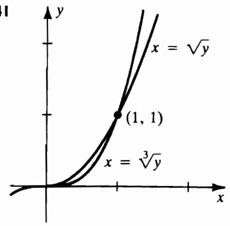

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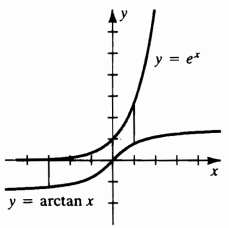


45 $\int_0^2 \int_0^{y/2} e^{(y^2)} dx\, dy = \frac{1}{4}(e^4 - 1) \approx 13.40$

47 $\int_0^4 \int_0^{\sqrt{x}} y \cos x^2\, dy\, dx = \frac{1}{4}\sin 16 \approx -0.07$

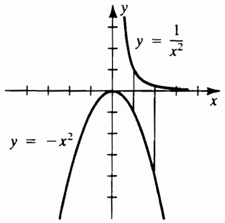
49 $\int_0^2 \int_0^{x^3} \frac{y}{\sqrt{16 + x^7}}\, dy\, dx = \frac{8}{7}$

51 1.10 **53** 2.35 **55** 0.62

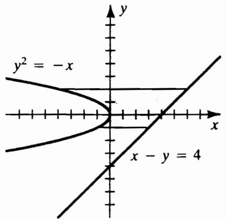
Exercises 13.2

1 $\int_0^5 \int_{-x}^{4x-x^2} dy\, dx$ **3** $\int_{-3}^3 \int_{-\sqrt{9-y^2}}^{(9-y)/2} dx\, dy$

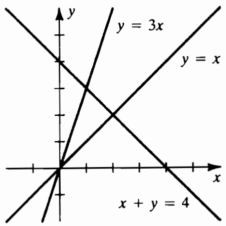
5 $\int_1^2 \int_{-x^2}^{1/x^2} dy\, dx = \frac{17}{6}$



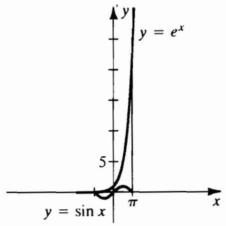
7 $\int_{-1}^2 \int_{-y^2}^{y+4} dx\, dy = \frac{33}{2}$



9 $\int_0^1 \int_x^{3x} dy\, dx + \int_1^2 \int_x^{4-x} dy\, dx = 2$



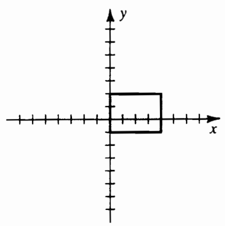
11 $\int_{-\pi}^{\pi} \int_{\sin x}^{e^x} dy\, dx = e^{\pi} - e^{-\pi} \approx 23.10$



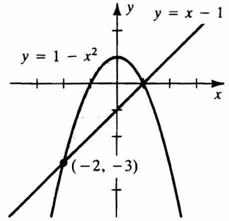
13 $\int_0^3 \int_0^{(12-4x)/3} \frac{1}{12}(60 - 20x - 15y)\, dy\, dx$

15 $\int_0^2 \int_0^{4-y^2} (6 - x)\, dx\, dy$

17 The plane $z = 3$

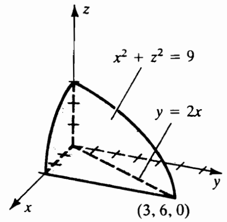


19 The paraboloid $z = x^2 + y^2$

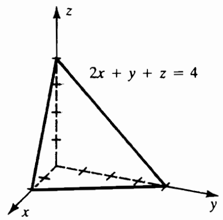


21 $\frac{34}{3}$

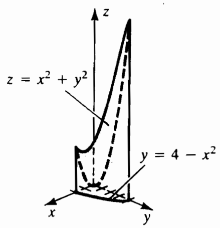
23 $\int_0^3 \int_0^{2x} (9 - x^2)^{1/2}\, dy\, dx = 18$



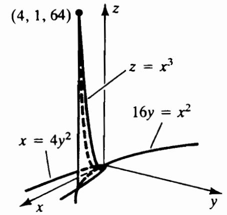
25 $\int_0^2 \int_0^{4-2x} (4 - 2x - y)\, dy\, dx = \frac{16}{3}$



27 $\int_0^2 \int_0^{4-x^2} (x^2 + y^2)\, dy\, dx = \frac{832}{35}$



29 $\int_0^4 \int_{x^2/16}^{\sqrt{x}/2} x^3\, dy\, dx = \frac{128}{9}$



31 $\int_{-1}^2 \int_{2-x}^{4-x^2} (x^2 + 4)\, dy\, dx = \frac{423}{20}$

33 0.417 **35** 0.344

37 0.791 **39** −1.281

Exercises 13.3

1 $2\int_0^{\pi/2} \int_0^{4\sin\theta} r\, dr\, d\theta$ **3** $2\int_0^{2\pi/3} \int_0^{1+2\cos\theta} r\, dr\, d\theta$

5 $4\int_0^{\pi/4} \int_0^{2\sqrt{\cos 2\theta}} r\, dr\, d\theta$ **7** $\int_0^{\pi/3} \int_0^{4\sin 3\theta} r\, dr\, d\theta = \frac{4\pi}{3}$

9 $2\int_{2\pi/3}^{\pi} \int_3^{2-2\cos\theta} r\, dr\, d\theta = \frac{9}{2}\sqrt{3} - \pi \approx 4.65$

11 $2\int_0^{\pi/4} \int_0^{3\sqrt{\cos 2\theta}} r\, dr\, d\theta = \frac{9}{2}$

13 $\int_0^{2\pi} \int_0^2 (r^3) r\, dr\, d\theta = \frac{64\pi}{5}$

15 $\int_0^{2\pi} \int_a^b (\cos^2\theta) r\, dr\, d\theta = \frac{\pi}{2}(b^2 - a^2)$

