Chapter 7

Annual Worth Analysis

Systematic Economic Analysis Technique

1. Identify the investment alternatives
2. Define the planning horizon
3. Specify the discount rate
4. Estimate the cash flows
5. Compare the alternatives
6. Perform supplementary analyses
7. Select the preferred investment

Annual Worth Analysis

Single Alternative

Annual Worth Method

- **converts all cash flows to a uniform annual series over the planning horizon using i=MARR**
- **a popular DCF method**

$$AW(i\%) = \left[\sum_{t=0}^{n} A_t (1+i)^{-t}\right]\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$AW(i\%) = PW(i\%)\,(A \mid P\ i\%, n)$$

Example 7.1

A \$500,000 investment in a surface mount placement machine is being considered. Over a 10-year planning horizon, it is estimated the SMP machine will produce net annual savings of \$92,500. At the end of 10 years, it is estimated the SMP machine will have a \$50,000 salvage value. Based on a 10% MARR and annual worth analysis, should the investment be made?

AW(10%) = -\$500K(A|P 10%,10) + \$92.5K

+ \$50K(A|F 10%,10)

= \$14,262.50

=PMT(10%,10,500000,-50000)+92500

= \$14,264.57

Example 7.2

How does annual worth change over the life of the investment? How does annual worth change when the salvage value decreases geometrically and as a gradient series?

Chapter 7 tables and figures (11-14-08) [Compatibility Mode] - Microsoft Excel

Home | Insert | Page Layout | Formulas | Data | Review | View | Acrobat

D12 f_x =PMT(B1,A12,-B3,-C12)+B12

	A	B	C	D	E	F	G
1	**MARR =**	**10%**					
2	**Planning Horizon**	**CF**	**SV**$_{\text{geometric}}$	***AW***$_{\text{geometric}}$	**SV**$_{\text{gradient}}$	***AW***$_{\textit{gradient}}$	**Excel Formula for *AW***$_{\text{gradient}}$
3	**0**	**-$500,000**	**$500,000**	**n/a**	**$500,000**	**n/a**	
4	**1**	**$92,500**	**$397,000**	**-$60,500.00**	**$455,000**	**-$2,500.00**	**=PMT(B1,A4,-B3,-E4)+B4**
5	**2**	**$92,500**	**$315,218**	**-$45,491.43**	**$410,000**	**-$357.14**	**=PMT(B1,A5,-B3,-E5)+B5**
6	**3**	**$92,500**	**$250,283**	**-$32,943.17**	**$365,000**	**$1,714.50**	**=PMT(B1,A6,-B3,-E6)+B6**
7	**4**	**$92,500**	**$198,725**	**-$22,416.01**	**$320,000**	**$3,715.26**	**=PMT(B1,A7,-B3,-E7)+B7**
8	**5**	**$92,500**	**$157,787**	**-$13,553.55**	**$275,000**	**$5,645.57**	**=PMT(B1,A8,-B3,-E8)+B8**
9	**6**	**$92,500**	**$125,283**	**-$6,066.06**	**$230,000**	**$7,506.01**	**=PMT(B1,A9,-B3,-E9)+B9**
10	**7**	**$92,500**	**$99,475**	**$282.45**	**$185,000**	**$9,297.27**	**=PMT(B1,A10,-B3,-E10)+B10**
11	**8**	**$92,500**	**$78,983**	**$5,684.59**	**$140,000**	**$11,020.15**	**=PMT(B1,A11,-B3,-E11)+B11**
12	**9**	**$92,500**	**$62,713**	**$10,297.92**	**$95,000**	**$12,675.58**	**=PMT(B1,A12,-B3,-E12)+B12**
13	**10**	**$137,500**	**$49,794**	**$14,251.63**	**$50,000**		
14	***AW* =**	**$14,264.57**	**=PMT(B1,10,500000,-50000)+92500**				

