Chapter 8

Rate of Return Analysis

Systematic Economic Analysis Technique

1. **Identify the investment alternatives**
2. **Define the planning horizon**
3. **Specify the discount rate**
4. **Estimate the cash flows**
5. **Compare the alternatives**
6. **Perform supplementary analyses**
7. **Select the preferred investment**

Internal Rate of Return Analysis

Single Alternative

Internal Rate of Return

- determines the interest rate (i^*) that yields a future worth equal to zero over the planning horizon[1]
- the rate of interest earned on the unrecovered balance of the investment
- a very popular DCF method

$$0 = \sum_{t=0}^{n} A_t (1 + i^*)^{n-t}$$

[1] **can also determine the interest rate that equates the present worth or annual worth to zero**

Internal Rate of Return

- Determining the value of i^* that satisfies the n-degree polynomial given below can be very challenging, since there can exist n distinct roots for an n-degree polynomial.

- *Descartes rule of signs* indicates an n-degree polynomial will have a single positive real root if there is a single sign change in the sequence of cash flows, A_1, A_2, …, A_{n-1}, A_n; if there are 2 sign changes, there will be either 2 or 0 positive real roots; if there are 3 sign changes, there will be either 3 or 1 positive real roots; if there are 4 sign changes, there will be 4, 2, or 0 positive real roots; …

- To determine if exactly one real and positive-valued root exists, use *Norstrom's criterion – if the cumulative cash flow begins with a negative value and changes only once to a positive-valued series, then there exists a unique positive real root*.

$$0 = \sum_{t=0}^{n} A_t (1+i^*)^{n-t}$$

Example 8.1
SMP Investment
Internal Rate of Return Analysis

EOY	CF
0	-$500,000
1 - 10	$92,500
10	$50,000

$$FW(i^*\%) = -\$500{,}000(F|P\ i^*\%,10) + \$50{,}000 + \$92{,}500(F|A\ i^*\%,10) = \$0$$

Example 8.1
SMP Investment
Internal Rate of Return Analysis

EOY	CF
0	-$500,000
1 - 10	$92,500
10	$50,000

i	FW
12%	$120,333.90
15%	-$94,684.90

$$FW(i^*\%) = -\$500{,}000(F|P\ i^*\%,10) + \$50{,}000 + \$92{,}500(F|A\ i^*\%,10) = \$0$$

$$i^* \approx 13.67893\%$$

The investment is recommended since i* > MARR

Example 8.2

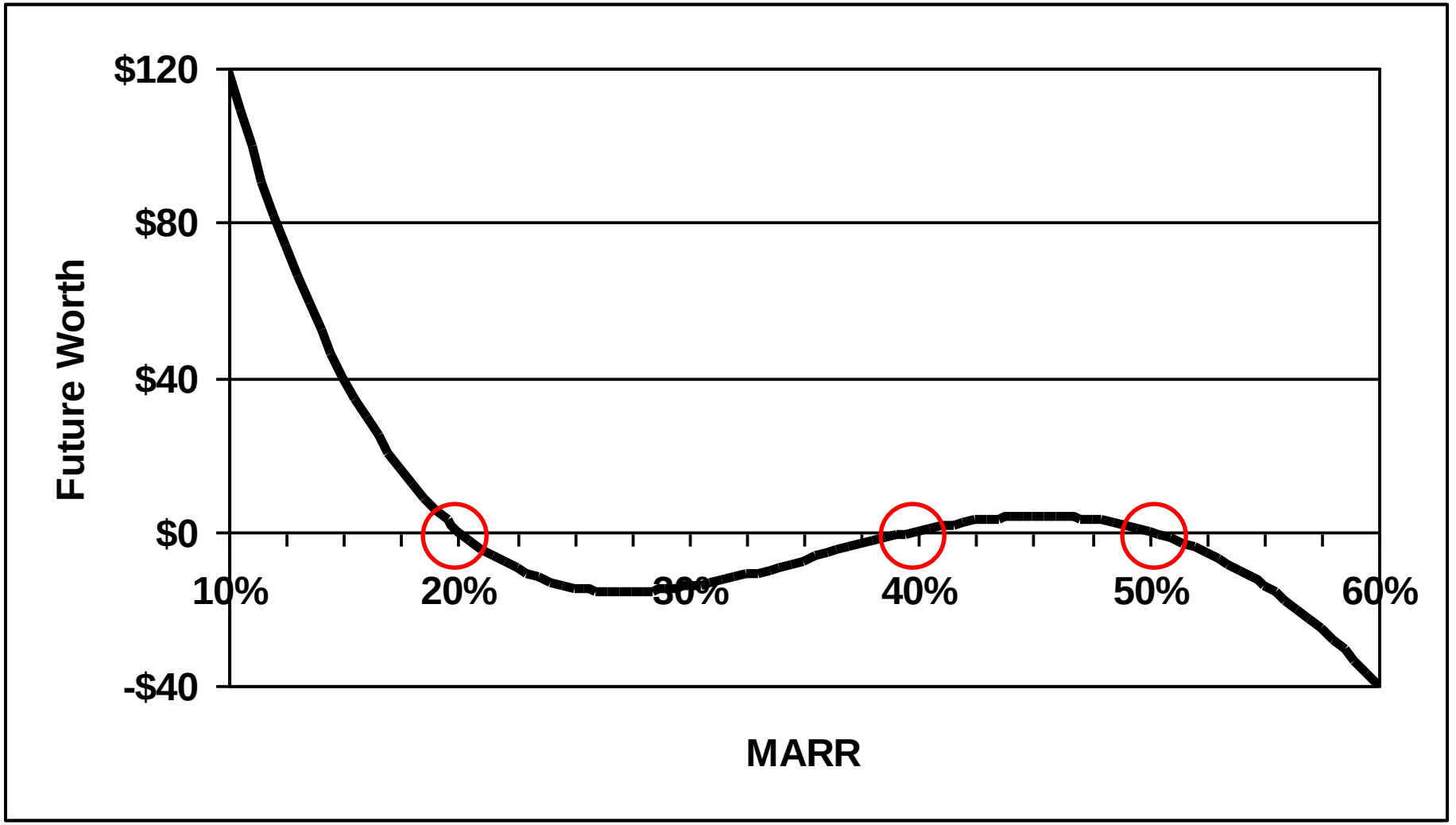
Consider the cash flow profile given below. The FW equals zero using a 20%, 40%, or 50% interest rate.

