Chapter 2

Student Version

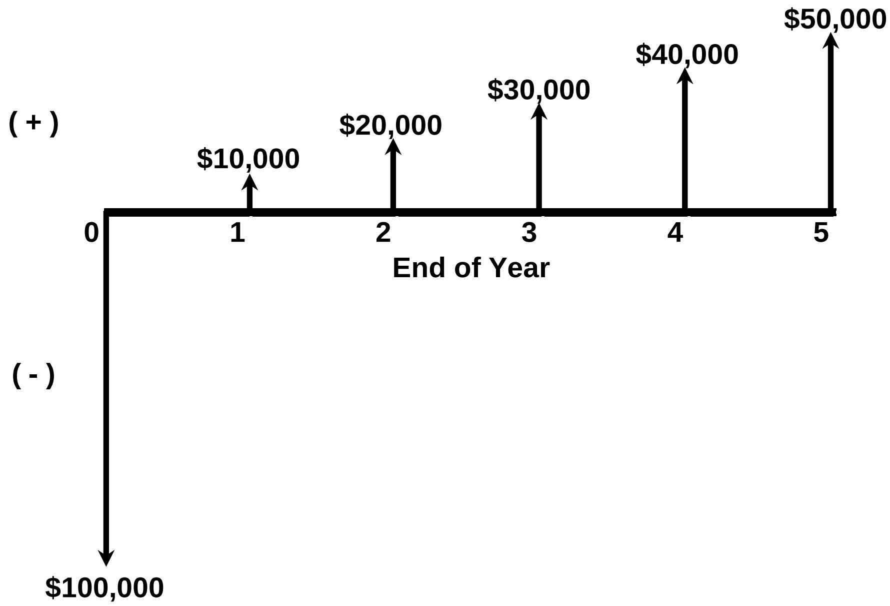
Example 2.1

Cash Flow Profiles for Two Investment Alternatives

End of Year (EOY)	CF(A)	CF(B)	CF(B-A)
0	-$100,000	-$100,000	$0
1	$10,000	$50,000	$40,000
2	$20,000	$40,000	$20,000
3	$30,000	$30,000	$0
4	$40,000	$20,000	-$20,000
5	$50,000	$10,000	-$40,000
Sum	$50,000	$50,000	$0

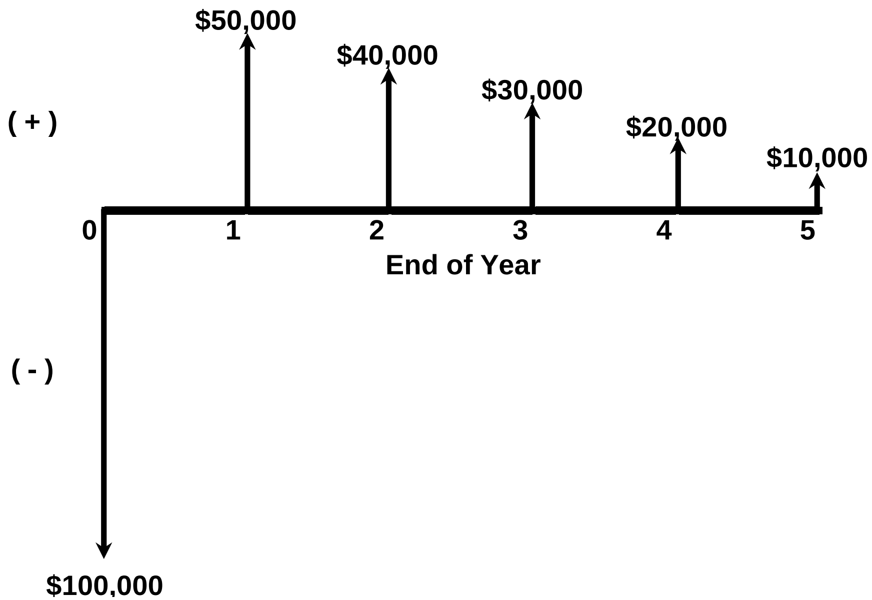
Although the two investment alternatives have the same "bottom line," there are obvious differences. Which would you prefer, A or B? Why?

Inv. A

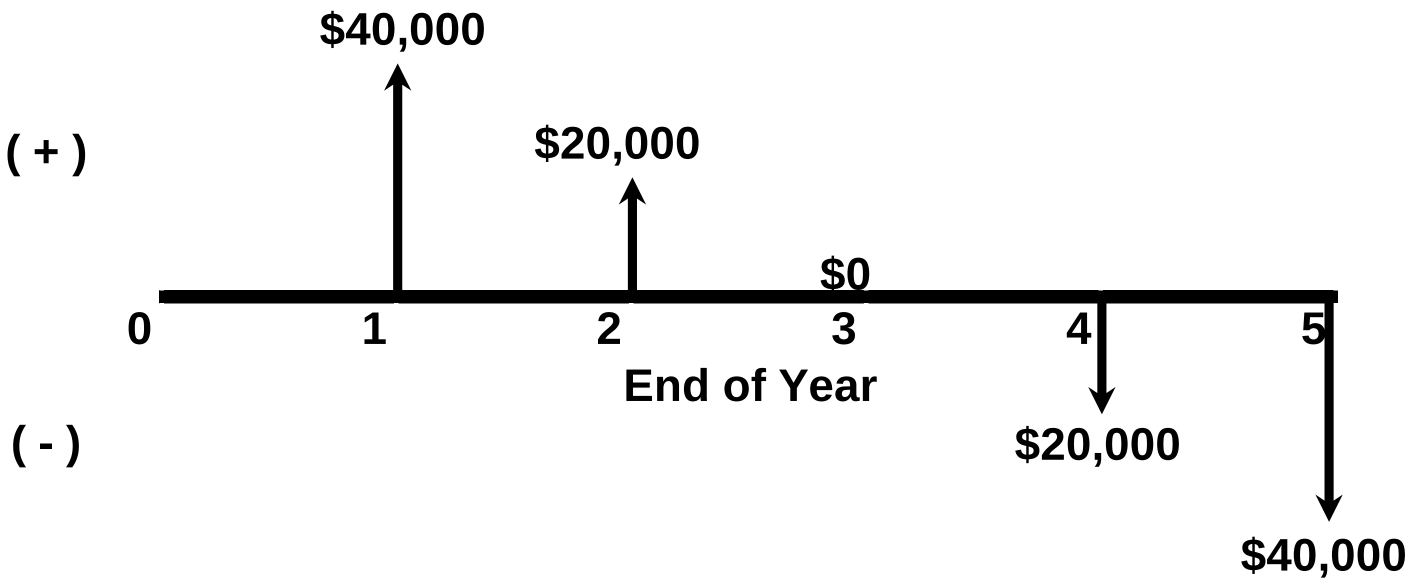


$50,000
$40,000
$30,000
$20,000
$10,000
(+)
0
1
2
3
4
5
End of Year
(-)
$100,000

Inv. B



$50,000
$40,000
$30,000
$20,000
$10,000
(+)
0
1
2
3
4
5
End of Year
(-)
$100,000

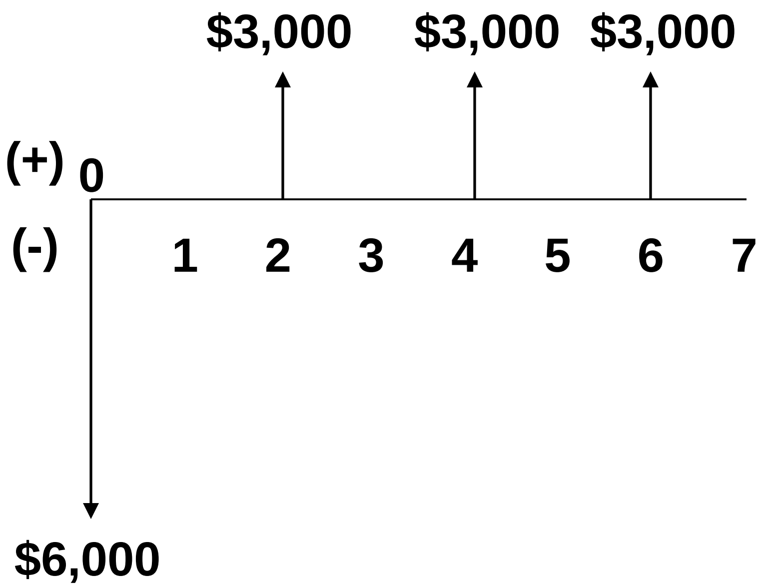


Inv. B – Inv. A

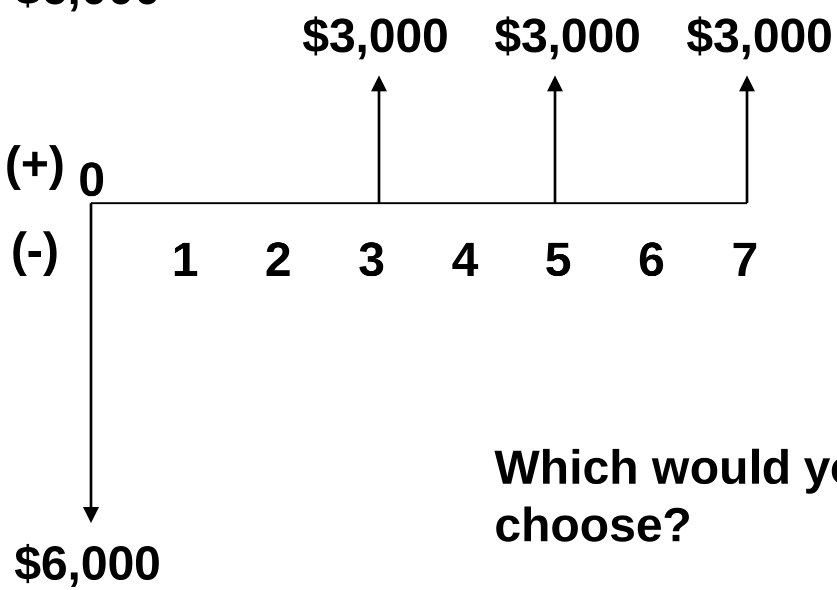
Based on the *TVOM* discussion in Chapter 1, we know Alternative B is preferred. Why? Given the reverse images of the positive-valued cash flows, we prefer to receive the $50,000 sooner rather than later. If we apply Principle #7 and examine the difference in cash flows for the two investment alternatives, it is easy to see why Alternative B is preferred. As shown in Figure 2.4, by choosing Alternative B, money is received quicker than it is with Alternative A.

Example 2.2

Alternative C



Alternative D



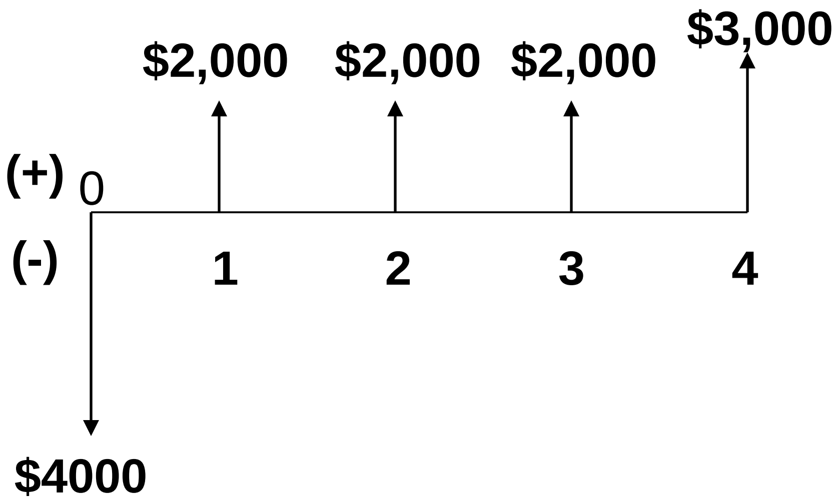
Which would you choose?