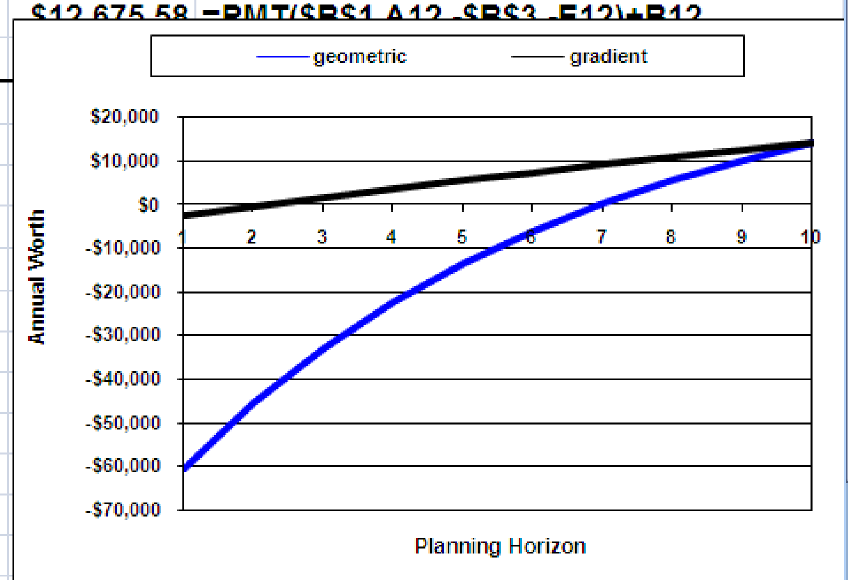


Figure 7.1 | Figure 7.2 | Figure 7.3 | Figure 7.4 | Figure 7.5 | Figure 7.6 | Figure 7.7

Chapter 7 tables and figures (11-14-08) [Compatibility Mode] - Microsoft Excel

Home Insert Page Layout Formulas Data Review View Acrobat

C24 =PMT(C23,10,500000,-50000)+92500

	A	B	C
1	*MARR*	*AW*	
2	0%	$47,500.00	=PMT(A2,10,500000,-50000)+9250
3	1%	$44,488.07	=PMT(A3,10,500000,-50000)+9250
4	2%	$41,403.06	=PMT(A4,10,500000,-50000)+9250
5	3%	$38,246.27	=PMT(A5,10,500000,-50000)+9250
6	4%	$35,019.08	=PMT(A6,10,500000,-50000)+9250
7	5%	$31,722.94	=PMT(A7,10,500000,-50000)+9250
8	6%	$28,359.42	=PMT(A8,10,500000,-50000)+9250
9	7%	$24,930.12	=PMT(A9,10,500000,-50000)+9250
10	8%	$21,436.73	=PMT(A10,10,500000,-50000)+925
11	9%	$17,880.96	=PMT(A11,10,500000,-50000)+925
12	10%	$14,264.57	=PMT(A12,10,500000,-50000)+925
13	11%	$10,589.36	=PMT(A13,10,500000,-50000)+92500
14	12%	$6,857.13	=PMT(A14,10,500000,-50000)+92500
15	13%	$3,069.70	=PMT(A15,10,500000,-50000)+92500
16	14%	-$771.09	=PMT(A16,10,500000,-50000)+92500
17	15%	-$4,663.43	=PMT(A17,10,500000,-50000)+92500
18	16%	-$8,605.49	=PMT(A18,10,500000,-50000)+92500
19	17%	-$12,595.47	=PMT(A19,10,500000,-50000)+92500
20	18%	-$16,631.59	=PMT(A20,10,500000,-50000)+92500
21	19%	-$20,712.09	=PMT(A21,10,500000,-50000)+92500
22	20%	-$24,835.24	=PMT(A22,10,500000,-50000)+92500
23		*MARR* =	13.800%
24		*AW* =	$0.00