EOY	CF
0	-$4,000
1	$16,400
2	-$22,320
3	$10,080

$$FW_1(20\%) = -\$4000(1.2)^3 + \$16{,}400(1.2)^2 - \$22{,}320(1.2) + \$10{,}080 = 0$$

$$FW_2(40\%) = -\$4000(1.4)^3 + \$16{,}400(1.4)^2 - \$22{,}320(1.4) + \$10{,}080 = 0$$

$$FW_3(50\%) = -\$4000(1.5)^3 + \$16{,}400(1.5)^2 - \$22{,}320(1.5) + \$10{,}080 = 0$$


$120
$80
$40
$0
-$40
Future Worth
10%
20%
30%
40%
50%
60%
MARR

Example 8.3

Julian Stewart invested $250,000 in a limited partnership to drill for natural gas. The investment yielded annual returns of $45,000 the 1st yr, followed by $10,000 increases until the 6th yr, at which time an additional $150,000 had to be invested for deeper drilling. Following the 2nd drilling, the annual returns decreased by $10,000 per year, from $85,000 to $5,000. Using Excel, the IRR = 19.12%.

Plot future worth as a function of MARR and determine the MARR that maximizes FW.

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E21

	A	B	C	D	E
1	EOY	CF	Cum CF	*i*	*FW*
2	0	-$250,000	-$250,000	-50%	$2.96
3	1	$45,000	-$205,000	-45%	$3.73
4	2	$55,000	-$150,000	-40%	$4.76
5	3	$65,000	-$85,000	-35%	$6.16
6	4	$75,000	-$10,000	-30%	$8.07
7	5	$85,000	$75,000	-25%	$10.71
8	6	-$55,000	$20,000	-20%	$14.33
9	7	$85,000	$105,000	-15%	$19.21
10	8	$75,000	$180,000	-10%	$25.63
11	9	$65,000	$245,000	-5%	$33.59
12	10	$55,000	$300,000	0%	$42.50
13	11	$45,000	$345,000	5%	$50.32
14	12	$35,000	$380,000	10%	$52.15
15	13	$25,000	$405,000	15%	$37.74
16	14	$15,000	$420,000	20%	-$12.83
17	15	$5,000	$425,000		
18	IRR =	19.12%			
19	*i* =	20.0000%	-$12.83	=FV(C19,15,,-NPV(C19,$B$3:$B$17)-$B$2)/10000	