Example 2.2

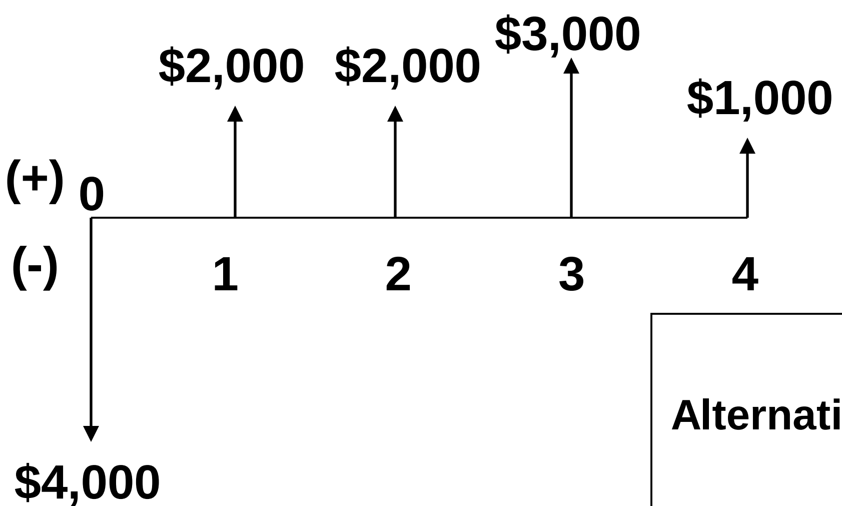
When faced with cash flows of equal magnitude occurring at different points in time, a corollary to the four *DCF* rules in Chapter 1 is *when receiving a given sum of money, we prefer to receive it sooner rather than later, and when paying a given sum of money, we prefer to pay it later rather than sooner*. Since we prefer to receive the $3,000 sooner, Alternative C is preferred to Alternative D.

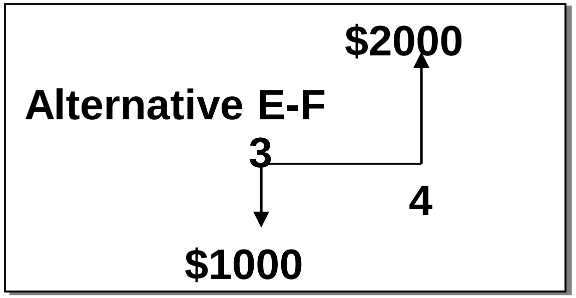
Example 2.3

Alternative E



Alternative F





Which would you choose?

Simple interest calculation:

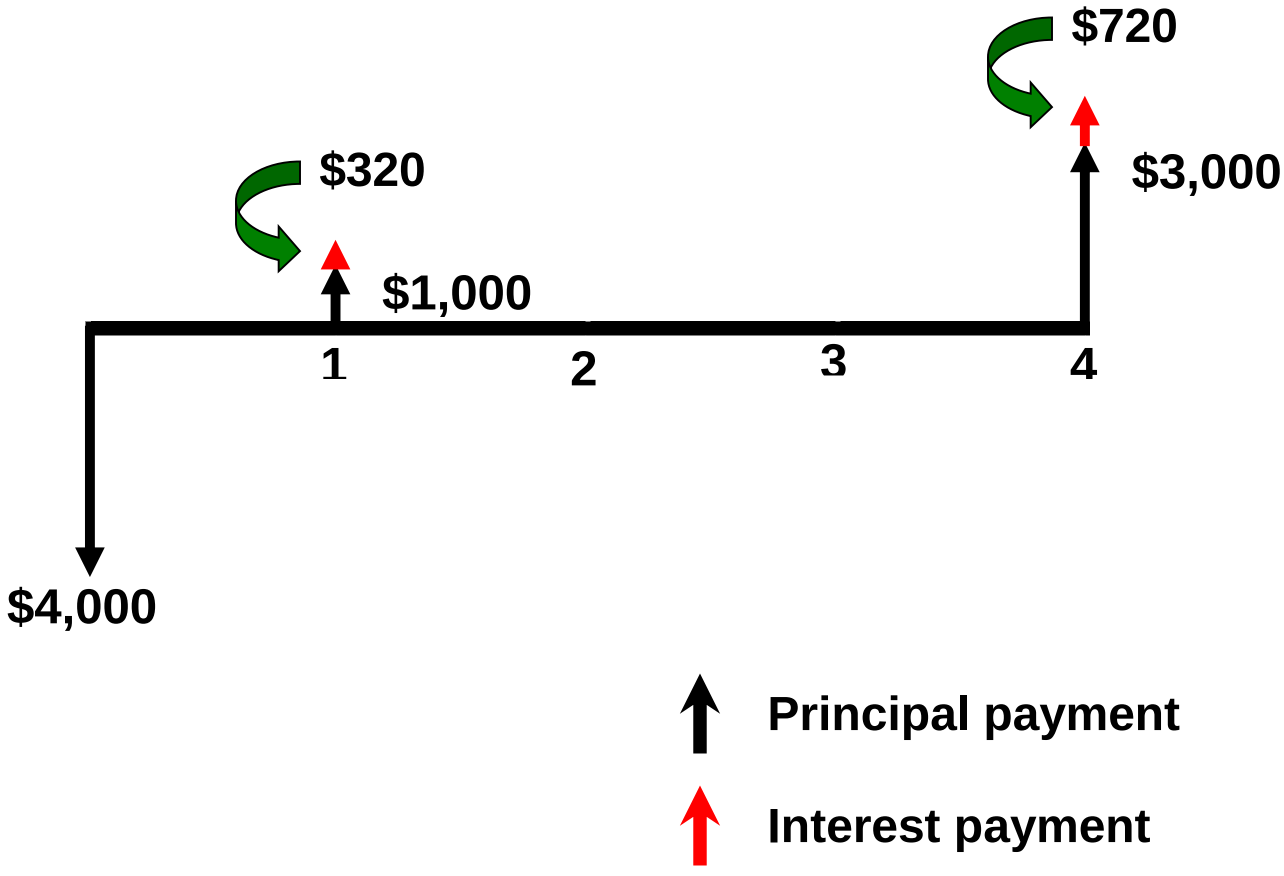
$$F_n = P(1 + in)$$

Example 2.7: simple interest calculation

Robert borrows $4,000 from Susan and agrees to pay $1,000 plus accrued interest at the end of the first year and $3,000 plus accrued interest at the end of the fourth year. What should be the size of the payments if 8% simple interest is used?

- **1st payment = $1,000 + 0.08($4,000)**
  = $1,320
- **2nd payment = $3,000 + 0.08($3,000)(3)**
  = $3,720

Simple Interest Cash Flow Diagram



Example 2.9

Dia St. John borrows $1,000 at 12% compounded annually. The loan is to be repaid after 5 years. How much must she repay in 5 years?

Example 2.9

Dia St. John borrows $1,000 at 12% compounded annually. The loan is to be repaid after 5 years. How much must she repay in 5 years?

F = P(F|P i%, n)

F = $1,000(F|P 12%,5)

F = $1,000(1.12)^5

F = $1,000(1.76234)

F = $1,762.34

Example 2.9

Dia St. John borrows \$1,000 at 12% compounded annually. The loan is to be repaid after 5 years. How much must she repay in 5 years?

F = P(F|P i%, n)

F = \$1,000(F|P 12%,5)

F = \$1,000(1.76234)

F = \$1,762.34

	Single Sums		Uniform Series				Gradient Series	
n	To Find F Given P $(F\|P\ i\%,n)$	To Find P Given F $(P\|F\ i\%,n)$	To Find F Given A $(F\|A\ i\%,n)$	To Find A Given F $(A\|F\ i\%,n)$	To Find P Given A $(P\|A\ i\%,n)$	To Find A Given P $(A\|P\ i\%,n)$	To Find P Given G $(P\|G\ i\%,n)$	To Find A Given G $(A\|G\ i\%,n)$
1	1.12000	0.89286	1.00000	1.00000	0.89286	1.12000	0.00000	0.00000
2	1.25440	0.79719	2.12000	0.47170	1.69005	0.59170	0.79719	0.47170
3	1.40493	0.71178	3.37440	0.29635	2.40183	0.41635	2.22075	0.92461
4	1.57352	0.63552	4.77933	0.20923	3.03735	0.32923	4.12731	1.35885
5	1.76234	0.56743	6.35285	0.15741	3.60478	0.27741	6.39702	1.77459

Example 2.10

How long does it take for money to <u>double</u> in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

I can think of six ways to solve this problem:

1) **Solve using the Rule of 72**
2) **Use the interest tables; look for F|P factor equal to 2.0**
3) **Solve numerically; n = log(2)/log(1+i)**
4) ~~**Solve using Excel® NPER function: =NPER(i%,,-1,2)**~~
5) ~~**Solve using Excel® GOAL SEEK tool**~~
6) ~~**Solve using Excel® SOLVER tool**~~

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

RULE OF 72

Divide 72 by interest rate to determine how long it takes for money to double in value.

(Quick, but not always accurate.)

Example 2.10

How long does it take for money to double in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest? **Rule of 72 solution**

(a) 72/2 = 36 yrs

(b) 72/3 = 24 yrs

(c) 72/4 = 18 yrs

(d) 72/6 = 12 yrs

(e) 72/8 = 9 yrs

(f) 72/12 = 6 yrs

Example 2.10

How long does it take for money to <u>double</u> in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest?

Using interest tables & interpolating

(a) 34.953 yrs

(b) 23.446 yrs

(c) 17.669 yrs

(d) 11.893 yrs

(e) 9.006 yrs

(f) 6.111 yrs

• interpolating for 3%

$$\frac{y-y_0}{x-x_0}=\frac{y_1-y_0}{x_1-x_0}$$

$$\frac{y-23}{2-1.97359}=\frac{24-23}{2.03279-1.97359}$$

$$y=23.44663$$

	Single Sums	
n	To Find F Given P ($F|P$ i%,n)	To Find P Given F ($P|F$ i%,n)
1	1.03000	0.97087
2	1.06090	0.94260
3	1.09273	0.91514
4	1.12551	0.88849
5	1.15927	0.86261
6	1.19405	0.83748
7	1.22987	0.81309
8	1.26677	0.78941
9	1.30477	0.76642
10	1.34392	0.74409
11	1.38423	0.72242
12	1.42576	0.70138
13	1.46853	0.68095
14	1.51259	0.66112
15	1.55797	0.64186
16	1.60471	0.62317
17	1.65285	0.60502
18	1.70243	0.58739
19	1.75351	0.57029
20	1.80611	0.55368
21	1.86029	0.53755
22	1.91610	0.52189
23	1.97359	0.50669
24	2.03279	0.49193
25	2.09378	0.47761

Example 2.10

How long does it take for money to <u>double</u> in value, if you earn (a) 2%, (b) 3%, (c) 4%, (d) 6%, (e) 8%, or (f) 12% annual compound interest? **Mathematical solution**

(a) log 2/log 1.02 = 35.003 yrs

(b) log 2/log 1.03 = 23.450 yrs

(c) log 2/log 1.04 = 17.673 yrs

(d) log 2/log 1.06 = 11.896 yrs

(e) log 2/log 1.08 = 9.006 yrs

(f) log 2/log 1.12 = 6.116 yrs

F|P Example

How long does it take for money to <u>triple</u> in value, if you earn (a) 4%, (b) 6%, (c) 8%, (d) 10%, (e) 12%, (f) 15%, (g) 18% interest?

F|P Example

How long does it take for money to <u>triple</u> in value, if you earn (a) 4%, (b) 6%, (c) 8%, (d) 10%, (e) 12%, (f) 15%, (g) 18% interest?

(a)

(b)

(c)

(d)

(e)

(f)

(g)

F|P Example

How long does it take for money to triple in value, if you earn (a) 4%, (b) 6%, (c) 8%, (d) 10%, (e) 12%, (f) 15%, (g) 18% interest?

(a) n =28.011

(b) n = 18.854

(c) n = 14.275

(d) n = 11.527

(e) n = 9.694

(f) n = 7.861

(g) n = 6.638

Example 2.11

How much must you deposit, today, in order to accumulate $10,000 in 4 years, if you earn 5% compounded annually on your investment?

Example 2.11

How much must you deposit, today, in order to accumulate \$10,000 in 4 years, if you earn 5% compounded annually on your investment?

P = F(P|F i, n)

P = \$10,000(P|F 5%,4)

P = \$10,000(1.05)$^{-4}$

P = \$10,000(0.82270)

P = \$8,227.00

Example 2.11

How much must you deposit, today, in order to accumulate $10,000 in 4 years, if you earn 5% compounded annually on your investment?