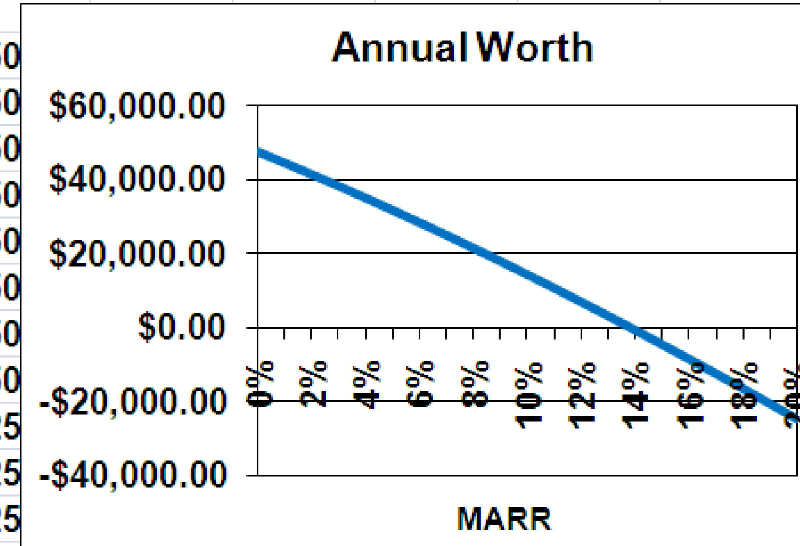


Figure 7.1 Figure 7.2 Figure 7.3 Figure 7.4 Figure 7.5 Figure 7.6 Figure 7.7

Annual Worth Analysis

Multiple Alternatives

Example 7.4

Recall the example involving two alternative designs for a new ride at a theme park: Alt. A costs \$300,000, has net annual after-tax revenue of \$55,000, and has a negligible salvage value at the end of the 10-year planning horizon; Alt. B costs \$450,000, has revenue of \$80,000/yr., and has a negligible salvage value. Based on an AW analysis and a 10% MARR, which is preferred?

$AW_A(10\%) = -\$300{,}000(A|P\ 10\%,10) + \$55{,}000$

$= -\$300{,}000(0.16275) + \$55{,}000 = \$6175.00$

=PMT(10%,10,300000)+55000 = \$6176.38

$AW_B(10\%) = -\$450{,}000(A|P\ 10\%,10) + \$80{,}000$

$= -\$450{,}000(0.16275) + \$80{,}000 = \$6762.50$

=PMT(10%,10,450000)+80000 = \$6764.57

Analyze the impact on AW based on salvage values decreasing geometrically to 1¢ after 10 years.

Chapter 7 tables and figures (11-14-08) [Compatibility Mode] - Microsoft Excel

Home Insert Page Layout Formulas Data Review View Acrobat

B16 =PMT(10%,B15,300000,-300000*(1+D1)^B15)+55000

Planning Horizon	Geometric Series Rate (A) = -85.800%			Geometric Series Rate (B) = -86.364%		
	CF(A)	SV(A)	AW(A)	CF(B)	SV(B)	AW(B)
0	-$300,000.00	$300,000.00	n/a	-$450,000.00	$450,000.00	n/a
1	$55,000.00	$42,599.95	-$232,400.05	$80,000.00	$61,360.84	-$353,639.16
2	$55,000.00	$6,049.19	-$114,976.58	$80,000.00	$8,367.00	-$175,301.43
3	$55,000.00	$858.98	-$65,374.93	$80,000.00	$1,140.90	-$100,606.98
4	$55,000.00	$121.98	-$39,614.96	$80,000.00	$155.57	-$61,928.34
5	$55,000.00	$17.32	-$24,136.41	$80,000.00	$21.21	-$38,705.39
6	$55,000.00	$2.46	-$13,881.90	$80,000.00	$2.89	-$23,322.95
7	$55,000.00	$0.35	-$6,621.61	$80,000.00	$0.39	-$12,432.43
8	$55,000.00	$0.05	-$1,233.20	$80,000.00	$0.05	-$4,349.80
9	$55,000.00	$0.01	$2,907.84	$80,000.00	$0.01	$1,861.76
10	$55,000.00	$0.00	$6,176.38	$80,000.00	$0.00	$6,764.57

