Vehicle Motion Equations:

$$v = at + v_\circ \qquad \rightarrow (2.2.4)$$

$$x - x_\circ = \frac{v^2 - v_\circ^2}{2a} \qquad \rightarrow (2.2.6)$$

$$v^2 = v_\circ^2 + 2a(x - x_\circ) \qquad \rightarrow (2.2.6)$$

$$x = \frac{1}{2}at^2 + v_\circ t + x_\circ \qquad \rightarrow (2.2.7)$$

$$D_b = x\cos\alpha \qquad \rightarrow (2.2.10)$$

$$x = vt$$

$$D_b = \frac{v_\circ^2 - v^2}{2g(f \mp G)} \qquad \rightarrow (2.2.14)$$

$$e + f_s = \frac{v^2}{gR}\cancel{(1 - f_s e)} \qquad \rightarrow (2.2.19)$$

$$\frac{v^2}{gR} = \frac{X + Ye}{Y - Xe} \qquad \rightarrow (2.2.20)$$

$$D_{PR} = v_\circ \delta \cos\alpha$$

$$D_s = D_{PR} + D_b$$

$$\frac{1km}{1hr} = \frac{1000m}{3600s}$$

$$\frac{1mile}{1hr} = \frac{5280ft}{3600s}$$

$$G = tan\alpha$$

A car collided with a telephone pole and left a 20 ft skid marks on the dry pavement. Based on the damages sustained, an engineer estimated that the speed at collision was 15 mph. if the roadway had a +3% grade, calculate the speed of the car at the onset of skidding.

$$v_f = 15mph = 15 \times \frac{5280}{3600} = 22ft/s$$

$$D_b = \frac{v_\circ^2 - v^2}{2g(f \mp G)}$$

$$v_\circ^2 = D_b 2g(f \mp G) + v^2$$

$$v_\circ^2 = 20 \times 2 \times 32.2 \times (0.6 + 0.03) + 22^2 = 1295.44$$

$$v_\circ = 35.99ft/s \ = 35.99 \times \frac{3600}{5280} = 24.5mph$$

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A vehicle crashed into an abutment wall of a bridge leaving a skid mark on the road (d=110m, f=0.6) followed by skid mark on the side slope (21 m, f=0.3) all the way to the wall. The road has an uphill slope of 2% and an equivalent of -5% on the side slope. The crash velocity was estimated to be 30 km/hr. Was the driver obeying the speed limit of 120 km/hr. before applying the breaks?

Side slope:

$$v = 30\frac{km}{hr} = 30 \times \frac{1000}{3600} = 8.33m/s$$

$$D_b = \frac{v_\circ^2 - v^2}{2g(f \mp G)}$$

$$v_\circ^2 = D_b 2g(f \mp G) + v^2$$

$$v_\circ^2 = 21 \times 2 \times 9.8 \times (0.3 - 0.05) + 8.33^2 = 172.34$$

$$v_\circ = 13.13m/s \ = 13.13 \times \frac{3600}{1000} = 47.26km/hr$$

Road: v_o (at side slope) = v (at road)

$$v_\circ^2 = D_b 2g(f \mp G) + v^2$$

$$v_\circ^2 = 110 \times 2 \times 9.8 \times (0.6 + 0.02) + 13.13^2 = 1509.12$$

$$v_\circ = 38.85m/s \ = 38.85 \times \frac{3600}{1000} = 139.85km/hr$$

➔ The driver did not obey the speed limit

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Check for safety against sliding and overturning on a curve with radios R=200 m, super elevation e=3% and f_s=0.2. the posted speed limit is 80 km/hr and vehicle's center of mass is at: X=1.1 m, Y = 1.5 m.

Sliding:

$$e + f_s = \frac{v^2}{gR}$$

$$v^2 = (e + f_s)gR$$

$$v = \sqrt{(e + f_s)gR}$$

$$v = \sqrt{(0.03 + 0.2) \times 9.8 \times 200} = 21.23m/s$$

$$v = 21.23m/s \ = 21.23 \times \frac{3600}{1000} = 76.4km/hr$$

➔ The curve is not safe because the maximum speed before sliding is less than the posted speed limit.