P = F(P|F i, n)

P = $10,000(P|F 5%,4)

P = $\$10,000(1.05)^{-4}$

P = $10,000(0.82270)

P = $8,227.00

P =PV(5%,4,,-10000)

P = $8,227.02

Homework # 1

8. Four proposals (A, B, C, and D) are available for investment. Proposals A and C cannot both be accepted; Proposal B is contingent upon the acceptance of either Proposal C or D; and Proposal A is contingent on D.

a. List all possible combinations of proposals and clearly show which are feasible.

b. Of the ten principles, which one(s) is well illustrated by this problem?

c. Of the systematic economic analysis technique's seven steps, which one(s) is well illustrated by this problem?

9. Five proposals (V, W, X, Y, and Z) are available for investment. At least two and no more than four must be chosen. Proposals X and Y are mutually exclusive. Proposal Z is contingent on either Proposal X or Y being funded. Proposal V cannot be pursued if either W, X, Y, or any combination of the three are pursued.

a. List all feasible mutually exclusive investment alternatives.

b. Of the ten principles, which one(s) is well illustrated by this problem?

c. Of the systematic economic analysis technique's seven steps, which one(s) is well illustrated by this problem?

10. Three proposals (P, Q, and R) are available for investment. Exactly one or two proposals must be chosen; Proposals P and Q are mutually exclusive. Proposal R is contingent on Proposal P being funded. List all feasible mutually exclusive investment alternatives.

Homework # 1 Solution

PROBLEM 1.8

ACCEPT PROPOSAL: 1
REJECT PROPOSAL: 0

a

ALT	PROP A	PROP B	PROP C	PROP D	WHY NOT FEASIBLE	
1	0	0	0	0		
2	0	0	0	1		
3	0	0	1	0		
4	0	0	1	1		
5	0	1	0	0	B CONTINGENT ON C OR D	
6	0	1	0	1		
7	0	1	1	0		
8	0	1	1	1		
9	1	0	0	0	A CONTINGENT ON D	
10	1	0	0	1		
11	1	0	1	0	NOT BOTH A AND C	A CONTINGENT ON D
12	1	0	1	1	NOT BOTH A AND C	
13	1	1	0	0	B CONTINGENT ON C OR D	A CONTINGENT ON D
14	1	1	0	1		
15	1	1	1	0	NOT BOTH A AND C	A CONTINGENT ON D
16	1	1	1	1	NOT BOTH A AND C	

b NONE

c 1. IDENTIFY THE INVESTMENT ALTERNATIVES

Homework # 1 Solution

PROBLEM 1.9

ACCEPT PROPOSAL: 1 REJECT PROPOSAL: 0

a

ALT	PROP V	PROP W	PROP X	PROP Y	PROP Z	WHY NOT FEASIBLE
1	0	0	0	0	0	AT LEAST 2 AND NO MORE THAN 4
2	0	0	0	0	1	AT LEAST 2 AND NO MORE THAN 4; Z CONTINGENT ON X OR Y
3	0	0	0	1	0	AT LEAST 2 AND NO MORE THAN 4
4	0	0	0	1	1	
5	0	0	1	0	0	AT LEAST 2 AND NO MORE THAN 4
6	0	0	1	0	1	
7	0	0	1	1	0	X AND Y MUTUALLY EXCLUSIVE
8	0	0	1	1	1	X AND Y MUTUALLY EXCLUSIVE
9	0	1	0	0	0	AT LEAST 2 AND NO MORE THAN 4
10	0	1	0	0	1	Z CONTINGENT ON X OR Y
11	0	1	0	1	0	
12	0	1	0	1	1	
13	0	1	1	0	0	
14	0	1	1	0	1	
15	0	1	1	1	0	X AND Y MUTUALLY EXCLUSIVE
16	0	1	1	1	1	X AND Y MUTUALLY EXCLUSIVE
17	1	0	0	0	0	AT LEAST 2 AND NO MORE THAN 4
18	1	0	0	0	1	Z CONTINGENT ON X OR Y
19	1	0	0	1	0	NOT V IF W, X, Y
20	1	0	0	1	1	NOT V IF W, X, Y
21	1	0	1	0	0	NOT V IF W, X, Y
22	1	0	1	0	1	NOT V IF W, X, Y
23	1	0	1	1	0	X AND Y MUTUALLY EXCLUSIVE; NOT V IF W, X, Y
24	1	0	1	1	1	X AND Y MUTUALLY EXCLUSIVE; NOT V IF W, X, Y
25	1	1	0	0	0	NOT V IF W, X, Y
26	1	1	0	0	1	Z CONTINGENT ON X OR Y
27	1	1	0	1	0	NOT V IF W, X, Y
28	1	1	0	1	1	NOT V IF W, X, Y
29	1	1	1	0	0	NOT V IF W, X, Y
30	1	1	1	0	1	NOT V IF W, X, Y
31	1	1	1	1	0	X AND Y MUTUALLY EXCLUSIVE; NOT V IF W, X, Y
32	1	1	1	1	1	AT LEAST 2 AND NO MORE THAN 4; X AND Y MUTUALLY EXCLUSIVE; NOT V IF W, X, Y

b NONE

c 1. IDENTIFY THE INVESTMENT ALTERNATIVES

Homework # 1 Solution

PROBLEM 1.10

ACCEPT PROPOSAL: 1 REJECT PROPOSAL: 0

ALTERNATIVE	PROPOSAL P	PROPOSAL Q	PROPOSAL R	WHY NOT FEASIBLE
1	0	0	0	EXACTLY 1 OR 2 CHOSEN
2	0	0	1	R CONTINGENT ON P
3	0	1	0	
4	0	1	1	R CONTINGENT ON P
5	1	0	0	
6	1	0	1	
7	1	1	0	P AND Q MUTUALLY EXCLUSIVE
8	1	1	1	EXACTLY 1 OR 2 CHOSEN; P AND Q MUTUALLY EXCLUSIVE

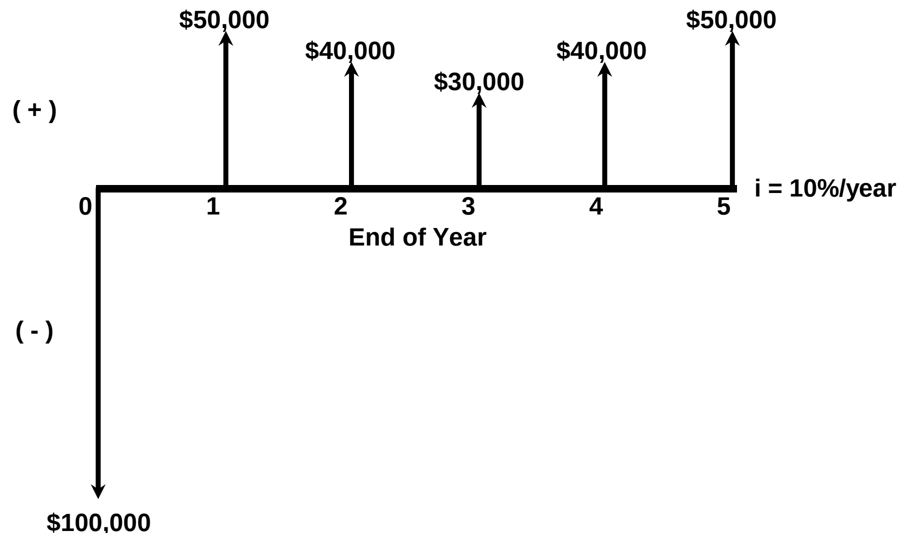
Computing the Present Worth of Multiple Cash flows

$$P = \sum_{t=0}^{n} A_t (1+i)^{-t} \qquad \textbf{(2.12)}$$

$$P = \sum_{t=0}^{n} A_t (P \mid F\ i\%, t) \qquad \textbf{(2.13)}$$

Example 2.12

Determine the present worth equivalent of the CFD shown below, using an interest rate of 10% compounded annually.

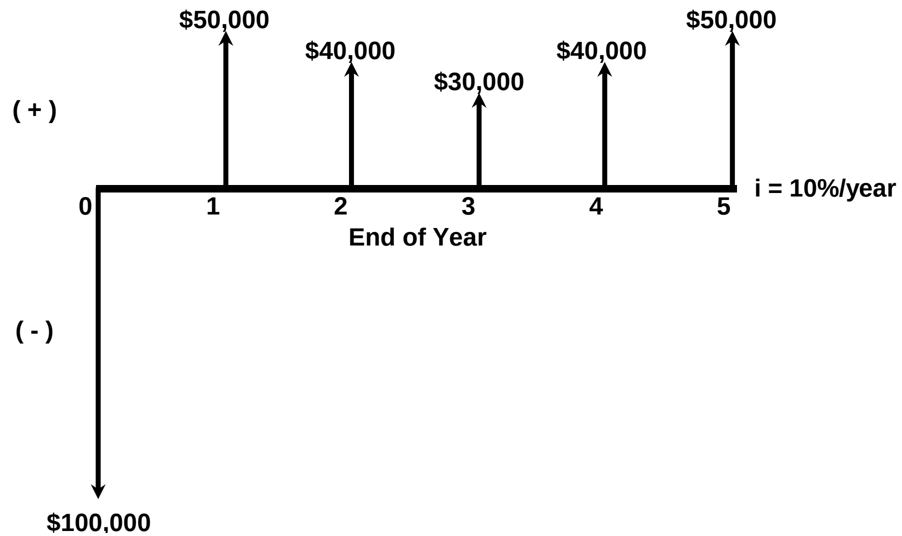


End of Year (n)	Cash Flow (CF)	(*P* |*F* 10%,n)	Present Worth	PV(10%,n,,-CF)	(*F* |*P* 10%,5-n)	Future Worth	FV(10%,5-n,,-CF)
0	-$100,000	1.00000	-$100,000.00	-$100,000.00	1.61051	-$161,051.00	-$161,051.00
1	$50,000	0.90909	$45,454.50	$45,454.55	1.46410	$73,205.00	$73,205.00
2	$40,000	0.82645	$33,058.00	$33,057.85	1.33100	$53,240.00	$53,240.00
3	$30,000	0.75131	$22,539.30	$22,539.44	1.21000	$36,300.00	$36,300.00
4	$40,000	0.68301	$27,320.40	$27,320.54	1.10000	$44,000.00	$44,000.00
5	$50,000	0.62092	$31,046.00	$31,046.07	1.00000	$50,000.00	$50,000.00
SUM			$59,418.20	$59,418.45		$95,694.00	$95,694.00

P= -\$100000+ 50000 $(1.1)^{-1}$ + 40000 $(1.1)^{-2}$ + 30000 $(1.1)^{-3}$ + 40000 $(1.1)^{-4}$ + 50000 $(1.1)^{-5}$ = <u>\$59,418.45</u>

Example 2.15

Determine the future worth equivalent of the CFD shown below, using an interest rate of 10% compounded annually.



End of Year (n)	Cash Flow (CF)	(*P* |*F* 10%,n)	Present Worth	PV(10%,n,,-CF)	(*F* |*P* 10%,5-n)	Future Worth	FV(10%,5-n,,-CF)
0	-$100,000	1.00000	-$100,000.00	-$100,000.00	1.61051	-$161,051.00	-$161,051.00
1	$50,000	0.90909	$45,454.50	$45,454.55	1.46410	$73,205.00	$73,205.00
2	$40,000	0.82645	$33,058.00	$33,057.85	1.33100	$53,240.00	$53,240.00
3	$30,000	0.75131	$22,539.30	$22,539.44	1.21000	$36,300.00	$36,300.00
4	$40,000	0.68301	$27,320.40	$27,320.54	1.10000	$44,000.00	$44,000.00
5	$50,000	0.62092	$31,046.00	$31,046.07	1.00000	$50,000.00	$50,000.00
SUM			$59,418.20	$59,418.45		$95,694.00	$95,694.00

F= -\$100000 $(1.1)^5$ + 50000 $(1.1)^4$ + 40000 $(1.1)^3$ + 30000 $(1.1)^2$ + 40000 $(1.1)^1$ + 50000 = <u>\$95,694.00</u>

Examples 2.13 & 2.16

Determine the present worth equivalent of the following series of cash flows. Use an interest rate of 6% per interest period.