$DPBP_A$ = 10.000 $DPBP_B$ = 10.000

AW_A = $6,176.38 AW_B = $6,764.57

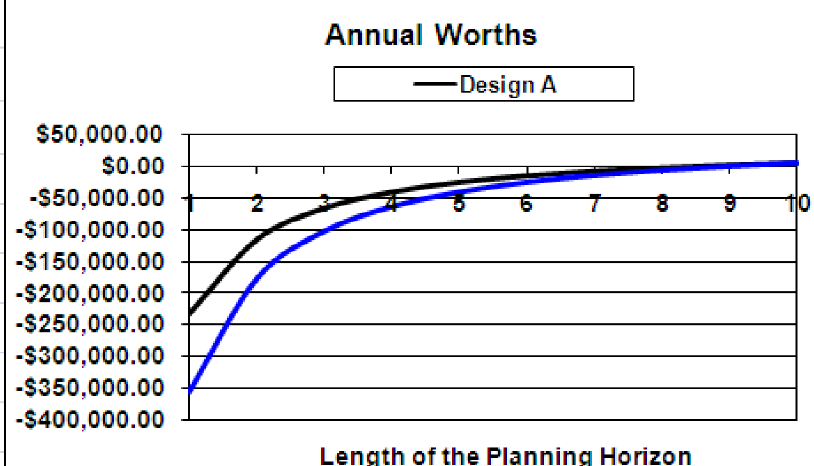
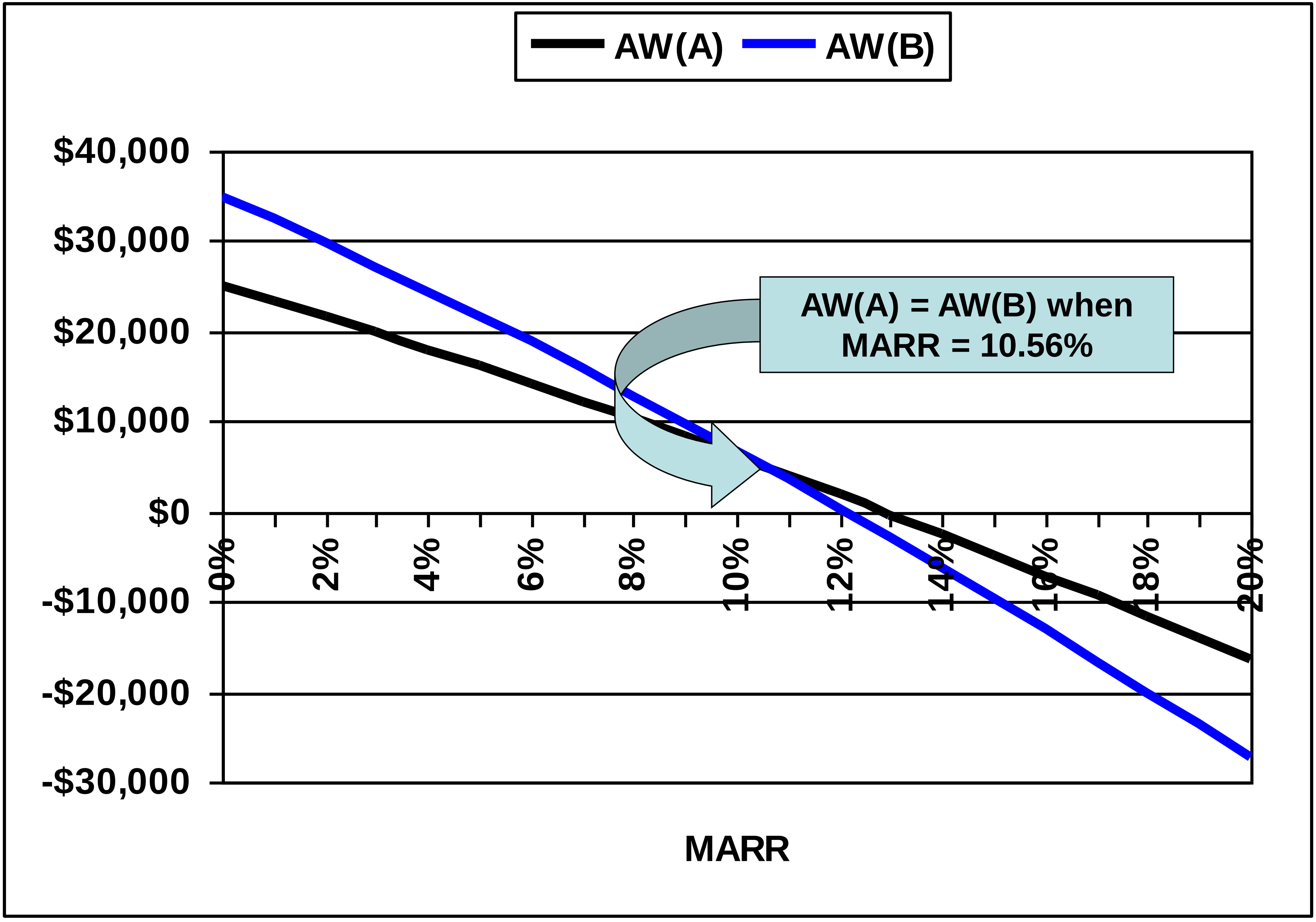


Figure 7.1 Figure 7.2 Figure 7.3 Figure 7.4 Figure 7.5 Figure 7.6 Figure 7.7



AW(A)
AW(B)
$40,000
$30,000
$20,000
$10,000
$0
-$10,000
-$20,000
-$30,000
0%
2%
4%
6%
8%
10%
12%
14%
16%
18%
20%
AW(A) = AW(B) when
MARR = 10.56%
MARR

Example 7.5

For The Scream Machine alternatives (A costing \$300,000, saving \$55,000, and having a negligible salvage value at the end of the 10-year planning horizon; B costing \$450,000, saving \$80,000, and having a negligible salvage value), using an incremental AW analysis and a 10% MARR, which is preferred?

