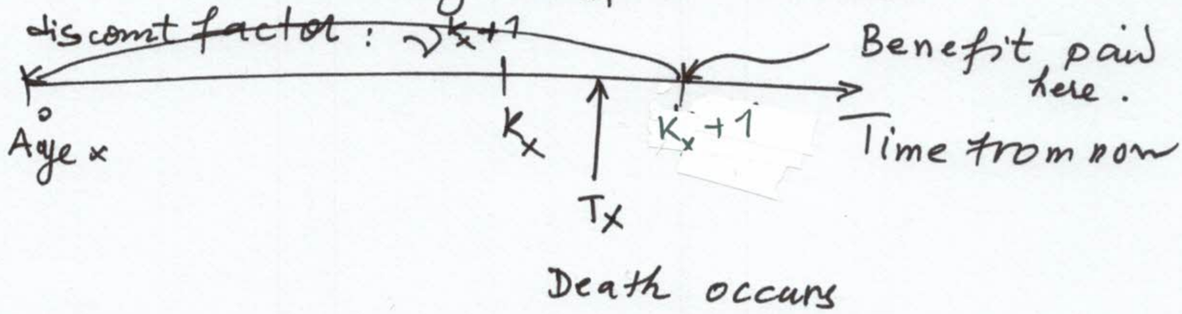
3°/ Discrete Life insurance

The difference between discrete life insurance and continuous life insurance is that the death benefit is payable at the end of the year of the death.



As the death benefit is payable at time $K_x + 1$, then we should use a discount factor v^{K_x+1} to discount it.

Level Benefit whole life Insurance

As before, we assume that the benefit is 1SR. In this case the APV is $Z = v^{K_x+1}$; $K_x = 0, 1, 2, \ldots$

Z depends on a discrete random variable (K_x).

$$\Rightarrow \quad E(Z) = \sum_{k=0}^{\infty} v^{k+1} \; {}_kp_x \, q_{x+k}$$

Present value random variable: $Z = v^{K_x+1}$

Probability function
$P(K_x = k) = {}_{k|}q_x$
$= {}_kp_x \, q_{x+k}$

$\Rightarrow$ $E(Z) = A_x$ (Notation). x indicates the age at inception.

We do not place bar above A because the benefit is not paid precisely at the moment of death.

If there is a limiting age, we replace ∞ by $\omega - x - 1$.

no one can survive to the age $\omega - x$ (ω integer)

$\Rightarrow$ $T_x < \omega - x$

the largest possible of K_x (the integral part of T_x) is $\omega - x - 1$.

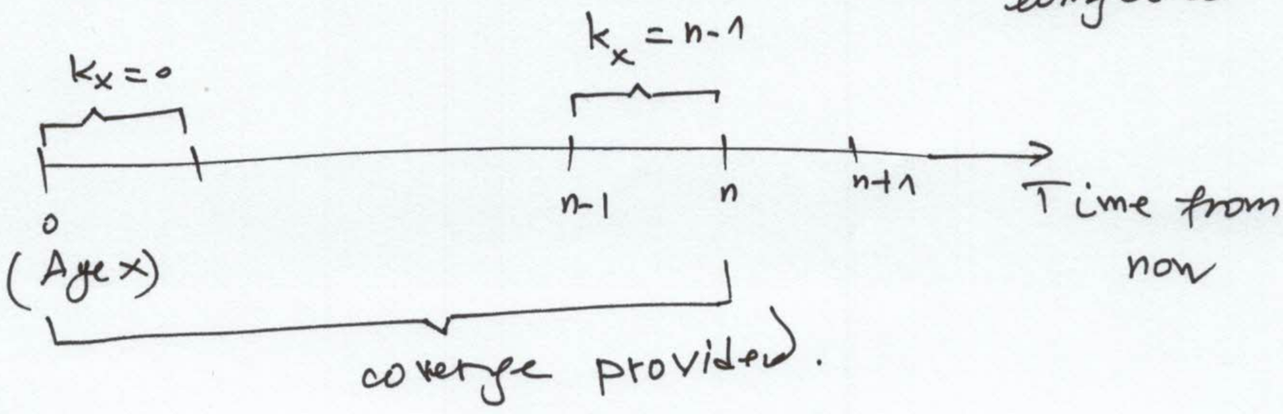
we used again the property of 0 or 1 benefit

$$\Rightarrow \quad E(Z^2) = {}^2A_x \quad \text{and} \quad Var(Z) = {}^2A_x - A_x^2$$

Level Benefit Term Life Insurance

An n-year life insurance covers the first n year from now. (the same as whole life insurance)

the upper limit is not longer ∞



$$\Rightarrow \quad E(Z) = \sum_{k=0}^{n-1} v^{k+1} {}_kp_x \, q_{x+k}$$

we denote by $A^{1}_{x:\overline{n}|}$ the $E(Z)$

$\overline{n}|$ indicates that this policy will last for at most n years, and 1 above x indicates that the policy is a term life insurance.

