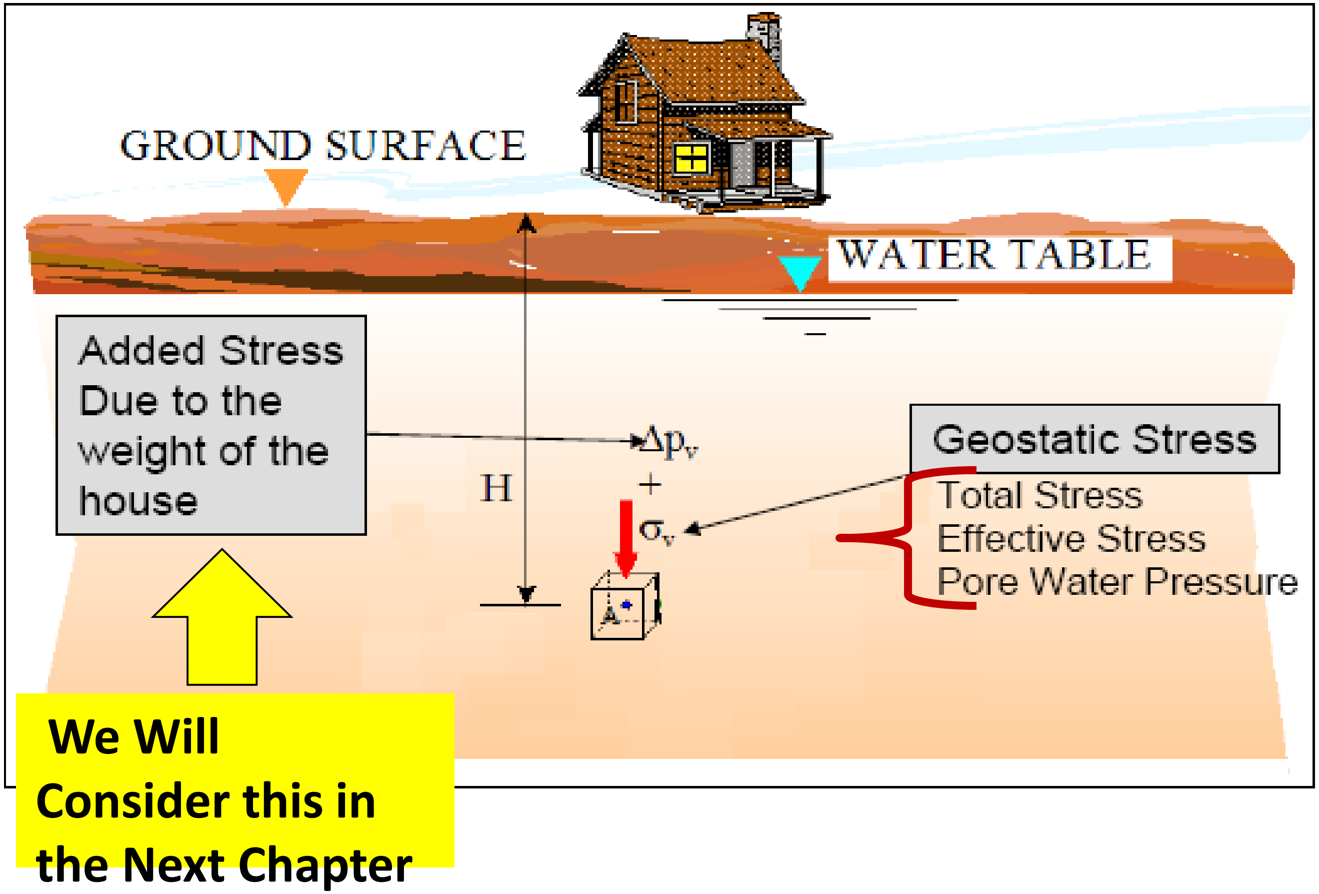
CHAPTER 9
IN SITU STRESSES

IN SITU STRESSES

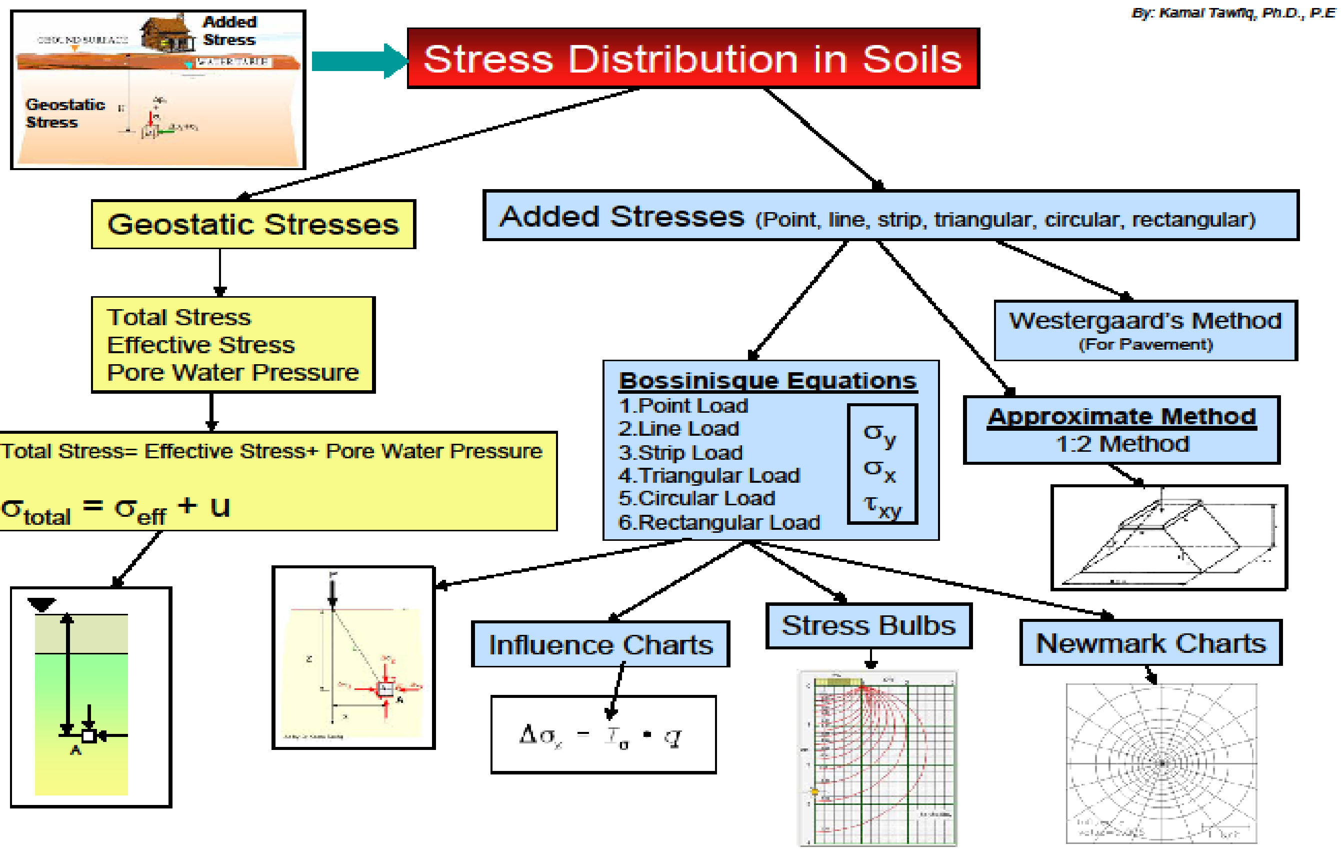
- Concept of effective stress
- Stresses in saturated soil without seepage, upward seepage, and downward seepage
- Seepage force per unit volume of soil
- Conditions for *heaving* or boiling for seepage under a hydraulic structure
- Use of filter to increase the stability against heaving or boiling
- Effective stress in partially saturated soil

TYPES OF STRESSES IN SOIL

There are two types of stresses in soil.

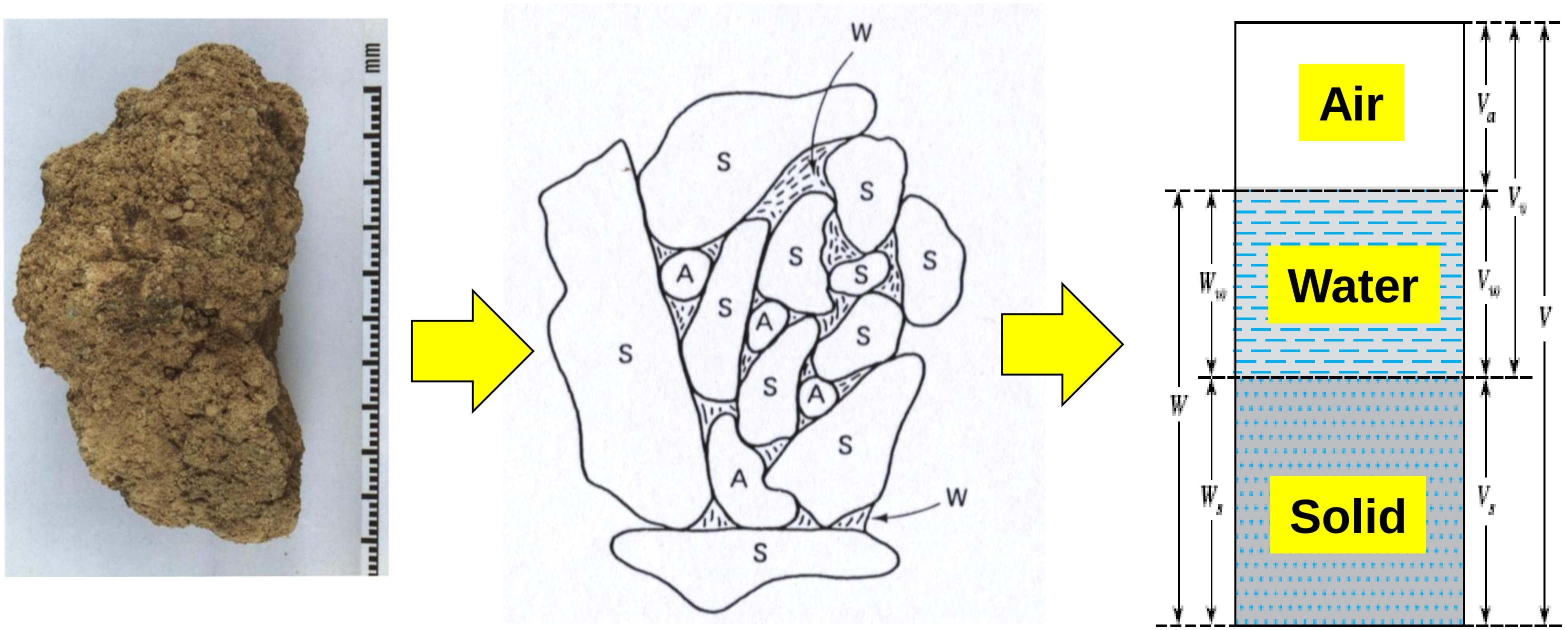


TYPES OF STRESSES IN SOIL



INTRODUCTION

- Soils are multiphase systems. They consists of solid particles enclosing continuous voids which contain water and/or air.



- When an external load is applied to soil it will be carried jointly by the three systems, solid, water, air.

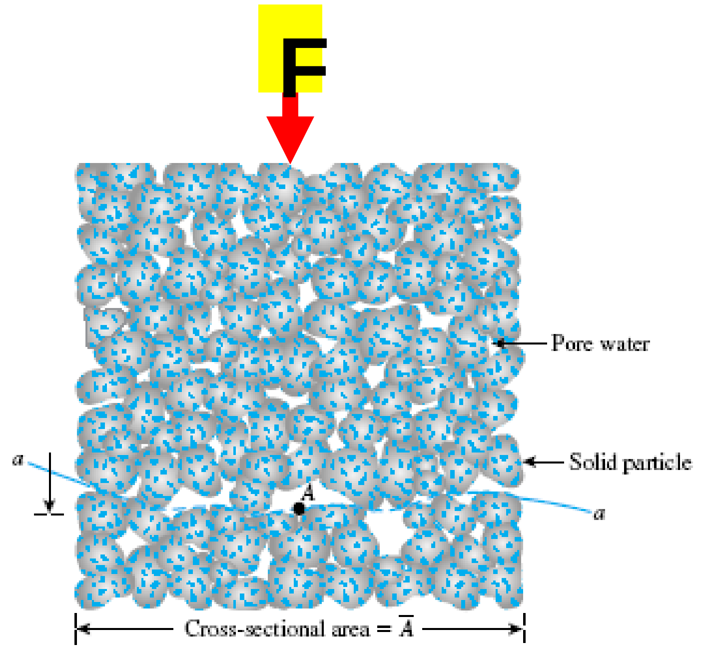
INTRODUCTION

- However, it is difficult, if not impossible to analyze such multiphase system.
- When the soil is fully saturated, then only water is presented in the voids and the problem becomes relatively simple.
- When an external load is applied to saturated soil it will be carried jointly by the two systems, solid grains and water in the pores.
- The increase in pressure within the pore water causes drainage (flow out of the soil), and the load is transferred to the solid grains.
- The rate of drainage depends on the permeability of the soil.

INTRODUCTION

- If by any mean we can find the part of the load carried by the water for a given external load, then we can find the part carried by the solids.

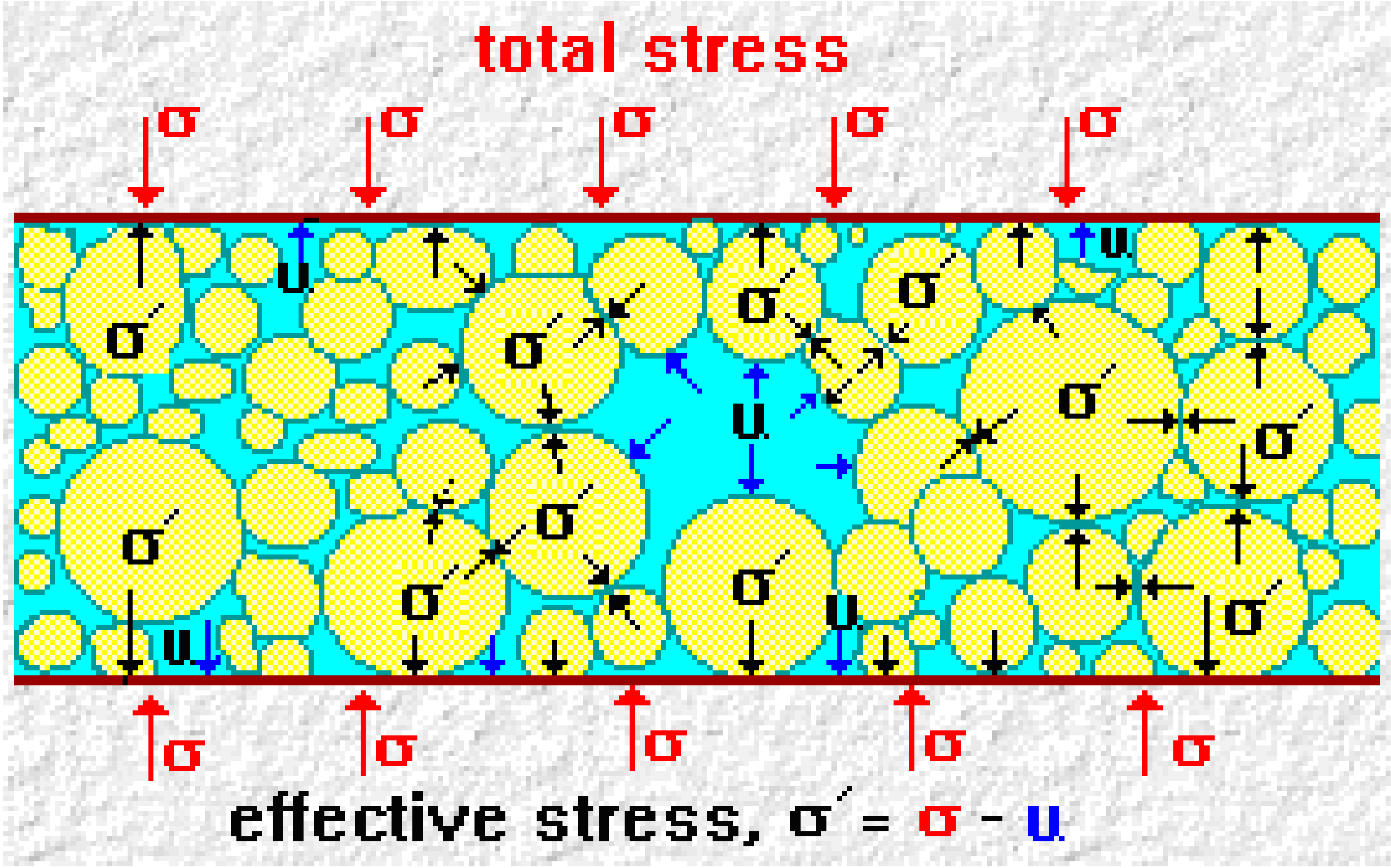
- This is the theme of the Principle of Effective Stress. In other word the principle of effective stress determines the effect of a pore pressure on the behavior of a soil with a given TOTAL STRESS.



- It is probably the single most important concept in soil mechanics and geotechnical engineering.

- The compressibility and shearing resistance of a soil depend to a great extent on the effective stress.

INTRODUCTION



STRESSES IN SATURATED SOILS

$$\sigma = H\gamma_w + (H_A - H)\gamma_{sat}$$

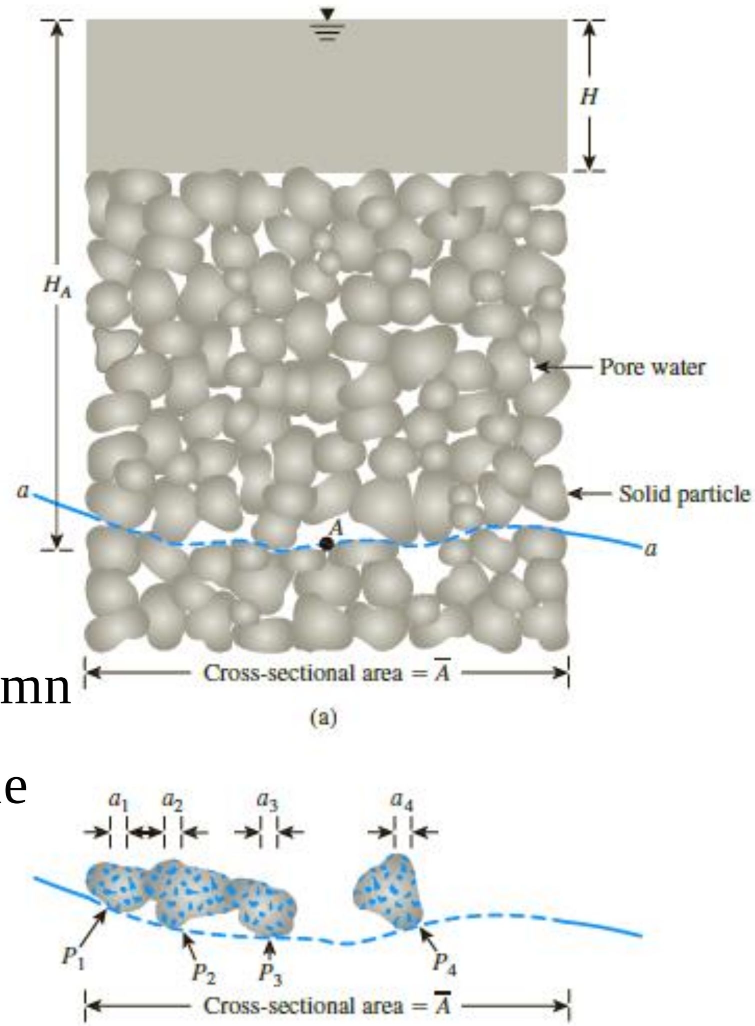
σ = Total stress @ the elevation of point A

γ_w = Unit weight of water

γ_{sat} = Saturated unit weight of the soil

H = Height of water table from the top of soil column

H_A = Distance between point A and the water table



STRESSES IN SATURATED SOILS

$$\sigma = \sigma' + u$$

$$\sigma = \text{Total normal stress}$$

$$\sigma' = \text{Effective normal stress}$$

$$\text{u} = \text{Pore water pressure}$$

The total stress can be divided into two parts:

1. **A portion is carried by water in the continuous void spaces. This portion acts with equal intensity in all directions.**

2. **The rest of the total stress is carried by the soil solids at their points of contact. The sum of the vertical components of the forces developed at the points of contact of the solid particles per unit cross-sectional area of the soil mass is called the *effective stress*.**