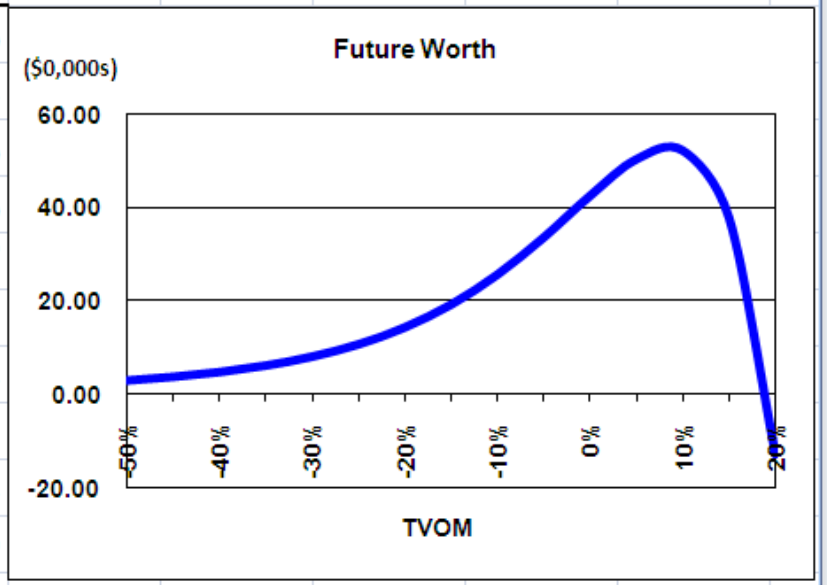

Figure 8.2 | Figure 8.3 | Figure 8.4 | Figure 8.5 | Figure 8.6 | Figure 8.7 | Figure 8.8

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B19 8.54688718606763%

	A	B	C	D	E
1	EOY	CF	Cum CF	*i*	*FW*
2	0	-$250,000	-$250,000	-50%	$2.96
3	1	$45,000	-$205,000	-45%	$3.73
4	2	$55,000	-$150,000	-40%	$4.76
5	3	$65,000	-$85,000	-35%	$6.16
6	4	$75,000	-$10,000	-30%	$8.07
7	5	$85,000	$75,000	-25%	$10.71
8	6	-$55,000	$20,000	-20%	$14.33
9	7	$85,000	$105,000	-15%	$19.21
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12	10	$55,000	$300,000	0%	$42.50
13	11	$45,000	$345,000	5%	$50.32
14	12	$35,000	$380,000	10%	$52.15
15	13	$25,000	$405,000	15%	$37.74
16	14	$15,000	$420,000	20%	-$12.83
17	15	$5,000	$425,000		
18	IRR =	19.12%			
19	*i* =	8.5469%	$52.70	=FV(C19,15,,-NPV(C19,$B$3:$B$17)-$B$2)/10000	

