5.2: The cost function of a large railroad corporation is $Y = 10^7 + 0.5T$, where Y is the total cost of shipping in US dollars and T is the tons shipped. Last year the company charged on average 88₡ for each ton of freight. Their annual shipments total 48 million tons. This year they are considering geographical expansion through the purchase of a smaller railroad corporation that last year shipped total of 21 million tons. Economists estimated that the total cost of function (for the merged corporation) will be $Y = 10.5^7 + 03.T$, while 10% more freight should be expected due to the better geographic coverage, at a price discounted by 8₡.

Show that the large and the merged railroad realize substantial economies of scale (EOS). Which railroad realizes greater EOS? Use a numerical example or a graphic for proof. Show numerically that the large railroad should merge with the smaller one.

Large railroad corporation:

Cost Function: $Y = 10^7 + 0.5T$.

Unit Revenue: 88₡/ton.

Total shipment: 48 million tons = 48×10^6 tons.

Total cost = $10^7 + (0.5) \times (48\times10^6) = \$34\times10^6$

Unit cost = $Y/T = \$34\times10^6 / 48\times10^6 = 71$₡/ton.

Unit cost for 20×10^6 tons shipped (u_{20}) =

$$u_{20} = \frac{Y_{20}}{T_{20}} = \frac{10^7 + 0.5(20 \times 10^6)}{20 \times 10^6} = \$1/ton$$

Unit cost for 10×10^6 tons shipped (u_{10}) =

$$u_{10} = \frac{Y_{10}}{T_{10}} = \frac{10^7 + 0.5(10 \times 10^6)}{10 \times 10^6} = \$1.5/ton$$

Percentage of cost reduction:

$$\% = \frac{u_{10} - u_{20}}{u_{10}} = \frac{1.5 - 1}{1.5} = 33.3\%$$

➔ large railroad corporation realizes EOS.

Merged railroad corporation:

Cost Function: $Y = 10.5^7 + 0.3T$.

Unit Revenue: 80₡/ton.

Total shipment:

48 million tons = 48×10^6 tons. From large corporation.

21 million tons = 21×10^6 tons. From small corporation.

10% increase

→ Total shipment = $1.1 \times (69\times10^6) = 75.9\times10^6$ tons

Total cost = $10.5^7 + (0.3) \times (75.9\times10^6) = \$36.841\times10^6$

Unit cost = $Y/T = \$36.841\times10^6 / 75.9\times10^6 = 49$₡/ton.

Unit cost for 20×10^6 tons shipped (u_{20}) =

$$u_{20} = \frac{Y_{20}}{T_{20}} = \frac{10.5^7 + 0.3(20 \times 10^6)}{20 \times 10^6} = \$1.003/ton$$

Unit cost for 10×10^6 tons shipped (u_{10}) =

$$u_{10} = \frac{Y_{10}}{T_{10}} = \frac{10.5^7 + 0.3(10 \times 10^6)}{10 \times 10^6} = \$1.707/ton$$

Percentage of cost reduction:

$$\% = \frac{u_{10} - u_{20}}{u_{10}} = \frac{1.707 - 1.003}{1.707} = 41.2\%$$

➔ Merged railroad corporation realizes EOS.

Since the merged corporation realizes more EOS, the large railroad should merge with the smaller one.

■■■■

5.9: A businessman residing in Chicago considers his options for a trip to Detroit. His options are private car, rental car, bus, or airplane. Given the following data, suggest the best mode for his travel.

Distance between cities (one way) = 425 km.

Estimated access travel at origin and destination = 38 km at each city; the access distance and access trips are the same for all modes.

Costs:

(1) private auto: 20₡/km (all costs combined); no access mode required;

(2) rental car (2 days): $50/day plus $12/day for insurance and tax; 11 *l*/100 km fuel efficiency and gas price is 40₡/*l*; no access mode required;

(3) bus: round-trip fare $55; access mode required;

(4) air: round-trip fare $100; access mode required.

Access modes and costs:

In Chicago taxies charge $2 plus 20₡/km, buses charge $1 per ride (assume two rides);

In Detroit taxies charge $2.40 plus 15₡/km, buses charge $1.5 per ride (assume two rides).