17 $\int_0^{\pi/4} \int_0^{3\sec\theta} (r) r\, dr\, d\theta = \frac{9}{2}[\sqrt{2} + \ln(\sqrt{2} + 1)] \approx 10.33$

19 $\int_0^{\pi} \int_0^a e^{-r^2} r\, dr\, d\theta = \frac{\pi}{2}(1 - e^{-a^2})$

21 $\int_0^{\pi/4} \int_{\sec\theta}^{2\sec\theta} \left(\frac{1}{r}\right) r\, dr\, d\theta = \ln(\sqrt{2} + 1) \approx 0.88$

23 $\int_0^{\pi/2} \int_0^2 \cos(r^2) r\, dr\, d\theta = \frac{\pi}{4}\sin 4 \approx -0.59$

25 $8\int_0^{\pi/2} \int_3^5 (25 - r^2)^{1/2} r\, dr\, d\theta = \frac{256\pi}{3}$

27 $2\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} (r) r\, dr\, d\theta = \frac{64}{9}$

29 $4\int_0^{\pi/2}\int_0^{4\sin\theta}(16-r^2)^{1/2}r\,dr\,d\theta = \frac{128}{9}(3\pi-4) \approx 77.15$
31 π

33 $\int_0^2\int_0^{\pi/2}\sqrt{1+r^4}\,r\,d\theta\,dr \approx 7.299$

35 $\int_0^3\int_0^{\pi/2}(\cos r)\,r\,d\theta\,dr \approx -2.461$

37 $\int_0^1\int_0^{2\pi}\sqrt{r^6+1}\,r\,d\theta\,dr \approx 3.492$

Exercises 13.4

1 $4\int_0^1\int_0^1\sqrt{\left(\frac{-x}{\sqrt{4-x^2-y^2}}\right)^2+\left(\frac{-y}{\sqrt{4-x^2-y^2}}\right)^2+1}\,dy\,dx$

3 $4\int_0^3\int_0^{\sqrt{9-x^2}}\sqrt{\left(\frac{8x}{3\sqrt{16x^2+9y^2+144}}\right)^2+\left(\frac{3y}{2\sqrt{16x^2+9y^2+144}}\right)^2+1}\,dy\,dx$

5 $\int_0^1\int_0^1\sqrt{(x)^2+(1)^2+1}\,dy\,dx$
$= \frac{1}{2}[\sqrt{3}+2\ln(1+\sqrt{3})-\ln 2] \approx 1.52$

7 $\pi ck^2\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}$

9 $\frac{\pi}{6}(5^{3/2}-1) \approx 5.33$

11 $2a^2(\pi-2)$ **13** 247.4 ft²
15 1.205 **17** 5.800 **19** 1.559

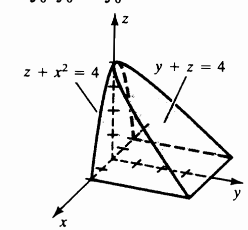
Exercises 13.5

1 $\frac{39}{2}$ **3** $-\frac{1}{12}$ **5** $\frac{513}{8}$

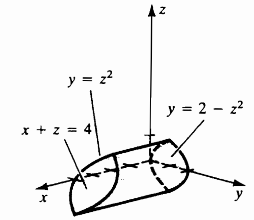
7 $\int_0^6\int_0^{(6-x)/2}\int_0^{(6-x-2y)/3} f(x,y,z)\,dz\,dy\,dx;$
$\int_0^3\int_0^{6-2y}\int_0^{(6-x-2y)/3} f(x,y,z)\,dz\,dx\,dy;$
$\int_0^3\int_0^{(6-2y)/3}\int_0^{6-2y-3z} f(x,y,z)\,dx\,dz\,dy;$
$\int_0^2\int_0^{(6-3z)/2}\int_0^{6-2y-3z} f(x,y,z)\,dx\,dy\,dz;$
$\int_0^6\int_0^{(6-x)/3}\int_0^{(6-x-3z)/2} f(x,y,z)\,dy\,dz\,dx;$
$\int_0^2\int_0^{6-3z}\int_0^{(6-x-3z)/2} f(x,y,z)\,dy\,dx\,dz$

9 $\int_{-3/2}^{3/2}\int_{-\sqrt{9-4x^2}}^{\sqrt{9-4x^2}}\int_0^{9-4x^2-y^2} f(x,y,z)\,dz\,dy\,dx;$
$\int_{-3}^{3}\int_{-\sqrt{9-y^2}/2}^{\sqrt{9-y^2}/2}\int_0^{9-4x^2-y^2} f(x,y,z)\,dz\,dx\,dy;$
$\int_{-3}^{3}\int_0^{9-y^2}\int_{-\sqrt{9-z-y^2}/2}^{\sqrt{9-z-y^2}/2} f(x,y,z)\,dx\,dz\,dy;$
$\int_0^9\int_{-\sqrt{9-z}}^{\sqrt{9-z}}\int_{-\sqrt{9-z-y^2}/2}^{\sqrt{9-z-y^2}/2} f(x,y,z)\,dx\,dy\,dz;$
$\int_{-3/2}^{3/2}\int_0^{9-4x^2}\int_{-\sqrt{9-4x^2-z}}^{\sqrt{9-4x^2-z}} f(x,y,z)\,dy\,dz\,dx;$
$\int_0^9\int_{-\sqrt{9-z}/2}^{\sqrt{9-z}/2}\int_{-\sqrt{9-4x^2-z}}^{\sqrt{9-4x^2-z}} f(x,y,z)\,dy\,dx\,dz$