End of Period	Cash Flow
0	$0
1	$300
2	$0
3	-$300
4	$200
5	$0
6	$400
7	$0
8	$200

P =$597.02

Computing the Future worth of Multiple cash Flows

$$F = \sum_{t=1}^{n} A_t (1+i)^{n-t} \qquad \textbf{(2.15)}$$

$$F = \sum_{t=1}^{n} A_t (F \mid P \quad i\%, n-t) \qquad \textbf{(2.16)}$$

Examples 2.14 & 2.16

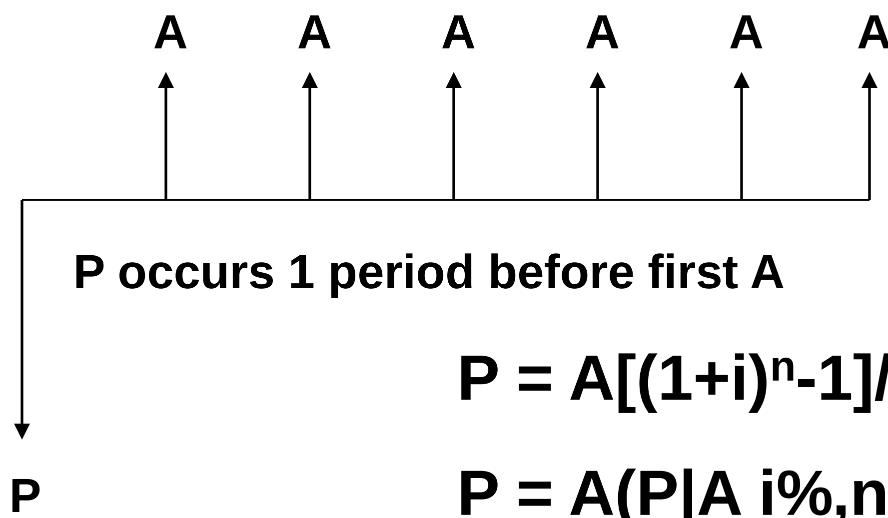
Determine the future worth equivalent of the following series of cash flows. Use an interest rate of 6% per interest period.

End of Period	Cash Flow
0	$0
1	$300
2	$0
3	-$300
4	$200
5	$0
6	$400
7	$0
8	$200

F =$951.56

(The 3¢ difference in the answers is due to round-off error in the tables in Appendix A.)

DCF Uniform Series Formulas



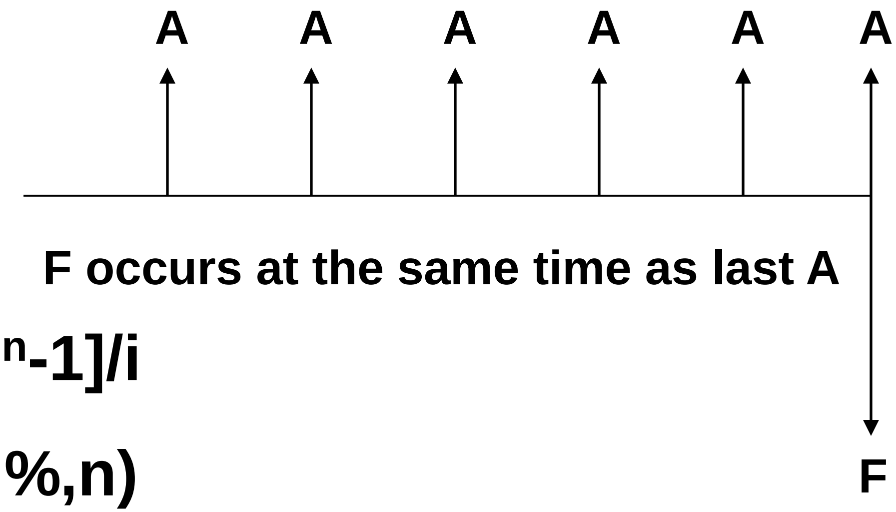
$$P = A[(1+i)^n-1]/[i(1+i)^n]$$

$$P = A(P|A\ i\%,n)$$

$$A = Pi(1+i)^n/[(1+i)^n-1]$$

$$A = P(A|P\ i\%,n)$$

DCF Uniform Series Formulas



$$F = A[(1+i)^n-1]/i$$

$$F = A(F|A\ i\%,n)$$

$$A = Fi/[(1+i)^n-1]$$

$$A = F(A|F\ i\%,n)$$

Uniform Series of Cash Flows
Discounted Cash Flow Formulas

$$P = A(P|A\ i\%,n) = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad (2.22)$$

$$A = P(A|P\ i\%,n) = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \qquad (2.25)$$

P occurs one period before the first A

$$F = A(F|A\ i\%,n) = A\left[\frac{(1+i)^n - 1}{i}\right] \qquad (2.28)$$

$$A = F(A|F\ i\%,n) = F\left[\frac{i}{(1+i)^n - 1}\right] \qquad (2.30)$$

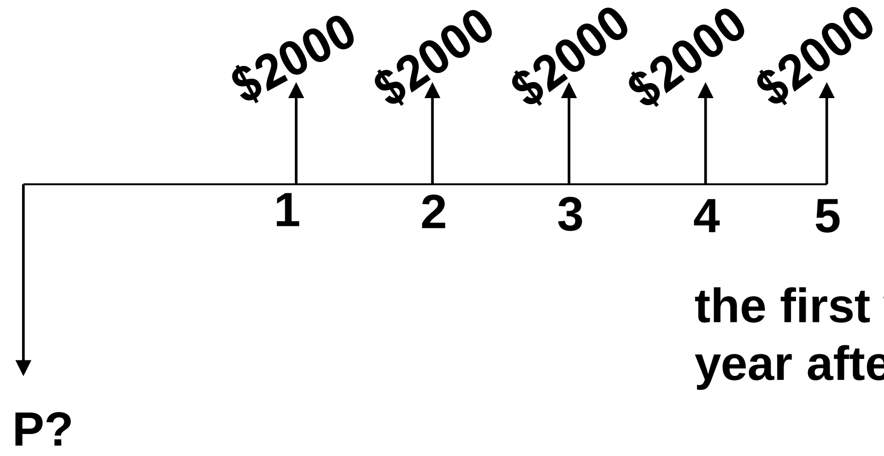
F occurs at the same time as the last A

Example 2. 17

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, with the first withdrawal occurring 1 year after the deposit?

Example 2. 17

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, with the first withdrawal occurring 1 year after the deposit?



the first withdrawal occurring 1 year after the deposit

Example 2. 17

P = \$2,000(P|A 5%,5)

P = \$2,000(4.32948) = \$8,658.96

TABLE A-a-11

n	Single Sums: To Find F Given P $(F\|P\ i\%,n)$	Single Sums: To Find P Given F $(P\|F\ i\%,n)$	Uniform Series: To Find F Given A $(F\|A\ i\%,n)$	Uniform Series: To Find A Given F $(A\|F\ i\%,n)$	Uniform Series: To Find P Given A $(P\|A\ i\%,n)$	Uniform Series: To Find A Given P $(A\|P\ i\%,n)$	Gradient Series: To Find P Given G $(P\|G\ i\%,n)$	Gradient Series: To Find A Given G $(A\|G\ i\%,n)$
1	1.05000	0.95238	1.00000	1.00000	0.95238	1.05000	0.00000	0.00000
2	1.10250	0.90703	2.05000	0.48780	1.85941	0.53780	0.90703	0.48780
3	1.15763	0.86384	3.15250	0.31721	2.72325	0.36721	2.63470	0.96749
4	1.21551	0.82270	4.31013	0.23201	3.54595	0.28201	5.10281	1.43905
5	1.27628	0.78353	5.52563	0.18097	4.32948	0.23097	8.23692	1.90252
6	1.34010	0.74622	6.80191	0.14702	5.07569	0.19702	11.96799	2.35790
7	1.40710	0.71068	8.14201	0.12282	5.78637	0.17282	16.23208	2.80523
8	1.47746	0.67684	9.54911	0.10472	6.46321	0.15472	20.96996	3.24451
9	1.55133	0.64461	11.02656	0.09069	7.10782	0.14069	26.12683	3.67579
10	1.62889	0.61391	12.57789	0.07950	7.72173	0.12950	31.65205	4.09909
11	1.71034	0.58468	14.20679	0.07039	8.30641	0.12039	37.49884	4.51444

5.00% Time Value of Money Fa

Example 2. 17

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, with the first withdrawal occurring 1 year after the deposit?

P = $2,000(P|A 5%,5)

P = $2,000(4.32948) = $8,658.96

$P = A[(1+i)^n-1]/[i(1+i)^n] = 2000\ [(1+0.05)^5-1]/[0.05(1+0.05)^5]$

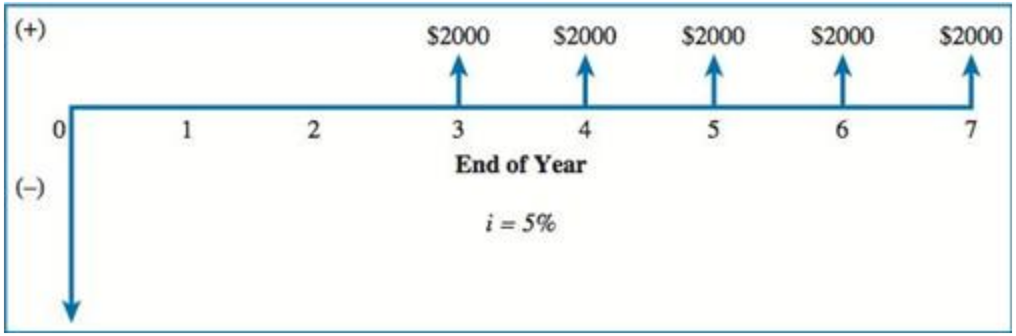
P= $8,658.96

Example 2. 18

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, ***with the first withdrawal occurring 3 years after the deposit?***

Example 2. 18

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, ***with the first withdrawal occurring 3 years after the deposit?***



Example 2. 18

P = $2,000(P|A 5%,5)(P|F 5%,2)

P = $2,000(4.32948)(0.90703) = $7,853.94

TABLE A-a-11

5.00% Time Value of Money Fa

	Single Sums		Uniform Series				Gradient Series	
n	To Find *F* Given *P* (*F*|*P* i%,n)	To Find *P* Given *F* (*P*|*F* i%,n)	To Find *F* Given *A* (*F*|*A* i%,n)	To Find *A* Given *F* (*A*|*F* i%,n)	To Find *P* Given *A* (*P*|*A* i%,n)	To Find *A* Given *P* (*A*|*P* i%,n)	To Find *P* Given *G* (*P*|*G* i%,n)	To Find *A* Given *G* (*A*|*G* i%,n)
1	1.05000	0.95238	1.00000	1.00000	0.95238	1.05000	0.00000	0.00000
2	1.10250	0.90703	2.05000	0.48780	1.85941	0.53780	0.90703	0.48780
3	1.15763	0.86384	3.15250	0.31721	2.72325	0.36721	2.63470	0.96749
4	1.21551	0.82270	4.31013	0.23201	3.54595	0.28201	5.10281	1.43905
5	1.27628	0.78353	5.52563	0.18097	4.32948	0.23097	8.23692	1.90252
6	1.34010	0.74622	6.80191	0.14702	5.07569	0.19702	11.96799	2.35790
7	1.40710	0.71068	8.14201	0.12282	5.78637	0.17282	16.23208	2.80523
8	1.47746	0.67684	9.54911	0.10472	6.46321	0.15472	20.96996	3.24451
9	1.55133	0.64461	11.02656	0.09069	7.10782	0.14069	26.12683	3.67579
10	1.62889	0.61391	12.57789	0.07950	7.72173	0.12950	31.65205	4.09909
11	1.71034	0.58468	14.20679	0.07039	8.30641	0.12039	37.49884	4.51444

Example 2. 18

Troy Long deposits a single sum of money in a savings account that pays 5% compounded annually. How much must he deposit in order to withdraw $2,000/yr for 5 years, ***with the first withdrawal occurring 3 years after the deposit?***

P = $2,000(P|A 5%,5)(P|F 5%,2)

P = $2,000(4.32948)(0.90703) = $7,853.94

$P = (2000\,[(1+0.05)^{5}-1]/[0.05(1+0.05)^{5}])\,(1+0.05)^{-2}$

P= $7,853.94

Example 2. 19

Rachel Townsley invests $10,000 in a fund that pays 8% compounded annually. If she makes 10 equal annual withdrawals from the fund, how much can she withdraw if the first withdrawal occurs 1 year after her investment?