$AW_A(10\%)$ = -\$300,000(*A*|*P* 10%,10) + \$55,000

= -\$300,000(0.16275) + \$55,000 = \$6175.00

=PMT(10%,10,300000)+55000 = \$6176.38 > \$0

(A is better than "do nothing")

$AW_{B-A}(10\%)$ = -\$150,000(*A*|*P* 10%,10) + \$25,000

= -\$150,000(0.16275) + \$25,000 = \$587.50

=PMT(10%,10,150000)+25000 = \$588.19 > \$0

(B is better than A)

Prefer B

Example 7.6

If an investor's MARR is 12%, which mutually exclusive investment alternative maximizes the investor's future worth, given the parameters shown below?

EOY	CF(1)	CF(2)	CF(3)
0	-$10,000	-$14,500	-$20,000
1	$5,000	$5,000	$0
2	$5,000	$5,000	$3,000
3	$10,000	$5,000	$6,000
4		$5,000	$9,000
5		$5,000	$12,000
6		$7,500	$15,000

Consider 3 scenarios: individual life cycles; least common multiple of lives; and "one-shot" investments

Example 7.6 (Continued)

Scenario 1: individual life cycles

$AW_1(12\%) = -\$10{,}000(A|P\ 12\%,3) + \$5000 + \$5000(A|F\ 12\%,3)$

$= \$2318.25$

=PMT(12%,3,10000,-5000)+5000 = $2318.26

$AW_2(12\%) = -\$14{,}500(A|P\ 12\%,6) + \$5000 + \$2500(A|F\ 5\%,6)$

$= \$1473.17$

=PMT(12%,6,14500,-2500)+5000 = $1473.23

$AW_3(12\%) = -\$20{,}000(A|P\ 12\%,6) + \$3000(A|G\ 12\%,6)$

$= \$1651.55$

=PMT(12%,6,-1000*NPV(12%,0,3,6,9,12,15)+20000)

= $1651.63

Example 7.6 (Continued)

Scenario 2: least common multiple of lives

EOY	CF(1')	CF(2)	CF(3)
0	-$10,000	-$14,500	-$20,000
1	$5,000	$5,000	$0
2	$5,000	$5,000	$3,000
3	$0	$5,000	$6,000
4	$5,000	$5,000	$9,000
5	$5,000	$5,000	$12,000
6	$10,000	$7,500	$15,000

$AW_1(12\%)$ = -$10,000(A|P 12%,6) + $5000 + $5000(A|F 12%,6)
- $5000(A|P 12%,3)(A|P 12%,6)
= $2318.22
=PMT(12%,6,10000,-5000)+5000
+PMT(12%,6,PV(12%,3,,-5000))= $2318.26

$AW_2(12\%)$ = $1473.17 = $1473.23
$AW_3(12\%)$ = $1651.55 = $1651.63

Identical results!

Example 7.6 (Continued)

Scenario 3: "one-shot" investments

EOY	CF(1)	CF(2)	CF(3)
0	-$10,000	-$14,500	-$20,000
1	$5,000	$5,000	$0
2	$5,000	$5,000	$3,000
3	$10,000	$5,000	$6,000
4	$0	$5,000	$9,000
5	$0	$5,000	$12,000
6	$0	$5,000	$15,000

$AW_1(12\%) = \{-\$10{,}000 + [\$5000(P|A\ 12\%,3)$
$+\ \$5000(P|F\ 12\%,3)]\}(A|P\ 12\%,6) = \$1354.32$

=PMT(12%,6,10000-PV(12%,3,-5000,-5000))

= $1354.29

$AW_2(12\%) = \$1473.17$ = $1473.23

$AW_3(12\%) = \$1651.55$ = $1651.63

Considering scenarios 1 and 2, is it reasonable to assume an investment alternative equivalent to Alt. 1 will be available in 3 years? If so, why was the MARR set equal to 12%?

Example 7.7

Three industrial mowers (Small, Medium, and Large) are being evaluated by a company that provides lawn care service. Determine the economic choice, based on the following cost and performance parameters:

	Small	Medium	Large
First Cost:	**$1,500**	**$2,000**	**$5,000**
Operating Cost/Hr	**$35**	**$50**	**$76**
Revenue/Hr	**$55**	**$75**	**$100**
Hrs/Yr	**1,000**	**1,100**	**1,200**
Useful Life (Yrs)	**2**	**3**	**5**

Use AW analysis to determine the preferred mower, based on a MARR of 12%.