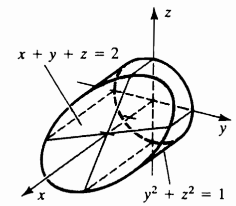
11 $2\int_0^4\int_0^{4-y}\int_0^{\sqrt{4-z}} dx\,dz\,dy = \frac{128}{5}$



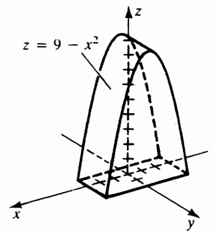
13 $\int_{-1}^{1}\int_{z^2}^{2-z^2}\int_0^{4-z} dx\,dy\,dz = \frac{32}{3}$



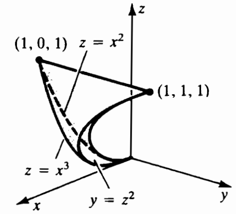
15 $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_0^{2-y-z} dx\,dy\,dz = 2\pi$



17 $\int_{-3}^{3}\int_{-1}^{2}\int_0^{9-x^2} dz\,dy\,dx = 108$



19 $\int_0^1\int_{x^3}^{x^2}\int_0^{z^2} dy\,dz\,dx = \frac{1}{70}$



21 $\frac{1}{6}abc$
23 The region bounded by the planes $z=0$, $z=1$, $x=2$, $x=3$ and the cylinders $y=\sqrt{1-z}$ and $y=\sqrt{4-z}$
25 The region under the plane $z=x+y$ and over the region in the xy-plane bounded by the parabola $y=x^2$ and the line $y=2x$
27 The region bounded by the paraboloid $z=x^2+y^2$ and the planes $z=1$ and $z=2$

29 $\int_0^1\int_0^{e^{-x}} y^2\,dy\,dx$

31 $\int_0^2\int_0^{4-2y}\int_0^{4-x-2y}(x^2+y^2)\,dz\,dx\,dy$

33 1.1685×10^9 kg **35** 0.77

Exercises 13.6

1 $m = \frac{2349}{20}$; $\bar{x} = \frac{1290}{203}$, $\bar{y} = \frac{38}{29}$

3 $m = 8k$ (k a proportionality constant); $\bar{x}=0$, $\bar{y}=\frac{8}{3}$

5 $m = \frac{1}{4}(1-e^{-2}) \approx 0.22$; $\bar{x}=0$, $\bar{y} = \frac{4(1-e^{-3})}{9(1-e^{-2})} \approx 0.49$

7 $m = 4\ln(\sqrt{2}+1) - 4\ln(\sqrt{2}-1) - \pi \approx 3.91$;
$\bar{x}=0$, $\bar{y} = \frac{16-\pi}{4m} \approx 0.82$

9 $3^5\left(\frac{31}{28}\right)$; $3^7\left(\frac{19}{8}\right)$; $3^5\left(\frac{1259}{56}\right)$ **11** $64k$; $\frac{32}{3}k$; $\frac{224}{3}k$

13 (δ = density) **(a)** $\frac{1}{3}a^4\delta$ **(b)** $\frac{1}{12}a^4\delta$ **(c)** $\frac{1}{6}a^4\delta$ **15** $\frac{a}{\sqrt{3}}$

17 $\bar{x}=\bar{y}=\bar{z}=\frac{7}{12}a$ (with fixed corner at O)

19 $m = \int_0^4\int_{-\sqrt{x}/2}^{\sqrt{x}/2}\int_{-\sqrt{x-4z^2}}^{\sqrt{x-4z^2}}(x^2+z^2)\,dy\,dz\,dx$; the integrals for M_{yz}, M_{xz}, and M_{xy} have the same limits, but the integrands are $x(x^2+z^2)$, $y(x^2+z^2)$, and $z(x^2+z^2)$, respectively.

21 $m = \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2-y^2}} dz\,dy\,dx$; the integral for M_{xy} has the same limits, but the integrand is z. By symmetry, $\bar{x}=\bar{y}=0$.

23 **(a)** $m = \int_0^3\int_0^{9-x^2}\int_0^{6-2x} dy\,dz\,dx$; the integrals for M_{yz}, M_{xz}, and M_{xy} have the same limits, but the integrands are x, y, and z, respectively.
(b) $m = \frac{135}{2}$; $M_{yz} = \frac{567}{10}$; $M_{xz} = \frac{729}{5}$; $M_{xy} = \frac{2673}{10}$;
$\bar{x} = \frac{21}{25}$, $\bar{y} = \frac{54}{25}$, $\bar{z} = \frac{99}{25}$

25 $I_z = \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}}(x^2+y^2)(x^2+y^2+z^2)\,dz\,dy\,dx$

27 $I_z = \int_0^a\int_0^{b(1-x/a)}\int_0^{c(1-x/a-y/b)}(x^2+y^2)\,\delta\,dz\,dy\,dx$

Exercises 13.7

1 **(a)** The right circular cylinder of radius 4 with axis along the z-axis **(b)** The yz-plane
(c) The plane parallel to the xy-plane with z-intercept 1
3 The plane parallel to the yz-plane with x-intercept -3
5 A paraboloid with vertex (0, 0, 0) and opening upward
7 The right circular cylinder with trace $x^2+(y-3)^2=9$ in the xy-plane
9 The cone $z^2 = 4x^2 + 4y^2$
11 The sphere with center at the origin and radius 3
13 The cylinder with trace $y^2 = 2x$ in the xy-plane and rulings parallel to the z-axis
15 $r^2+z^2=4$ **17** $3r\cos\theta + r\sin\theta - 4z = 12$
19 $r=2$ **21** $r=2z$ **23** $r^2\sin^2\theta + z^2 = 9$

25 $\int_0^{2\pi}\int_0^4\int_0^{16-r^2} f(r,\theta,z)\,r\,dz\,dr\,d\theta$

27 $\int_0^{2\pi}\int_0^3\int_0^4 f(r,\theta,z)\,r\,dz\,dr\,d\theta +$
$\int_0^{2\pi}\int_3^4\int_0^{\sqrt{25-r^2}} f(r,\theta,z)\,r\,dz\,dr\,d\theta$