Example 2. 19

A = $10,000(A|P 8%,10)

A = $10,000(0.14903) = $1,490.30

Money Factors Discrete Compounding 8.00%

TABLE A-a-14

	Single Sums		Uniform Series				Gradient Series	
n	To Find *F* Given *P* (*F*|*P* i%,n)	To Find *P* Given *F* (*P*|*F* i%,n)	To Find *F* Given *A* (*F*|*A* i%,n)	To Find *A* Given *F* (*A*|*F* i%,n)	To Find *P* Given *A* (*P*|*A* i%,n)	To Find *A* Given *P* (*A*|*P* i%,n)	To Find *P* Given *G* (*P*|*G* i%,n)	To Find *A* Given *G* (*A*|*G* i%,n)
1	1.08000	0.92593	1.00000	1.00000	0.92593	1.08000	0.00000	0.00000
2	1.16640	0.85734	2.08000	0.48077	1.78326	0.56077	0.85734	0.48077
3	1.25971	0.79383	3.24640	0.30803	2.57710	0.38803	2.44500	0.94874
4	1.36049	0.73503	4.50611	0.22192	3.31213	0.30192	4.65009	1.40396
5	1.46933	0.68058	5.86660	0.17046	3.99271	0.25046	7.37243	1.84647
6	1.58687	0.63017	7.33593	0.13632	4.62288	0.21632	10.52327	2.27635
7	1.71382	0.58349	8.92280	0.11207	5.20637	0.19207	14.02422	2.69366
8	1.85093	0.54027	10.63663	0.09401	5.74664	0.17401	17.80610	3.09852
9	1.99900	0.50025	12.48756	0.08008	6.24689	0.16008	21.80809	3.49103
10	2.15892	0.46319	14.48656	0.06903	6.71008	0.14903	25.97683	3.87131
11	2.33164	0.42888	16.64549	0.06008	7.13896	0.14008	30.26566	4.23950
12	2.51817	0.39711	18.97713	0.05270	7.53608	0.13270	34.63391	4.59575
13	2.71962	0.36770	21.49530	0.04652	7.90378	0.12652	39.04629	4.94021
14	2.93719	0.34046	24.21492	0.04130	8.24424	0.12130	43.47228	5.27305
15	3.17217	0.31524	27.15211	0.03683	8.55948	0.11683	47.88566	5.59446
16	3.42594	0.29189	30.32428	0.03298	8.85137	0.11298	52.26402	5.90463
17	3.70002	0.27027	33.75023	0.02963	9.12164	0.10963	56.58832	6.20375
18	3.99602	0.25025	37.45024	0.02670	9.37189	0.10670	60.84256	6.49203

Example 2. 19

Rachel Townsley invests $10,000 in a fund that pays 8% compounded annually. If she makes 10 equal annual withdrawals from the fund, how much can she withdraw if the first withdrawal occurs 1 year after her investment?

$$A = Pi(1+i)^n/[(1+i)^n-1]$$

$$= 10000*0.08(1.08)^{10}/[(1.08)^{10}-1] = \$1,490.30$$

Example 2.22 *(note the skipping)*

Suppose Rachel delays the first withdrawal for 2 years. How much can be withdrawn each of the 10 years?

Example 2.22

A = $10,000(F|P 8%,2)(A|P 8%,10)

A = $10,000(1.16640)(0.14903)

A = $1,738.29

Money Factors Discrete Compounding 8.00%

TABLE A-a-14

	Single Sums		Uniform Series				Gradient Series	
n	To Find *F* Given *P* (*F*|*P* i%,n)	To Find *P* Given *F* (*P*|*F* i%,n)	To Find *F* Given *A* (*F*|*A* i%,n)	To Find *A* Given *F* (*A*|*F* i%,n)	To Find *P* Given *A* (*P*|*A* i%,n)	To Find *A* Given *P* (*A*|*P* i%,n)	To Find *P* Given *G* (*P*|*G* i%,n)	To Find *A* Given *G* (*A*|*G* i%,n)
1	1.08000	0.92593	1.00000	1.00000	0.92593	1.08000	0.00000	0.00000
2	1.16640	0.85734	2.08000	0.48077	1.78326	0.56077	0.85734	0.48077
3	1.25971	0.79383	3.24640	0.30803	2.57710	0.38803	2.44500	0.94874
4	1.36049	0.73503	4.50611	0.22192	3.31213	0.30192	4.65009	1.40396
5	1.46933	0.68058	5.86660	0.17046	3.99271	0.25046	7.37243	1.84647
6	1.58687	0.63017	7.33593	0.13632	4.62288	0.21632	10.52327	2.27635
7	1.71382	0.58349	8.92280	0.11207	5.20637	0.19207	14.02422	2.69366
8	1.85093	0.54027	10.63663	0.09401	5.74664	0.17401	17.80610	3.09852
9	1.99900	0.50025	12.48756	0.08008	6.24689	0.16008	21.80809	3.49103
10	2.15892	0.46319	14.48656	0.06903	6.71008	0.14903	25.97683	3.87131
11	2.33164	0.42888	16.64549	0.06008	7.13896	0.14008	30.26566	4.23950
12	2.51817	0.39711	18.97713	0.05270	7.53608	0.13270	34.63391	4.59575
13	2.71962	0.36770	21.49530	0.04652	7.90378	0.12652	39.04629	4.94021
14	2.93719	0.34046	24.21492	0.04130	8.24424	0.12130	43.47228	5.27305
15	3.17217	0.31524	27.15211	0.03683	8.55948	0.11683	47.88566	5.59446
16	3.42594	0.29189	30.32428	0.03298	8.85137	0.11298	52.26402	5.90463
17	3.70002	0.27027	33.75023	0.02963	9.12164	0.10963	56.58832	6.20375
18	3.99602	0.25025	37.45024	0.02670	9.37189	0.10670	60.84256	6.49203

Example 2.22

Suppose Rachel delays the first withdrawal for 2 years. How much can be withdrawn each of the 10 years?

$$A = (P\,(1 + i)^{n}\,)\; i(1+i)^{n}/[(1+i)^{n}-1]$$

$$= (10000\,(1.08)^{2})*0.08(1.08)^{10}/[(1.08)^{10}-1] = \$1{,}490.30$$

Example 2.20

A firm borrows $2,000,000 at 12% annual interest and pays it back with 10 equal annual payments. What is the payment?

Money Factors Discrete Compounding 12.00%

TABLE A-a-18

	Single Sums		Uniform Series				Gradient Series	
n	To Find F Given P (F|P i%,n)	To Find P Given F (P|F i%,n)	To Find F Given A (F|A i%,n)	To Find A Given F (A|F i%,n)	To Find P Given A (P|A i%,n)	To Find A Given P (A|P i%,n)	To Find P Given G (P|G i%,n)	To Find A Given G (A|G i%,n)
1	1.12000	0.89286	1.00000	1.00000	0.89286	1.12000	0.00000	0.00000
2	1.25440	0.79719	2.12000	0.47170	1.69005	0.59170	0.79719	0.47170
3	1.40493	0.71178	3.37440	0.29635	2.40183	0.41635	2.22075	0.92461
4	1.57352	0.63552	4.77933	0.20923	3.03735	0.32923	4.12731	1.35885
5	1.76234	0.56743	6.35285	0.15741	3.60478	0.27741	6.39702	1.77459
6	1.97382	0.50663	8.11519	0.12323	4.11141	0.24323	8.93017	2.17205
7	2.21068	0.45235	10.08901	0.09912	4.56376	0.21912	11.64427	2.55147
8	2.47596	0.40388	12.29969	0.08130	4.96764	0.20130	14.47145	2.91314
9	2.77308	0.36061	14.77566	0.06768	5.32825	0.18768	17.35633	3.25742
10	3.10585	0.32197	17.54874	0.05698	5.65022	0.17698	20.25409	3.58465
11	3.47855	0.28748	20.65458	0.04842	5.93770	0.16842	23.12885	3.89525
12	3.89598	0.25668	24.13313	0.04144	6.19437	0.16144	25.95228	4.18965
13	4.36349	0.22917	28.02911	0.03568	6.42355	0.15568	28.70237	4.46830
14	4.88711	0.20462	32.39260	0.03087	6.62817	0.15087	31.36242	4.73169
15	5.47357	0.18270	37.27971	0.02682	6.81086	0.14682	33.92017	4.98030
16	6.13039	0.16312	42.75328	0.02339	6.97399	0.14339	36.36700	5.21466
17	6.86604	0.14564	48.88367	0.02046	7.11963	0.14046	38.69731	5.43530
18	7.68997	0.13004	55.74971	0.01794	7.24967	0.13794	40.90798	5.64274

Example 2.21

Suppose the firm pays back the loan ($2,000,000) over 15 years in order to obtain a 10% interest rate. What would be the size of the annual payment?

Example 2.21

A = $2,000,000(A|P 10%,15)

A = $2,000,000(0.13147)

A = $262,940

of Money Factors Discrete Compounding 10.00%

TABLE A-a-16

	Single Sums		Uniform Series				Gradient Series	
n	To Find *F* Given *P* (*F*|*P* i%,n)	To Find *P* Given *F* (*P*|*F* i%,n)	To Find *F* Given *A* (*F*|*A* i%,n)	To Find *A* Given *F* (*A*|*F* i%,n)	To Find *P* Given *A* (*P*|*A* i%,n)	To Find *A* Given *P* (*A*|*P* i%,n)	To Find *P* Given *G* (*P*|*G* i%,n)	To Find *A* Given *G* (*A*|*G* i%,n)
1	1.10000	0.90909	1.00000	1.00000	0.90909	1.10000	0.00000	0.00000
2	1.21000	0.82645	2.10000	0.47619	1.73554	0.57619	0.82645	0.47619
3	1.33100	0.75131	3.31000	0.30211	2.48685	0.40211	2.32908	0.93656
4	1.46410	0.68301	4.64100	0.21547	3.16987	0.31547	4.37812	1.38117
5	1.61051	0.62092	6.10510	0.16380	3.79079	0.26380	6.86180	1.81013
6	1.77156	0.56447	7.71561	0.12961	4.35526	0.22961	9.68417	2.22356
7	1.94872	0.51316	9.48717	0.10541	4.86842	0.20541	12.76312	2.62162
8	2.14359	0.46651	11.43589	0.08744	5.33493	0.18744	16.02867	3.00448
9	2.35795	0.42410	13.57948	0.07364	5.75902	0.17364	19.42145	3.37235
10	2.59374	0.38554	15.93742	0.06275	6.14457	0.16275	22.89134	3.72546
11	2.85312	0.35049	18.53117	0.05396	6.49506	0.15396	26.39628	4.06405
12	3.13843	0.31863	21.38428	0.04676	6.81369	0.14676	29.90122	4.38840
13	3.45227	0.28966	24.52271	0.04078	7.10336	0.14078	33.37719	4.69879
14	3.79750	0.26333	27.97498	0.03575	7.36669	0.13575	36.80050	4.99553
15	4.17725	0.23939	31.77248	0.03147	7.60608	0.13147	40.15199	5.27893
16	4.59497	0.21763	35.94973	0.02782	7.82371	0.12782	43.41642	5.54934
17	5.05447	0.19784	40.54470	0.02466	8.02155	0.12466	46.58194	5.80710
18	5.55992	0.17986	45.59917	0.02193	8.20141	0.12193	49.63954	6.05256
19	6.11591	0.16351	51.15909	0.01955	8.36492	0.11955	52.58268	6.28610
20	6.72750	0.14864	57.27500	0.01746	8.51356	0.11746	55.40691	6.50808

Example 2. 23

Luis Jimenez deposits $1,000/yr in a savings account that pays 6% compounded annually. How much will be in the account immediately after his 30th deposit?

Example 2. 23

Luis Jimenez deposits $1,000/yr in a savings account that pays 6% compounded annually. How much will be in the account immediately after his 30th deposit?

F = $1,000(F|A 6%,30)

F = $1,000(79.05819) = $79,058.19

$$F = A[(1+i)^n-1]/i = \$1000\ [(1.06)^{30}-1]/0.06$$

$$= \$79{,}058.19$$

Example 2. 24

Andrew Brewer invests $5,000/yr and earns 6% compounded annually. How much will he have in his investment portfolio after 15 yrs? 20 yrs? 25 yrs? 30 yrs?