Example 7.7 (Continued)

AW_{small} = -\$1500(*A*|*P* 12%,2) + \$20(1000) = \$19,112.45
=PMT(12%,2,1500)+20*1000 = \$19,112.45

AW_{med} = -\$2000(*A*|*P* 12%,3) + \$25(1100) = \$26,667.30
=PMT(12%,3,2000)+25*1100 = \$26,667.30

AW_{large} = -\$5000(*A*|*P* 12%,5) + \$24(1200) = \$27,412.95
=PMT(12%,5,5000)+24*1200 = \$27,412.95

What did we assume when solving the example?

Example 7.8

If a 5-year planning horizon were used, what salvage values are required to have the same AW as before? The small mower will be replaced at the end of year 4; the medium mower will be replaced at the end of year 3. One year of service of the small mower will have the following cash flows:

SV_{small} = \$19,112.45(*F*|*A* 12%,1) - \$20,000(*F*|*A* 12%,1)
+ \$1500(*F*|*P* 12%,1) = \$792.45
=FV(12%,1,-19112.45)-FV(12%,1,-20000,1500)
= \$792.45

SV_{med} = \$26,667.30(*F*|*A* 12%,2) - \$27,500(*F*|*A* 12%,2)
+ \$2000(F|P 12%,2) = \$743.48
=FV(12%,2,-26667.3)-FV(12%,2,-27500,2000)
= \$743.48

SV_{large} = \$0

Example 7.9

Suppose a 6-yr planning horizon is used and the large mower is continued in service for an additional year at a penalty of a 15% increase in operating cost the 6th year. Which mower is preferred?

$$\left.\begin{aligned} AW_{small} &= \$19{,}112.45 \\ AW_{med} &= \$26{,}667.30 \end{aligned}\right\} \quad \textbf{From Example 7.7}$$

AW_{large} = -\$5000($A|P$ 12%,6) + \$24(1200)

- 0.15(\$76)(1200)($A|F$ 12%,6) = \$25,898.06

=PMT(12%,6,5000,.15*76*1200)+24*1200

= \$25,898.14

Example 7.10

Suppose a 6-yr planning horizon is used and a large mower is leased for the 6th year at a \$1500 beginning of year cost and an end of year operating cost of \$77. Which mower is preferred?

AW_{small} = \$19,704.15
AW_{med} = \$26,667.30

From Example 7.7

AW_{large} = -\$5000(*A*|*P* 12%,6) + \$24(1200)
- [\$1(1200) + \$1500(*F*|*P* 12%,1)](*A*|*F* 12%,6)
= \$27,228.95

=PMT(12%,6,5000,1200-FV(12%,1,,1500))
+24*1200
= \$27,228.98

Principle #8

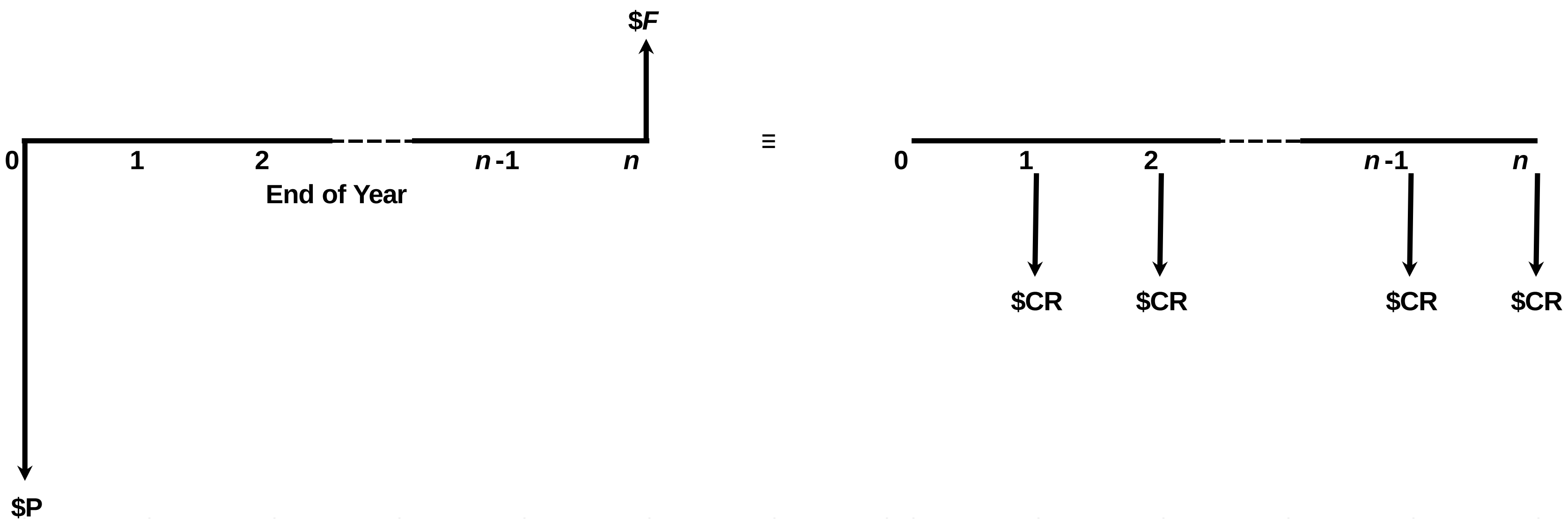
Compare investment alternatives over a common period of time