29 **(a)** 8π **(b)** $\left(0,0,\frac{4}{3}\right)$

31 **(a)** $\frac{1}{2}\pi ha^4\delta$ **(b)** $\pi ha^2\left(\frac{1}{4}a^2+\frac{1}{3}h^2\right)\delta$ **33** $\frac{1}{4}k\pi^2a^4$

35 $\frac{1}{8}k\pi^2a^6$ **37** 215,360 kg **39** $\frac{7\pi}{16}$

41 (a) $\int_0^{2\pi}\int_0^{2}\int_0^{4} ze^{-(r^2+5r^2\cos\theta\sin\theta)}r\,dz\,dr\,d\theta$ **(b)** 973.947

43 (a) $\int_0^{\pi/2}\int_0^{2}\int_0^{4}\sqrt{r^2+2r^2\cos\theta\sin\theta+z^3}\,r\,dz\,dr\,d\theta$
(b) 48.848

Exercises 13.8

1 (a) $(0, 2, 2\sqrt{3})$ **(b)** $\left(2, \frac{\pi}{2}, 2\sqrt{3}\right)$

3 (a) $\left(\sqrt{10}, \cos^{-1}\left(\frac{-2}{\sqrt{5}}\right), \frac{\pi}{4}\right)$ **(b)** $\left(\sqrt{2}, \frac{\pi}{4}, -2\sqrt{2}\right)$

5 (a) The sphere of radius 3 and center O
(b) A half-cone with vertex O and vertex angle $\pi/3$
(c) A half-plane with edge on the z-axis and making an angle of $\pi/3$ with the xz-plane
7 The sphere of radius 2 and center (0, 0, 2)
9 The plane $z = 3$
11 The sphere of radius 3 and center (3, 0, 0)
13 The plane $y = 5$
15 The right circular cylinder of radius 5 with axis along the z-axis
17 The cone $x^2 + y^2 = 4z^2$
19 The paraboloid $6z = x^2 + y^2$ **21** $\rho = 2$
23 $\rho(3\sin\phi\cos\theta + \sin\phi\sin\theta - 4\cos\phi) = 12$
25 $\rho = 2\csc\phi$ **27** $\tan^2\phi = 4$
29 $\rho^2(\sin^2\phi\sin^2\theta + \cos^2\phi) = 9$

31 $\frac{1}{2}k\pi a^4$ (k a proportionality constant); center of mass is $\frac{2}{5}a$ from base along the axis of symmetry.

33 $\frac{2}{9}k\pi a^6$ **35** $\frac{16\pi}{3}$ **37** $\frac{124}{5}k\pi$

39 $\frac{256\pi}{5}(\sqrt{2}-1) \approx 66.63$

41 (a) $(-3\sqrt{2}, 3\sqrt{6}, -6\sqrt{2})$
(b) θ should be increased by 105°, ϕ decreased by $\left(45 + \arctan\frac{8}{\sqrt{128}}\right) \approx 80.26°$, and L increased by $\sqrt{192} - 12 \approx 1.86$ in.

43 (a) $\int_0^{\pi}\int_0^{\pi/2}\int_0^{3}\sqrt{\rho^2}\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$ **(b)** 63.617

Exercises 13.9

1 (a) Vertical lines; horizontal lines **(b)** $x = \frac{1}{3}u$, $y = \frac{1}{5}v$

3 (a) Lines with slopes 1 and $-\frac{2}{3}$

(b) $x = \frac{3}{5}u + \frac{1}{5}v$, $y = -\frac{2}{5}u + \frac{1}{5}v$

5 (a) Vertical lines; lines with slope -1
(b) $x = u^{1/3}$, $y = v - u^{1/3}$
7 (a) Vertical lines; horizontal lines **(b)** $x = \ln u$, $y = \ln v$
9 (a) The rectangle with vertices (0, 0), (6, 0), (6, 5), (0, 5)
(b) The ellipse $\frac{u^2}{9} + \frac{v^2}{25} = 1$
11 (a) The triangle with vertices (0, 0), (−1, 3), (2, 4)
(b) The line $-u + 3v = 5$
13 $4u^2 + 4v^2$ **15** $2u(uv - 1)e^{-(2u+v)}$ **17** −6

19 $\int_0^2\int_0^{2-u}(v-u)\,2\,dv\,du$ **21** $\int_{-1}^{1}\int_{-\sqrt{1-v^2}}^{\sqrt{1-v^2}}(u^2+v^2)6\,du\,dv$

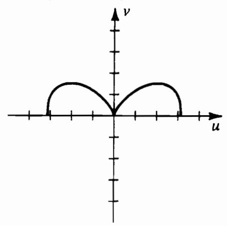
23 $\int_1^3\int_{-1}^{1}(u^2\cos^2 v)\frac{1}{2}du\,dv = \frac{1}{3} + \frac{1}{12}\sin 6 - \frac{1}{12}\sin 2 \approx 0.23$

25 $\int_1^2\int_{\sqrt{u}}^{\sqrt{2u}}\left(\frac{u^2}{v^2} + 2v^2\right)\frac{1}{v}dv\,du = \frac{15}{8}$

27 $-\int_2^4\int_{-u/2}^{2u}\left(\frac{v}{u}\right)\left(-\frac{1}{5}\right)dv\,du = \frac{9}{4}$ **29** 1.08×10^{12} km³

31 (a) (1, 1, 1) **(b)** $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\sqrt{2}\right)$
(c) $\left(\frac{1}{\sqrt{2}} - 1, \frac{1}{\sqrt{2}} + 1, 0\right)$