The door-to-door travel times by mode are as follows:

	Private auto	Rental car	Bus	Air
Best	4.5	5.0	6.0	1.5
Worst	6.0[a]	6.5[a]	8.0[b]	3.0[b]

[a] Accounts for potentially congested conditions.
[b] Use of bus for access.

In order to make his selection, the businessman assumed a disutility function (a measure of discomfort due to the cost and travel time encountered):

Disutility = (total trip cost)/5 + 8 (one-way travel time)

Which mode did the businessman select? (Round out all the cost estimates to the nearest integer)

ANSWER IS BASED ON THE BEST DOOR-TO-DOOR TRAVEL TIME ONLY.

Private Auto:

Outbound distance:

- Access distance out of Chicago = 38 km driving
- Intercity distance = 425 km driving
- Access distance into Detroit = 38 km driving

Return distance:

- Access distance out of Detroit = 38 km driving
- Intercity distance = 425 km driving
- Access distance into Chicago = 38 km driving

→ Total distance = 1002 km driving

Cost per km driving = \$0.2/km

→ Total cost of private auto = \$0.2/km × 1002km = \$200

Disutility = (total trip cost)/5 + 8 (one-way travel time) = 200/5 + 8 × 2.25 = 58

Rental Car:

Outbound distance:

- Access distance out of Chicago = 38 km driving
- Intercity distance = 425 km driving
- Access distance into Detroit = 38 km driving

Return distance:

- Access distance out of Detroit = 38 km driving
- Intercity distance = 425 km driving
- Access distance into Chicago = 38 km driving

→ Total distance = 1002 km driving

Fuel consumed = 1002km × 11 *l*/100km = 110 *l*

→ Cost of fuel = 110 *l* × \$0.4/*l* = \$44

Cost of car rental = 2 × (\$50 +\$12) = \$124

→ Total cost of rental car = \$124 + \$44 = \$168

Disutility = (total trip cost)/5 + 8 (one-way travel time) = 168/5 + 8 × 2.5 = 53.6

Bus:

Outbound distance:

- Access distance out of Chicago = 38 km taxi
- Intercity distance = 425 km bus
- Access distance into Detroit = 38 km taxi

Return distance:

- Access distance out of Detroit = 38 km taxi
- Intercity distance = 425 km bus
- Access distance into Chicago = 38 km taxi

→ Total distance = 76 km Chicago taxi
76 km Detroit taxi
850 km bus

Cost of Chicago taxi = \$2 +(\$0.2/km × 76 km) = \$17
Cost of Detroit taxi = \$2.4 +(\$0.15/km × 76 km) = \$14
Cost of round-trip bus fare = \$55
→ Total cost of bus trip = \$17 + \$14 + \$55 = \$86
Disutility = (total trip cost)/5 + 8 (one-way travel time) = 86/5 + 8 × 3 = 41.2

Air:

Outbound distance:

- Access distance out of Chicago = 38 km taxi
- Intercity distance = 425 km air
- Access distance into Detroit = 38 km taxi

Return distance:

- Access distance out of Detroit = 38 km taxi
- Intercity distance = 425 km air
- Access distance into Chicago = 38 km taxi

→ Total distance = 76 km Chicago taxi
76 km Detroit taxi
850 km bus

Cost of Chicago taxi = \$2 +(\$0.2/km × 76 km) = \$17
Cost of Detroit taxi = \$2.4 +(\$0.15/km × 76 km) = \$14
Cost of round-trip airline fare = \$100
→ Total cost of bus trip = \$17 + \$14 + \$100 = \$131
Disutility = (total trip cost)/5 + 8 (one-way travel time) = 131/5 + 8 × 0.75 = 32.2

	Private Auto	Rental Car	Bus	Air
Total Cost	$200	$168	$86	$131
One-way Time	2.25hr	2.5hr	3ht	0.75hr
Disutility	58	53.6	41.2	32.2

Assuming the best conditions, the businessman selected to travel by air since it had the least disutility.

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