THE PRINCIPLE OF EFFECTIVE STRESS

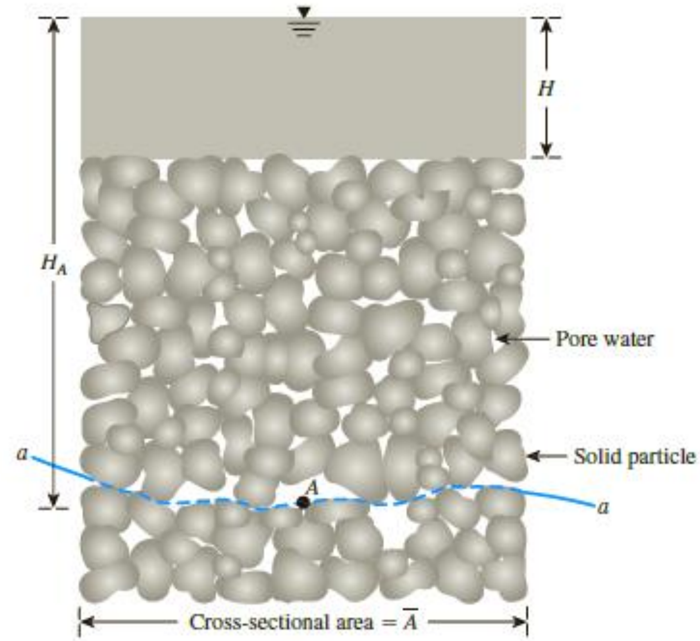
$$\sigma = \sigma' + u$$

$$\sigma' = \sigma - u$$

$$\sigma' = [H\gamma_w + (H_A - H)\gamma_{sat}] - H_A\gamma_w$$

$$\sigma' = (H_A - H)(\gamma_{sat} - \gamma_w)$$

$$\sigma' = (\text{Height of soil column}) \text{ x } \gamma'$$

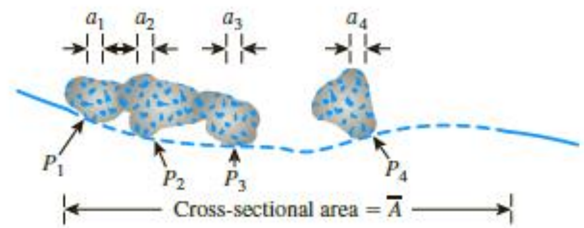


(a)

γ' = Submerged unit weight of the soil

σ' = Effective stress at point A

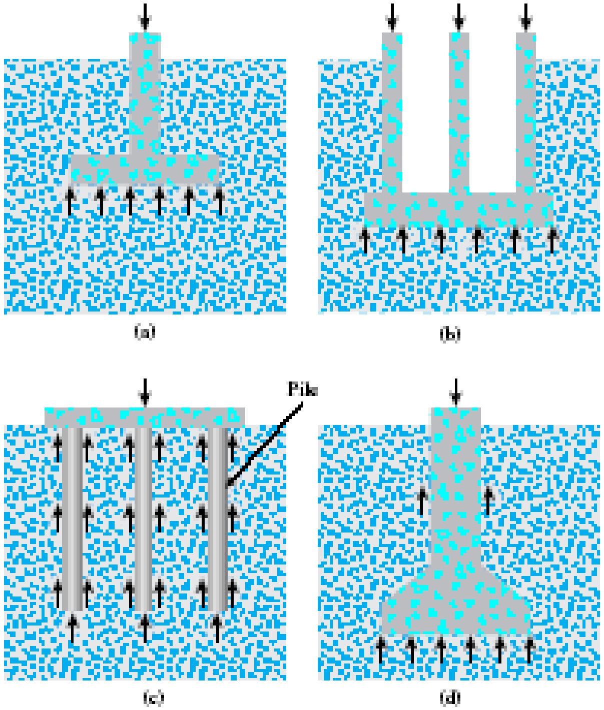
u = Pore water pressure at point A (Neutral stress)



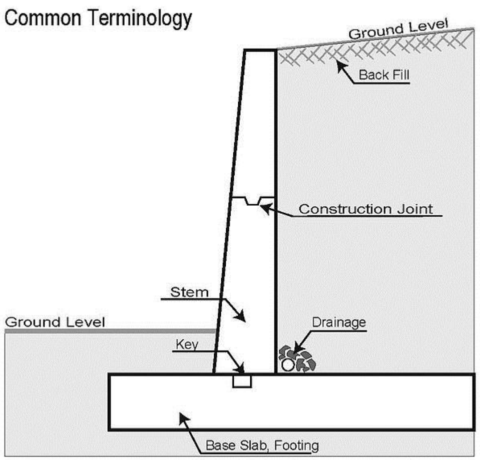
THE PRINCIPLE OF EFFECTIVE STRESS

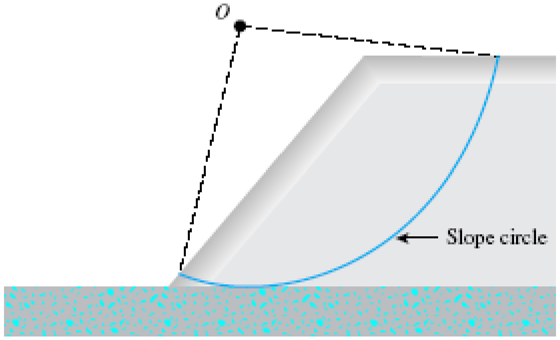
- The concept of effective stress is significant in solving geotechnical engineering problems, such as:

- **Bearing capacity and settlement of foundations**



- **Lateral earth pressure on retaining structures**





Stability of earth slopes

EVALUATION OF EFFECTIVE STRESS

- **We now know the importance of effective stress and that it is responsible for soil behavior and the dilemma that we cannot measure it.**
- **However, if we can find σ and *u*, then**

$$\sigma^{/} = \sigma - u \qquad (*)$$

- **Therefore, our job now is to find σ and u.**

I. Determination of total stress, σ

$$\sigma = \gamma . z \qquad (**)$$

Where

γ = Unit weight of the soil, the unit weight of the soil may be, wet, saturated, dry. (This depends on the degree of saturation of soil).

z = The vertical distance from the surface to the point at which σ is evaluated.

EVALUATION OF EFFECTIVE STRESS

II. Determination of p.w.p, u

$$u = \gamma_w . z \qquad (***)$$

Where

γ_w = Unit weight of water

Therefore, by substituting Eq. (**) and Eq.(***) into Eq.(*), we get

$$\sigma' = (\gamma - \gamma_w) z$$

Stresses in Saturated Soil in the Static Case

(Under Hydrostatic Conditions)

HYDROSTATIC PRESSURE

- The pressure exerted by water at a given point within the soil medium, due to the force of gravity.

- Hydrostatic pressure increases in proportion to depth measured from the surface because of the increasing weight of fluid exerting downward force from above.
- The level in the ground at which the pore pressure is zero (equal to atmospheric) is defined as the water table or phreatic surface.

- When there is no flow, the water surface will be at exactly the same level in any stand pipe placed in the ground below the water table.

- This is called a hydrostatic pressure condition.
- Pore water pressure (sometimes abbreviated to pwp) refers to the pressure of groundwater held within a soil or rock, in gaps between particles (pores).

STRESSES IN SATURATED SOILS WITHOUT UPWARD SEEPAGE

<u>**At point A,**</u>

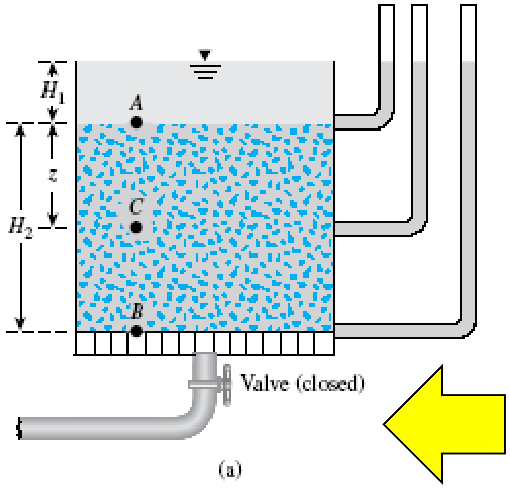
- **Total Stress:** $\sigma_A = H_1\,\gamma_w$
- **Pore water pressure:** $u_A = H_1\,\gamma_w$
- **Effective stress:** $\sigma'_A = 0$

<u>**At point B,**</u>

- **Total Stress:** $\sigma_B = H_1\,\gamma_w + H_2\,\gamma_{sat}$
- **Pore water pressure:** $u_B = (H_1 + H_2)\,\gamma_w$
- **Effective stress:** $\sigma'_B = H_2(\gamma_{sat} - \gamma_w) = H_2\,\gamma'$

<u>**At point C,**</u>

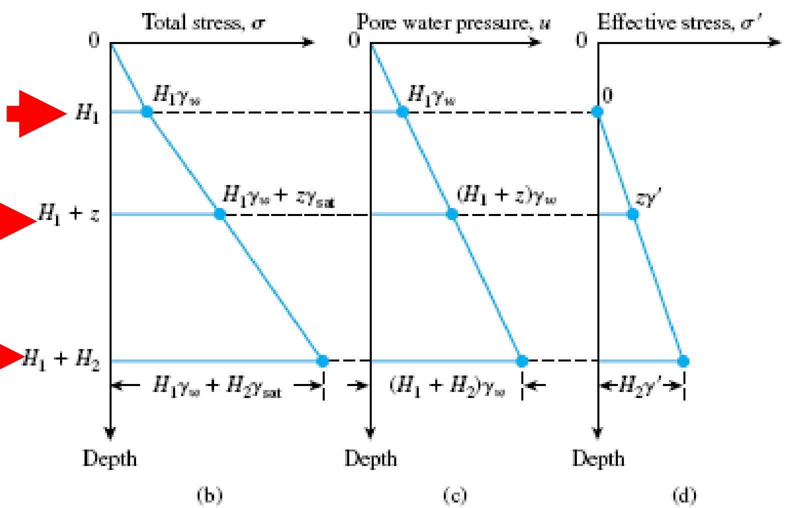
- **Total Stress:** $\sigma_C = H_1\,\gamma_w + z\,\gamma_{sat}$
- **Pore water pressure:** $u_C = (H_1 + z)\,\gamma_w$
- **Effective stress:** $\sigma'_C = z(\gamma_{sat} - \gamma_w) = z\,\gamma'$



at Point A

at Point C

at Point B



EXAMPLE 9.1

Example 9.1

A soil profile is shown in Figure 9.3. Calculate the total stress, pore water pressure, and effective stress at points A, B, and C.

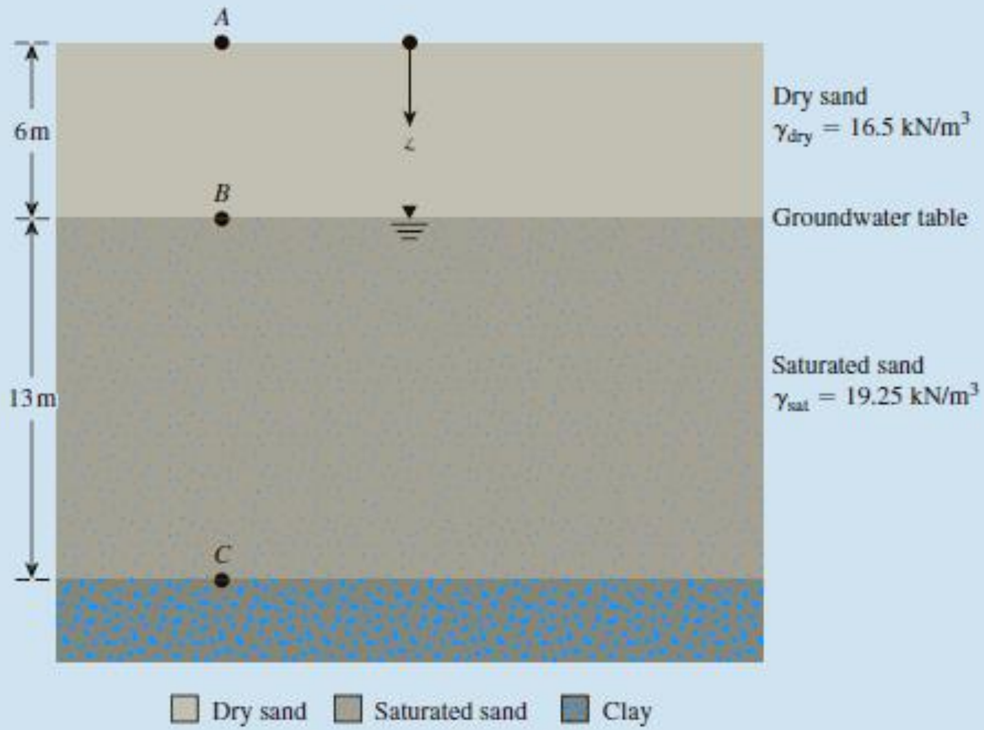


Figure 9.3 Soil profile

EXAMPLE 9.1

Solution

At Point A,

$$\text{Total stress: } \sigma_A = 0$$
$$\text{Pore water pressure: } u_A = 0$$
$$\text{Effective stress: } \sigma'_A = 0$$

At Point B,

$$\sigma_B = 6\gamma_{\text{dry(sand)}} = 6 \times 16.5 = \mathbf{99\ kN/m^2}$$
$$u_B = \mathbf{0\ kN/m^2}$$
$$\sigma'_B = 99 - 0 = \mathbf{99\ kN/m^2}$$

At Point C,

$$\sigma_C = 6\gamma_{\text{dry(sand)}} + 13\gamma_{\text{sat(sand)}}$$
$$= 6 \times 16.5 + 13 \times 19.25$$
$$- 99 + 250.25 - \mathbf{349.25\ kN/m^2}$$
$$u_C = 13\gamma_w = 13 \times 9.81 = \mathbf{127.53\ kN/m^2}$$
$$\sigma'_C = 349.25 - 127.53 = \mathbf{221.72\ kN/m^2}$$

EXAMPLE 9.2

Example 9.2

Refer to Example 9.1. How high should the water table rise so that the effective stress at C is 190 kN/m^2? Assume γ_{sat} to be the same for both layers (i.e., 19.25 kN/m^2).

Solution

Let the groundwater table rise be h above the present groundwater table shown in Figure 9.3 with

$$\sigma_C = (6 - h)\gamma_{dry} + h\gamma_{sat} + 13\gamma_{sat}$$

$$u = (h + 13)\gamma_w$$

So

$$\sigma'_C = \sigma_c - u = (6 - h)\gamma_{dry} + h\gamma_{sat} + 13\gamma_{sat} - h\gamma_w - 13\gamma_w$$

$$= (6 - h)\gamma_{dry} + h(\gamma_{sat} - \gamma_w) + 13(\gamma_{sat} - \gamma_w)$$

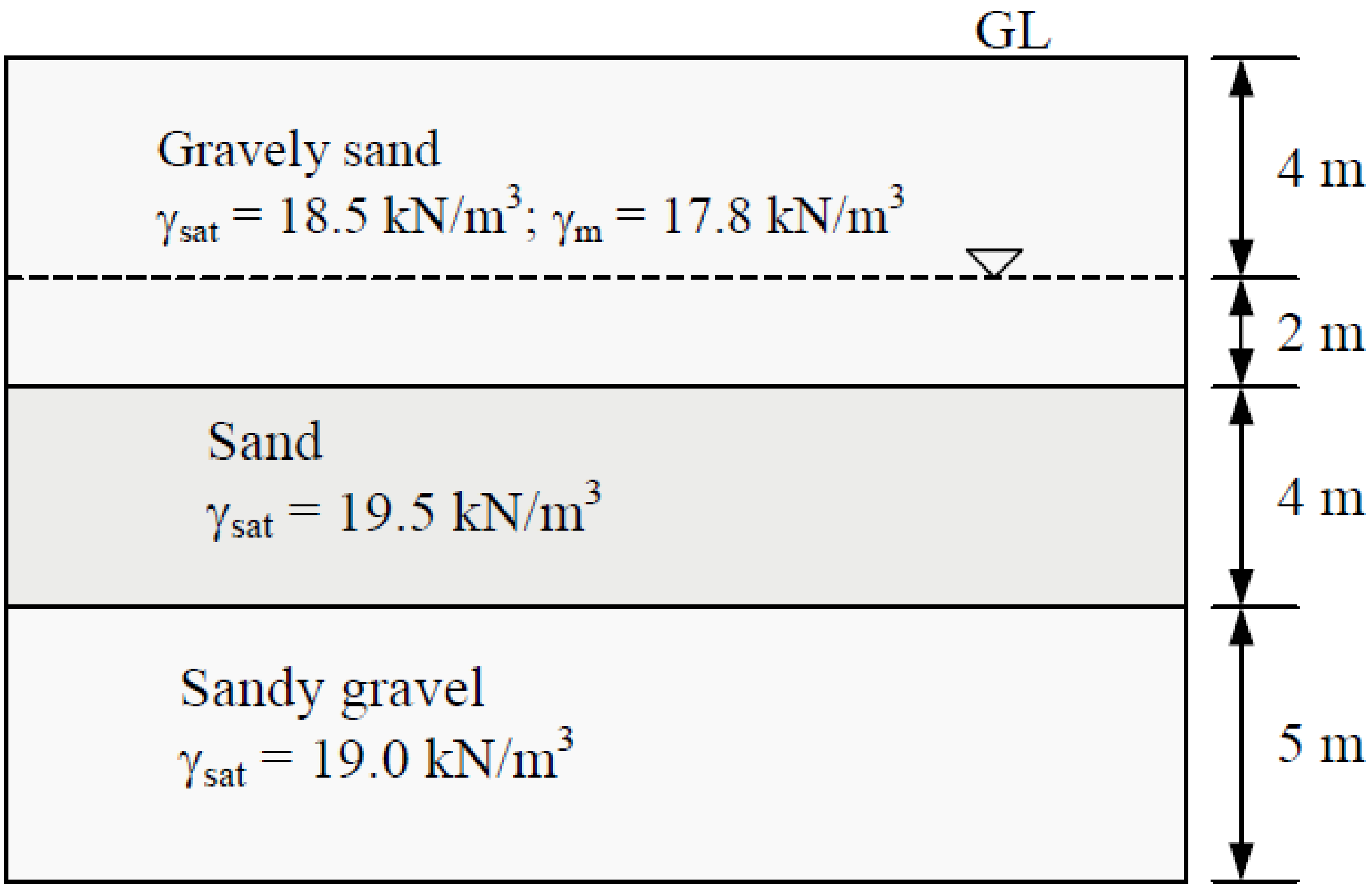
or

$$190 = (6 - h)16.5 + h(19.25 - 9.81) + 13(19.25 - 9.81)$$

$$h = \mathbf{4.49\ m}$$

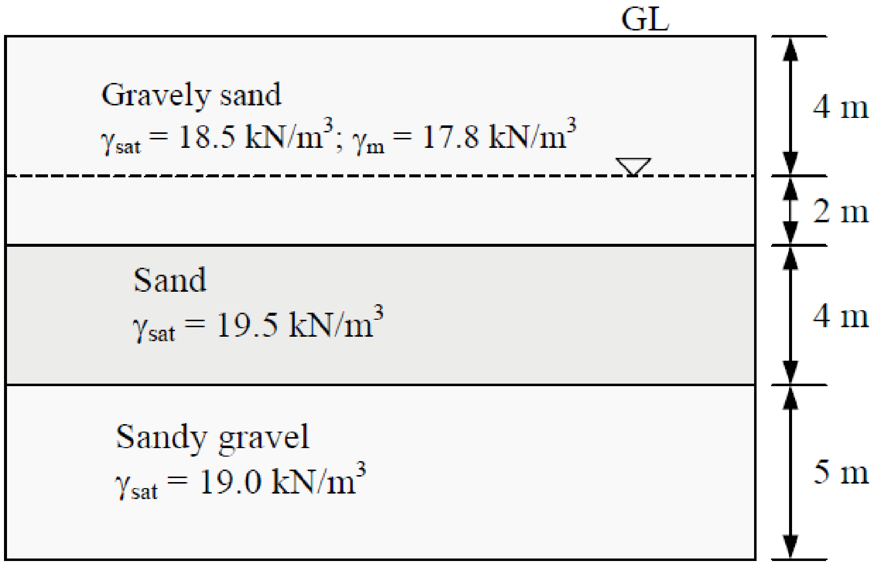
EXAMPLE

- **Plot the variation of total, effective vertical stresses, and pore water pressure with depth for the soil profile shown below.**