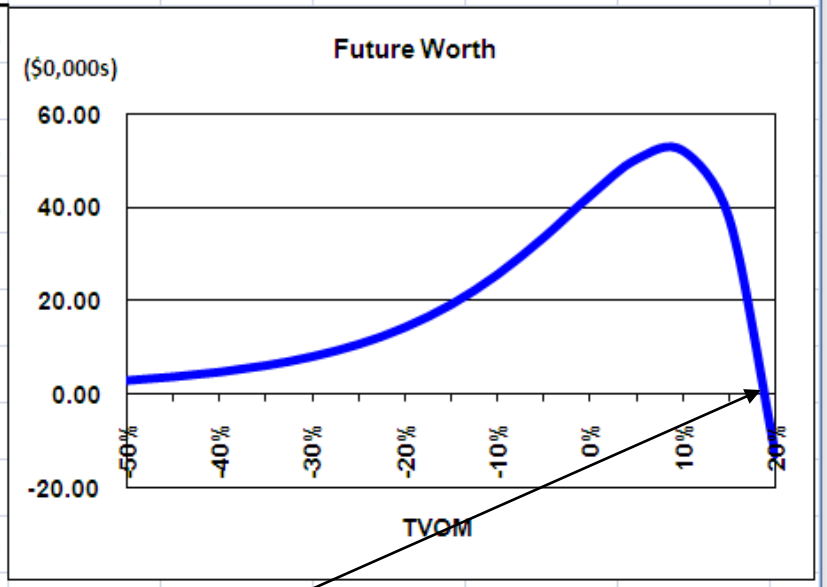

Table 8.9 Table 8.10 Figure 8.1 Figure 8.2 Figure 8.3 Figure 8.4 Figure 8.5

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B19 8.54688718606763%

	A	B	C	D	E
1	EOY	CF	Cum CF	*i*	*FW*
2	0	-$250,000	-$250,000	-50%	$2.96
3	1	$45,000	-$205,000	-45%	$3.73
4	2	$55,000	-$150,000	-40%	$4.76
5	3	$65,000	-$85,000	-35%	$6.16
6	4	$75,000	-$10,000	-30%	$8.07
7	5	$85,000	$75,000	-25%	$10.71
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12	10	$55,000	$300,000	0%	$42.50
13	11	$45,000	$345,000	5%	$50.32
14	12	$35,000	$380,000	10%	$52.15
15	13	$25,000	$405,000	15%	$37.74
16	14	$15,000	$420,000	20%	-$12.83
17	15	$5,000	$425,000		
18	IRR =	19.12%			
19	*i* =	8.5469%	$52.70	=FV(C19,15,,-NPV(C19,$B$3:$B$17)-$B$2)/10000	

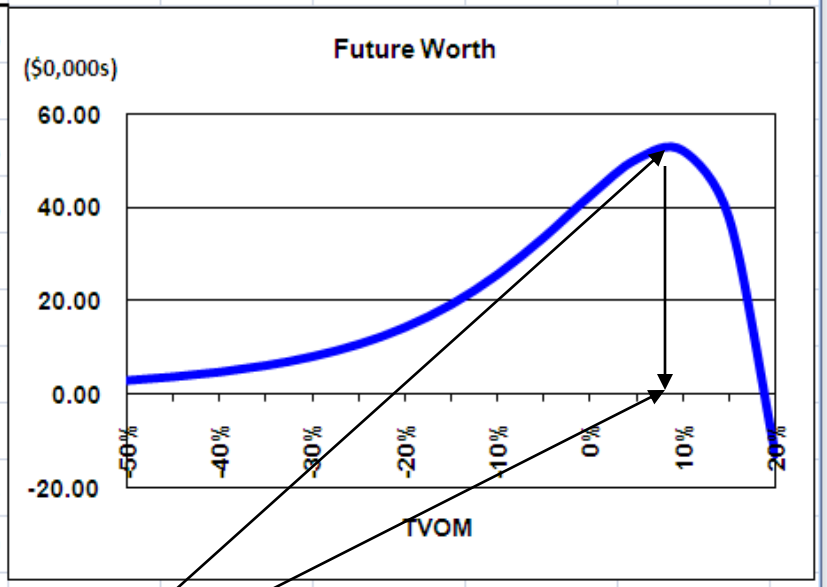

Table 8.9 Table 8.10 Figure 8.1 Figure 8.2 Figure 8.3 Figure 8.4 Figure 8.5

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B19 8.54688718606763%

	A	B	C	D	E
1	EOY	CF	Cum CF	*i*	*FW*
2	0	-$250,000	-$250,000	-50%	$2.96
3	1	$45,000	-$205,000	-45%	$3.73
4	2	$55,000	-$150,000	-40%	$4.76
5	3	$65,000	-$85,000	-35%	$6.16
6	4	$75,000	-$10,000	-30%	$8.07
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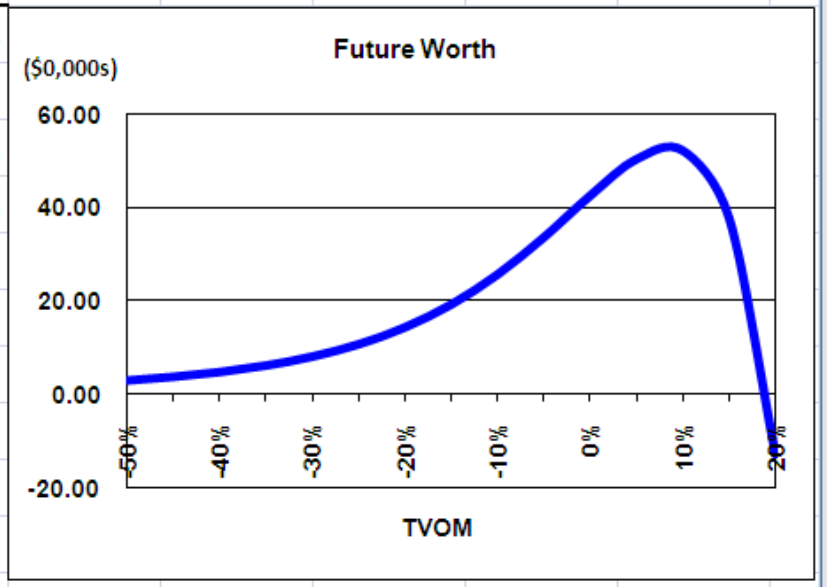
13	11	$45,000	$345,000	5%	$50.32



Notice, the cumulative cash flow series changes from negative to positive and remains positive. Therefore, a unique positive real root exists. (Interestingly, if the interim investment is $200,000, Norstrom's criterion is not met, but a unique positive real root exists. Recall, Norstrom's criterion is a sufficient condition, not a necessary condition.)