Example 2. 24

Andrew Brewer invests $5,000/yr and earns 6% compounded annually. How much will he have in his investment portfolio after 15 yrs? 20 yrs? 25 yrs? 30 yrs?

F = $5,000(F|A 6%,15) = $5,000(23.27597) = $116,379.85

F = $5,000(F|A 6%,20) = $5,000(36.78559) = $183,927.95

F = $5,000(F|A 6%,25) = $5,000(54.86451) = $274,322.55

F = $5,000(F|A 6%,30) = $5,000(79.05819) = $395,290.95

Example 2. 24

Andrew Brewer invests \$5,000/yr and earns 6% compounded annually. How much will he have in his investment portfolio after 15 yrs? 20 yrs? 25 yrs? 30 yrs? (What if he earns 3%/yr?)

F = \$5,000(F|A 6%,15) = \$5,000(23.27597) = \$116,379.85

F = \$5,000(F|A 6%,20) = \$5,000(36.78559) = \$183,927.95

F = \$5,000(F|A 6%,25) = \$5,000(54.86451) = \$274,322.55

F = \$5,000(F|A 6%,30) = \$5,000(79.05819) = \$395,290.95

F = \$5,000(F|A 3%,15) = \$5,000(18.59891) = \$92,994.55

F = \$5,000(F|A 3%,20) = \$5,000(26.87037) = \$134,351.85

F = \$5,000(F|A 3%,25) = \$5,000(36.45926) = \$182,296.30

F = \$5,000(F|A 3%,30) = \$5,000(47.57542) = \$237,877.10

Example 2. 24

F = $5,000(F|A 6%,15) = $5,000(23.27597) = $116,379.85

F = $5,000(F|A 6%,20) = $5,000(36.78559) = $183,927.95

F = $5,000(F|A 6%,25) = $5,000(54.86451) = $274,322.55

F = $5,000(F|A 6%,30) = $5,000(79.05819) = $395,290.95

F = $5,000(F|A 3%,15) = $5,000(18.59891) = $92,994.55

F = $5,000(F|A 3%,20) = $5,000(26.87037) = $134,351.85

F = $5,000(F|A 3%,25) = $5,000(36.45926) = $182,296.30

F = $5,000(F|A 3%,30) = $5,000(47.57542) = $237,877.10

The single sum, compound amount factor, $(1+i)^n$, answers the question regarding the impact of the interest earned versus the investment's duration. Because the duration (n) is in the exponent, time is far more critical than the interest rate earned on investments. For that reason, it is essential that an individual begin investing sooner rather than later.

Twice the time at half the rate is best! $(1 + i)^n$

Example 2.25

If Coby Durham earns 7% on his investments, how much must he invest annually in order to accumulate $1,500,000 in 25 years?

A = $1,500,000(A|F 7%,25)

A = $1,500,000(0.01581)

A = $23,715

Example 2.26

If Crystal Wilson earns 10% on her investments, how much must she invest annually in order to accumulate $1,000,000 in 40 years?

Example 2.26

If Crystal Wilson earns 10% on her investments, how much must she invest annually in order to accumulate $1,000,000 in 40 years?

A = $1,000,000(A|F 10%,40)

A = $1,000,000(0.0022594)

A = $2,259.40

Example 2.27

$500,000 is spent for a SMP machine in order to reduce annual expenses by $92,500/yr. At the end of a 10-year planning horizon, the SMP machine is worth $50,000. Based on a 10% TVOM, a) what single sum at t = 0 is equivalent to the SMP investment? b) what single sum at t = 10 is equivalent to the SMP investment? c) what uniform annual series over the 10-year period is equivalent to the SMP investment?

Example 2.27 (Solution)

a) P = -$500,000 + $92,500(P|A 10%,10) + $50,000(P|F 10%,10)

P = -$500,000 + $92,500(6.14457) + $50,000(0.38554)

P = $87,649.73

b) F = -$500,000(F|P 10%,10) + $92,500(F|A 10%,10) + $50,000

F = -$500,000(2.59374) + $92,500(15.93742) + $50,000

F = $227,341.40

Example 2.27 (Solution)

c)

A = -$500,000(A|P 10%,10) + $92,500 +

$50,000(A|F 10%,10)

A = -$500,000(0.16275) + $92,500 + $50,000(0.06275)

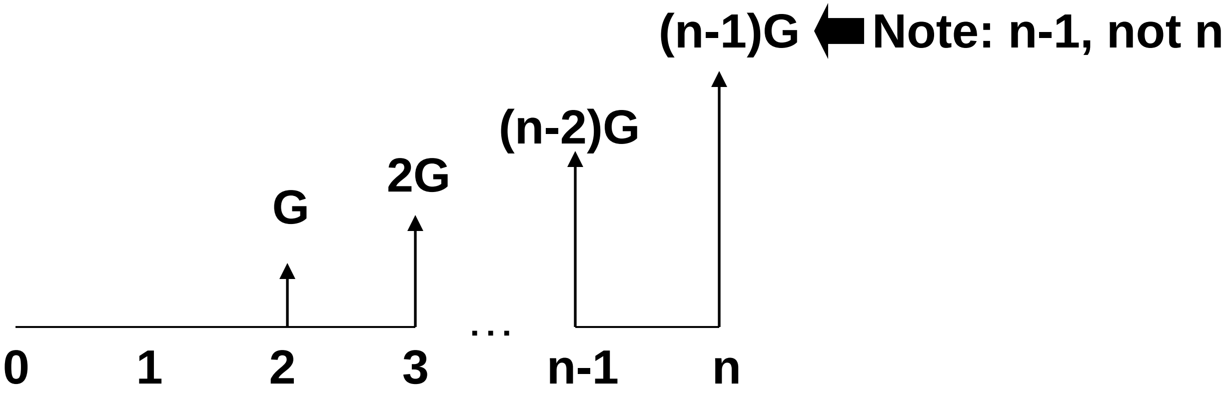
A = $14,262.50

Gradient Series

$$A_t = \begin{cases} 0 & t = 1 \\ A_{t-1} + G & t = 2, \ldots, n \end{cases}$$

or

$$A_t = (t-1)G \qquad t = 1, \ldots, n$$



Converting Gradient Series

Converting gradient series to present worth

$$P = G \left[\frac{1-(1+ni)(1+i)^{-n}}{i^2} \right] \qquad (2.35)$$

$$P = G \left[\frac{(P|A\ i\%,n)-n(P|F\ i\%,n)}{i} \right] \qquad (2.36)$$

$$P = G(P|G\ i\%, n) \qquad (2.37)$$

Converting Gradient Series - II

Converting gradient series to annual worth

$$A = G \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1} \right]$$

$$A = G \left[\frac{1 - n(A \mid F\ i\%, n)}{i} \right]$$

$$A = G(A|G\ i\%, n) \qquad (2.38)$$

Converting Gradient Series - III

Converting gradient series to future worth

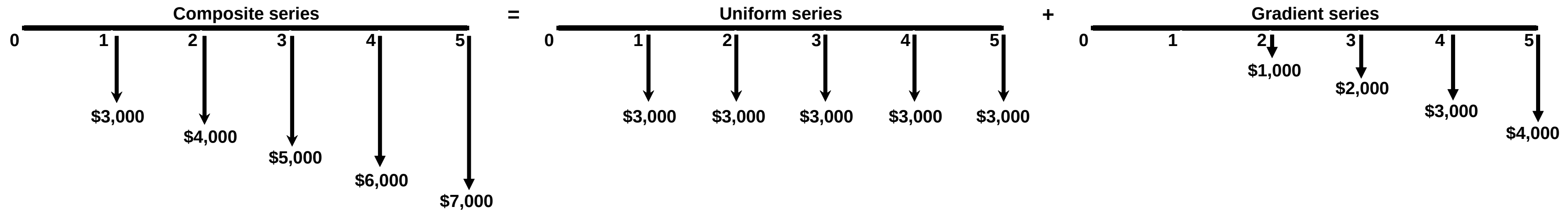
$$F = G\left[\frac{(1+i)^n - (1+ni)}{i^2}\right] \qquad \textbf{(2.39)}$$

$$F = G\left[\frac{(F\,|\,A\ i\%, n) - n}{i}\right]$$

F = G(F|G i%, n) (not provided in the tables)

Example 2.28

Maintenance costs for a particular production machine increase by $1,000/year over the 5-yr life of the machine. The initial maintenance cost is $3,000. Using an interest rate of 8% compounded annually, determine the present worth equivalent for the maintenance costs.



Example 2.28

P = \$3,000(P|A 8%,5) + \$1,000(P|G 8%,5)

P = \$3,000(3.99271) + \$1,000(7.372.43) = \$19,350.56 or

P = (\$3,000 + \$1,000(A|G 8%,5))(P|A 8%,5)

P = (\$3,000 + \$1,000(1.846.47))(3.99271) = \$19,350.55

Money Factors Discrete Compounding 8.00%

TABLE A-a-14

	Single Sums		Uniform Series				Gradient Series	
n	To Find *F* Given *P* (*F*|*P* i%,n)	To Find *P* Given *F* (*P*|*F* i%,n)	To Find *F* Given *A* (*F*|*A* i%,n)	To Find *A* Given *F* (*A*|*F* i%,n)	To Find *P* Given *A* (*P*|*A* i%,n)	To Find *A* Given *P* (*A*|*P* i%,n)	To Find *P* Given *G* (*P*|*G* i%,n)	To Find *A* Given *G* (*A*|*G* i%,n)
1	1.08000	0.92593	1.00000	1.00000	0.92593	1.08000	0.00000	0.00000
2	1.16640	0.85734	2.08000	0.48077	1.78326	0.56077	0.85734	0.48077
3	1.25971	0.79383	3.24640	0.30803	2.57710	0.38803	2.44500	0.94874
4	1.36049	0.73503	4.50611	0.22192	3.31213	0.30192	4.65009	1.40396
5	1.46933	0.68058	5.86660	0.17046	3.99271	0.25046	7.37243	1.84647
6	1.58687	0.63017	7.33593	0.13632	4.62288	0.21632	10.52327	2.27635
7	1.71382	0.58349	8.92280	0.11207	5.20637	0.19207	14.02422	2.69366
8	1.85093	0.54027	10.63663	0.09401	5.74664	0.17401	17.80610	3.09852
9	1.99900	0.50025	12.48756	0.08008	6.24689	0.16008	21.80809	3.49103
10	2.15892	0.46319	14.48656	0.06903	6.71008	0.14903	25.97683	3.87131
11	2.33164	0.42888	16.64549	0.06008	7.13896	0.14008	30.26566	4.23950
12	2.51817	0.39711	18.97713	0.05270	7.53608	0.13270	34.63391	4.59575
13	2.71962	0.36770	21.49530	0.04652	7.90378	0.12652	39.04629	4.94021
14	2.93719	0.34046	24.21492	0.04130	8.24424	0.12130	43.47228	5.27305
15	3.17217	0.31524	27.15211	0.03683	8.55948	0.11683	47.88566	5.59446
16	3.42594	0.29189	30.32428	0.03298	8.85137	0.11298	52.26402	5.90463
17	3.70002	0.27027	33.75023	0.02963	9.12164	0.10963	56.58832	6.20375
18	3.99602	0.25025	37.45024	0.02670	9.37189	0.10670	60.84256	6.49203

Example 2.28

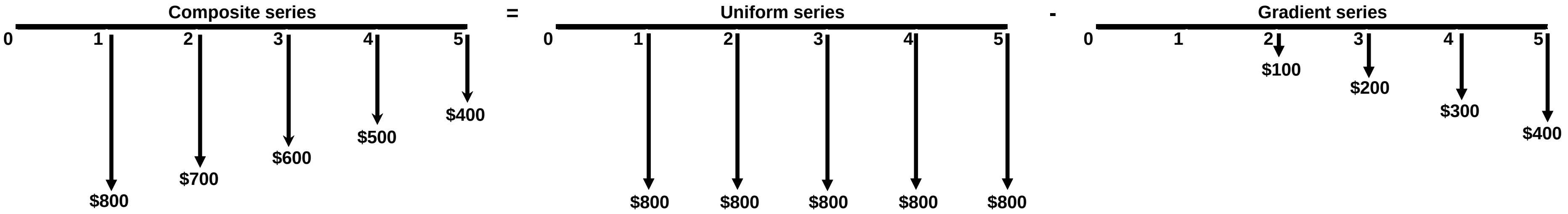
$P = \$3,000\ [(1.08)^5 - 1]/[0.08(1.08)^5]$

$+ \$1,000[(1-(1+5*0.08)(1.08)^{-5})/(0.08)^2]$

$P = \$19,350.56$

Example 2.29

Amanda Dearman made 5 annual deposits into a fund that paid 8% compound annual interest. Her first deposit was $800; each successive deposit was $100 less than the previous deposit. How much was in the fund immediately after the 5th deposit?