Fundamentals of Engineering Examination

Even though you might not encounter a situation in your professional practice that requires the least common multiple of lives assumption to be used, it is very likely you will have problems of this type on the FE Exam. Therefore, you need to be familiar with how to solve such problems. Specifically, on the FE Exam, *unless instructions are given to do otherwise*, **calculate the annual worth for a life cycle of each alternative and recommend the one that has the greatest annual worth.**

Capital Recovery Cost

CFD for Capital Recovery Cost (CR).



CR = P(A|P i,n) – F(A|F i, n)

Remember: **(A|F i, n)= (A|P i,n) - i**

Capital Recovery Cost Formulas

CR = P(A|P i,n) – F(A|F i, n)

CR = (P-F)(A|F i, n) + Pi

CR = (P-F)(A|P i,n) + Fi

CR =PMT(*i,n,-P,F*)

Example

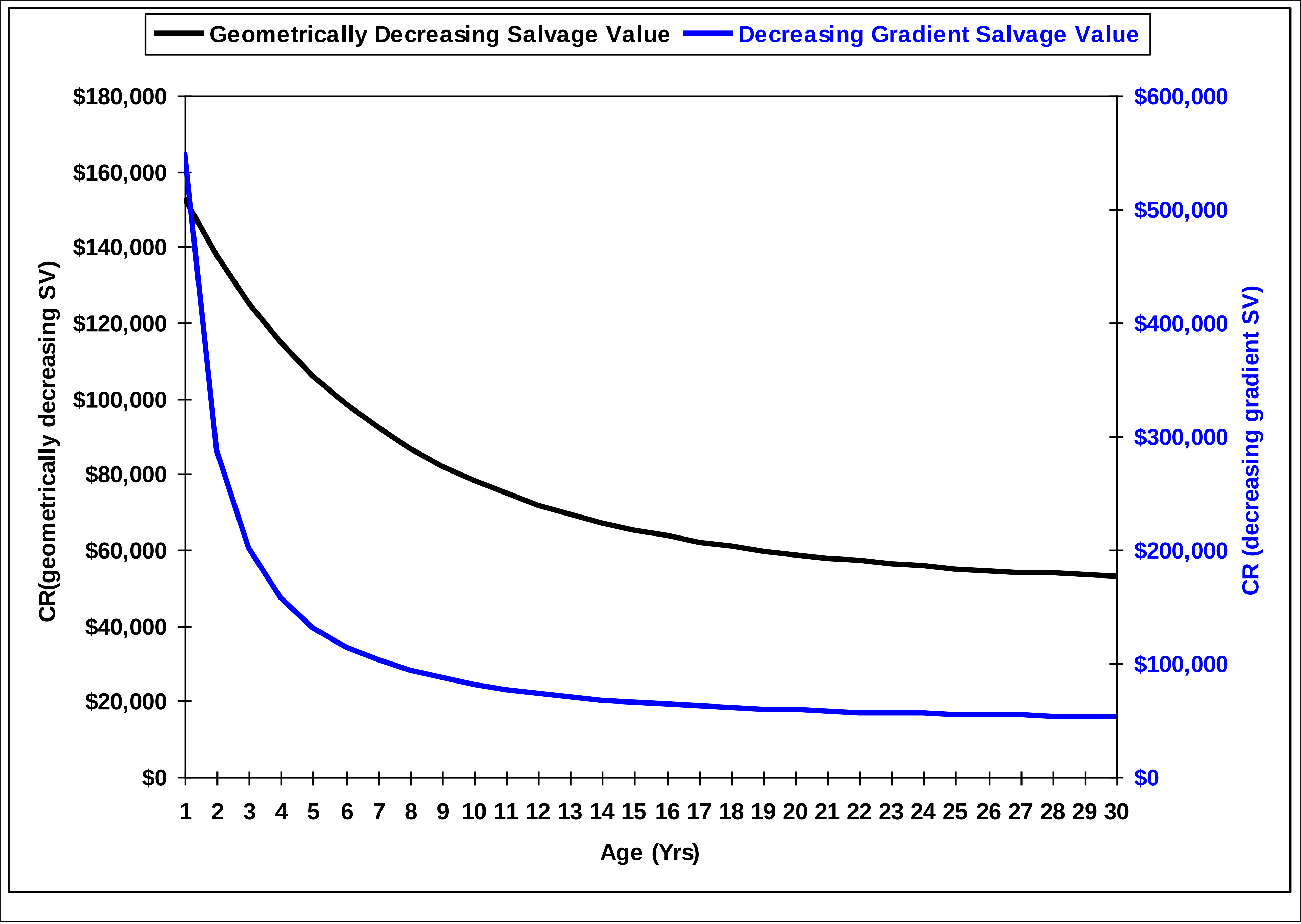
P = \$500,000   F = \$50,000   i = 10%   n = 10 yrs