Overturning:

$$\frac{v^2}{gR} = \frac{X + Ye}{Y - Xe}$$

$$v^2 = gR\frac{X + Ye}{Y - Xe}$$

$$v = \sqrt{gR\frac{X + Ye}{Y - Xe}}$$

$$v = \sqrt{9.8 \times 200 \times \frac{1.1 + 1.5 \times 0.03}{1.5 - 1.1 \times 0.03}} = 39.13m/s$$

$$v = 39.13m/s \ = 39.13 \times \frac{3600}{1000} = 140.9km/hr$$

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What should be the speed for a 1,000 ft curve with super elevation of 2% ensuring no skidding nor overturning on wet conditions (f=0.15). The vehicles have a center of mass at X=4.5 ft and Y=5.5 ft.

Sliding:

$$e + f_s = \frac{v^2}{gR}$$

$$v^2 = (e + f_s)gR$$

$$v = \sqrt{(e + f_s)gR}$$

$$v = \sqrt{(0.02 + 0.15) \times 32.2 \times 1000} = 73.99 ft/s$$

$$v = 73.99 ft/s \quad = 73.99 \times \frac{3600}{5280} = 50.44 mph$$

Overturning:

$$\frac{v^2}{gR} = \frac{X + Ye}{Y - Xe}$$

$$v^2 = gR\frac{X + Ye}{Y - Xe}$$

$$v = \sqrt{gR\frac{X + Ye}{Y - Xe}}$$

$$v = \sqrt{32..2 \times 1000 \times \frac{4.5 + 5.5 \times 0.02}{5.5 - 4.5 \times 0.02}} = 165.65 ft/s$$

$$v = 165.65 ft/s \quad = 165.65 \times \frac{3600}{5280} = 112.9 mph$$

The proposed speed limit must be less than both results. Thus, an appropriate speed limit is *v* = 45 mph

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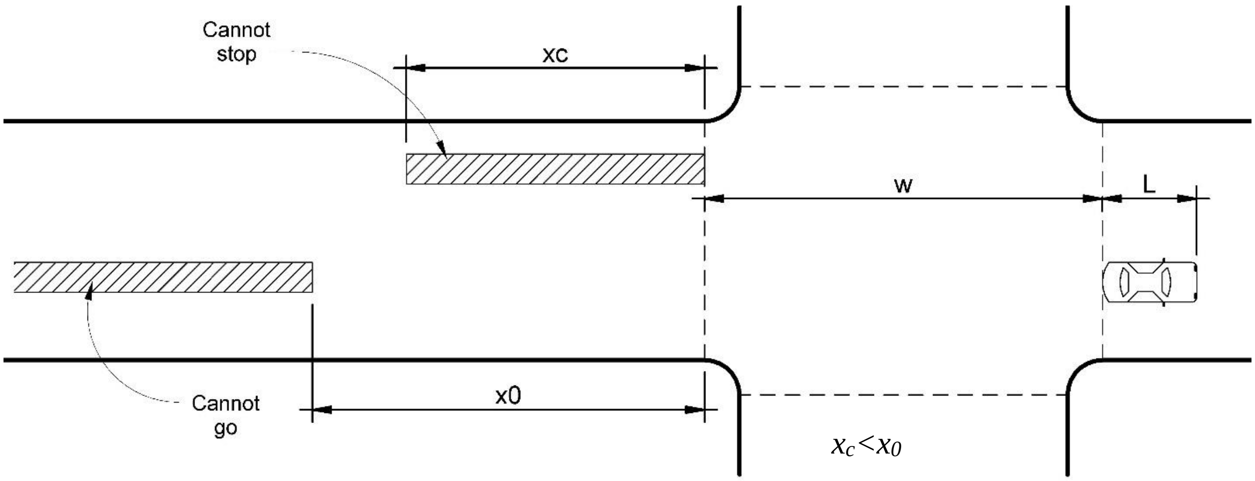
Human Factors

x_c: is the critical distance required for the car to stop before the intersection.
x_o: is the maximum distance the car can be from the intersection of the yellow interval and still clear the intersection.

$$x_c = v_0\delta_2 + \frac{v_0^2}{2a_2^*} \qquad \rightarrow (2.3.3)$$

$$x_0 = v_0\tau - (w + L) \qquad \rightarrow (2.3.6)$$

$$\tau_{min} = \delta_2 + \frac{v_0}{2a_2^*} + \frac{w + L}{v_0} \qquad \rightarrow (2.3.7)$$