Example 2.29

Amanda Dearman made 5 annual deposits into a fund that paid 8% compound annual interest. Her first deposit was $800; each successive deposit was $100 less than the previous deposit. How much was in the fund immediately after the 5th deposit?

A = $800 - $100(A|G 8%,5) = $800 - $100(1.84647) = $615.35

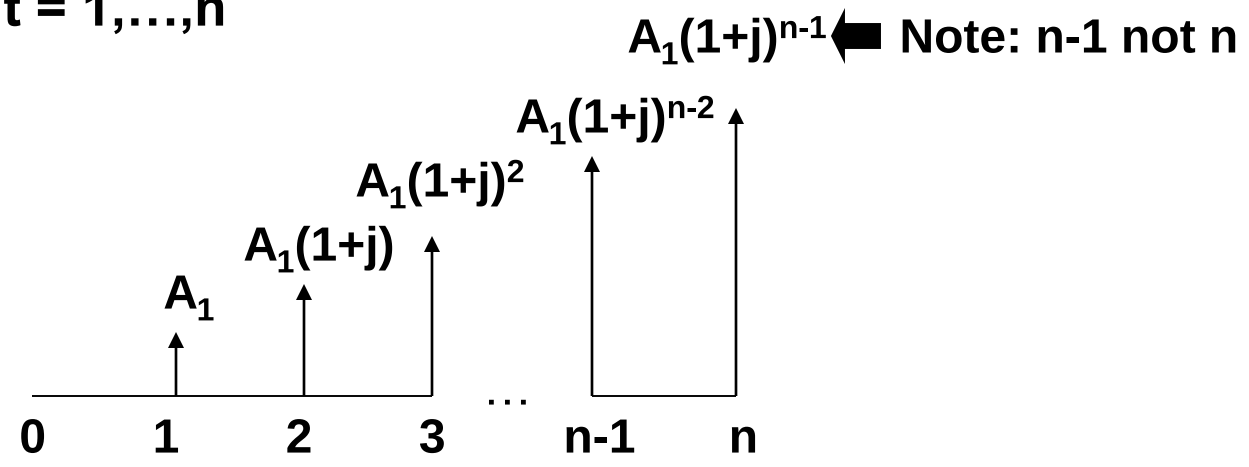
F = $615.35(F|A 8%,5) = $615.35(5.86660) = $3,610.01

Geometric Series

$A_t = A_{t-1}(1+j) \quad t = 2,\ldots,n$

or

$A_t = A_1(1+j)^{t-1} \quad t = 1,\ldots,n$



Converting Geometric Series – I

Converting a geometric series to a present worth

$$P = A_1\left[\frac{1-(1+j)^n(1+i)^{-n}}{i-j}\right] \qquad i \neq j \qquad \textbf{(2.42)}$$

$$P = A_1\left[\frac{1-(F\,|\,P\ j\%,n)(P\,|\,F\ i\%,n)}{i-j}\right] \qquad i \neq j \quad j > 0 \qquad \textbf{(2.44)}$$

$$P = nA_1/(1+i) \qquad i = j \qquad \textbf{(2.42)}$$

$$P = A_1(P\,|\,A_1\ \ i\%, j\%, n) \qquad \textbf{(2.43)}$$

Converting Geometric Series – II

Converting a geometric series to a future worth

$$F = A_1\left[\frac{(1+i)^n-(1+j)^n}{i-j}\right] \qquad i \neq j \qquad \textbf{(2.45)}$$

$$F = A_1\left[\frac{(F\,|\,P\ i\%,n)-(F\,|\,P\ j\%,n)}{i-j}\right] \qquad i \neq j \quad j > 0$$

$$\boldsymbol{F = nA_1(1+i)^{n-1}} \qquad \boldsymbol{i = j}$$

$$\boldsymbol{F = A_1(F|A_1\ i\%,j\%,n)}$$

Note: $(F|A_1\ i\%,j\%,n) = (F|A_1\ j\%,i\%,n)$

Notice the symmetry

Example 2.30

A firm is considering purchasing a new machine. It will have maintenance costs that increase 8% per year. An initial maintenance cost of $1,000 is expected. Using a 10% interest rate, what present worth cost is equivalent to the cash flows for maintenance of the machine over its 15-year expected life?

Example 2.30

A_1 = \$1,000, i = 10%, j = 8%, n = 15, P = ?

P = \$1,000(P|$A_1$10%,8%,15) = \$1,000(12.03040) = \$12,030.40

$$P = A_1\left[\frac{1-(1+j)^n(1+i)^{-n}}{i-j}\right] \qquad i \neq j$$

P = \$1000[(1-(1+0.08) ^{15} (1+0.1) $^{-15}$)/(0.1-0.08)]
= \$12,030.395

Example 2.31

Mattie Bookhout deposits her annual bonus in a savings account that pays 8% compound annual interest. Her annual bonus is expected to increase by 10% each year. If her initial deposit is $500, how much will be in her account immediately after her 10^{th} deposit?

Example 2.31

A_1 = \$500, i = 8%, j = 10%, n = 10, F = ?

F = \$500 (F|A_1 8%,10%,10) = \$500(21.74087)

F = \$10,870.44

$$F = A_1\left[\frac{(1+i)^n-(1+j)^n}{i-j}\right] \qquad i \neq j$$

F = \$500[((1+0.08) ^{10} - (1+0.1) ^{10})/(0.08-0.1)]

= \$10,870.437

Example 2.32

Julian Stewart invested \$100,000 in a limited partnership in a natural gas drilling project. His net revenue the 1st year was \$25,000. Each year, thereafter, his revenue decreased 10%/yr. Based on a 12% TVOM, what is the present worth of his investment over a 20-year period?

A_1 = \$25,000, i = 12%, j = -10%, n = 20, P = ?

P = -\$100,000 + \$25,000(P|A_1 12%,-10%,20)

$$P = A_1\left[\frac{1-(1+j)^n(1+i)^{-n}}{i-j}\right] \qquad i \neq j$$

P = -\$100,000 + \$25,000[1 – (0.90)^{20}(1.12$^{)-20}$]/(0.12 + 0.10)

P = \$12,204.15

Multiple Compounding Periods in a Year

Example: credit card debt

Example 2.36

Rebecca Carlson purchased a car for $25,000 by borrowing the money at 8% per year compounded monthly. She paid off the loan with 60 equal monthly payments, the first of which was paid one month after receiving the car. How much was her monthly payment?

A = $25,000(A|P ⅔%,60) = $25,000(0.02028) = $507.00

Compounding and Timing

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 \qquad = \qquad \left(F \mid P \;\; \frac{r\%}{m}, m\right) - 1$$

- **r = nominal annual interest rate**
- **m = number of compound periods per year**
- i_{eff} **=EFFECT(r,m)**

Example 2.37

What is the effective interest rate for 12% per annum compounded annually?

$r = 12\%$, $m = 1$, $r/m = 12\%$

$i_{eff} = (F|P\ 12\%,1)-1 = 1.1200-1.0 = 12.00\%$

=EFFECT(12%,1) = 12%

What is the effective interest rate for 12% per annum compounded semiannually?

$r = 12\%$, $m = 2$, $r/m = 6\%$

$i_{eff} = (F|P\ 6\%,2)-1 = 1.1236-1.0 = 12.36\%$

=EFFECT(12%,2) = 12.36%

Example 2.37

What is the effective interest rate for 12% per annum compounded quarterly?

$r = 12\%$, $m = 4$, $r/m = 3\%$

$i_{eff} = (F|P\ 3\%,4)-1 = 1.12551-1.0 = 12.551\%$

=EFFECT(12%,4) = 12.551%

What is the effective interest rate for 12% per annum compounded monthly?

$r = 12\%$, $m = 12$, $r/m = 1\%$

$i_{eff} = (F|P\ 1\%,12)-1 = 1.12683-1.0 = 12.683\%$

=EFFECT(12%,12) = 12.683%

Example 2.37

What is the effective interest rate for 12% per annum compounded weekly?

$r = 12\%$ $\quad\quad\quad$ $m = 52,$

$i_{eff} = (1 + 0.12/52)^{52} - 1 = 1.1273409872 - 1.0$

$= 12.73409872\%$

=EFFECT(12%,52) = 12.73409872%

What is the effective interest rate for 12% per annum compounded daily?

$r = 12\%$ $\quad\quad\quad$ $m = 365$

$i_{eff} = (1 + 0.12/365)^{365} - 1 = 1.1274746156 - 1.0$

$= 12.74746156\%$

=EFFECT(12%,365) = 12.74746156%

Example 2.37

What is the effective interest rate for 12% per annum compounded hourly?

$r = 12\%$ $m = 8760,$

$i_{eff} = (1 + 0.12/8760)^{8760} - 1 = 1.1274959248782 - 1.0$

$= 12.74959248782\%$

=EFFECT(12%,8760) = 12.74959248776% (!!!)

What is the effective interest rate for 12% per annum compounded every minute?

$r = 12\%$ $m = 525{,}600,$

$i_{eff} = (1 + 0.12/525{,}600)^{525{,}600} - 1 = 1.127496836146 - 1.0$

$= 12.7496836146\%$

=EFFECT(12%,525600) = 12.7496836133% (!!!)

Example 2.37

What is the effective interest rate for 12% per annum compounded every second?

r = 12% m = 31,536,000

$i_{eff} = (1 + 0.12/31{,}536{,}000)^{31{,}536{,}000} - 1 = 1.127496851431 - 1.0$

$= 12.7496851431\%$

=EFFECT(12%,31536000) = 12.7496852242% (!!!)

What is the effective interest rate for 12% per annum compounded continuously?

r = 12% m = infinitely large

From Appendix 2A, we'll find that

$i_{eff} = e^{0.12} - 1 = 12.7496851579\%$

Example 2.38

Wenfeng Li borrowed $1,000 and paid off the loan in 4.5 years with a single lump sum payment of $1,500. Based on a 6-month interest period, what was the annual effective interest rate paid?

Example 2.38

Wenfeng Li borrowed \$1,000 and paid off the loan in 4.5 years with a single lump sum payment of \$1,500. Based on a 6-month interest period, what was the annual effective interest rate paid?

n = 9 6-mo periods, P = \$1,000, F = \$1,500, i = ?

\$1,500 = \$1,000(F|P i%,9)

$F = P(1 + i)^n$

$1500 = 1000(1 + i)^9$ **or** $1.5 = (1 + i)^9$

i = 0.046 or 4.6%

$i_{eff} = (1 + 0.046)^2 - 1 = 0.0943$ **or 9.43%**

Example 2.39

Greg Wilhelm borrowed $100,000 to purchase a house. It will be repaid with 360 equal monthly payments at a nominal annual rate of 6% (6%/year/month). Determine the monthly payments.

A = $100,000(*A*|*P* 0.5%,360)

= $100,000(0.0059955)

= $599.55/month

Example 2.39

In addition to the usual interest charges, Mr. Wilhelm had to pay $2,000 in closing costs to the lending firm. If the closing costs are financed, what would be the size of the monthly payments?

A = $102,000($A|P$ 0.5%,360)

= $102,000(0.0059955)

= $611.54/month

Example 2.39

When closing costs are financed, what is the overall annual effective interest rate on the loan? With 360 payments of $611.54/mo to borrow $100,000, then the annual effective interest rate is

i_{eff} **=EFFECT(12*RATE(360,611.54,-100000),12)**

i_{eff} **= 6.364%**

When Compounding and Cash Flow Frequencies Differ

When compounding frequency and cash flow frequency differ

The value of *i* is obtained as follows,

$$i = (1 + r/m)^{m/k} - 1 \qquad (2.49)$$

- **Let *r* denote the nominal annual interest rate for money and *m* denote the number of compounding periods in a year; and let *k* denote number of cash flows in a year and *i* denote the interest rate per cash flow period.**

compounding frequency of money equal to the effective annual interest rate for the cash flow frequency.

$$(1 + i)^k - 1 = (1 + r/m)^m - 1$$

Example 2.40

What size monthly payments should occur when \$10,000 is borrowed at 8% compounded quarterly (8%/year/quarter) and the loan is repaid with 36 equal monthly payments?