37

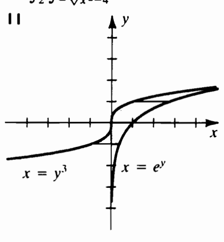


Chapter 13 Review Exercises

1 $-\frac{5}{84}$ **3** $\frac{63}{4}$ **5** $-\frac{107}{210}$

7 $\int_2^4\int_{-\sqrt{x^2-4}}^{\sqrt{x^2-4}}f(x, y)\,dy\,dx$ **9** $\int_{-2}^{2}\int_{y^2-4}^{4-y^2}f(x, y)\,dx\,dy$

11



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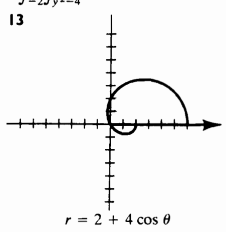


15 $\frac{1}{4}(1 - e^{-81}) \approx 0.25$ **17** 13 **19** $4\sqrt{2\pi} \approx 17.77$

21 $2\ln 2 - \frac{3}{4} \approx 0.64$

23 $\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{4\sqrt{2}}(\rho\cdot\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta = 64(2-\sqrt{2})\pi \approx 117.78$

25 $9k$ (k a proportionality constant); $\left(\frac{9}{4}, \frac{27}{8}\right)$

27 $2\pi k$ **29** $\frac{1}{20}ab^4k$ **31** $\frac{256}{15}$; $\left(0, \frac{8}{7}, \frac{12}{7}\right)$

33 $I_y = \int_0^4\int_{-\sqrt{1+y^2}}^{\sqrt{1+y^2}}\int_{-\sqrt{1-x^2+y^2}}^{\sqrt{1-x^2+y^2}}k(x^2+z^2)\sqrt{x^2+z^2}\,dz\,dx\,dy$

35 πa^4k
37 Rectangular: $(-3\sqrt{2}, 3\sqrt{2}, 6\sqrt{3})$;
cylindrical: $\left(6, \frac{3\pi}{4}, 6\sqrt{3}\right)$
39 $z = 9 - 3x^2 - 3y^2$; a paraboloid with vertex (0, 0, 9) and opening downward
41 $x^2 + y^2 = 16$; a right circular cylinder of radius 4 with axis along the z-axis
43 $\sqrt{x^2+y^2+z^2}(\sqrt{x^2+y^2+z^2} - 3) = 0$; the sphere of radius 3 with center at the origin, together with its center
45 (a) $z = r^2\cos 2\theta$ **(b)** $\cos\phi = \rho\sin^2\phi\cos 2\theta$
47 (a) $2r\cos\theta + r\sin\theta - 3z = 4$
(b) $2\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta - 3\rho\cos\phi = 4$

49 (a) $\int_0^4\int_0^{\sqrt{25-x^2}}dy\,dx$ **(b)** $\int_0^3\int_0^4 dx\,dy + \int_3^5\int_0^{\sqrt{25-y^2}}dx\,dy$

(c) $\int_0^{\arctan(3/4)}\int_0^{4\sec\theta}r\,dr\,d\theta + \int_{\arctan(3/4)}^{\pi/2}\int_0^5 r\,dr\,d\theta$

51 (a) $\int_0^5\int_0^{\sqrt{25-x^2}}\int_0^{\sqrt{25-x^2-y^2}}dz\,dy\,dx - \int_0^4\int_0^{\sqrt{16-x^2}}\int_3^{\sqrt{25-x^2-y^2}}dz\,dy\,dx$

(b) $\int_0^{\pi/2}\int_0^4\int_0^3 r\,dz\,dr\,d\theta + \int_0^{\pi/2}\int_4^5\int_0^{\sqrt{25-r^2}}r\,dz\,dr\,d\theta$

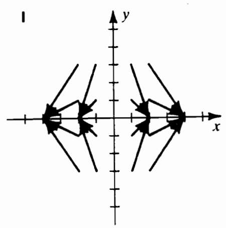
(c) $\int_0^{\pi/2}\int_0^{\arctan(4/3)}\int_0^{3\sec\phi}\rho^2\sin\phi\,d\rho\,d\phi\,d\theta + \int_0^{\pi/2}\int_{\arctan(4/3)}^{\pi/2}\int_0^5\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

53 $e - 1 \approx 1.72$

CHAPTER ■ 14

Exercises 14.1

1



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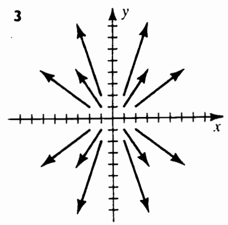

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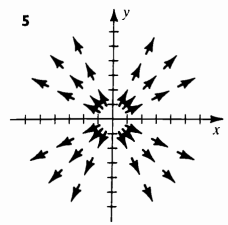

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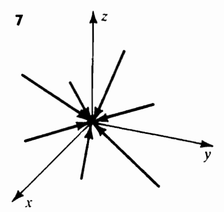

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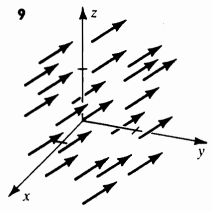


13 $\mathbf{F}(x, y, z) = 2x\mathbf{i} - 6y\mathbf{j} + 8z\mathbf{k}$
15 $\mathbf{F}(x, y) = \frac{1}{1 + x^2y^2}(y\mathbf{i} + x\mathbf{j})$
17 $\mathbf{i} + x^2\mathbf{j} + y^2\mathbf{k}$; $2xz + 2xy + 2$
19 $-y^2\cos z\,\mathbf{i} + (6xyz - e^{2z})\mathbf{j} - 3xz^2\mathbf{k}$; $3yz^2 + 2y\sin z + 2xe^{2z}$
39 0.145 **41** −1.807