Cannot
stop
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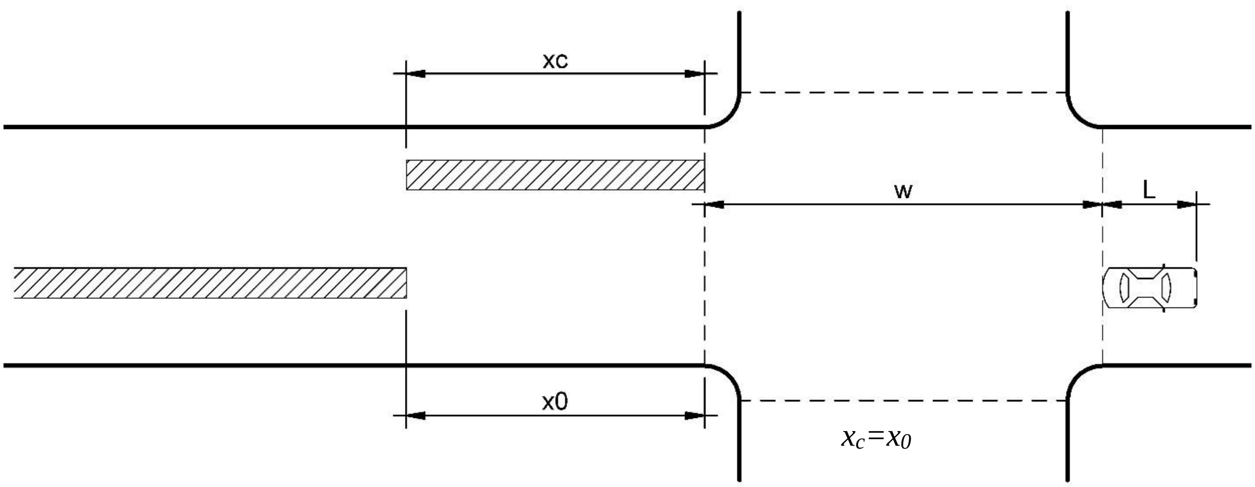
x0
$x_c < x_0$


xc
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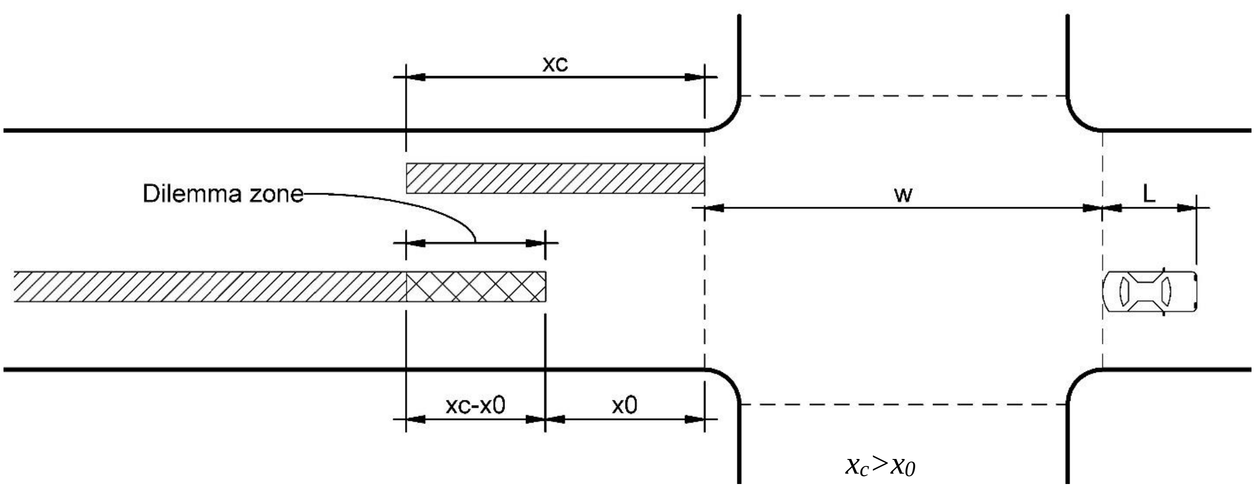
x0
$x_c = x_0$


xc
Dilemma zone
w
L
xc-x0
x0
$x_c > x_0$

Determine if the following intersection has a dilemma zone:

- Comfortable deceleration a_2 =8 ft/s2,
- intersection width w =65 ft,
- vehicle length L =15 ft,
- amber duration τ =4.5 sec,
- PRT δ_2 =1 sec
- and an approach speed of 60 mph.

Note, if there exists a dilemma zone, determine its length.