EXAMPLE

Solution



- At the ground level,

  $\sigma_v = 0$; $\sigma_v' = 0$; and $u=0$

- At 4 m depth,

  $\sigma_v = (4)(17.8) = 71.2$ kPa; $u = 0$

  $\therefore \sigma_v' = 71.2$ kPa

- At 6 m depth,

  $\sigma_v = (4)(17.8) + (2)(18.5) = 108.2$ kPa

  $u = (2)(9.81) = 19.6$ kPa

  $\therefore \sigma_v' = 108.2 - 19.6 = 88.6$ kPa

- At 10 m depth,

  $\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) = 186.2$ kPa

  $u = (6)(9.81) = 58.9$ kPa

  $\therefore \sigma_v' = 186.2 - 58.9 = 127.3$ kPa

- At 15 m depth,

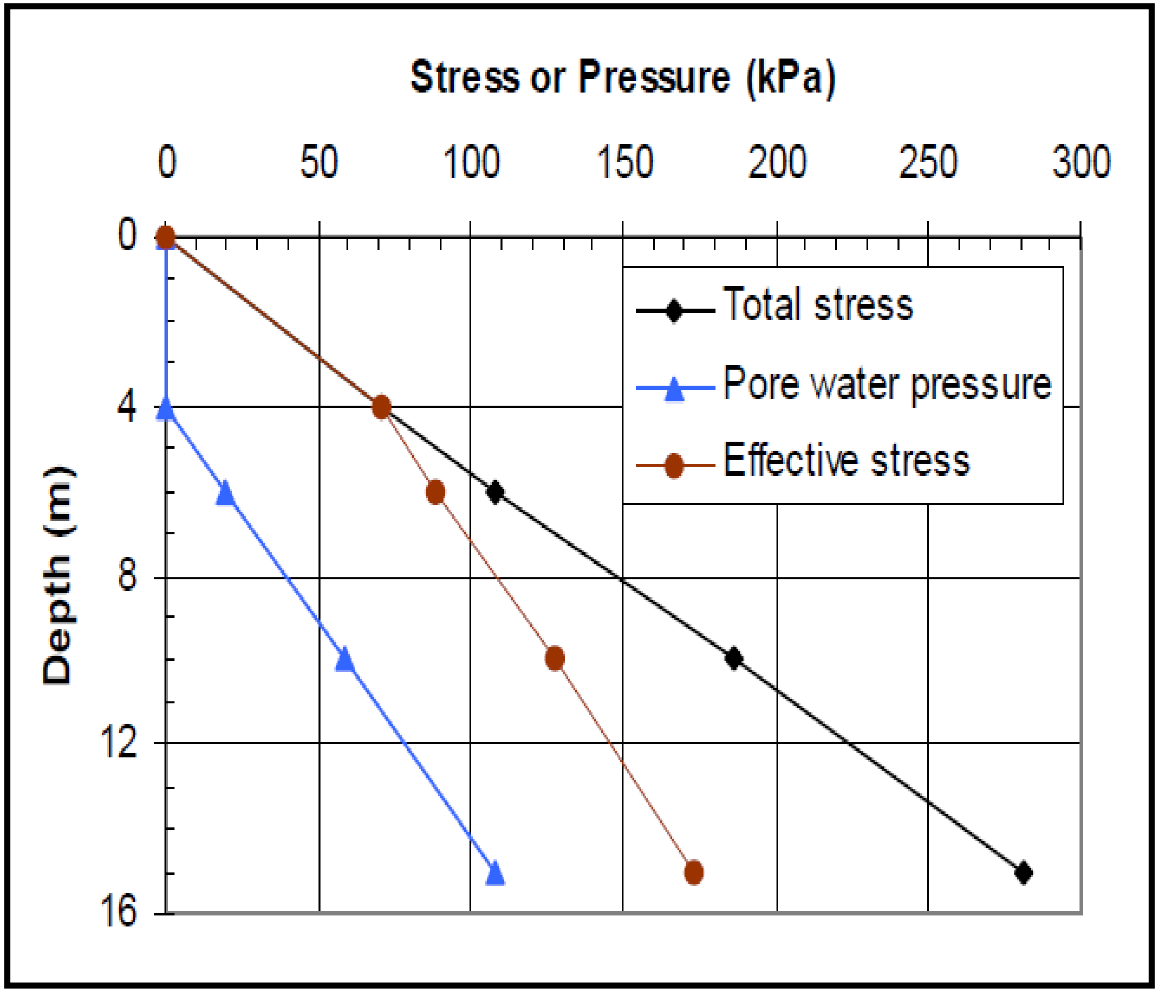
  $\sigma_v = (4)(17.8) + (2)(18.5) + (4)(19.5) + (5)(19.0) = 281.2$ kPa

  $u = (11)(9.81) = 107.9$ kPa

  $\therefore \sigma_v' = 281.2 - 107.9 = 173.3$ kPa

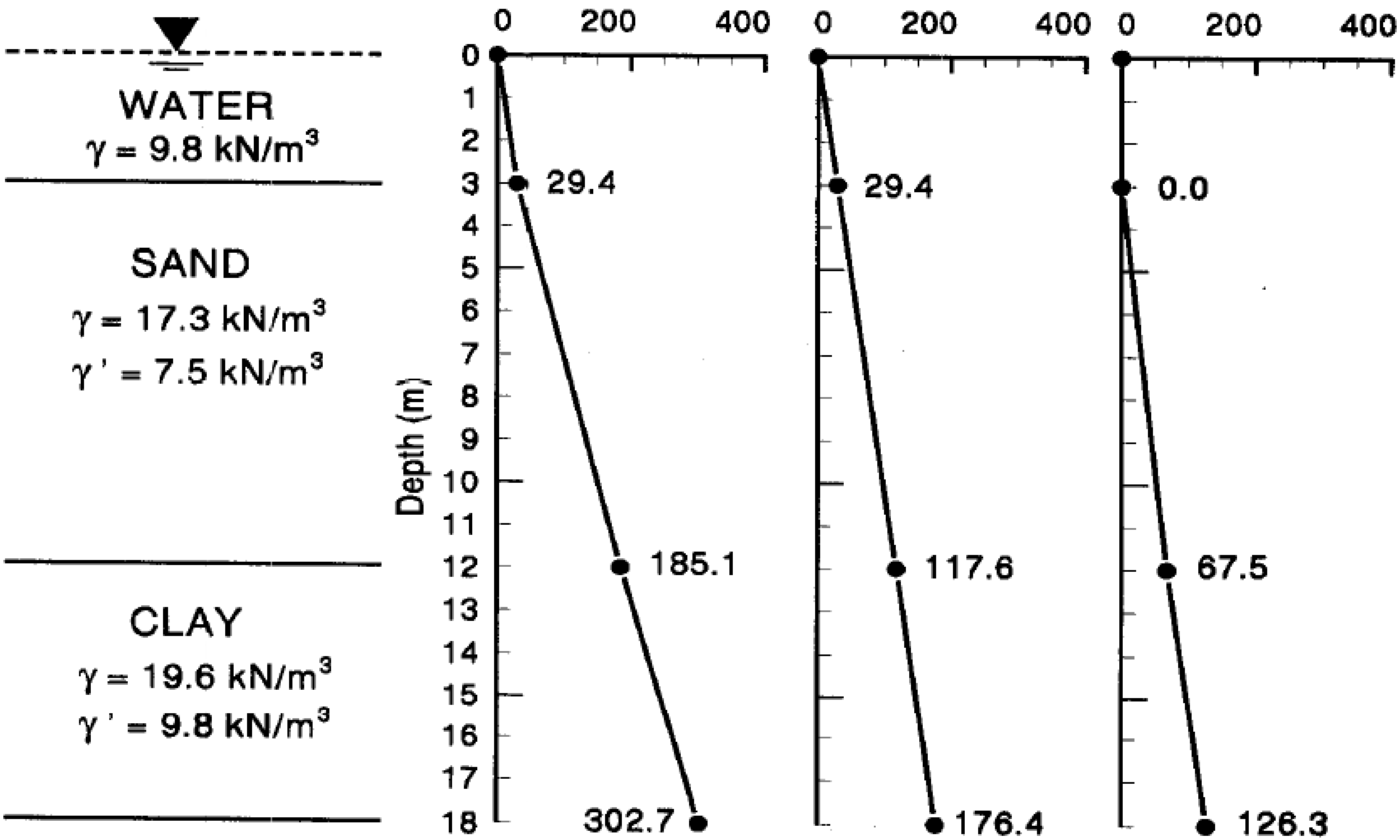
EXAMPLE

Solution



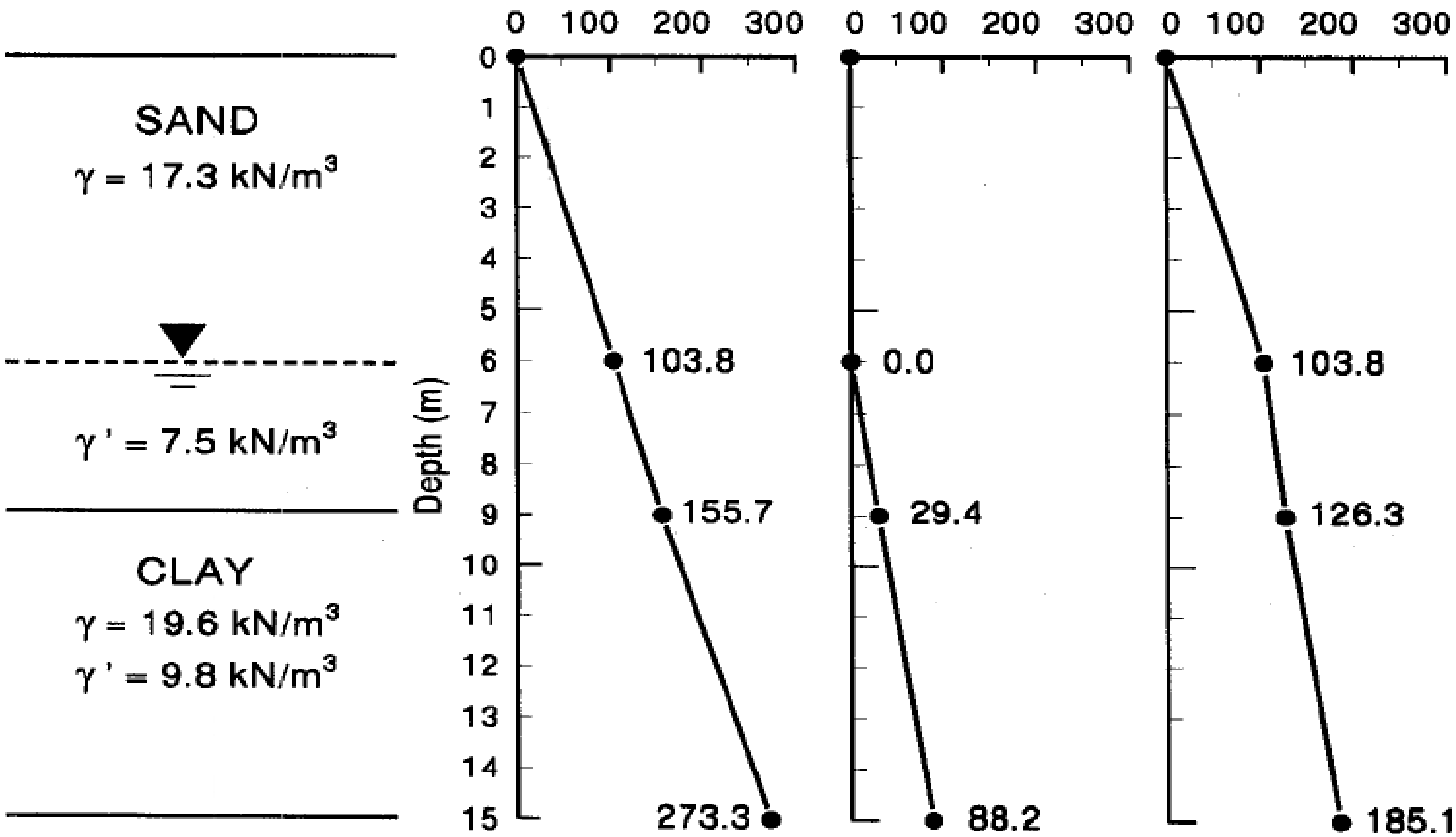
EXAMPLE

σ (kPa) − u (kPa) = σ' (kPa)



EXAMPLE

σ (kPa) − u (kPa) = σ' (kPa)

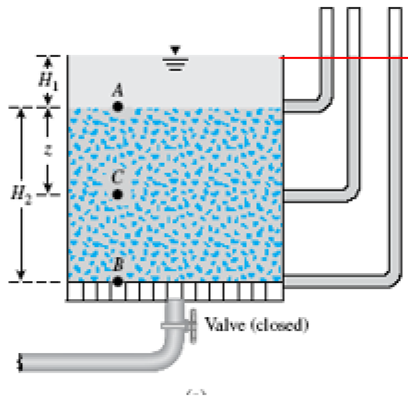


Stresses in Saturated Soil

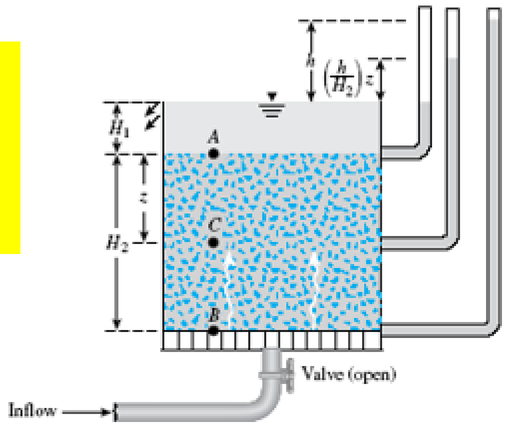
Case of Seepage

ONE-DIMENSIONAL FLOW

Static Case (Hydrostatic Conditions)	Case of Upward Seepage



All piezometers are at the same level

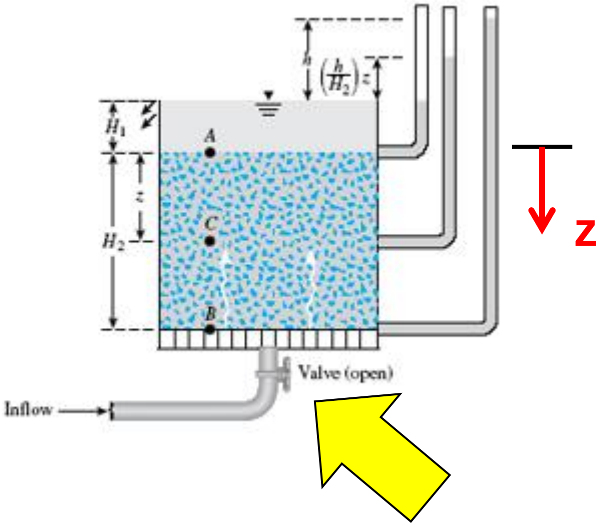


- **Note: the static case is a limit for the upward seepage case.**
- **In other word, if the is no head difference then the upward seepage will seize and we come to static case.**

UPWARD SEEPAGE

- **In case of seepage the effective stress at any point in a soil mass will differ from that in the static case.**

Note: We are given the pressure head



At A,

- Total stress: $\sigma_A = H_1\gamma_w$
- Pore water pressure: $u_A = H_1\gamma_w$
- Effective stress: $\sigma'_A = \sigma_A - u_A = 0$

At B,

- Total stress: $\sigma_B = H_1\gamma_w + H_2\gamma_{sat}$
- Pore water pressure: $u_B = (H_1 + H_2 + h)\gamma_w$
- Effective stress: $\sigma'_B = \sigma_B - u_B$

$$= H_2(\gamma_{sat} - \gamma_w) - h\gamma_w$$
$$= H_2\gamma' - h\gamma_w$$

<u>Note</u>: For illustration we consider simple 1D case
For 1D flow the pressure head at any point can be found through interpolation.

UPWARD SEEPAGE

At C,

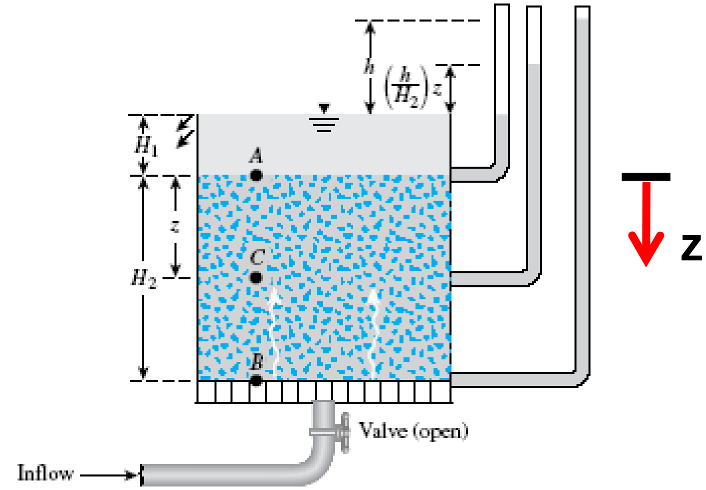
- Total stress: $\sigma_C = H_1\gamma_w + z\gamma_{sat}$
- Pore water pressure: $u_C = \left(H_1 + z + \frac{h}{H_2}z\right)\gamma_w$
- Effective stress: $\sigma'_C = \sigma_C - u_C$

$$= z(\gamma_{sat} - \gamma_w) - \frac{h}{H_2}z\gamma_w$$

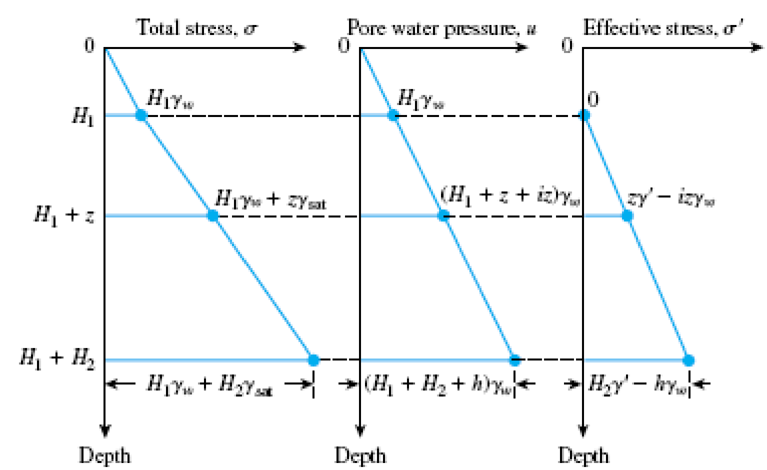
$$= z\gamma' - \frac{h}{H_2}z\gamma_w$$

$$\boxed{\sigma'_C = z\gamma' - iz\gamma_w}$$

- **Therefore the effective stress <u>decreases</u> when there is upward seepage. It decreases by exactly the increase in p.w.p.**

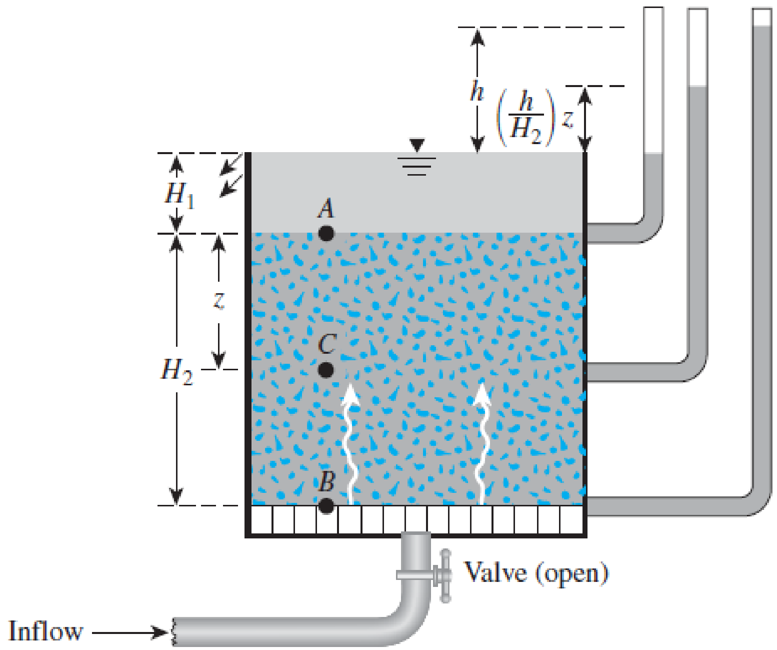


(a)



(b) (c) (d)