Exercises 14.2

1 $14(2^{3/2} - 1) \approx 25.60$; 21; 14 **3** 3.8185; 3.1918; 0.2550

5 $\frac{34}{7}$ **7** $-\frac{16}{3}$ **9** −0.060

11 (a) $\frac{15}{2}$ **(b)** 6 **(c)** 7 **(d)** $\frac{29}{4}$

13 $\frac{1}{12}(3e^4 + 6e^{-2} - 12e + 8e^3 - 5) \approx 23.97$

15 (a) 19 **(b)** 35 **(c)** 27 **17** $\frac{3}{2}\sqrt{14}$

19 $\frac{9}{2}$ (for all paths) **21** 0 **23** $\frac{412}{15}$ **27** $\bar{x} = 0$, $\bar{y} = \frac{1}{4}\pi a$

31 $I_x = \frac{4}{3}ka^4$; $I_y = \frac{2}{3}ka^4$

33 If the density at (x, y, z) is $\delta(x, y, z)$, then
$I_x = \int_C (y^2 + z^2)\,\delta(x, y, z)\,ds$,
$I_y = \int_C (x^2 + z^2)\,\delta(x, y, z)\,ds$, and
$I_z = \int_C (x^2 + y^2)\,\delta(x, y, z)\,ds$.
35 −0.1584 **37** 18.8815

Exercises 14.3

1 $f(x, y) = x^3y + 2x + y^4 + c$
3 $f(x, y) = x^2\sin y + 4e^x + c$

5 $f(x, y) = 2y^3 \cos x + 5y + c$
7 $f(x, y, z) = 4x^2z + y - 3y^2z^3 + c$
9 $f(x, y, z) = y \tan x - ze^x + c$

Exer. 11–14: A potential function f is given along with the value of the integral.

11 $f(x, y) = xy^2 + x^2y$; 14
13 $f(x, y, z) = 3x^2y^3 + 2xz^2 + z$; -31
15 $\frac{\partial}{\partial y}(4xy^3) \neq \frac{\partial}{\partial x}(2xy^3)$ **17** $\frac{\partial}{\partial y}(e^x) \neq \frac{\partial}{\partial x}(3 - e^x \sin y)$
21 $\frac{\partial N}{\partial z} \neq \frac{\partial P}{\partial y}$
23 $f(x, y, z) = -\frac{1}{2}c \ln(x^2 + y^2 + z^2) + d$, where $c > 0$ and d is a constant
27 This does not violate Theorem (14.16) since D is not simply connected. M and N are not continuous at (0, 0).
29 $W = c\frac{d_2 - d_1}{d_1d_2}$

Exercises 14.4

1 $-\frac{7}{60}$ **3** $\frac{2}{3}$ **5** π **7** -3 **9** 0 **11** 0
13 -3π **15** $\frac{128}{3}$ **17** $6 - \frac{5}{2}\ln 5 \approx 1.98$ **19** $\frac{3}{8}\pi a^2$
23 Green's theorem does not apply since M and N are undefined at (0, 0) and hence are not continuous everywhere inside the unit circle.
27 $\bar{x} = 0, \bar{y} = \frac{4a}{3\pi}$

Exercises 14.5

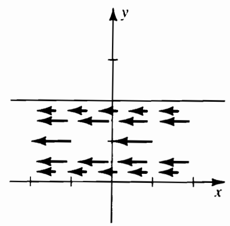
1 $\frac{2}{3}\pi a^4$ **3** $5\sqrt{14}$
5 (a) $\int_0^4 \int_0^{(12-3y)/4} \frac{1}{2}(12 - 3y - 4z)y^2z^3\left(\frac{1}{2}\sqrt{29}\right) dz\, dy$
(b) $\int_0^3 \int_0^{6-2z} x\left[\frac{1}{3}(12 - 2x - 4z)\right]^2 z^3\left(\frac{1}{3}\sqrt{29}\right) dx\, dz$
7 (a) $\int_0^8 \int_0^6 \left(4 - 3y + \frac{1}{16}y^2 + z\right)\left(\frac{1}{4}\sqrt{17}\right) dz\, dy$
(b) $\int_0^2 \int_0^6 [x^2 - 2(8 - 4x) + z]\sqrt{17}\, dz\, dx$
9 Since $\iint_S g(x, y, z)\, dS = c\iint_R dA$, the value of the integral equals the volume of a cylinder of altitude c, with rulings parallel to the z-axis, whose base is the projection of S on the xy-plane.
11 $2\pi a^3$ **13** 3π **15** 18 **17** 8
21 (a) $\frac{255}{2}\pi\sqrt{2}; \bar{x} = \bar{y} = 0, \bar{z} = \frac{1364}{425}$ **(b)** $1365\pi\sqrt{2}$

Exercises 14.6

1 24 **3** 20π **5** 0 **7** $136\pi/3$ **9** 24
11 Both integrals equal $4\pi a^3$. **13** Both integrals equal 4π.
27 625π lb upward

Exercises 14.7

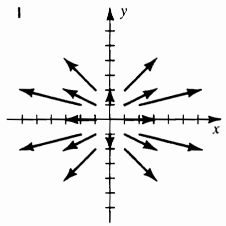
1 Both integrals equal -1. **3** Both integrals equal πa^2.
5 0 **7** -8π
9 The curl meter rotates counterclockwise for $0 < y < 1$ and clockwise for $1 < y < 2$. There is no rotation if $y = 1$. curl $\mathbf{F} = 2(1 - y)\mathbf{k}$; $|(\text{curl } \mathbf{F}) \cdot \mathbf{k}| = |2(1 - y)|$ has a maximum value 2 at $y = 0$ and $y = 2$ and a minimum value 0 at $y = 1$.