(there is no bar above A, because the policy is discrete rather than continous)

we used again the 0 or 1 benefit criterion

we can evalute $E(Z)$ at 2δ.

$$\Rightarrow \quad Var(Z) = {}^2A^{1}_{x:\overline{n}|} - A^{1\,2}_{x:\overline{n}|}$$

Level Benefit Endowment Insurance

the same in the continous case, the n-year endowment insurance is just a combination of an n-year life insurance and an n-year pure endowment. Hence, we have

$$E(Z) = A^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\;\;1}$$

we denote $E(Z)$ by $A_{x:\overline{n}|}$. the criterion that the benefit is either 0 or 1 is satisfied, $\Rightarrow$ $E(Z^2)$ is the same as $E(Z)$ evaluated at 2δ $\Rightarrow$

$$Var(Z) = {}^2A_{x:\overline{n}|} - A_{x:\overline{n}|}^2$$

<u>Level Benefit Deferred whole life insurance</u>

An n-year deferred whole life insurance does not cover the first n years from now, but will provide coverage for life thereafter. $\Rightarrow$

$$E(Z) = \sum_{k=n}^{\infty} v^{k+1} \, {}_kp_x \, q_{x+k}$$

[$E(Z)$ is unit-benefit n-year deferred whole life insurance]

We denote $E(Z)$ by ${}_{n|}A_x$

the same we have here the criterion of 0 or 1 is satisfied so:

$E(Z^2)$ can be evaluated at 2δ.

$$\Rightarrow Var(Z) = {}_{n|}^{2}A_x - {}_{n|}A_x^2$$

In the same way for continunous non-level-benefit insurances, we can construct a whole life insurance policy with a benefit of 1SR and increases by an amount of 1SR annually. We assume that the benefit is payable at the end of year of death, then the APV of the benefit is given by:

$$(IA)_x = \sum_{k=0}^{\infty} (k+1)\, v^{k+1}\, {}_kp_x\, q_{x+k}$$

there is no bar above A which mean that the benefit is not payable precisely at the moment of death.

Formula

Discrete Insurance

Policy	ADV: Notation	ADV: Formula
whole life	A_x	$\sum_{k=0}^{\infty} v^{k+1} \, {}_kp_x \, q_{x+k}$
n-year term life	$A^{1}_{x:\overline{n}\rceil}$	$\sum_{k=0}^{n-1} v^{k+1} \, {}_kp_x \, q_{x+k}$
n-year endowment	$A_{x:\overline{n}\rceil}$	$A^{1}_{x:\overline{n}\rceil} + A_{x:\overset{1}{\overline{n}\rceil}}$
n-year deferred whole life	${}_{n\rceil}A_x$	$\sum_{k=n}^{\infty} v^{k+1} \, {}_kp_x \, q_{x+k}$
Annually increasing n-year term life	$(IA)^{1}_{x:\overline{n}\rceil}$	$\sum_{k=0}^{n-1} (k+1) v^{k+1} \, {}_kp_x \, q_{x+k}$
Annually increasing whole life	$(IA)_x$	$\sum_{k=0}^{\infty} (k+1) v^{k+1} \, {}_kp_x \, q_{x+k}$
Annually increasing n-year endowment	$(IA)_{x:\overline{n}\rceil}$	$(IA)^{1}_{x:\overline{n}\rceil} + n \, A_{x:\overset{1}{\overline{n}\rceil}}$
Annually decreasing n-year term life	$(DA)^{1}_{x:\overline{n}\rceil}$	$\sum_{k=0}^{k-1} (n-k) v^{k+1} \, {}_kp_x \, q_{x+k}$

Example

You are given:

(I) the following life table

x	90	91	92	93
ℓ_x	100	72	39	0
d_x	28	33	39	–

(II) $i = 0.06$

calculate:

a) A_{90} , b) $A^{1}_{90:\overline{1}|}$, c) $A_{90:\overline{1}|}$, d) ${}_{1|}A_{90}$

e) $(IA)_{90}$

Solution

a°/ We have $\omega = 93$, $(\ell_{93} = 0)$

$$\Rightarrow \quad A_{90} = \sum_{k=0}^{93-90-1} v^{k+1} \, {}_kp_{90} \, q_{90+k}$$

$${}_tp_x = \frac{\ell_{x+t}}{\ell_x}$$

$$= \sum_{k=0}^{2} v^{k+1} \frac{\ell_{x+90}}{\ell_{90}} \frac{d_{90+k}}{\ell_{90+k}}$$

$${}_tq_x = \frac{{}_td_x}{\ell_x} = \frac{\ell_x - \ell_{x+t}}{\ell_x}$$

$$= \sum_{k=0}^{2} v^{k+1} \frac{d_{90+k}}{\ell_{90}} .$$

$$v = \frac{1}{1+i}$$

$$= \sum_{k=0}^{2} \frac{1}{(1+i)^{k+1}} \frac{d_{90+k}}{\ell_{90}}$$

$$= \frac{1}{1.06}\frac{28}{100} + \frac{1}{(1.06)^2}\frac{33}{100} + \frac{1}{(1.06)^3}\frac{39}{100}$$

$$= 0.885301$$

b/

$$A^{1}_{90:\overline{1}|} = \sum_{k=0}^{1-1} v^{k+1} {}_kP_x \, q_{x+k} = \sum_{k=0}^{0} \frac{1}{(1+i)^{k+1}} \frac{d_{90+k}}{l_{90}}$$

$$= \frac{1}{1+i}\frac{d_{90}}{l_{90}}$$

in just one term expression for $A^{1}_{90:\overline{1}|}$

$$= \frac{1}{1.06}\frac{28}{100}$$

$$= 0.264151$$

c/ we have $A_{90:\overline{1}|} = A^{1}_{90:\overline{1}|} + A_{90:\overset{1}{\overline{1}|}}$

$\Rightarrow$ we have from b/ $A^{1}_{90:\overline{1}|} = 0.264151$

and ${}_1E_{90} = A_{90:\overset{1}{\overline{1}|}} = v \, p_{90} = \frac{1}{1.06}\frac{72}{100}$

$$= 0.679245$$

$$\Rightarrow \quad A_{90:\overline{1}|} = 0.943396$$

d/ we have

$${}_{1|}A_{90} = \sum_{k=1}^{93-90-1} v^{k+1} {}_kP_{90} \, q_{90+k}$$

$$= \sum_{k=1}^{2} v^{k+1} \frac{d_{90+k}}{l_{90}}$$

$$= \frac{1}{1.06^2}\frac{33}{100} + \frac{1}{1.06^3}\frac{39}{100}$$

$$= 0.621150.$$

e) $$(IA)_{90} = \sum_{k=0}^{2} (k+1) v^{k+1} \frac{d_{90+k}}{l_{90}}$$

$$= \frac{1}{1.06}\frac{28}{100} + 2\,\frac{1}{1.06^2}\frac{33}{100} + 3 \cdot \frac{1}{1.06^3}\frac{39}{100}$$

$$= 1.83390.$$

Example 2

For a special 3 year term life insurance on (x), you are given:

(I) Z is the present value random variable for the death benefits

(II) $q_{x+k} = 0.02(k+1)$; $k = 0, 1, 2$

(III) The following death benefits payable at the end of the year of death:

k	b_{k+1}
0	300,000
1	350,000
2	400,000

(IV) $i = 0.06$

Calculate $E(Z)$.

Solution

this policy is caled a special policy (notstandard policy),

We can solve that by using basic concepts.

We have 3 years term life insurance, which means that there is no payment if death occurs after three years from now.

- If the death occurs during the 1° year, the benefit $b_1 = 300{,}000$ and we have the associated probability is $q_x = 0.02(1) = 0.02$.

- If the death occurs during the second year, the death benefit is 350 000, and $q_{x+1} = 0.02(2) = 0.04$

  in this case the associated probabily is

  $p_x\, q_{x+1} = (1-0.02)(0.04) = 0.0392$

$$\boxed{v^k = \frac{b_k}{(1+i)^k}}$$

- If the death occurs during the third year, the death benefit is 400,000, and the associated probability is ${}_2p_x\, q_{x+2} = (1-0.02)(0.06) = 0.056448$

$$\Rightarrow \quad E(Z) = \sum_{k=0}^{2} v^{k+1}\, {}_kp_x \cdot q_{x+k} = \frac{300\,000}{1.06}\, 0.02 + \frac{350\,000}{1.06^2}\, 0.0392 + \cdots$$

$$= \frac{300000}{1.06} \, 0.02 + \frac{350000}{1.06^2} \, 0.392 + \frac{400000}{1.06^3} \, 0.56448$$

$$= 368283.885$$