UPWARD SEEPAGE



So we directly can apply

$u = \gamma_w h_p$

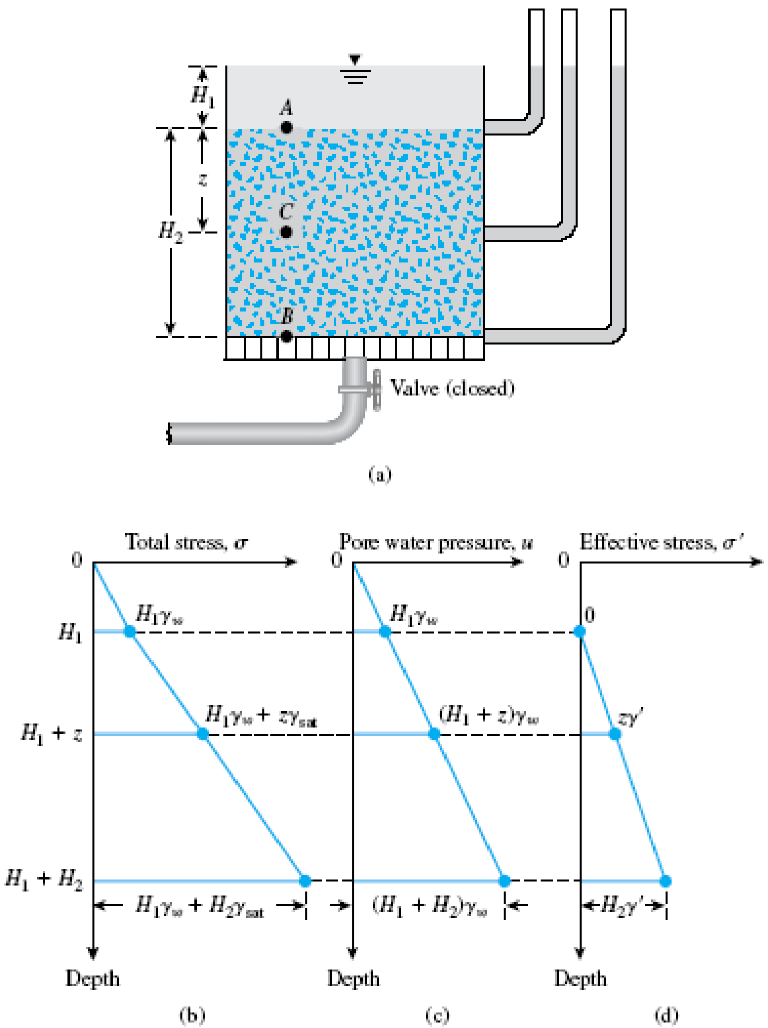
$u_A = \gamma_w h_p = \gamma_w H_1$

$u_B = \gamma_w h_p = \gamma_w (H_2 + H_1 + h)$

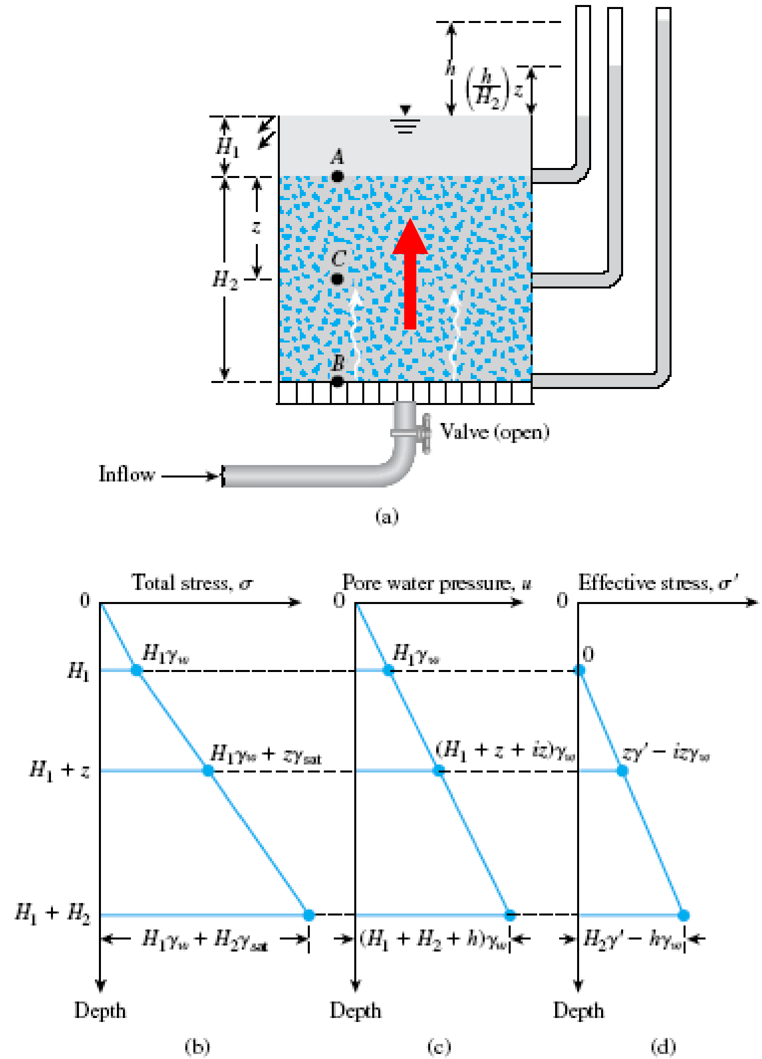
$$u_c = \gamma_w h_p = \gamma_w \left(z + H_1 + \frac{h}{H_2} z\right)$$

ONE-DIMENSIONAL FLOW

WITHOUT SEEPAGE



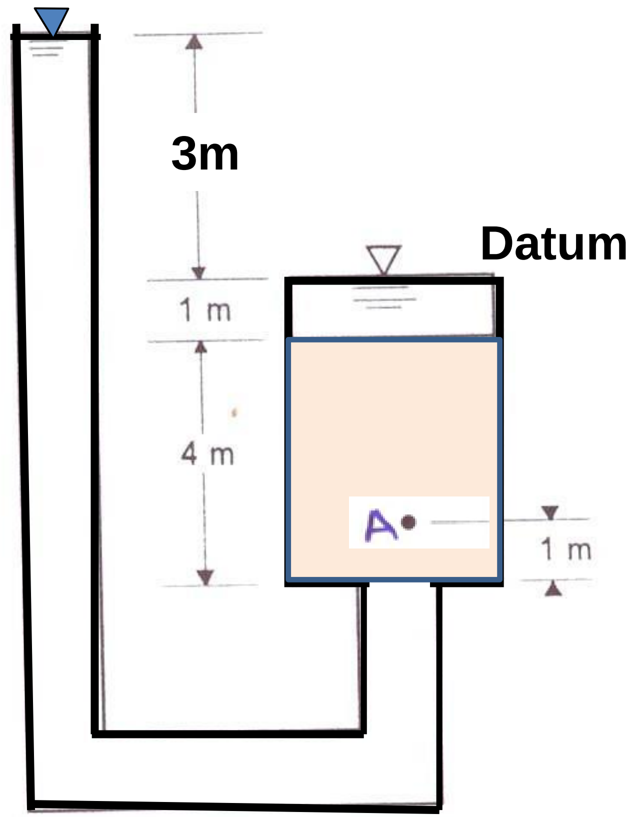
UPWARD SEEPAGE



EXAMPLE

For the case shown, determine;-

The effective stress at point (A)



The unit weight is 18.0 kN/m³

Solution

$$\sigma'_{(A)} = \sigma_{(A)} - u_{(A)}$$

$$\sigma_{(A)} = 9.81 \times 1 + 18 \times 3 = 63.81 kPa$$

$$h_{T(A)} = \frac{h}{L} \times z = \frac{3}{4} \times 3 = 2.25 m$$

$$u_{(A)} = (h_{T(A)} - Z_{(A)}) \times \gamma_w = (2.25 - (-4)) \times$$

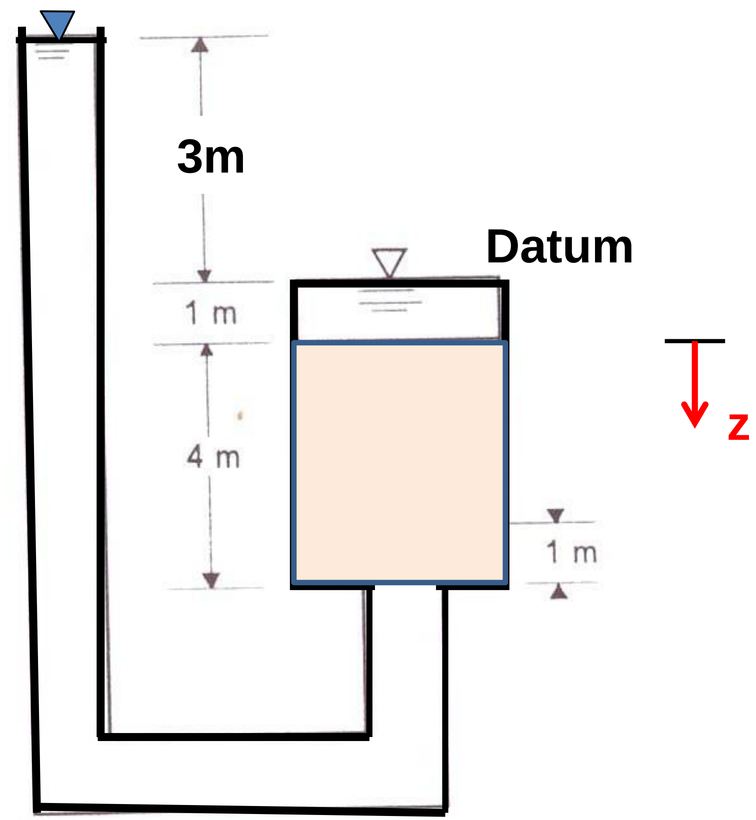
$$\sigma'_{(A)} = 63.8 - 61.3 = \underline{2.5\ kPa}$$

EXAMPLE

We can apply the following equation directly

$$\sigma'_C = z\gamma' - iz\gamma_w$$

$$\sigma'_{(A)} = 3 \times (18 - 9.81) - \frac{3}{4} \times 3 \times 9.81 = 2.5\ kPa$$



Quick Condition (Boiling)

- For the case of upward seepage, what happen if the hydraulic gradient gradually increased. At a certain value of hydraulic gradient the effective stress σ' will be zero (Note that σ' cannot be less than zero).

$$\sigma'_C = z\gamma' - iz\gamma_w$$

$$z\gamma' - iz\gamma_w = 0 \quad \rightarrow \quad i_c = \frac{\gamma'}{\gamma_w}$$

- i_c is called the CRITICAL HYDRAULIC GRADIENT. It is the value of *i* when a quick condition occurs.

- Recall that

$$\gamma' = \gamma_{sub} = \frac{G_s - 1}{1+e}\gamma_w$$

Note: fully saturated soil

$$i_c = \frac{G_s - 1}{1+e}$$

EXAMPLE 9.3

Example 9.3

A 9-m-thick layer of stiff saturated clay is underlain by a layer of sand (Figure 9.5). The sand is under artesian pressure. Calculate the maximum depth of cut H that can be made in the clay.

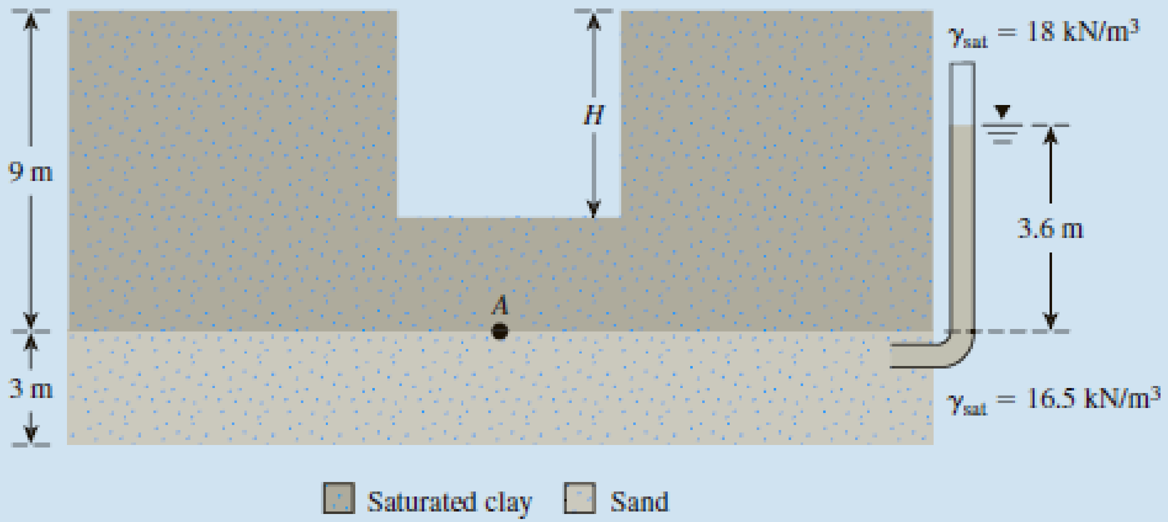


Figure 9.5

EXAMPLE 9.3

Solution

Due to excavation, there will be unloading of the overburden pressure. Let the depth of the cut be H, at which point the bottom will heave. Let us consider the stability of point A at that time:

$$\sigma_A - (9 - H)\gamma_{sat(clay)}$$

$$u_A = 3.6\gamma_w$$

For heave to occur, σ'_A should be 0. So

$$\sigma_A - u_A = (9 - H)\gamma_{sat(clay)} - 3.6\gamma_w$$

or

$$(9 - H)18 - (3.6)9.81 = 0$$

$$H = \frac{(9)18 - (3.6)9.81}{18} = \mathbf{7.04\ m}$$

EXAMPLE 9.4

Example 9.4

A cut is made in a stiff, saturated clay that is underlain by a layer of sand (Figure 9.6). What should be the height of the water, h, in the cut so that the stability of the saturated clay is not lost?

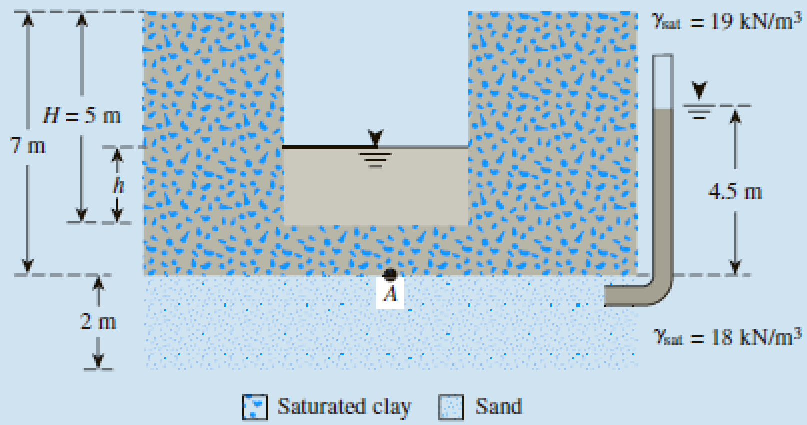


Figure 9.6

Solution

At point A,

$$\sigma_A = (7-5)\gamma_{sat(clay)} + h\gamma_w = (2)(19) + (h)(9.81) = 38 + 9.81h \text{ (kN/m}^2\text{)}$$

$$u_A = 4.5\gamma_w = (4.5)(9.81) = 44.15 \text{ kN/m}^2$$

For loss of stability, $\sigma_A' = 0$. So,

$$\sigma_A - u_A = 0$$

$$38 + 9.81h - 44.15 = 0$$

$$h = \mathbf{0.63\ m}$$

DOWNWARD SEEPAGE

At A,

- **Total Stress:** $\sigma_A = H_1 \gamma_w$
- **Pore water pressure:** $u_A = H_1 \gamma_w$
- **Effective stress:** $\sigma'_A = \sigma_A - uA = 0$

At B,

- **Total Stress:** $\sigma_B = H_1 \gamma_w + H_2 \gamma_{sat}$
- **Pore water pressure:** $u_B = (H_1 + H_2 - h)\, \gamma_w$
- **Effective stress:** $\sigma'_B = H_2(\gamma_{sat} - \gamma_w) + h\, \gamma_w$

<u>At C,</u>

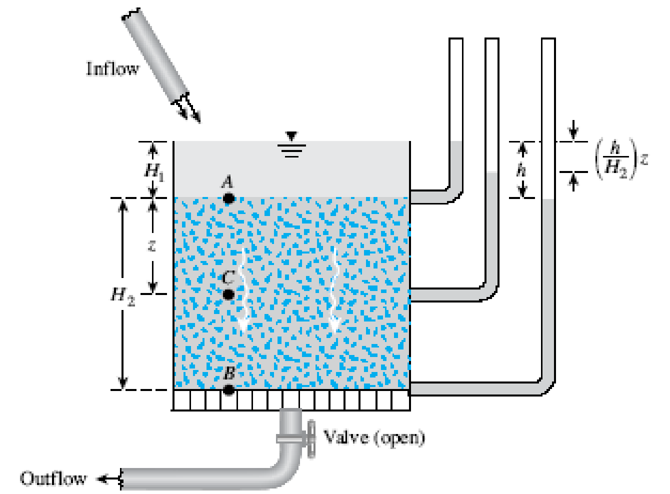
- **Total Stress:** $\sigma_C = H_1 \gamma_w + z\, \gamma_{sat}$
- **Pore water pressure:** $u_C = (H_1 + z - h/H_2\, z)\, \gamma_w$

$$= (H_1 + z - i\, z)\, \gamma_w$$

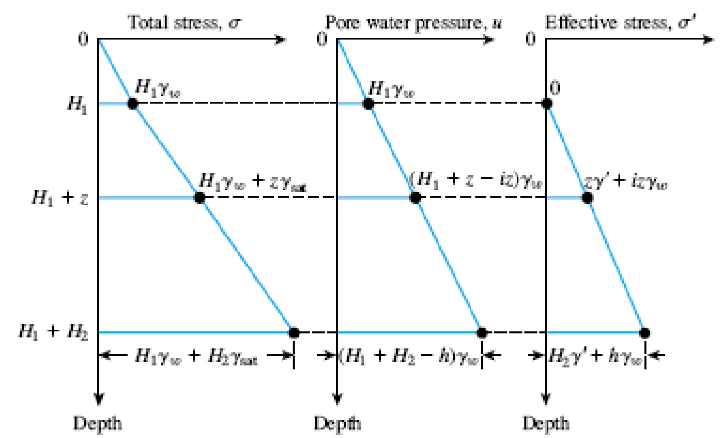
- **Effective stress:** $\sigma'_C = z(\gamma_{sat} - \gamma_w) + iz\, \gamma_w$

$$= z\gamma' + iz\, \gamma_w$$

$$= z\gamma' + iz\gamma_w$$



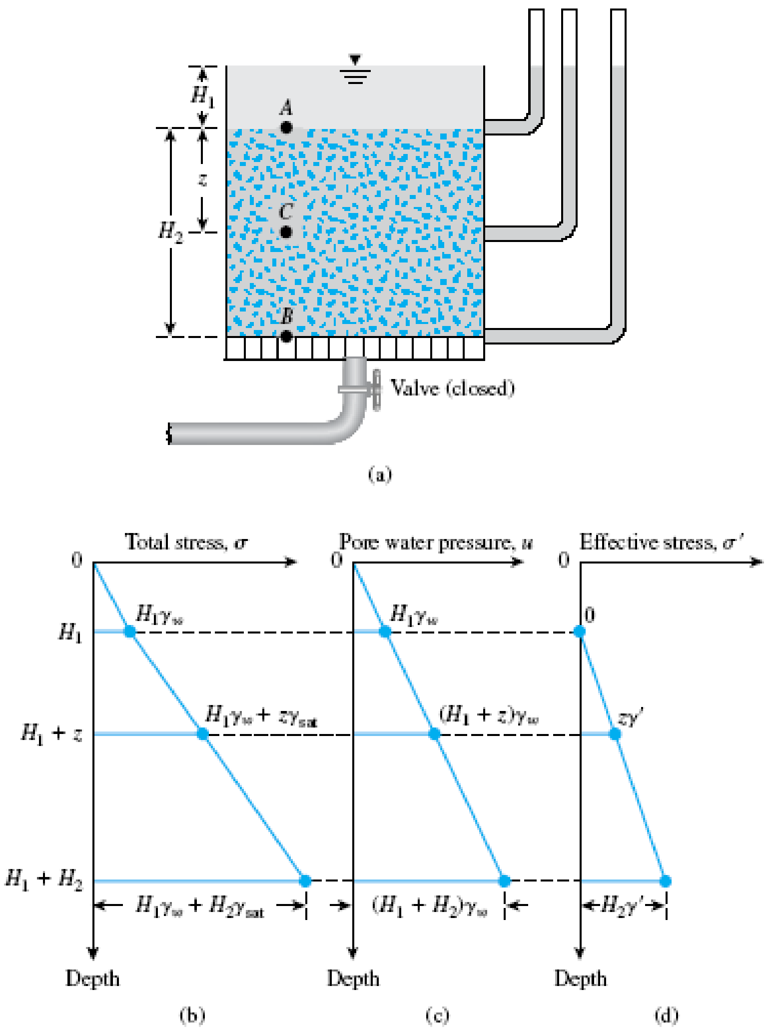
(a)



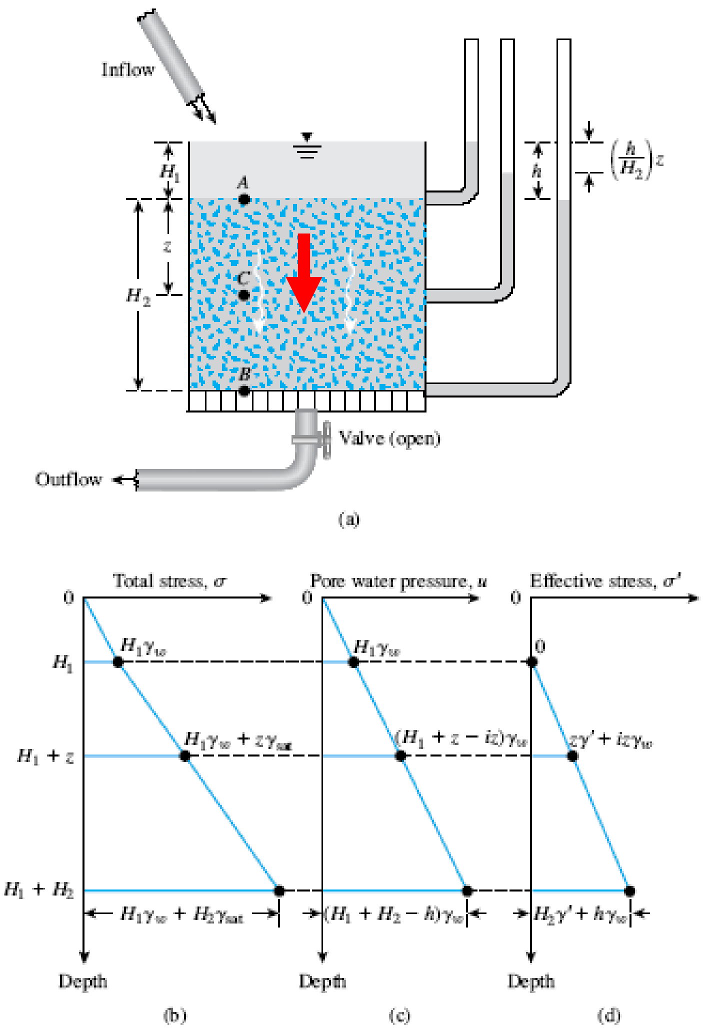
(b) (c) (d)