CR = \$500,000(0.16275) - \$50,000(0.06275) = \$78,237.50

CR = \$450,000(0.06275) + \$500,000(0.10) = \$78,237.50

CR = \$450,000(0.16275) + \$50,000(0.10) = \$78,237.50

CR =PMT(10%,10,-500000,50000) = \$78,235.43



Geometrically Decreasing Salvage Value
Decreasing Gradient Salvage Value
$180,000
$160,000
$140,000
$120,000
$100,000
$80,000
$60,000
$40,000
$20,000
$0
$600,000
$500,000
$400,000
$300,000
$200,000
$100,000
$0
CR(geometrically decreasing SV)
CR (decreasing gradient SV)
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
Age (Yrs)

Pit Stop #7—No Time to Coast

1. True or False: Annual worth analysis is the most popular *DCF* measure of economic worth.
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest annual worth over the planning horizon.
3. True or False: The capital recovery cost is the uniform annual cost of the investment less the uniform annual worth of the salvage value.
4. True or False: If *AW* > 0, then *PW*>0, and *FW*>0.
5. True or False: If *AW*(A) > *AW*(B), then *PW*(A) > *PW*(B).
6. True or False: If *AW* (A) < *AW*(B), then AW(B-A) > 0.
7. True or False: If *AW*(A) > *AW*(B), then *CW*(A) > *CW*(B) and *DPBP*(A) < *DPBP*(B).
8. True or False: *AW* can be applied as either a ranking method or as an incremental method.
9. True or False: To compute capital recovery cost using Excel, enter =PMT(i%,n,-P,F) in any cell in a spreadsheet.
10. True or False: When using annual worth analysis with mutually exclusive alternatives having unequal lives, always use a planning horizon equal to the least common multiple of lives.

Pit Stop #7—No Time to Coast

1. True or False: Annual worth analysis is the most popular *DCF* measure of economic worth. FALSE
2. True or False: Unless non-monetary considerations dictate otherwise, choose the mutually exclusive investment alternative that has the greatest annual worth over the planning horizon. TRUE
3. True or False: The capital recovery cost is the uniform annual cost of the investment less the uniform annual worth of the salvage value. TRUE
4. True or False: If $AW > 0$, then $PW > 0$, and $FW > 0$. TRUE
5. True or False: If $AW(A) > AW(B)$, then $PW(A) > PW(B)$. TRUE
6. True or False: If $AW(A) < AW(B)$, then $AW(B-A) > 0$. TRUE
7. True or False: If $AW(A) > AW(B)$, then $CW(A) > CW(B)$ and $DPBP(A) < DPBP(B)$. FALSE
8. True or False: *AW* can be applied as either a ranking method or as an incremental method. TRUE
9. True or False: To compute capital recovery cost using Excel, enter =PMT(i%,n,-P,F) in any cell in a spreadsheet. TRUE
10. True or False: When using annual worth analysis with mutually exclusive alternatives having unequal lives, always use a planning horizon equal to the least common multiple of lives. FALSE