To determine if there is a dilemma zone, check the relations $x_0=x_c$, $x_0>x_c$, $x_0<x_c$,

$$v_0 = 60mph = 60 \times \frac{5280}{3600} = 88ft/s$$

$$x_c = v_0\delta_2 + \frac{v_0^2}{2a_2^*}$$

$$x_c = 88 \times 1 + \frac{88^2}{2 \times 8} = 572ft$$

$$x_0 = v_0\tau - (w + L)$$

$$x_0 = 88 \times 4.5 - (65 + 15) = 316ft$$

Since $x_0<x_c$, ➔ there is a dilemma zone.

$$x_0 - x_c = v_0\tau - (w + L) - \left(v_0\delta_2 + \frac{v_0^2}{2a_2^*}\right)$$

$$x_0 - x_c = 88 \times 4.5 - (65 + 15) - \left(88 \times 1 + \frac{88^2}{2 \times 8}\right)$$

$$x_0 - x_c = -256ft$$

➔ The length of the dilemma zone is 256 *ft*.

Note: you can directly calculate x_0-x_c and determine if there is a dilemma zone:

- x_0-$x_c \geq 0$, there is no dilemma zone.
- x_0-$x_c < 0$, there is a dilemma zone, and the difference is its length.

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Calculate the length of the dilemma zone in the following intersection. Moreover, select an appropriate yellow interval for the intersection. Driver and intersection properties:

- Comfortable deceleration rate a_2 =2.5 m/s^2
- Intersection width w =35 m
- Design vehicle length L =3 m
- Yellow duration τ =4.5 sec
- PRT δ_2 =1 sec
- Speed limit: 60 km/h

$$v_0 = 60km/hr \ = 60 \times \frac{1000}{3600} = 16.67m/s$$

$$x_0 - x_c = v_0\tau - (w + L) - \left(v_0\delta_2 + \frac{v_0^2}{2a_2^*}\right)$$

$$x_0 - x_c = 16.67 \times 4.5 - (35 + 3) - \left(16.67 \times 1 + \frac{16.67^2}{2 \times 2.5}\right)$$

$$x_0 - x_c = -35.23m$$

➔ The length of the dilemma zone is 35.23m.

$$\tau_{min} = \delta_2 + \frac{v_0}{2a_2^*} + \frac{w + L}{v_0}$$

$$\tau_{min} = 1 + \frac{16.67}{2 \times 2.5} + \frac{35 + 3}{16.67}$$

$$\tau_{min} = 6.61 \ s$$

To be sure that x_0-$x_c \geq 0$ and eliminate dilemma zone, the minimum amber time should be 6.61 seconds.

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Vehicles must reduce speed from 100 km/h to 60 km/h to negotiate a tight curve on a rural highway. A warning sign is clearly visible for a person with 6/6 from a distance of 50m. Calculate the distance at which the sign should be place before the curve for the design driver. Given the design driver has 3/6 vision, PRT δ= 2 sec and decelerates comfortably at the rate of a=3 m/s^2.

Distance of sign to be clearly visible for the design driver:

$$\frac{6}{6} \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \ 50m$$

$$\frac{3}{6} \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \ x$$

➔ x = 25m

Slowing distance:

$$D_s = D_{PR} + D_b$$

$$D_s = v_0\delta + \frac{v_\circ^2 - v^2}{2a}$$

$$D_s = 27.78 \times 2 + \frac{27.78^2 - 16.67^2}{2 \times 3}$$

$$D_s = 137.86m$$

The deceleration starts when the drivers sees the sign, so the sign should be placed before the deceleration starts by the distance *x*.

➔ Sign distance from the curve = 137.86 – 25 = 112.86 m round up = 113m

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A driver with 20/40 vision and a six-grade education needs 2 seconds to read a directional sign. The sign can be read by a person with 20/20 vision from a distance of 70 meters. Does the subject driver have enough time to read the sign at the speed of 50 km/h?

Distance of sign to be clearly visible for the subject driver:

$$\frac{20}{20} \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow 70m$$

$$\frac{20}{40} \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow x$$

➔ x = 35m.

$$d = vt \rightarrow t = \frac{d}{v}$$

$$t = \frac{35}{13.89} = 2.5\ sec$$

Time available for the subject driver to read the sign is 2.5 seconds which is more the time he needs to read it. This conclude the driver have enough time to read the sign at the speed of 50 km/h.

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