ONE-DIMENSIONAL FLOW

WITHOUT SEEPAGE

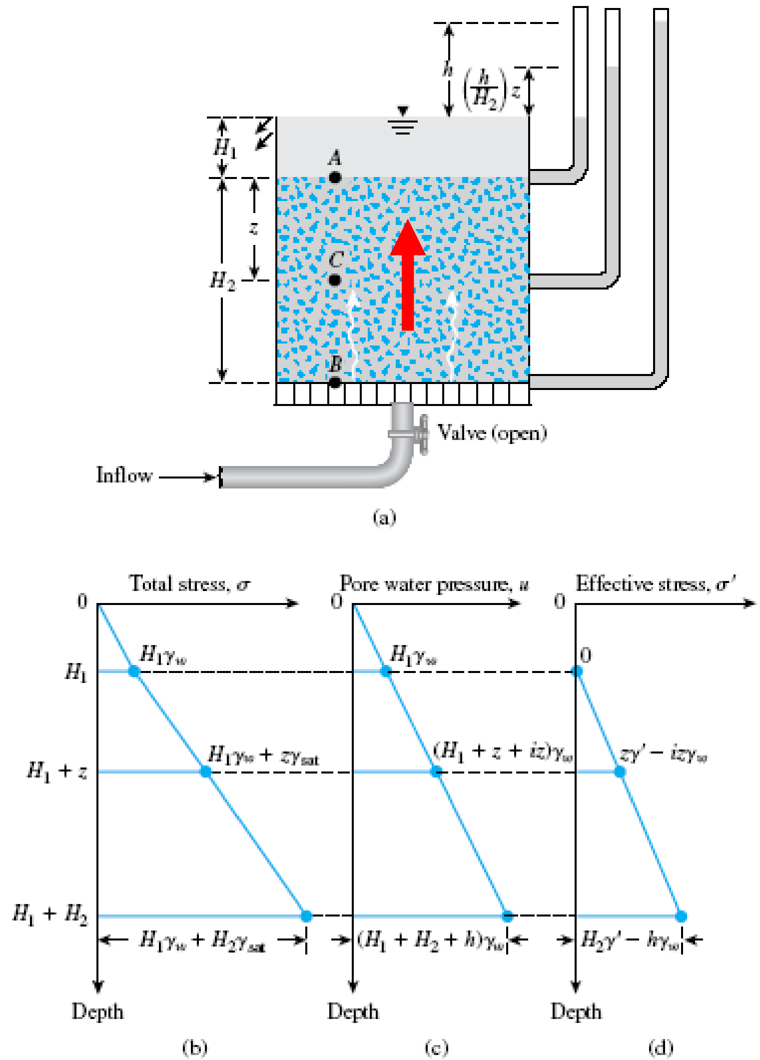


DOWNWORD SEEPAGE

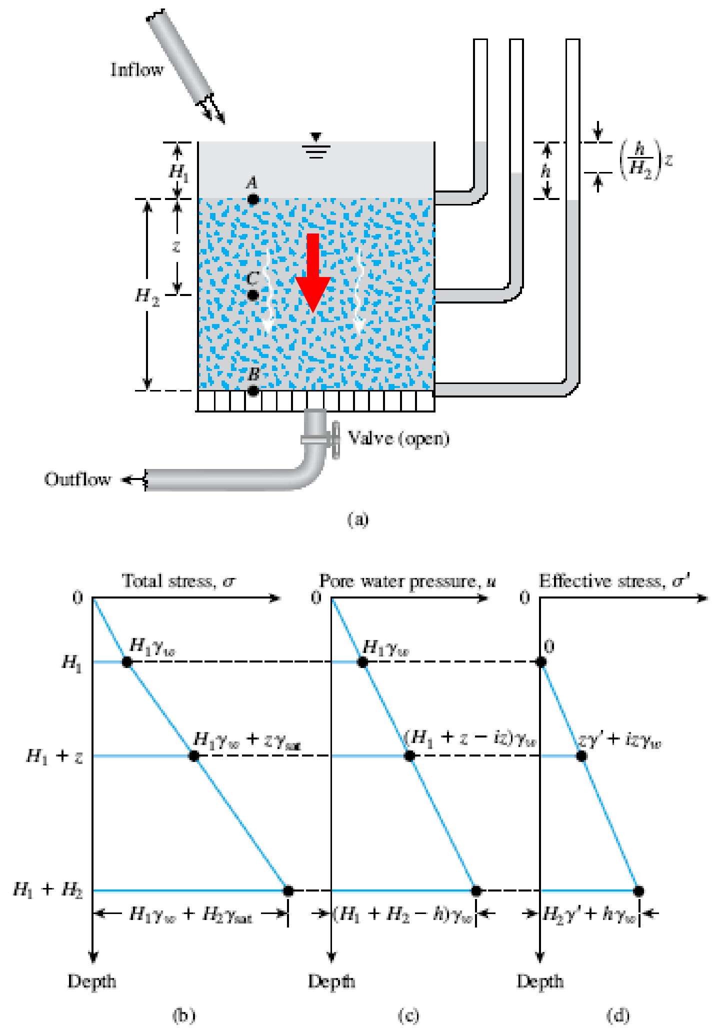


ONE-DIMENSIONAL FLOW

UPWARD SEEPAGE

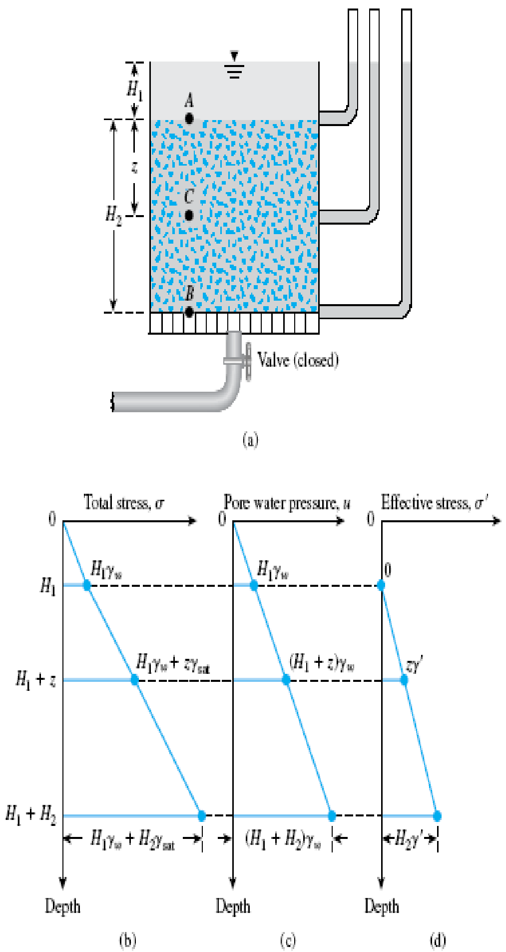


DOWNWORD SEEPAGE

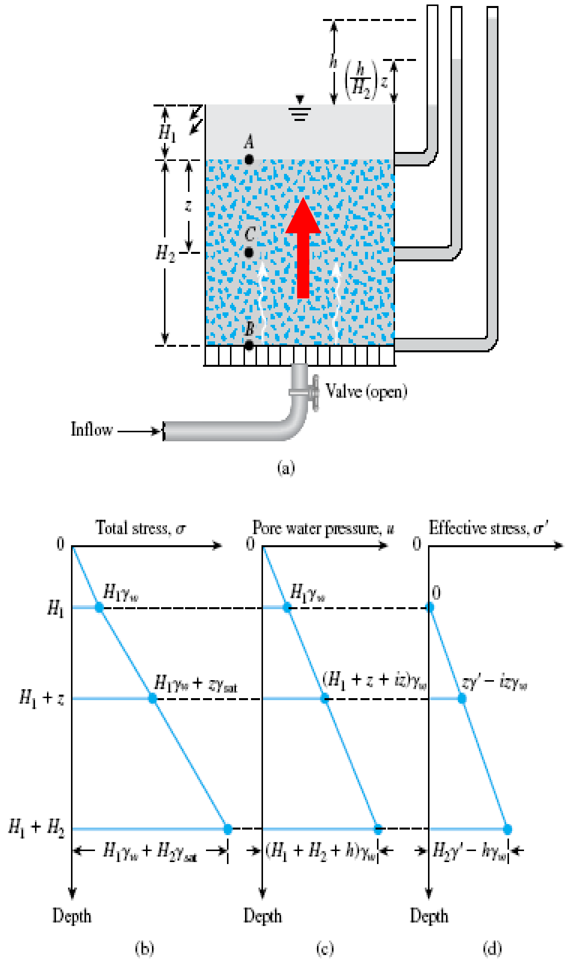


SUMMARY

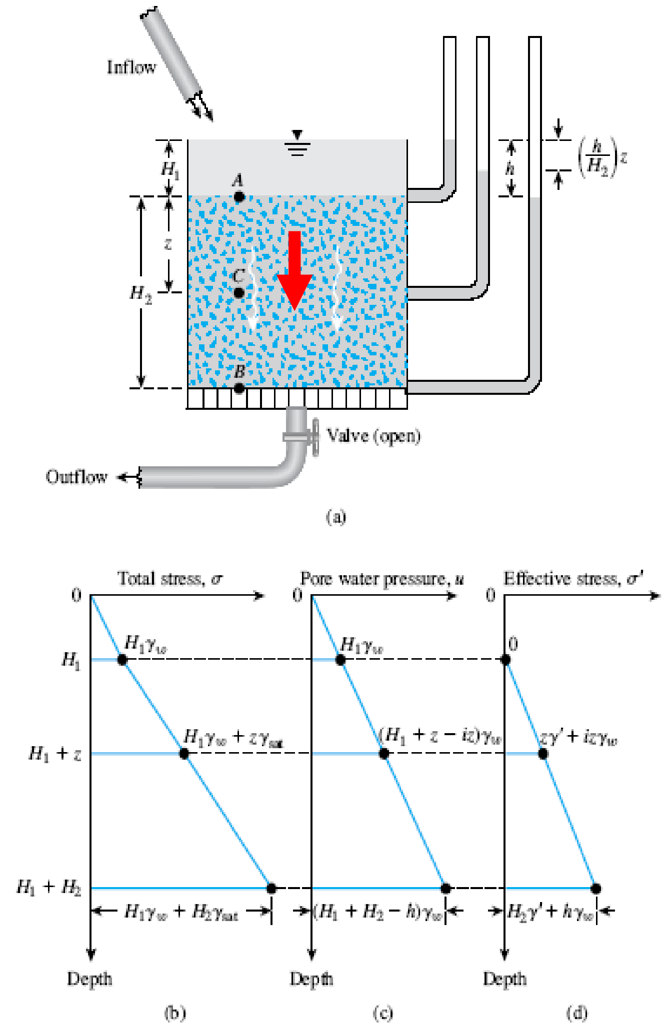
WITHOUT SEEPAGE



UPWARD SEEPAGE



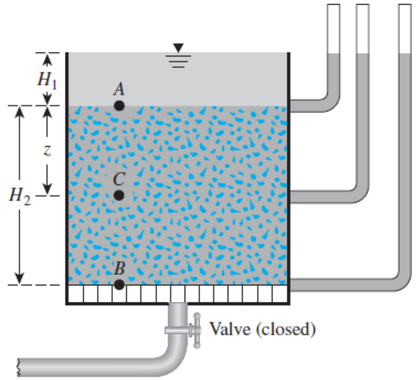
DOWNWARD SEEPAGE



SUMMARY

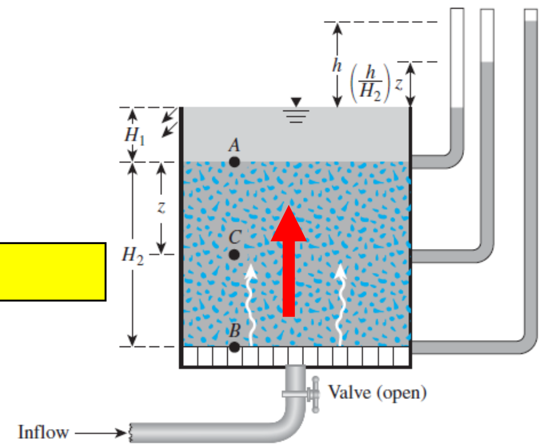
Static Case

$$\sigma'_C = z\,\gamma'$$



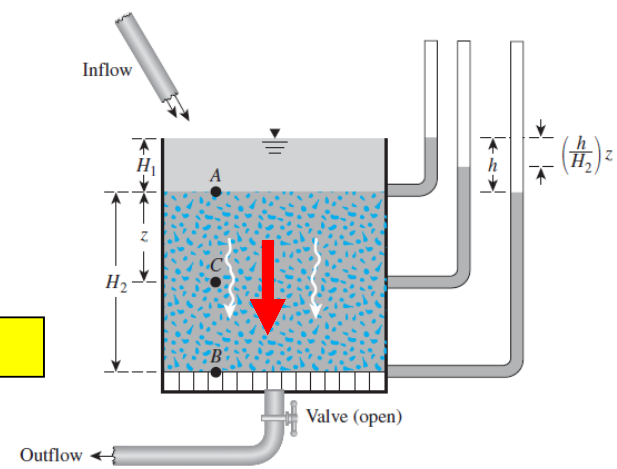
Upward Seepage

$$\sigma'_C = z\gamma' - iz\gamma_w$$



Downward Seepage

$$\sigma'_C = z\gamma' + iz\gamma_w$$



SUMMARY

EFFECT OF SEEPAGE

UPWARD SEEPAGE — **DECREASES EFFECTIVE STRESS**

DOWNWARD SEEPAGE — **INCREASES EFFECTIVE STRESS**

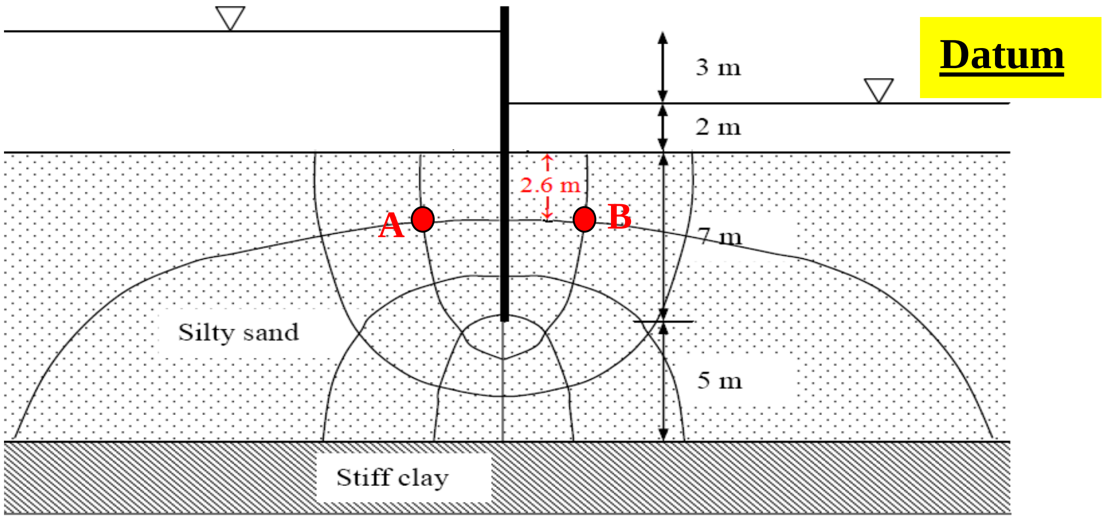
TWO-DIMENSIONAL FLOW

- We have seen from the previous chapter how the total head is obtained by solving Laplace equation, for the specific boundary conditions.
- The problem can be solved graphically using flow net.
- Knowing the total head, the pressure head can be calculated from Bernoulli's equation.
- Effective stress is obtained from effective stress principle.

$$\sigma' = \sigma - u$$

EXAMPLE

Find the effective stress at points A and B (both are at a depth of 2.6 m) if the unit of the silty sand is 19.0 kN/m³.



$$\sigma' = \sigma - u$$

$$\sigma_A = 5 \text{ x } 9.81 + 2.6 \text{ x } 19 = 98.45 \text{ kPa}$$

$$h_{TA} = 3 - 1 \times \frac{3}{8} = 2.625 \text{ m}$$

$$2.625 = \frac{u_A}{9.81} - 4.6$$

$$u_A = 70.9\ kPa$$

$$\sigma'_A = 98.45 - 70.9 = 27.57 kPa$$

$$\sigma_B = 2 \text{ x } 9.81 + 2.6 \text{ x } 19 = 69.02 \text{ kPa}$$

$$h_{TB} = 3 - 7 \times \frac{3}{8} = 0.375 \text{ m}$$

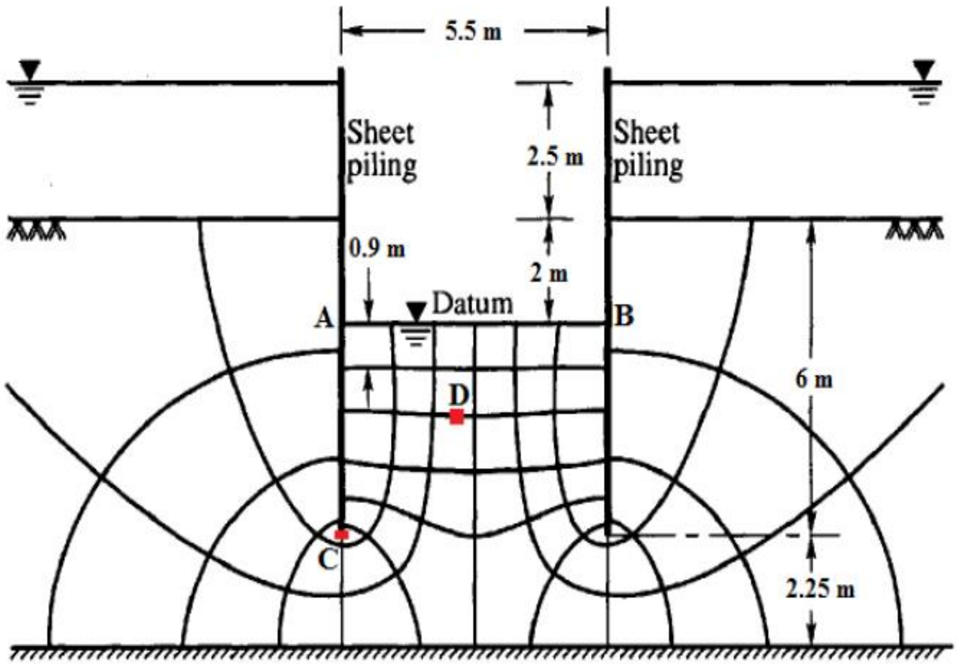
$$0.375 = \frac{u_B}{9.81} - 4.6$$

$$u_B = 48.8\ kPa$$

$$\sigma'_B = 69.02 - 48.8 = 20.22\ kPa$$

EXAMPLE

a) Find the effective stress at point D (located 1.8 m below line AB) if the unit of the sand is 18.0 kN/m^3.



b) Repeat part (a) if the water level behind the wall is lowered to the ground surface and the water level rises 2.0 above the river bed.