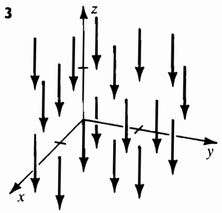
11 Typical field vectors are shown in Figure 14.5. A curl meter rotates counterclockwise for every $(x, y) \neq (0, 0)$. curl $\mathbf{F} = 2\mathbf{k}$; $|(\text{curl } \mathbf{F}) \cdot \mathbf{k}| = 2$ for every (x, y).

Chapter 14 Review Exercises

1



3

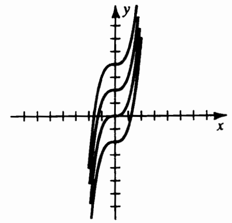


5 $\mathbf{F}(x, y) = (y^2 \sec^2 x)\mathbf{i} + (2y \tan x)\mathbf{j}$
9 -8 **11** $-\frac{56}{5}$ **13** $\frac{1}{144}(1025^{3/2} - 17^{3/2}) \approx 227.40$
15 70 **17** 0 **19** 25.905; 12.168; 22.744 **21** 2.361
23 $\frac{25}{2}$ **25** $f(x, y, z) = x^2e^{2y} + y^2 \cot z + c$ **27** $\frac{1}{6}$
29 $3x^2z^4 + xz^2$; $(2x^2y - 2xyz)\mathbf{i} + (4x^3z^3 - 2xy^2)\mathbf{j} + (yz^2)\mathbf{k}$
31 $\frac{1}{5}\sqrt{3}$ **33** $\frac{5\pi}{2}$ **35** Both integrals equal 8π.

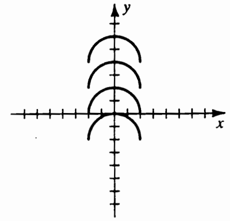
CHAPTER 15

Exercises 15.1

1 (a) $y = x^3 + C$



3 (a) $y = \sqrt{4 - x^2} + C$



(b) $y = x^3 + 2$ **(b)** $y = \sqrt{4 - x^2}$
11 $y = Ce^{2\sin x}$ **13** $y = Cx$ **15** $x^3e^yy^5 = C$
17 $y = -1 + Ce^{(x^2/2)-x}$ **19** $y = -\frac{1}{3}\ln(3C + 3e^{-x})$
21 $y^2 = C(1 + x^3)^{-2/3} - 1$
23 $x \sin x + \cos x - \ln|\sin y| = C$
25 $\sec x + e^{-y} = C$ **27** $y^2 + \ln y = 3x - 8$
29 $y = \ln(2x + \ln x + e^2 - 2)$ **31** $y = 2e^{2-\sqrt{4+x^2}} - 1$
33 $\tan^{-1} y - \ln|\sec x| = \frac{\pi}{4}$ **35** $xy = k$; hyperbolas
37 $2x^2 + y^2 = k$; ellipses **39** $2x^2 + 3y^2 = k$; ellipses

41 (a)

x	y
1.0	0.5600
1.2	0.4747
1.4	0.4063
1.6	0.3528
1.8	0.3107
2.0	0.2772
2.2	0.2500
2.4	0.2276
2.6	0.2088
2.8	0.1928
3.0	0.1791

43 (a)

x	y
0.0	0.1200
0.2	0.1800
0.4	0.2414
0.6	0.3092
0.8	0.3913
1.0	0.5006
1.2	0.6565
1.4	0.8867
1.6	1.2116
1.8	1.5702
2.0	1.7418

45 (a)

x	y
1.0	0.5600
1.2	0.4832
1.4	0.4213
1.6	0.3720
1.8	0.3323
2.0	0.2999
2.2	0.2732
2.4	0.2507
2.6	0.2316
2.8	0.2151
3.0	0.2009

47 (a)

x	y
0.0	0.1200
0.2	0.1807
0.4	0.2453
0.6	0.3204
0.8	0.4173
1.0	0.5540
1.2	0.7578
1.4	1.0507
1.6	1.3772
1.8	1.5824
2.0	1.6256

Exercises 15.2

1 $y = \frac{1}{4}e^{2x} + Ce^{-2x}$ **3** $y = \frac{1}{2}x^5 + Cx^3$
5 $y = \frac{e^x}{x} - \frac{1}{2}x + \frac{C}{x}$ **7** $y = \frac{e^x + C}{x^2}$
9 $y = \frac{4}{3}x^3 \csc x + C \csc x$ **11** $y = 2 \sin x + C \cos x$
13 $y = x \sin x + Cx$ **15** $y = \left(\frac{1}{3}x + \frac{C}{x^2}\right)e^{-3x}$
17 $y = \frac{3}{2} + Ce^{-x^2}$ **19** $y = \frac{1}{2}\sin x + \frac{C}{\sin x}$
21 $y = \frac{1}{3} + (x + C)e^{-x^3}$ **23** $y = x(x + \ln x + 1)$
25 $y = e^{-x}(1 - x^{-1})$ **27** $Q = CV(1 - e^{-t/RC})$
29 $f(t) = \frac{80}{3}(1 - e^{-0.075t}) + Ke^{-0.075t}$
31 (a) $f(t) = M + (A - M)e^{k(1-t)}$ (k a constant)
(b) 28 items
33 $y = y_L(1 - ce^{-kt}), k > 0$
35 (b) $y = \frac{I}{k}(1 - e^{-kt})$; $\frac{I}{k}$ **(c)** 0.58 mg/min
37 -3.23103

39 (a)

x	y
1.0	0.5600
1.4	0.4209
1.8	0.3318
2.2	0.2728
2.6	0.2312
3.0	0.2006

41 (a)

x	y
0.0	0.1200
0.4	0.2445
0.8	0.4148
1.2	0.7547
1.6	1.3881
2.0	1.6540

43

n	y	E_n	$E_{n/2}/E_n$
4	7.38624259	5.2×10^{-5}	—
8	7.38629082	3.5×10^{-6}	15
16	7.38629413	2.3×10^{-7}	15
32	7.38629435	1.5×10^{-8}	15