19	*i* =	8.5469%	$52.70	=FV(C19,15,,-NPV(C19,$B$3:$B$17)-$B$2)/10000

Table 8.9 | Table 8.10 | Figure 8.1 | Figure 8.2 | Figure 8.3 | Figure 8.4 | Figure 8.5

Internal Rate of Return Analysis

Multiple Alternatives

Example 8.4

You have available $70,000 to invest and have been presented with 5 equal-lived, mutually exclusive investment alternatives with cash flows as depicted below. Currently, you are earning 18% on your investment of the $70,000. Hence, you will not choose to invest in either of the alternatives if it does not provide a return on investment greater than 18%.

Using the internal rate of return method, which (if either) would you choose? What is its rate of return?

Data for Example 8.4

Investment	1	2	3	4	5
Initial Investment	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
Annual Return	$3,750.00	$5,000.00	$9,250.00	$11,250.00	$14,250.00
Salvage Value	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
Internal Rate of Return	25.00%	20.00%	23.13%	22.50%	20.36%

With an 18% MARR, which investment would you choose?

When the salvage value equals the initial investment and annual returns are a uniform annual series, the internal rate of return equals the quotient of the annual return and the initial investment

Solution to Example 8.4

Investment	1	2 - 1	3 - 1	4 - 3	5 - 4
Δ Investment Δ Annual Return Δ Salvage Value					
Δ *IRR*	25.00%	12.50%	22.00%	20.00%	15.00%
≥ *MARR* ? Defender	Yes 1	No 1	Yes 3	Yes 4	No 4

Portfolio Solution to Example 8.4

"Do Nothing"

$70,000(0.18) = $12,600/year

Invest in 1

$3,750 + $55,000(0.18) = $13,650

Prefer 1 to "Do Nothing"

Invest in 2

$5,000 + $45,000(0.18) = $13,100

Prefer 1 to 2

Portfolio Solution to Example 8.4

Invest in 3

$9,250 + $30,000(0.18) = $14,650

Prefer 3 to 1

Invest in 4

$11,250 + $20,000(0.18) = $14,850

Prefer 4 to 3

Invest in 5

$14,250

Prefer 4 to 5

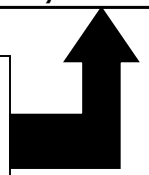
Choose 4

Present Worths with 10-Year Planning Horizon

Investment	1	2	3	4	5
Initial Investment	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
Annual Return	$3,750.00	$5,000.00	$9,250.00	$11,250.00	$14,250.00
Salvage Value	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
Present Worth	$4,718.79	$2,247.04	$9,212.88	$10,111.69	$7,415.24

Present Worths with 10-Year Planning Horizon

Investment	1	2	3	4	5
Initial Investment	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
Annual Return	$3,750.00	$5,000.00	$9,250.00	$11,250.00	$14,250.00
Salvage Value	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
Present Worth	$4,718.79	$2,247.04	$9,212.88	$10,111.69	$7,415.24



$PW = \$11{,}250(P|A\ 18\%,10) + \$50{,}000(P|F\ 18\%,10) - \$50{,}000$

=PV(18%,10,-11250,-50000)-50000

Principle #6

Continue to invest as long as each additional increment of investment yields a return that is greater than the investor's TVOM

"The object of management is not necessarily the highest rate of return on capital, but … to assure profit with each increment of volume that will at least equal the economic cost of additional capital required."

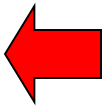
Donald Brown
Chief Financial Officer
General Motors
1924

Example 8.5

Recall the theme park example involving two designs for the new ride, The Scream Machine: A costs $300,000, has $55,000/yr revenue, and has a negligible salvage value at the end of the 10-year planning horizon; B costs $450,000, has $80,000/yr revenue, and has a negligible salvage value. Based on an IRR analysis and a 10% MARR, which is preferred?