SEEPAGE FORCE

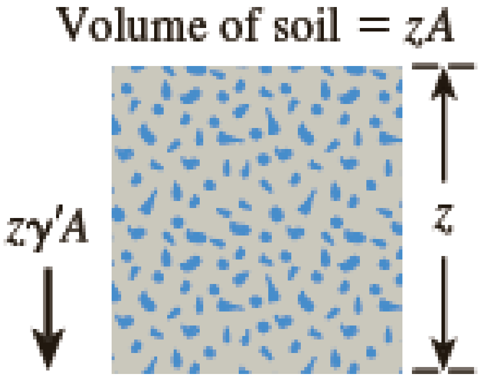
- **We have seen in the preceding sections that the effective stress at a point in a soil layer would increase or decrease due to seepage.**

- **For the hydrostatic case the effective stress at depth z measured from the surface of the soil layer is given by**

$$\sigma' = \gamma' . z$$

- **Therefore, the effective force on an area A is**

$$P_1' = \gamma' . z . A$$



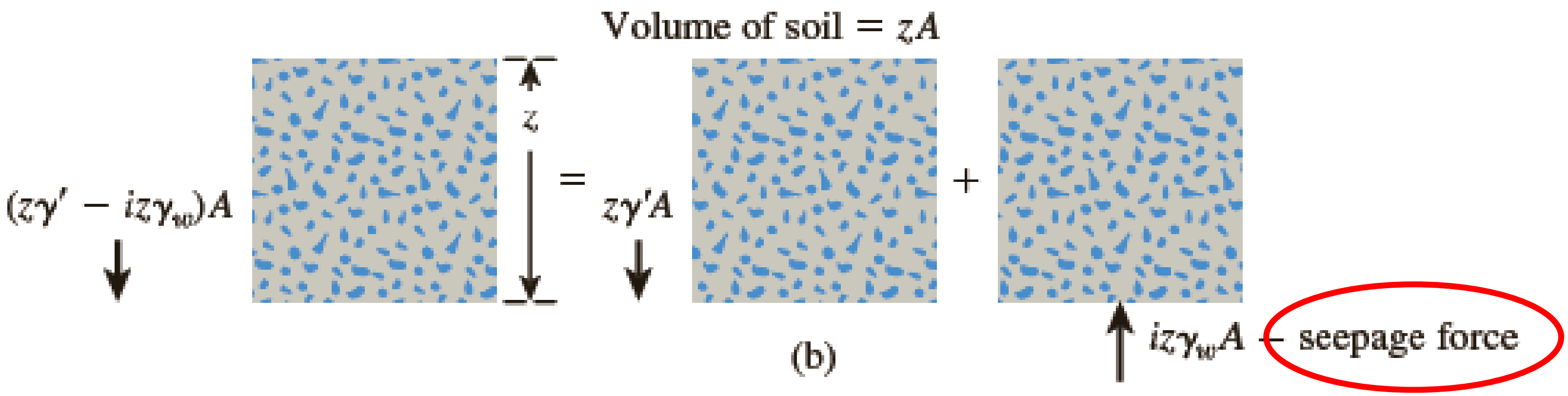
(a)

SEEPAGE FORCE

- If there is an upward seepage of water in the vertical direction through the same soil layer, the effective force on an area A at a depth z is given by,

$$\sigma' = \gamma' z - iz\gamma_w$$

$$P_2' = (\gamma'.z - i.z.\gamma_w)A$$



SEEPAGE FORCE

- Hence, the decrease in the total force because of seepage is

$$P_1' - P_2' = iz\gamma_w A$$

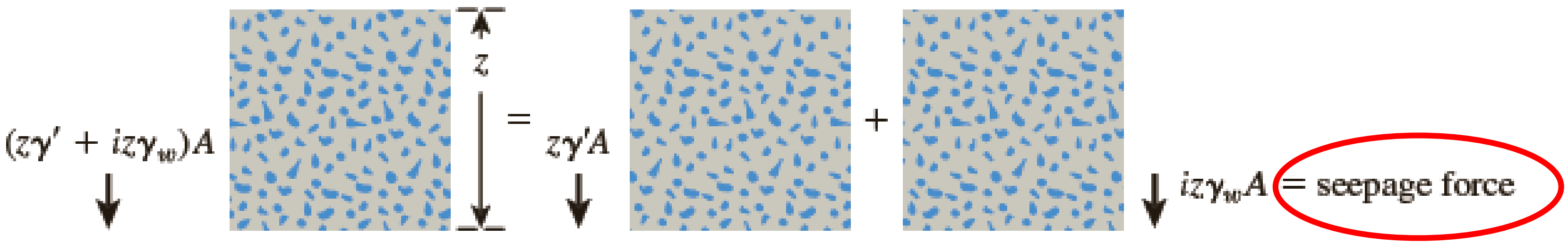
- It is often convenient to express the seepage force per unit volume. Hence the seepage force per unit volume of soil is

$$\frac{P_1' - P_2'}{(\text{Volume of soil})} = \frac{iz\gamma_w A}{zA} = i\gamma_w$$

- The force per unit volume, $i\gamma_w$, for this case acts in the upward direction—that is, in the direction of flow.

SEEPAGE FORCE

- **Similarly, for downward seepage, it can be shown that the seepage force in the downward direction per unit volume of soil is $i\gamma_w$.**



(c)

Remarks

- **Flow nets can be used to find *i* at any point and, thus, seepage force per unit volume of soil. This is important in analyzing the stability of structures where heave is of a problem.**

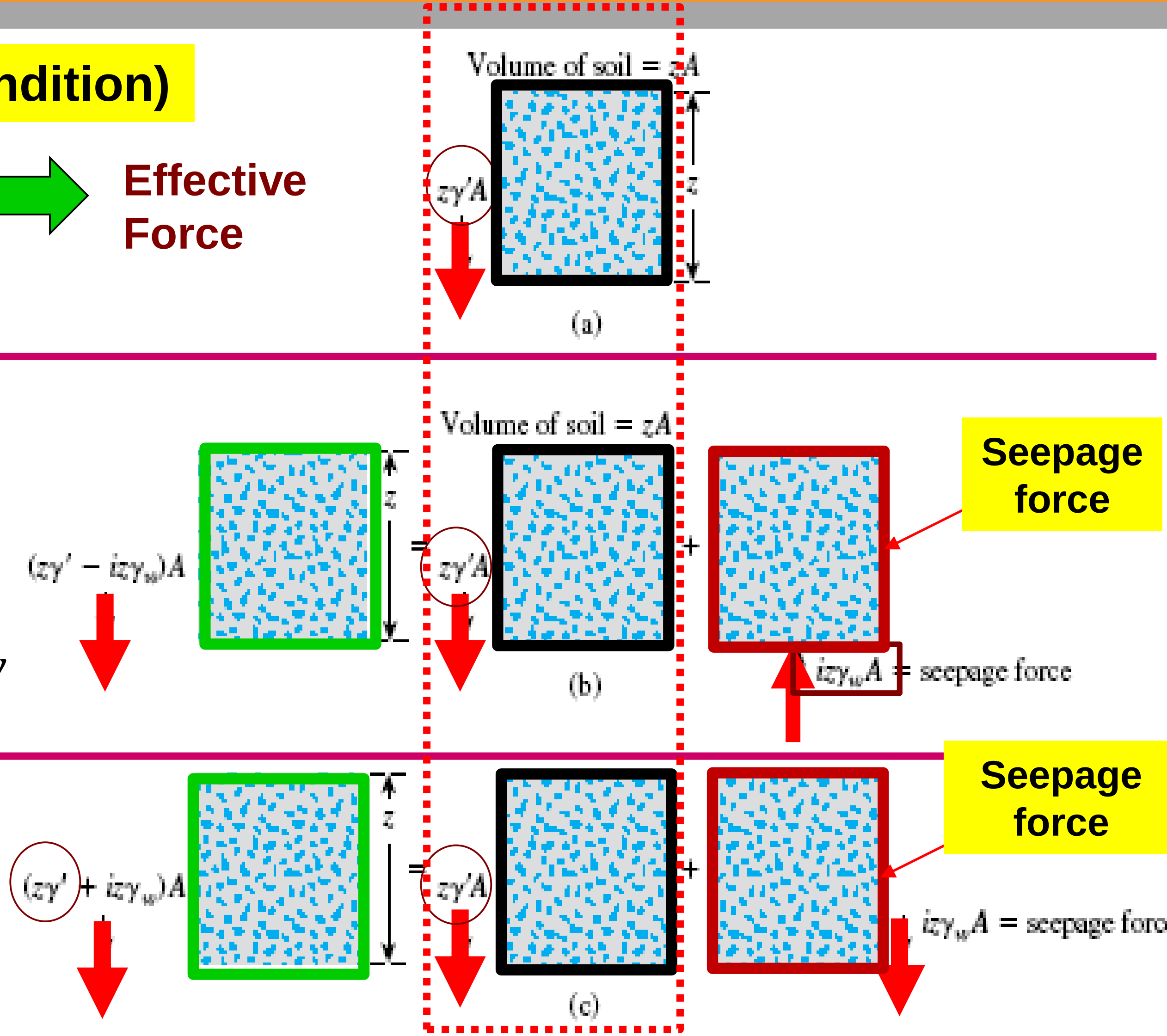
- **In an isotropic soils, the force acts in the same direction as the direction of flow.**

SUMMARY

Static (Hydrostatic condition)

$$\sigma' = z\,\gamma'$$

Effective Force



Upward Seepage

$$\sigma' = \gamma' z - iz\gamma_w$$

Downward Seepage

$$\sigma' = \gamma' z + iz\gamma_w$$

Figure 9.7 Force due to (a) no seepage; (b) upward seepage; (c) downward seepage on a volume of soil

SEEPAGE FORCE

Seepage force per unit volume — determination from flow net

- From the preceding discussions, we can conclude that the seepage force per unit volume of soil is equal to $i.\gamma_w$, and in isotropic soils the force acts in the same direction as the direction of flow (i.e. upward or downward).

- This statement is true for flow in any direction.

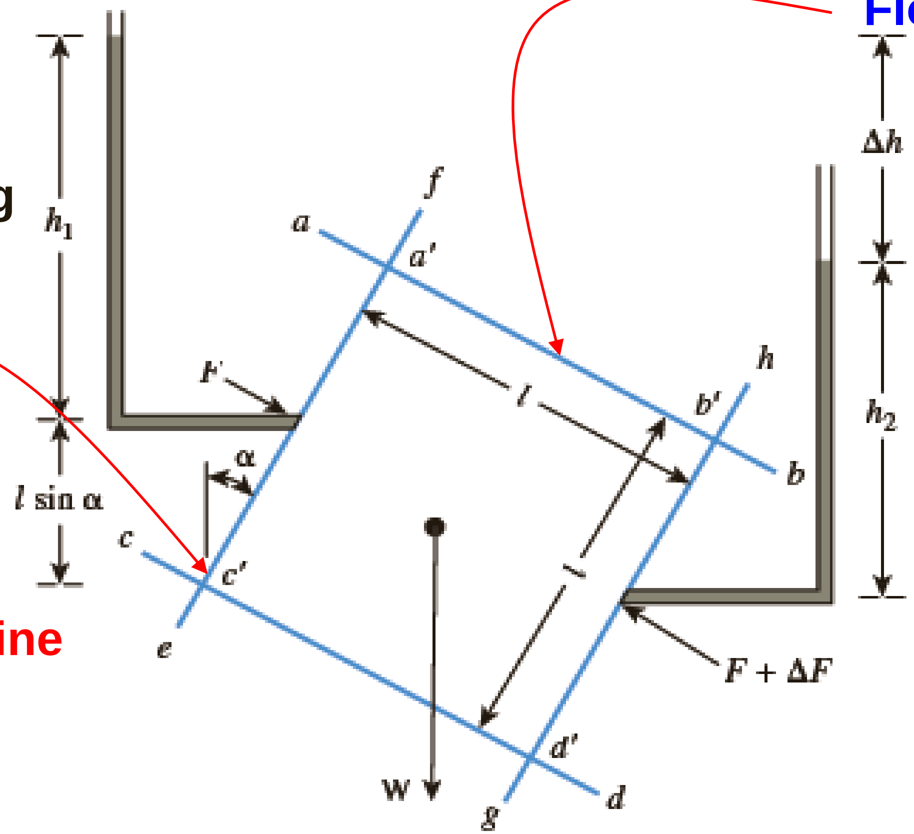
- Flow nets can be used to find the hydraulic gradient at any point and, thus, the seepage force per unit volume of soil.

SEEPAGE FORCE

This is taken from a flow net

Average piezometric elevation along the face $a^{/}$ $c^{/}$

Flow line

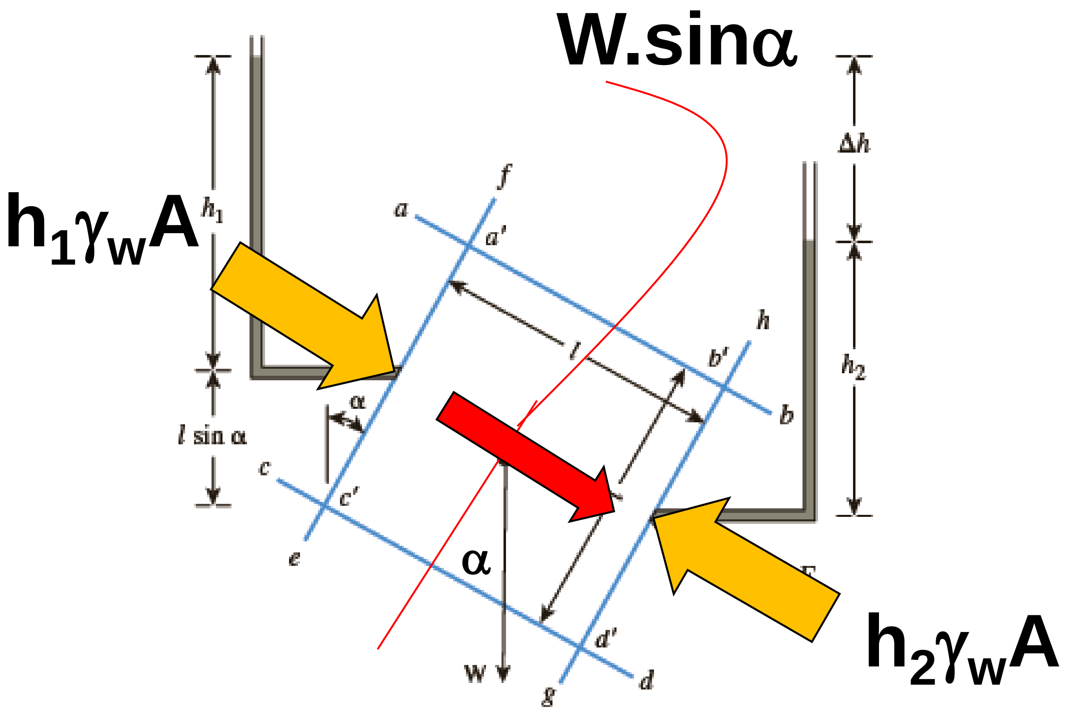


Average piezometric elevation along the face $b^{/}$ $d^{/}$

Equipotential line

- The soil mass has a unit thickness at right angles to the section shown.

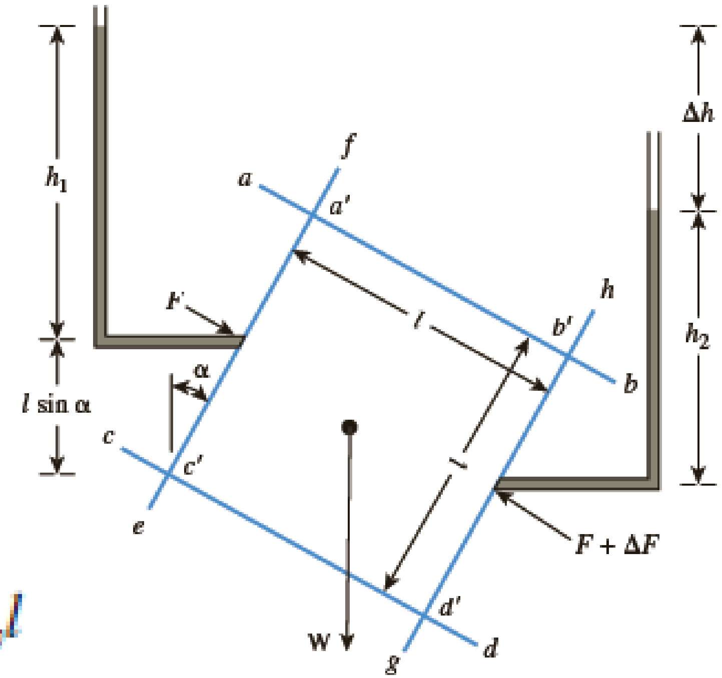
SEEPAGE FORCE



$$W = (l)(l)(1)\gamma_{sat}$$

Hydrostatic Forces

$$\Delta F = h_1\,\gamma_w l + l^2\,\gamma_{sat}\sin\alpha - h_2\,\gamma_w l$$



From geometry

$$h_2 = h_1 + l\sin\alpha - \Delta h$$

$$\Delta F = h_1\,\gamma_w l + l^2\,\gamma_{sat}\sin\alpha$$

$$- (h_1 + l\sin\alpha - \Delta h)\gamma_w l$$

$$\Delta F = l^2(\gamma_{sat} - \gamma_w)\sin\alpha + \Delta h\gamma_w l$$

$$= \underbrace{l^2\gamma'\sin\alpha}_{\text{component of the effective weight of soil in direction of flow}} + \underbrace{\Delta h\gamma_w l}_{\text{seepage force}}$$

$$\text{Seepage force/unit volume} = \frac{\Delta h\gamma_w l}{l^2} = \gamma_w i$$

where *i* hydraulic gradient along the direction of flow.

Exactly the same found before

EXAMPLE 9.5

Example 9.5

Consider the upward flow of water through a layer of sand in a tank as shown in Figure 9.10. For the sand, the following are given: void ratio (e) = 0.52 and specific gravity of solids = 2.67.

a. Calculate the total stress, pore water pressure, and effective stress at points A and B.
b. What is the upward seepage force per unit volume of soil?

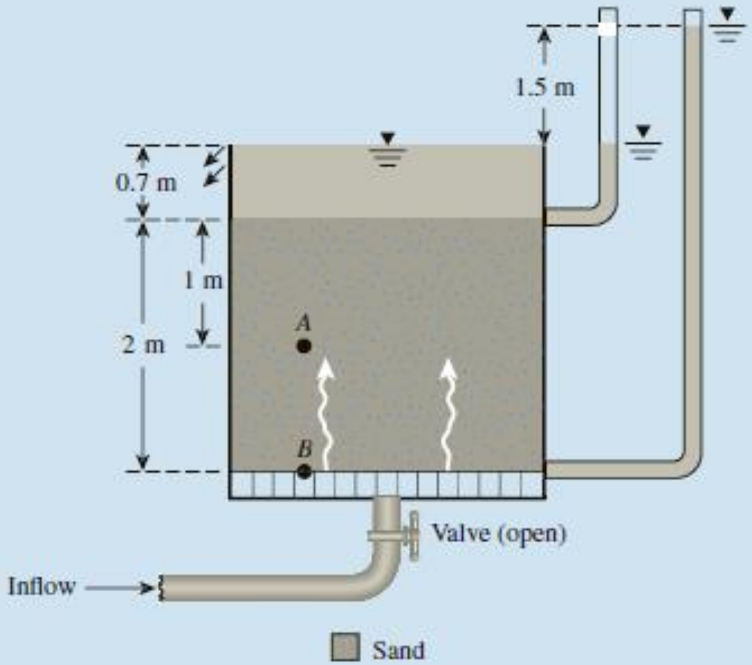


Figure 9.10 Upward flow of water through a layer of sand in a tank

EXAMPLE 9.5

Solution

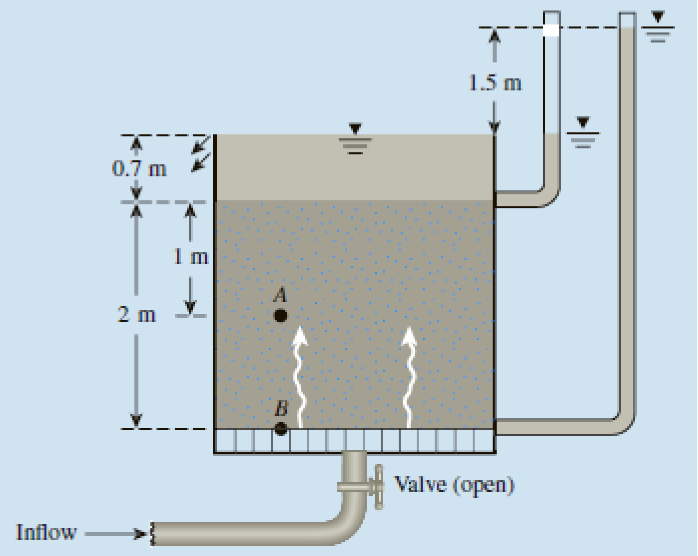
Part a

The saturated unit weight of sand is calculated as follows:

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.67 + 0.52)9.81}{1 + 0.52} = 20.59 \text{ kN/m}^3$$

Now, the following table can be prepared:

Point	Total stress Total stress, σ (kN/m²)	p.w.p Pore water pressure, u (kN/m²)	Effective stress, $\sigma' = \sigma - u$ (kN/m²)
A	$0.7\gamma_w + 1\gamma_{sat} = (0.7)(9.81) + (1)(20.59) = \mathbf{27.46}$	$\left[(1 + 0.7) + \left(\frac{1.5}{2}\right)(1)\right]\gamma_w = (2.45)(9.81) = \mathbf{24.03}$	3.43
B	$0.7\gamma_w + 2\gamma_{sat} = (0.7)(9.81) + (2)(20.59) = \mathbf{48.05}$	$(2 + 0.7 + 1.5)\gamma_w = (4.2)(9.81) = \mathbf{41.2}$	6.85



Part b

Hydraulic gradient (i) = 1.5/2 = 0.75. Thus, the seepage force per unit volume can be calculated as

$$i\gamma_w = (0.75)(9.81) = 7.36 \text{ kN/m}^3$$

HEAVING

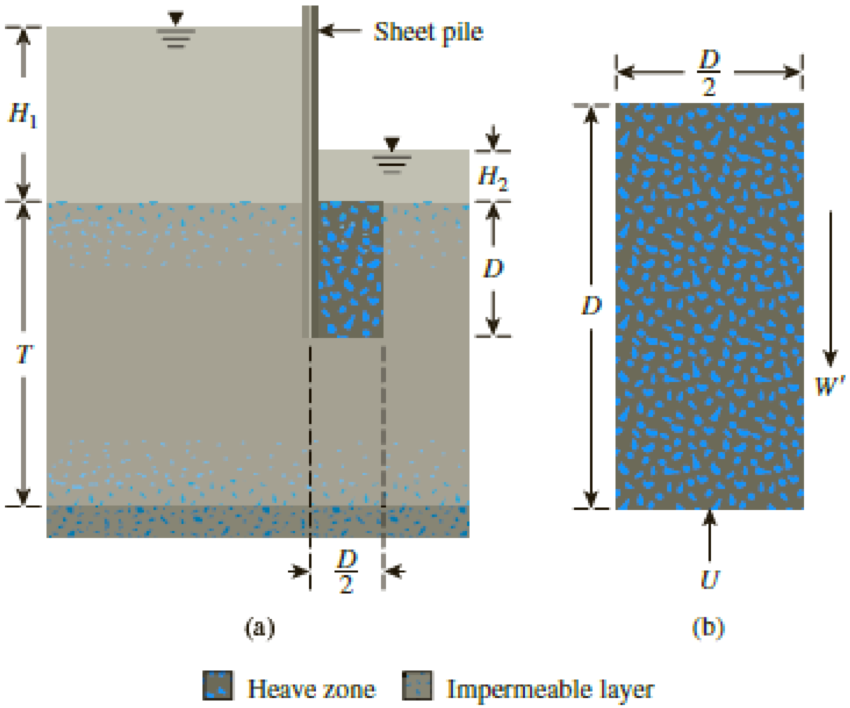
Heaving in Soil Due to Flow around Sheet Piles

$$FS = \frac{\gamma'}{i_{av}\gamma_w}$$

$$FS = \frac{D\gamma'}{C_o \gamma_w (H_1 - H_2)}$$

Table 9.1 Variation of C_o with D/T

D/T	C_o
0.1	0.385
0.2	0.365
0.3	0.359
0.4	0.353
0.5	0.347
0.6	0.339
0.7	0.327
0.8	0.309
0.9	0.274