From Equation 2.49, $r = 0.08$, $m = 4$, and $k = 12$. Therefore,

$$i = (1 + 0.08/4)^{4/12} - 1 = 0.006623 \text{ or } 0.6623\%/\text{month}$$

Knowing the monthly interest rate, the monthly payment can be determined,

$$A = \$10{,}000(A|P\ 0.6623\%,36)$$

$$= \$10{,}000[(0.006623)(1.006623)^{36}]/[(1.006623)^{36} - 1]$$

$$= \$313.12$$

Summary of discrete compounding interest factors.

To Find	Given	Factor	Symbol	Name
P	F	$(1+i)^{-n}$	$(P \mid F\ i\%,n)$	Single sum, present worth factor
F	P	$(1+i)^{n}$	$(F \mid P\ i\%,n)$	Single sum, compound amount factor
P	A	$\dfrac{(1+i)^n - 1}{i(1+i)^n}$	$(P \mid A\ i\%,n)$	Uniform series, present worth factor
A	P	$\dfrac{i(1+i)^n}{(1+i)^n - 1}$	$(A \mid P\ i\%,n)$	Uniform series, capital recovery factor
F	A	$\dfrac{(1+i)^n - 1}{i}$	$(F \mid A\ i\%,n)$	Uniform series, compound amount factor
A	F	$\dfrac{i}{(1+i)^n - 1}$	$(A \mid F\ i\%,n)$	Uniform series, sinking fund factor
P	G	$\dfrac{[1-(1+ni)(1+i)^{-n}]}{i^2}$	$(P \mid G\ i\%,n)$	Gradient series, present worth factor
A	G	$\dfrac{(1+i)^n - (1+ni)}{i[(1+i)^n - 1]}$	$(A \mid G\ i\%,n)$	Gradient series, uniform series factor
P	A_1,j	$\dfrac{1-(1+j)^n(1+i)^{-n}}{i-j}$ for $i \neq j$	$(P \mid A_1\ i\%,j\%,n)$	Geometric series, present worth factor
F	A_1,j	$\dfrac{(1+i)^n - (1+j)^n}{i-j}$ for $i \neq j$	$(F \mid A_1\ i\%,j\%,n)$	Geometric series, future worth factor

Pit Stop #2 — Hang On!

1. True of False: If money is worth 5% compounded annually to you, then you should prefer to receive $2,750 today than to receive $3,500 five years from today.

2. True or False: If money is worth 7% compounded annually to you, then you should prefer to receive $1,000,000 thirty years from now than to receive $200,000 today.

3. True of False: If money is worth 6% compounded annually to you, then you should prefer to receive $1,750 per year for 5 years than to receive $1,000 per year for 10 years, assuming the first receipt occurs one year from today in both cases.

4. True or False: If money is worth 7% compounded annually to you, then you would prefer to receive $2,200 each year for 5 years than to receive $4,000 the first year, $3,000 the second year, $2,000 the third year, $1,000 the fourth year, and $0 the fifth year.

5. True of False: If money has a time value of 8% compounded annually, you should prefer to receive a uniform series of ten $1,000 cash flows over the interval [1,10] to receiving a uniform series of ten $1,260 cash flows over the interval [4,13].

Pit Stop #2 — Hang On!

1. **True of False: If money is worth 5% compounded annually to you, then you should prefer to receive $2,750 today than to receive $3,500 five years from today.** **True PV(5%,5,,-3500) = $2,742.34 < $2,750.00**

2. **True or False: If money is worth 7% compounded annually to you, then you should prefer to receive $1,000,000 thirty years from now than to receive $200,000 today.** **False $1,000,000.00 < FV(7%,30,,-200000) = $1,522,451.01**

3. **True of False: If money is worth 6% compounded annually to you, then you should prefer to receive $1750 per year for 5 years than to receive $1,000 per year for 10 years, assuming the first receipt occurs one year from today in both cases.** **True PV(6%,5,-1750) = $7,371.64 > PV(6%,10,-1000) = $7,360.09**

4. **True or False: If money is worth 7% compounded annually to you, then you would prefer to receive $2,200 each year for 5 years than to receive $4,000 the first year, $3,000 the second year, $2,000 the third year, $1,000 the fourth year, and $0 the fifth year.** **True PV(7%,5,-2200) = $9,020.43 > NPV(7%,4000,3000,2000,1000) = $8,754.12**

5. **True of False: If money has a time value of 8% compounded annually, you should prefer to receive a uniform series of ten $1,000 cash flows over the interval [1,10] to receiving a uniform series of ten $1,260 cash flows over the interval [4,13].** **False PV(8%,10,-1000) = $6,710.08 < PV(8%,3,,PV(8%,10,1260)) = $6,711.62**

Continuous Compounding

$$\lim_{m\to\infty}(1+r/m)^{mn} = e^{rn}$$

Hence,

$F = Pe^{rn}$

$F = P(F|P\ r\%,\ n)_{\infty}$

$P = Fe^{-rn}$ and

$P = F(P|F\ r\%,\ n)_{\infty}$

Continuous Compounding

TABLE 2.A.1 Summary of Continuous Compounding Interest Factors for Discrete Flows.			
To Find	Given	Factor	Symbol
P	F	e^{-rn}	$(P\|F\ r,n)_\infty$
F	P	e^{rn}	$(F\|P\ r,n)_\infty$
F	A	$\dfrac{e^{rn}-1}{e^{r}-1}$	$(F\|A\ r,n)_\infty$
A	F	$\dfrac{e^{r}-1}{e^{rn}-1}$	$(A\|F\ r,n)_\infty$
P	A	$\dfrac{e^{rn}-1}{e^{rn}(e^{r}-1)}$	$(P\|A\ r,n)_\infty$
A	P	$\dfrac{e^{rn}(e^{r}-1)}{e^{rn}-1}$	$(A\|P\ r,n)_\infty$
P	G	$\dfrac{e^{rn}-1-n(e^{r}-1)}{e^{rn}(e^{r}-1)^2}$	$(P\|G\ r,n)_\infty$
A	G	$\dfrac{1}{e^{r}-1}-\dfrac{n}{e^{rn}-1}$	$(A\|G\ r,n)_\infty$
P	A_1, c	$\dfrac{1-e^{(c-r)n}}{e^{r}-e^{c}}$	$(P\|A_1\ r,c,n)_\infty$*
F	A_1, c	$\dfrac{e^{rn}-e^{cn}}{e^{r}-e^{c}}$	$(F\|A_1\ r,c,n)_\infty$*

Continuous Compounding

$i_{eff} = e^r - 1$ or

$$i_{eff} = (F|P\ r\%,1)^{\infty} - 1$$

Example 2.A.1

If $2,000 is invested in a fund that pays interest at a rate of 12% compounded continuously, after 5 years how much will be in the fund?

$$F = P(F|P\ 12\%,5)_{\infty} = \$2{,}000(1.82212) = \$3{,}644.24$$

Example 2.A.2

If $1,000 is deposited annually in an account that pays interest at a rate of 12% compounded continuously, after the 10th deposit how much will be in the fund?

$$F = \$1{,}000(F|A\ 12\%,10)_\infty$$

$$F = \$1{,}000(18.19744)$$

$$F = \$18{,}197.44$$

Example 2.A.2

$1,000 is deposited annually in an account that pays interest at a rate of 12% compounded continuously. What is the present worth of the 10-year investment?

$$P = \$1{,}000(P|A\ 12\%,10)_{\infty}$$

$$P = \$1{,}000(5.48097)$$

$$P = \$5{,}480.97$$

Example 2.A.3

Annual bonuses are deposited in a savings account that pays 8 percent compounded continuously. The size of the bonus increases at a rate of 10% compounded continuously; the initial bonus was \$500. How much will be in the account immediately after the 10th deposit?

A_1 = \$500, r = 8%, c = 10%, n = 10, F = ?

$F = \$500(F|A_1\ 8\%,10\%,10)_{\infty}$

$F = \$500(22.51619)$

$F = \$11,258.09$

Continuous Compounding, Continuous Flow

(A) equivalent to an annual continuous cash flow ($\bar{A}$), based on an annual nominal interest rate of r, is given by

$A = \bar{A}(e^r - 1)/r$

Summary of continuous compounding interest factors for continuous flows.

Find	Given	Factor	Symbol
P	$\bar{A}$	$(e^{rn}-1)/(re^{rn})$	$(P\|\bar{A}\ r\%, n)$
$\bar{A}$	P	$re^{rn}/(e^{rn}-1)$	$(\bar{A}\|P\ r\%, n)$
F	$\bar{A}$	$(e^{rn}-1)/r$	$(F\|\bar{A}\ r\%, n)$
$\bar{A}$	F	$r/(e^{rn}-1)$	$(\bar{A}\|F\ r\%, n)$

Example 2.A.4

Determine the present worth equivalent of a uniform series of continuous cash flows totaling $10,000/yr for 10 years when the interest rate is 20% compounded continuously.

Example 2.A.4

Determine the present worth equivalent of a uniform series of continuous cash flows totaling $10,000/yr for 10 years when the interest rate is 20% compounded continuously.

$P = \$10{,}000(P|\bar{A}\ 20\%,10)$

$P = \$10{,}000(4.32332)$

$P = \$43{,}233.20$

Example 2.A.4

Determine the future worth equivalent of a uniform series of continuous cash flows totaling $10,000/yr for 10 years when the interest rate is 20% per year compounded continuously.

Example 2.A.4

Determine the future worth equivalent of a uniform series of continuous cash flows totaling $10,000/yr for 10 years when the interest rate is 20% per year compounded continuously.

$F = \$10,000(F|\bar{A}\ 20\%,10)$

$F = \$10,000(31.94528)$

$F = \$319,452.80$

Homework #2

6. Today you borrow \$10,000 to pay for your expected college costs over the next 4 years, including a master's degree. Two years from now, you determine that you need an additional \$4,000, so you borrow this amount. Starting 4 years from the original loan (2 years from the second loan), you begin to repay your combined debt by making annual payments of \$2,880. You will make these payments for 10 years. Draw a cash flow diagram of this situation from your perspective.

9. Mercruiser purchases a used, recently upgraded computer numerical control (CNC) machine for turning operations. It costs \$50,000, and since the machine will increase productivity, the company expects to increase sales by \$7,000 per year. Maintenance costs are \$1,000 per year starting 1 year after purchase. Every 5 years, the machine will require a software upgrade costing \$5,000. Draw the cash flow diagram for the scenario described if Mercruiser uses a 10-year planning horizon.

Homework #2

14. You want to withdraw a single sum of $8,000 from an account at the end of 12 years. This withdrawal will deplete the account.

a. What single sum of money must you deposit today if the account earns 5 percent simple interest? (2.3)

b. What single sum of money must you deposit today if the account earns 5 percent compound interest? (2.4)

15. You have $10,000 to invest for 5 years. Your local bank has the following accounts available:

Account 1 for $500 or over: 5.5 percent per year simple interest

Account 2 for $2,000 or over: 5 percent per year compound interest

Account 3 for $6,000 or over: 6.5 percent per year simple interest

Account 4 for $10,000 or over 6 percent per year compound interest

Construct a table showing the projected amount of money in each account at the end of each of the 5 years. Which do you prefer? (2.3 and 2.4)

Homework #2

56. The cash flow profile for an investment is given below, and the interest rate is 6.5 percent compounded annually.

EOY	0	1	2	3	4	5	6	7
Cash Flow	$0	–$500	$200	$400	–$300	$500	–$200	$100

a. Find the future worth of this cash flow series using the actual cash flows.
b. Find the present worth of this series using the actual cash flows.
c. Find the present worth using the future worth.

59. Ken loans his grandson Rex $20,000 at 5.5 percent per year to help pay for executive chef schooling in Florida. Rex requires 3 years of schooling before beginning to earn a salary. He agrees to pay Ken back the loan following the schedule below:

End of Year	0	1	2	3	4	5	6	7	8
Cash Flow	$20,000	$0	$0	$0	–$X	–$2X	–$3X	–$4X	–$5X

a. Draw the cash flow diagram from Ken's perspective.
b. Find the value of *X* such that the loan is fully repaid with the last payment.
c. What is the dollar amount of each of the 5 payments?