$PW_A(12\%) = -\$300{,}000 + \$55{,}000(P|A\ 12\%,10) = \$10{,}762.10$
$PW_A(15\%) = -\$300{,}000 + \$55{,}000(P|A\ 15\%,10) = -\$23{,}967.65$
interpolating,
$IRR_A = 12\% + 3\%(\$10{,}762.10)/(\$10{,}762.10 + \$23{,}967.65) = 12.93\%$
IRR_A =RATE(10,-55000,300000)
= 12.87% > MARR = 10% (Alt. A is acceptable)

$PW_{B-A}(12\%) = -\$150{,}000 + \$25{,}000(P|A\ 12\%,10) = -\$8744.50$
$PW_{B-A}(10\%) = -\$150{,}000 + \$25{,}000(P|A\ 10\%,10) = \$37{,}951.35$
interpolating,
$IRR_{B-A} = 10\% + 2\%(\$8744.50)/(\$8744.50 + \$37{,}951.35) = 10.375\%$
IRR_{B-A} =RATE(10,-25000,150000)
= 10.56% > MARR = 10% (Alt. B is preferred)

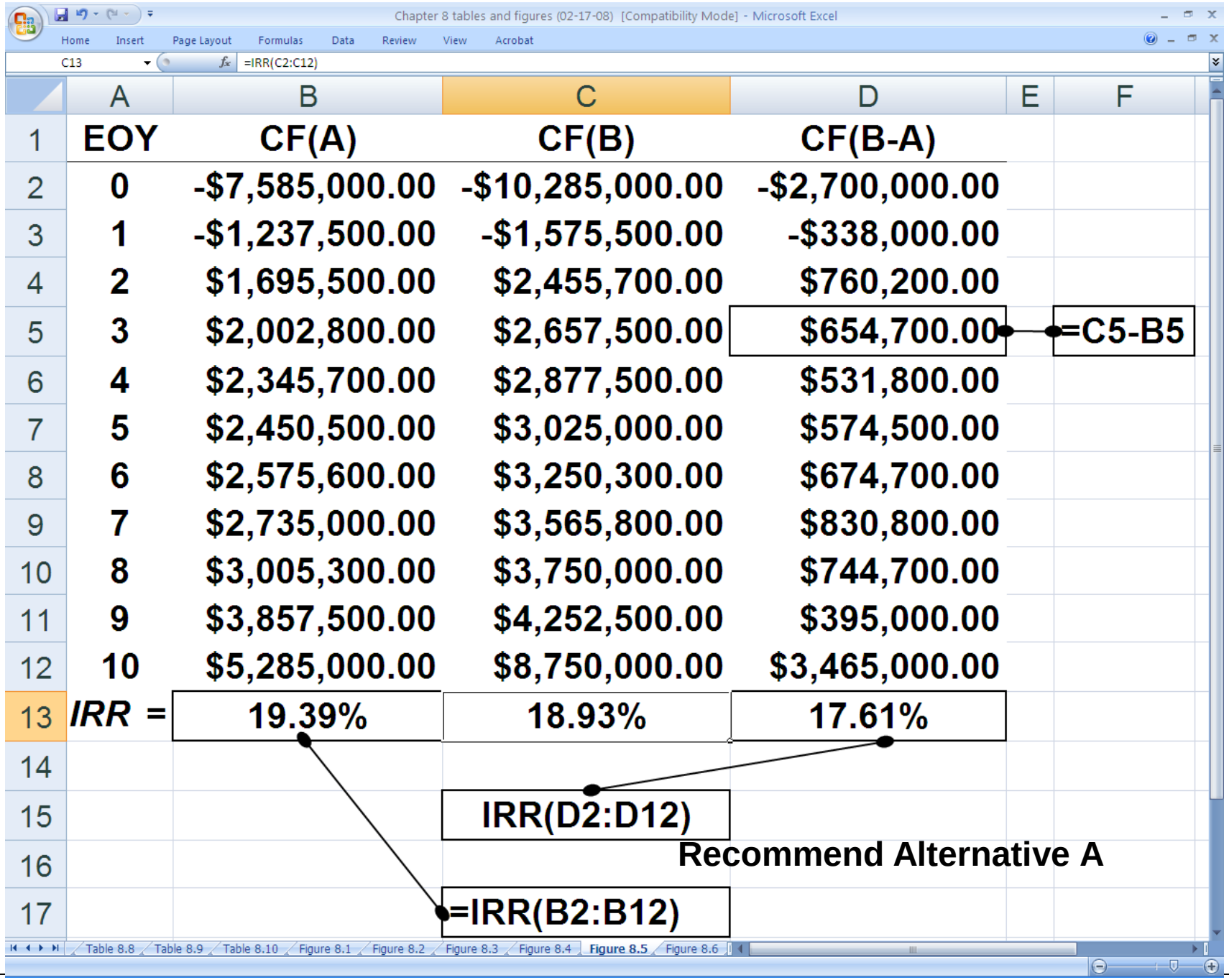


IRR_B =RATE(10,-80000,450000)
= 12.11%

Example 8.6

A batch chemical processing company is adding centrifuges. Two alternatives are in consideration. The estimated cash flow profiles are shown below. Using a MARR of 18.5%, which should be chosen?