EXAMPLE 9.6

Example 9.6

Figure 9.14 shows the flow net for seepage of water around a single row of sheet piles driven into a permeable layer. Calculate the factor of safety against downstream heave, given that γ_{sat} for the permeable layer = 17.7 kN/m³. (*Note:* Thickness of permeable layer T = 18 m)

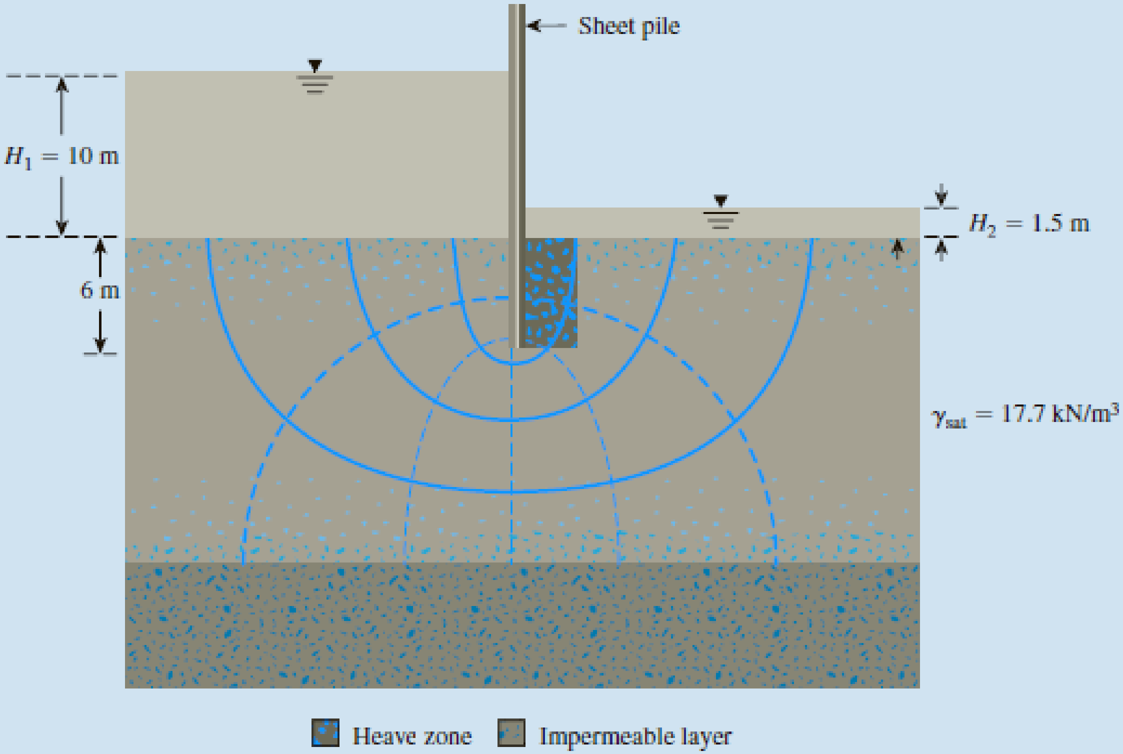


Figure 9.14 Flow net for seepage of water around sheet piles driven into permeable layer

EXAMPLE 9.6

Solution

From the dimensions given in Figure 9.14, the soil prism to be considered is 6 m × 3 m in cross section.

The soil prism is drawn to an enlarged scale in Figure 9.15. By use of the flow net, we can calculate the head loss through the prism.

At b,

$$\text{Driving head} = \frac{3}{6}(H_1 - H_2)$$

At c,

$$\text{Driving head} \approx \frac{1.6}{6}(H_1 - H_2)$$

Similarly, for other intermediate points along bc, the approximate driving heads have been calculated and are shown in Figure 9.15.

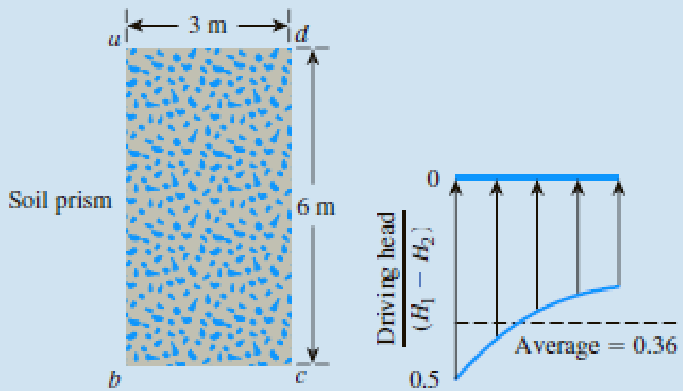


Figure 9.15 Soil prism—enlarged scale

EXAMPLE 9.6

The average value of the head loss in the prism is $0.36(H_1 - H_2)$, and the average hydraulic gradient is

$$i_{av} = \frac{0.36(H_1 - H_2)}{D}$$

Thus, the factor of safety [Eq. (9.20)] is

$$FS = \frac{\gamma'}{i_{av}\gamma_w} = \frac{\gamma' D}{0.36(H_1 - H_2)\gamma_w} = \frac{(17.7 - 9.81)6}{0.36(10 - 1.5) \times 9.81} = \mathbf{1.58}$$

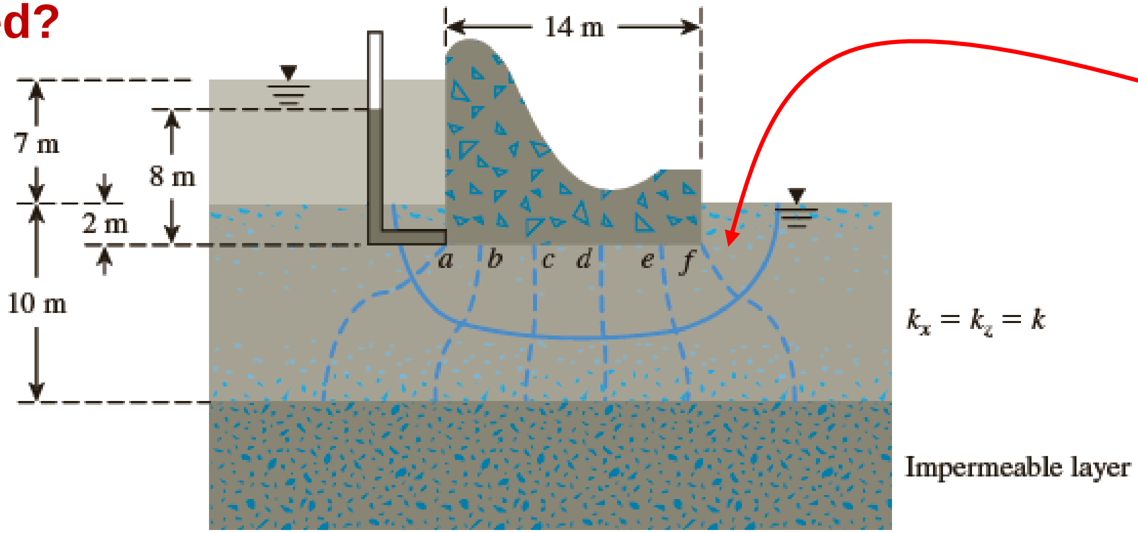
Alternate Solution

For this case, $D/T = 1/3$. From Table 9.1, for $D/T = 1/3$, the value of $C_o = 0.357$. Thus, from Eq. (9.22),

$$FS = \frac{D\gamma'}{C_o\gamma_w(H_1 - H_2)} = \frac{(6)(17.7 - 9.81)}{(0.357)(9.81)(10 - 1.5)} = \mathbf{1.59}$$ **Low**

PIPING

What are of concerned?

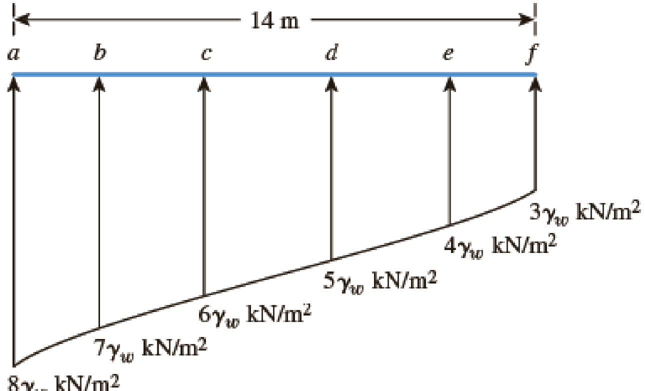


(a)

Exit gradient

$$F_{piping} = \frac{i_c}{i_{exit}}$$

Uplift Pressure

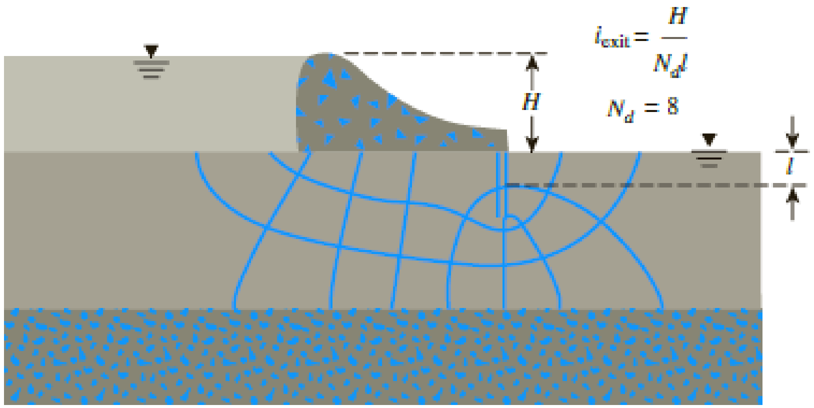


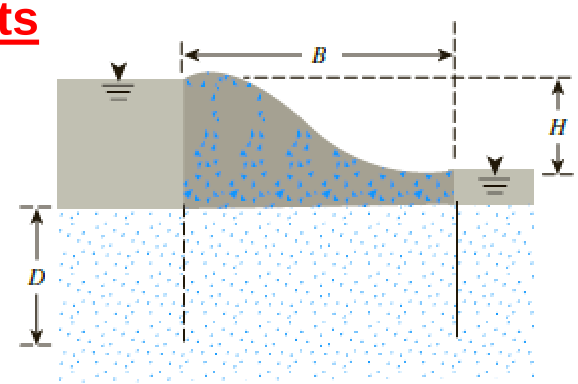
(b)

The uplift force per unit length measured along the axis of the weir can be calculated by finding the area of the pressure diagram.

HEAVING

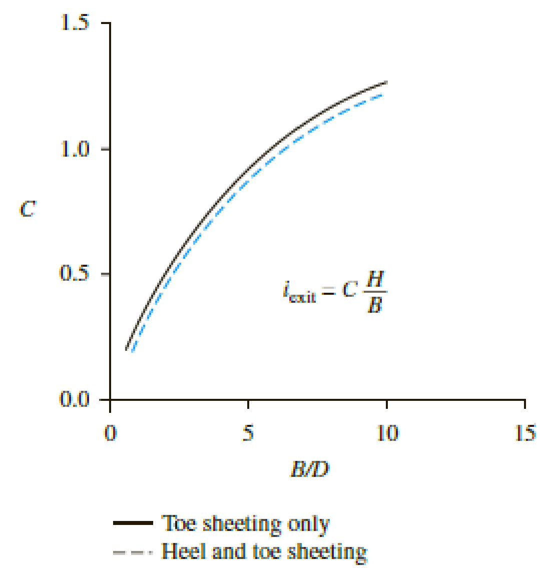
Dams constructed over deep homogeneous deposits





Deep homogeneous soil

$$FS = \frac{i_{cr}}{i_{exit}}$$



$$i_c = \frac{G_s - 1}{1 + e}$$

$$i_{exit} = C\frac{H}{B}$$

EXAMPLE 9.7

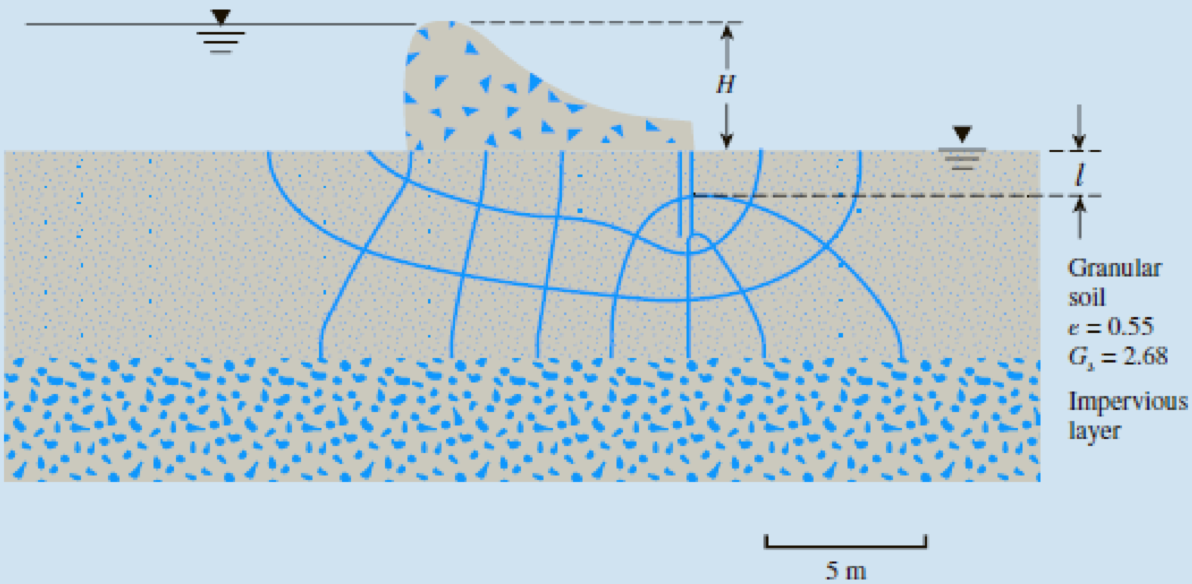
Example 9.7

Refer to Figure 9.16. For the flow under the weir, estimate the factor of safety against piping.

Solution

We can scale the following:

$$H = 4.2 \text{ m}$$
$$l = 1.65 \text{ m}$$



EXAMPLE 9.7

From the flow net, note that $N_d = 8$. So

$$\Delta h = \frac{H}{N_d} = \frac{4.2}{8} = 0.525 \text{ m}$$

$$i_{exit} = \frac{\Delta h}{l} = \frac{0.525}{1.65} = 0.318$$

From Eq. (9.24),

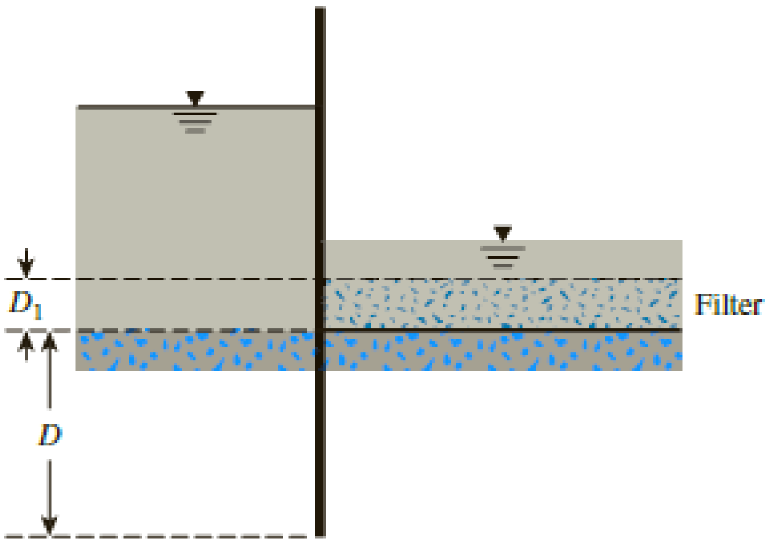
$$i_{cr} = \frac{G_s - 1}{1 + e} = \frac{2.68 - 1}{1 + 0.55} = 1.08$$

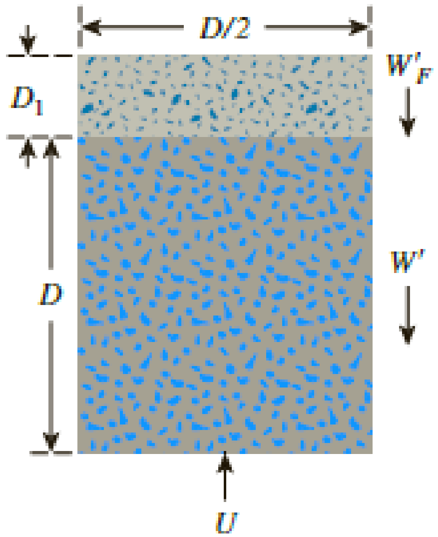
From Eq. (9.23),

$$FS = \frac{i_{cr}}{i_{exit}} = \frac{1.08}{0.318} = \mathbf{3.14}$$

FILTERS

USE OF FILTERS TO INCREASE FS AGAINST HEAVING





$$FS = \frac{D\gamma' + D_1\gamma'_F}{C_o\gamma_w(H_1 - H_2)}$$

FILTERS

USE OF FILTERS TO INCREASE FS AGAINST HEAVING

WITH FILTER

$$FS = \frac{\gamma' + (\frac{D_1}{D})\gamma_F'}{i_{av}\gamma_w}$$

$$FS = \frac{D\gamma' + D_1\gamma_F'}{C_o\gamma_w(H_1 - H_2)}$$

NO FILTER

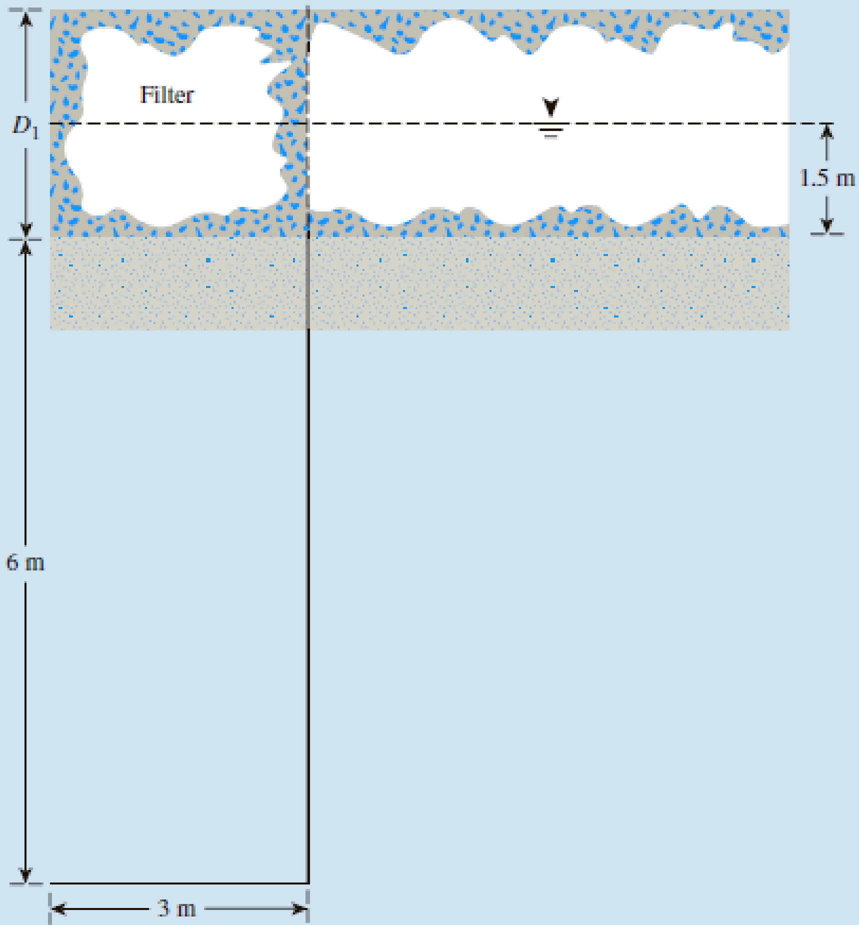
$$FS = \frac{\gamma'}{i_{av}\gamma_w}$$

$$FS = \frac{D\gamma'}{C_o\gamma_w(H_1 - H_2)}$$

EXAMPLE 9.8

Example 9.8

Refer to Example 9.6. If the factor of safety against heaving needs to be increased to 2.5 by laying a filter layer on the downstream side, what should be the thickness of the layer? Given: dry and saturated unit weights of the filter material are 16 kN/m^3 and 20 kN/m^3, respectively.