Exercises 15.3

1 $y = C_1e^{2x} + C_2e^{3x}$ **3** $y = C_1 + C_2e^{3x}$
5 $y = C_1e^{-2x} + C_2xe^{-2x}$ **7** $y = C_1e^{(2+\sqrt{3})x} + C_2e^{(2-\sqrt{3})x}$
9 $y = C_1e^{-\sqrt{2}x} + C_2xe^{-\sqrt{2}x}$ **11** $y = C_1e^{-3x/2} + C_2e^{5x/4}$
13 $y = C_1e^{4x/3} + C_2xe^{4x/3}$
15 $y = C_1e^{(2+\sqrt{2})x/2} + C_2e^{(2-\sqrt{2})x/2}$
17 $y = e^x(C_1 \cos x + C_2 \sin x)$
19 $y = e^{2x}(C_1 \cos 3x + C_2 \sin 3x)$
21 $y = C_1e^{(-3+\sqrt{7})x} + C_2e^{(-3-\sqrt{7})x}$ **23** $y = -2e^x + 2e^{2x}$
25 $y = \cos x + 2 \sin x$ **27** $y = e^{-4x}(2 + 9x)$
29 $y = \frac{1}{2}e^x \sin 2x$

Exercises 15.4

1 $y = C_1 \cos x + C_2 \sin x - \cos x \ln|\sec x + \tan x|$

3 $y = \left(C_1 + C_2x + \frac{1}{12}x^4\right)e^{3x}$

5 $y = C_1e^x + C_2e^{-x} + \frac{2}{5}e^x \sin x - \frac{1}{5}e^x \cos x$

7 $y = \left(C_3 + \frac{1}{6}x\right)e^{3x} + C_2e^{-3x}$, where $C_3 = C_1 - \frac{1}{36}$

9 $y = C_1e^{-x} + C_2e^{4x} - \frac{1}{2}$ **11** $y = C_1e^x + C_2e^{2x} + \frac{2}{3}e^{-x}$

13 $y = C_1 + C_2e^{-2x} + \frac{1}{8}\sin 2x - \frac{1}{8}\cos 2x$

15 $y = C_1e^x + C_2e^{-x} + \frac{1}{9}(-4 + 3x)e^{2x}$

17 $y = e^{3x}(C_1 \cos 2x + C_2 \sin 2x) + \frac{1}{65}e^x(7 \cos x - 4 \sin x)$

21 0.776805

Exercises 15.5

1 $y = -\frac{1}{3}\cos 8t$ **3** $y = \frac{1}{8}\sqrt{2}e^{-8t}(e^{4\sqrt{2}t} - e^{-4\sqrt{2}t})$

5 $y = \frac{1}{3}e^{-8t}(\sin 8t + \cos 8t)$

7 If m is the mass of the weight, then the spring constant is $24m$ and the damping force is $-4m\frac{dy}{dt}$. The motion is begun by releasing the weight from 2 ft above the equilibrium position with an initial velocity of 1 ft/sec in the upward direction.

9 $-6\sqrt{2}\frac{dy}{dt}$

11 (a) Overdamped: 2.3, 2.4; underdamped: 1.7, 1.8, 1.9, 2.0, 2.1, 2.2

Chapter 15 Review Exercises

1 $\sin x - x \cos x + e^{-y} = C$ **3** $y = \tan(\sqrt{1 - x^2} + C)$

5 $y = \frac{2x - 2\cos x + C}{\sec x + \tan x}$ **7** $y = 2 \sin x + C \cos x$

9 $\sqrt{1 - y^2} + \sin^{-1} x = C$ **11** $\csc y = e^{-x} + C$

13 $y = \frac{1}{2} + Ce^{-2\sin x}$ **15** $y = (C_1 + C_2x)e^{4x}$

17 $y = C_1 + C_2e^{2x}$

19 $y = C_1e^{-x} + C_2e^x - \frac{1}{5}e^x(2 \cos x + \sin x)$

21 $y = \frac{1}{5}e^{4x} + Ce^{-x}$ **23** $y = C_1e^x + C_2e^{2x} + \frac{1}{12}e^{5x}$

25 $y = \frac{(x-2)^3}{3x} + \frac{C}{x}$

27 $y = e^{-5x/2}\left[C_1 \cos\left(\frac{1}{2}\sqrt{3}x\right) + C_2 \sin\left(\frac{1}{2}\sqrt{3}x\right)\right]$

29 $\frac{1}{2}e^x(\sin x - \cos x) + e^{-y} = C$

31 $y = \frac{1}{2}\cos x + C \sec x$

33 $y = \frac{\ln|\sec x + \tan x| - x + C}{\csc x - \cot x}$

35 $y = \frac{3500}{3}e^{(3/(2\pi))(1-\cos 2\pi t)} - \frac{2000}{3}$; 2366

37 $f(t) = \frac{ab[e^{k(b-a)t} - 1]}{be^{k(b-a)t} - a}$

39 $y\,dy + x\,dx = 0$; a circle with center at the origin

41 (a)

x	y
0.0	1.0200
0.2	1.1212
0.4	1.1909
0.6	1.2532
0.8	1.3237
1.0	1.4071
1.2	1.4958
1.4	1.5704
1.6	1.5993
1.8	1.5423
2.0	1.3781

43 (a)

x	y
0.0	1.0200
0.2	1.1068
0.4	1.1776
0.6	1.2478
0.8	1.3256
1.0	1.4110
1.2	1.4939
1.4	1.5526
1.6	1.5537
1.8	1.4680
2.0	1.3077

45 (a) 0.132956, 0.140085, 0.144016
(b) 0.149392, 0.148477, 0.148247
(c) 0.148170, 0.148170, 0.148170

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