EOY	CF(A)	CF(B)
0	-$7,585,000.00	-$10,285,000.00
1	-$1,237,500.00	-$1,575,500.00
2	$1,695,500.00	$2,455,700.00
3	$2,002,800.00	$2,657,500.00
4	$2,345,700.00	$2,877,500.00
5	$2,450,500.00	$3,025,000.00
6	$2,575,600.00	$3,250,300.00
7	$2,735,000.00	$3,565,800.00
8	$3,005,300.00	$3,750,000.00
9	$3,857,500.00	$4,252,500.00
10	$5,285,000.00	$8,750,000.00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C13
=IRR(C2:C12)
EOY CF(A) CF(B) CF(B-A)
0 -$7,585,000.00 -$10,285,000.00 -$2,700,000.00
1 -$1,237,500.00 -$1,575,500.00 -$338,000.00
2 $1,695,500.00 $2,455,700.00 $760,200.00
3 $2,002,800.00 $2,657,500.00 $654,700.00
=C5-B5
4 $2,345,700.00 $2,877,500.00 $531,800.00
5 $2,450,500.00 $3,025,000.00 $574,500.00
6 $2,575,600.00 $3,250,300.00 $674,700.00
7 $2,735,000.00 $3,565,800.00 $830,800.00
8 $3,005,300.00 $3,750,000.00 $744,700.00
9 $3,857,500.00 $4,252,500.00 $395,000.00
10 $5,285,000.00 $8,750,000.00 $3,465,000.00
IRR = 19.39% 18.93% 17.61%
IRR(D2:D12)
Recommend Alternative A
=IRR(B2:B12)
Table 8.8 Table 8.9 Table 8.10 Figure 8.1 Figure 8.2 Figure 8.3 Figure 8.4 Figure 8.5 Figure 8.6

Example 8.7

Three mutually exclusive investment alternatives are being considered; the cash flow profiles are shown below. Based on a 15% MARR, which should be chosen?

EOY	CF(1)	CF(2)	CF(3)
0	-$100,000	-$125,000	-$150,000
1	$20,000	-$25,000	-$35,000
2	$20,000	$75,000	$75,000
3	$20,000	$70,000	$75,000
4	$20,000	$60,000	$75,000
5	$120,000	$55,000	$95,000

Example 8.7 (Continued)