EXAMPLE 9.8

Solution

Refer to Figure 9.18. The filter material has a thickness of D_1. The top (D_1 − 1.5 m) of the filter is dry, and the bottom 1.5 m of the filter is submerged. Now, from Eq. (9.27),

$$FS = \frac{D\gamma' + (D_1 - 1.5)\gamma_{d(F)} + 1.5\gamma'_F}{C_o\gamma_w(H_1 - H_2)}$$

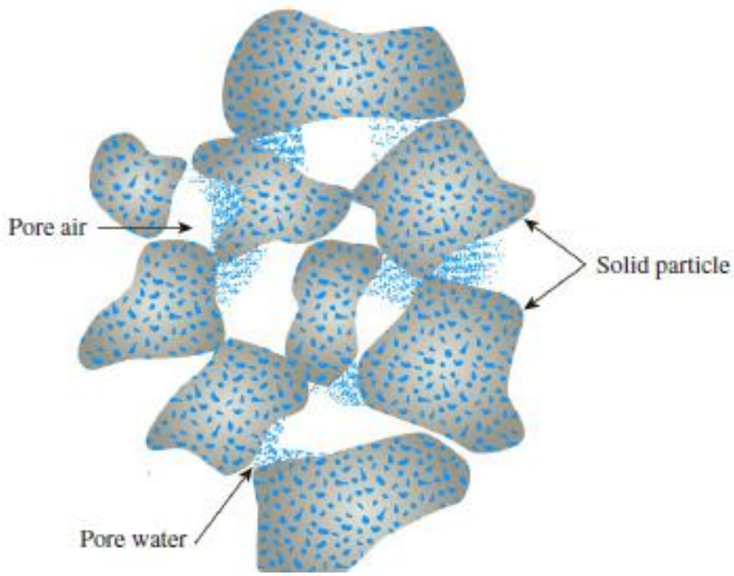
or

$$2.5 = \frac{(6)(17.7 - 9.81) + (D_1 - 1.5)(16) + (1.5)(20 - 9.81)}{(0.375)(9.81)(10 - 1.5)}$$

$D_1 \approx$ **2.47 m**

EFFECTIVE STRESS IN PARTIALLY SATURATED SOIL

$$\sigma' = \sigma - u_a + x(u_a - u_w)$$



$\sigma' =$ Effective stress

$\sigma =$ Total stress

$u_a =$ Pore air pressure

$u_w =$ Pore water pressure

$x =$ fraction of cross - sectional area of the soil occupied by water

$x = 0$ dry soil

$x = 1$ saturated soil

CAPILLARY RISE IN SOILS

- The continuous void spaces in soil can behave as bundles of capillary tubes of variable cross section.
- Because of surface tension force, water may rise above the phreatic surface.

Height of Rise in a Capillary Tube

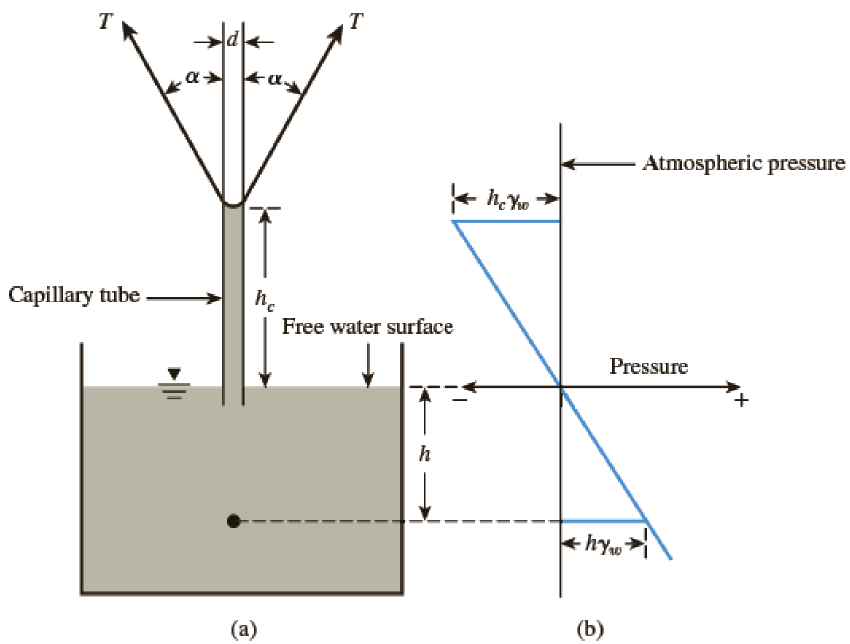
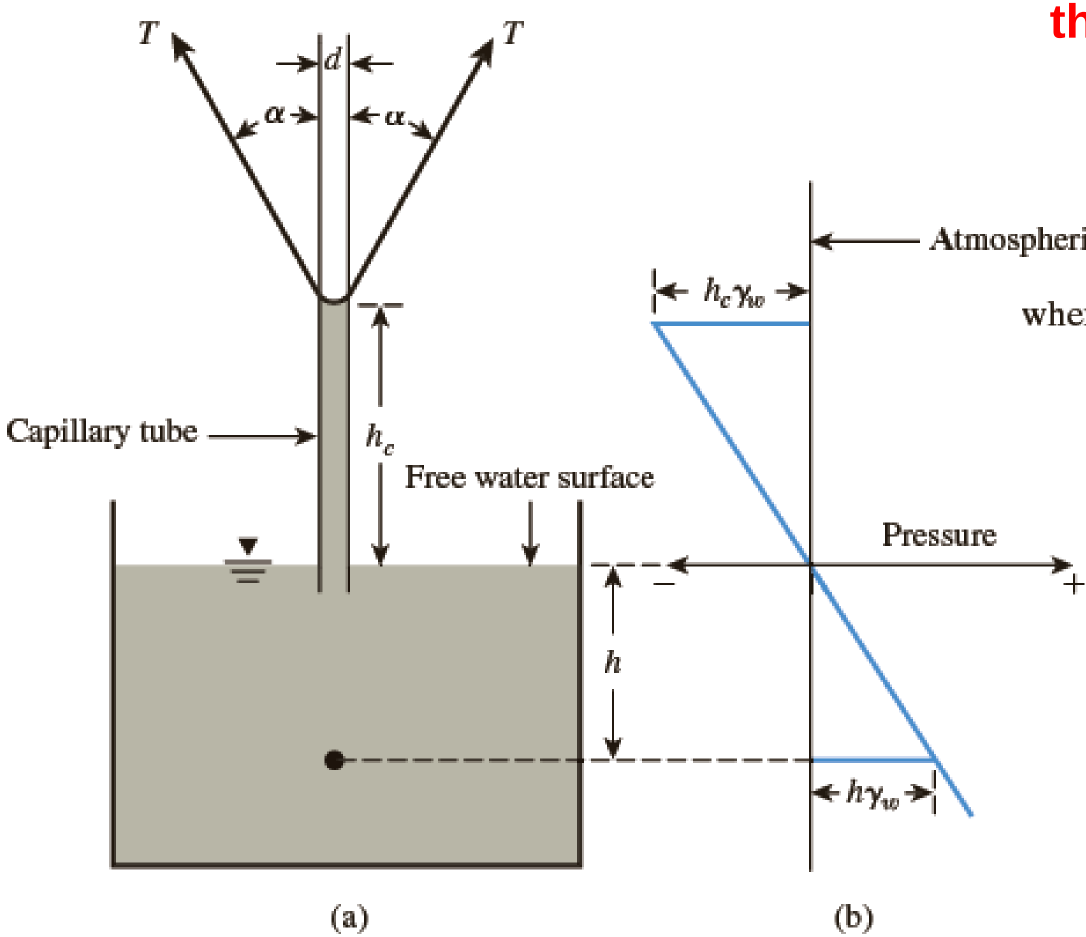


Figure 9.21 (a) Rise of water in the capillary tube; (b) pressure within the height of rise in the capillary tube (atmospheric pressure taken as datum)

CAPILLARY RISE IN SOILS



Summing the forces in the vertical direction

$$\left(\frac{\pi}{4} d^2\right) h_c \gamma_w = \pi d T \cos\alpha$$

where T = surface tension (force/length)
α = angle of contact
d = diameter of capillary tube
γ_w = unit weight of water

$$h_c = \frac{4T\cos\alpha}{d\gamma_w}$$

For pure water and clean glass, $\alpha = 0$.

$$h_c = \frac{4T}{d\gamma_w}$$

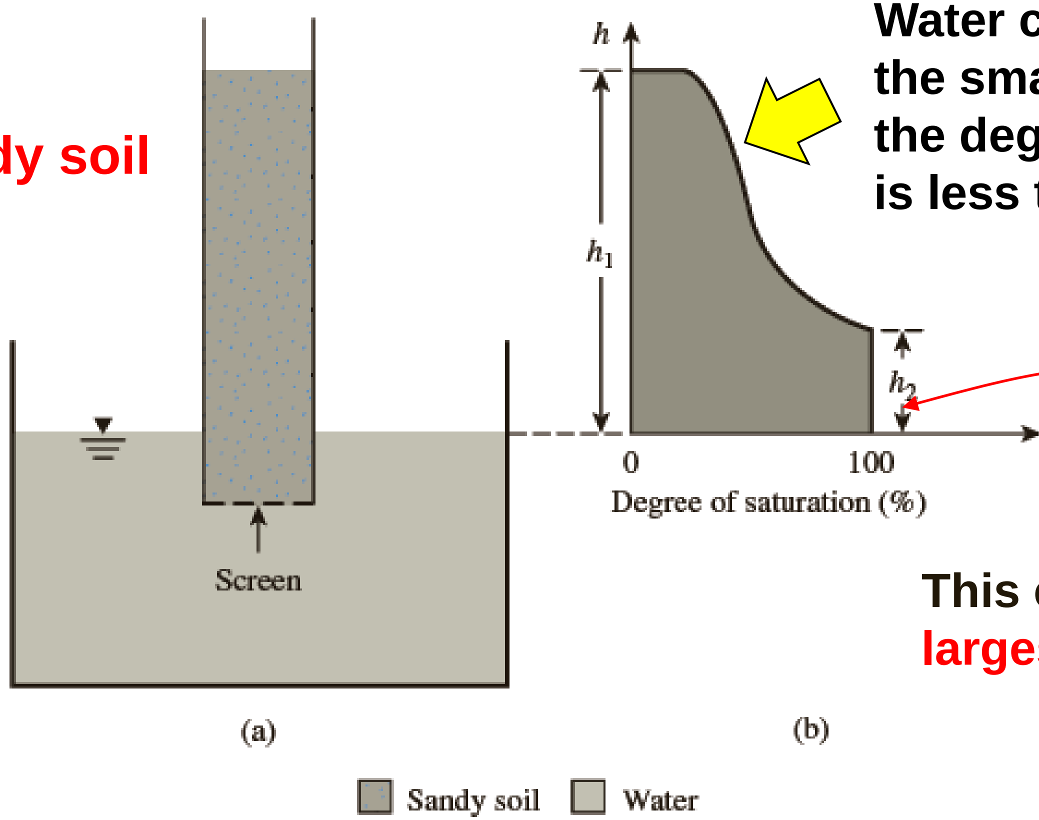
$$h_c \propto \frac{1}{d}$$

Figure 9.21 (a) Rise of water in the capillary tube; (b) pressure within the height of rise in the capillary tube (atmospheric pressure taken as datum)

CAPILLARY RISE IN SOILS

- The capillary tubes formed in soils because of the continuity of voids have variable cross sections.

Sandy soil



Water can occupy only the smaller voids; hence, the degree of saturation is less than 100%.

$$h_1\ (\text{mm}) = \frac{C}{eD_{10}}$$

This corresponds to the largest voids.

Figure 9.22 Capillary effect in sandy soil: (a) a soil column in contact with water; (b) variation of degree of saturation in the soil column

CAPILLARY RISE IN SOILS

$$h_1\,(\text{mm}) = \frac{C}{eD_{10}}$$

C_1 = a constant that varies from 10 to 50 mm^2

Table 9.2 Approximate Range of Capillary Rise in Soils

Soil type	Range of capillary rise (m)
Coarse sand	0.1–0.2
Fine sand	0.3–1.2
Silt	0.75–7.5
Clay	7.5–23

Effective Stress in the Zone of Capillary Rise

$$\sigma = \sigma' + u$$

- The pore water pressure u at a point in a layer of soil fully saturated by capillary rise is equal to

$$u = -\gamma_w h$$

- Where h is the height of the point under consideration measured from the groundwater table.

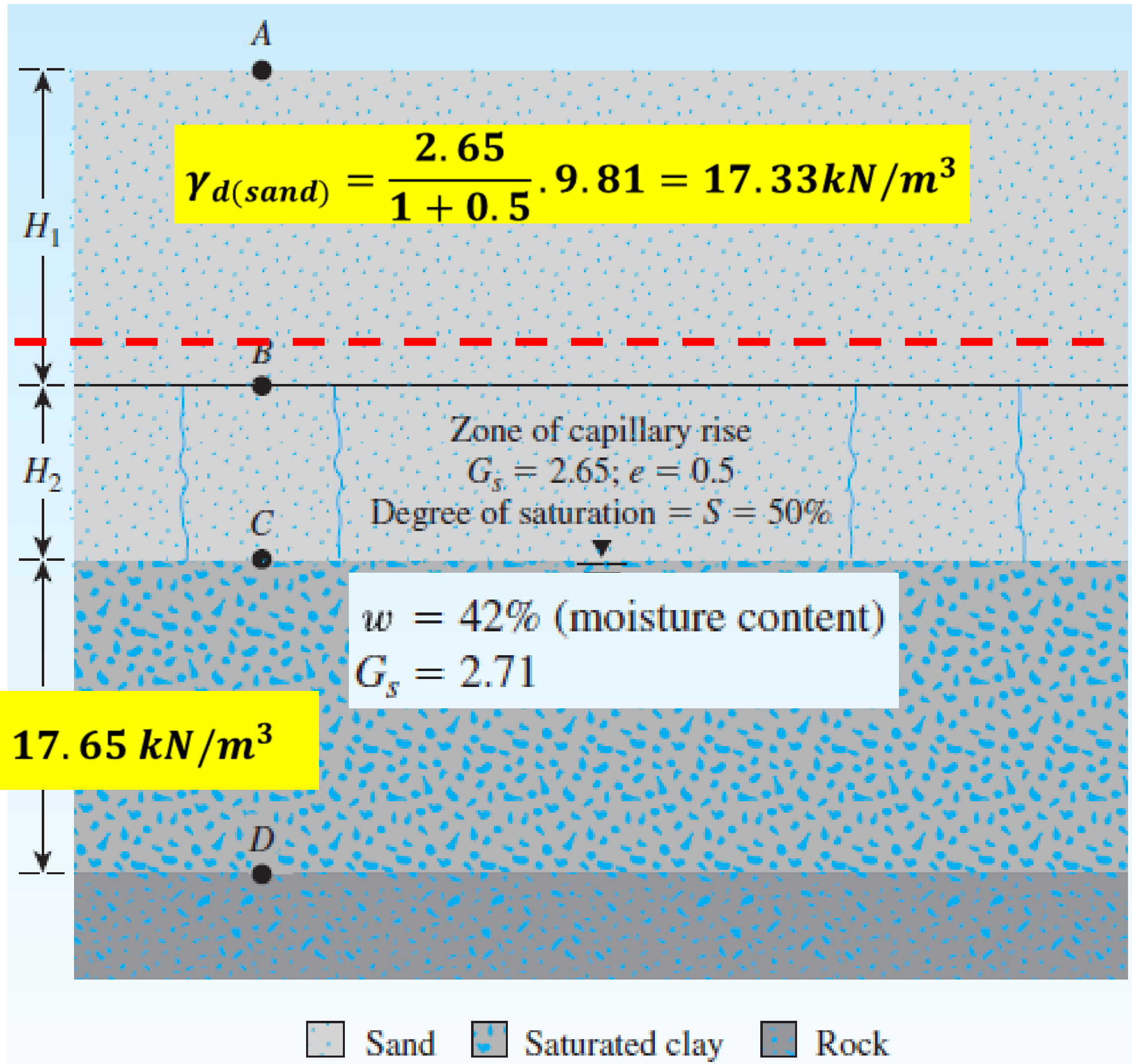
- In case of partial saturation, p.w.p. is approximately given by

$$u = -\left(\frac{S}{100}\right)\gamma_w h$$

where *S* degree of saturation, in percent.

EXAMPLE 9.8

A soil profile is shown in Figure 9.23. Given: $H_1 = 1.83$ m, $H_2 = 0.91$ m, $H_3 = 1.83$ m. Plot the variation of σ, u, and σ' with depth.

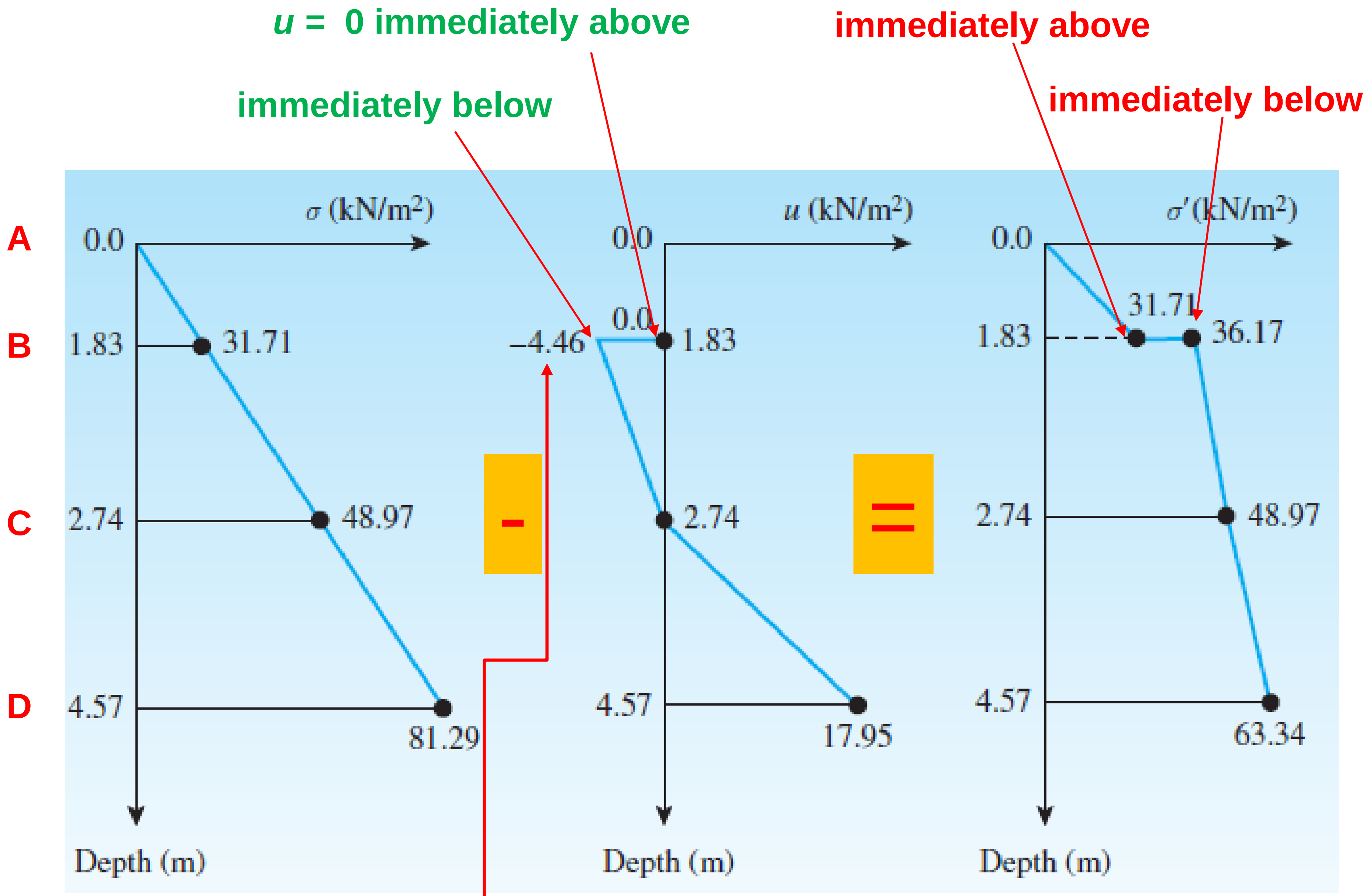


$$\gamma_{d(sand)} = \frac{2.65}{1+0.5} .9.81 = 17.33 kN/m^3$$

$$\gamma_{sand} = \frac{2.65 + 0.5x0.5}{1+0.5} .9.81$$
$$= 18.97 kN/m^3$$

$$e = \frac{wG_s}{.S} = \frac{0.42 \times 2.71}{1}$$

$$\gamma_{s(clay)} = \frac{2.71 + 1x1.14}{1+1.14} .9.81 = 17.65\ kN/m^3$$



$$u = -\left(\frac{S}{100}\right)\gamma_w h = -\left(\frac{50}{100}\right) X\ 9.81\ X\ 0.91 = -4.46$$

THE END