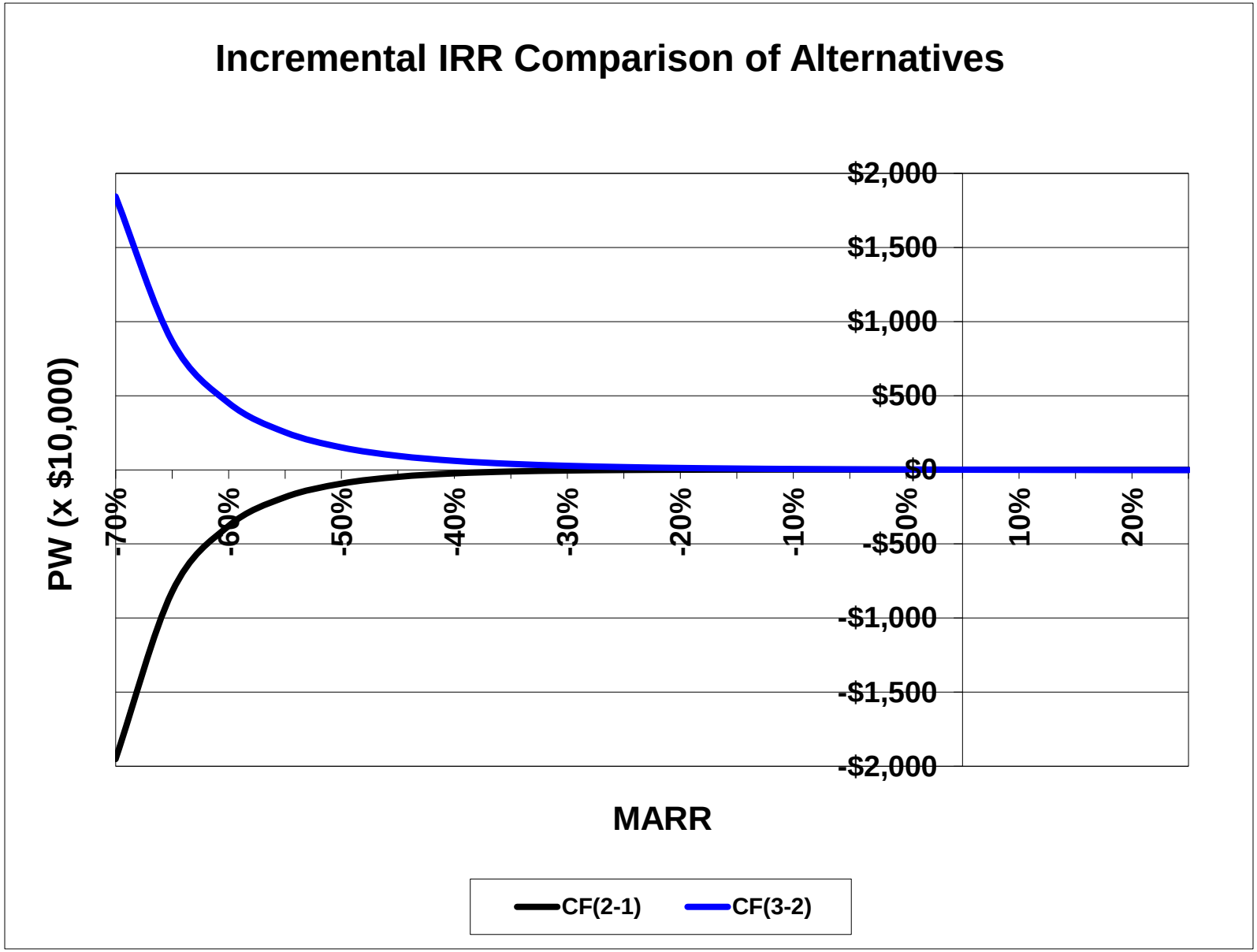
EOY	CF(1)	CF(2)	CF(3)	CF(2-1)	CF(3-2)
0	-$100,000	-$125,000	-$150,000	-$25,000	-$25,000
1	$20,000	-$25,000	-$35,000	-$45,000	-$10,000
2	$20,000	$75,000	$75,000	$55,000	$0
3	$20,000	$70,000	$75,000	$50,000	$5,000
4	$20,000	$60,000	$75,000	$40,000	$15,000
5	$120,000	$55,000	$95,000	-$65,000	$40,000
***IRR* =**	20.00%	19.39%	18.01%	16.41%	13.41%

Recommend Alternative 2

$PW_1(15\%)$ =PV(0.15,5,-20000,-100000)-100000 = $16,760.78

$PW_2(15\%)$ =NPV(0.15,-25,75,70,60,55)*1000-125000 = $17,647.70

$PW_3(15\%)$ =NPV(0.15,-35,75,75,75,95)*1000-150000 = $15,702.99


Incremental IRR Comparison of Alternatives
PW (x $10,000)
$2,000
$1,500
$1,000
$500
$0
-$500
-$1,000
-$1,500
-$2,000
-70%
-60%
-50%
-40%
-30%
-20%
-10%
0%
10%
20%
MARR
CF(2-1)
CF(3-2)

External Rate of Return Analysis

Single Alternative

External Rate of Return Method

- equates the future worth of positive cash flows using the MARR to the future worth of negative cash flows using the ERR, i'
- not a popular DCF method

$$\sum_{t=0}^{n} R_t(1+r)^{n-t} = \sum_{t=0}^{n} C_t(1+i')^{n-t}$$

R_t is positive-valued cash flow and C_t is the absolute value of a negative-valued cash flow; r is the MARR

(useful way to avoid the "multiple root" problem of the IRR)

Relationships among *MARR*, *IRR*, and *ERR*

- If *IRR* < *MARR*, then *IRR* < *ERR* < *MARR*
- If *IRR* > *MARR*, then *IRR* > *ERR* > *MARR*
- If *IRR* = *MARR*, then *IRR* = *ERR* = *MARR*

Example 8.8
SMP Investment
External Rate of Return Solution

EOY	CF			
0	-$500,000			
1 - 10	$92,500			
10	$50,000			
	$500,000(F|P i'%, 10) = $92,500(F|A 10%,10)			
			+ $50,000	

Example 8.8
SMP Investment
External Rate of Return Solution

EOY	CF			
0	-$500,000			
1 - 10	$92,500			
10	$50,000			
	$500,000(F|P i'%,10)= $92,500(F|A 10%,10)			
	+ $50,000			
	(F|P i'%,10) =	3.048423		
	i' = 11.79117%			
	i' = 11.79118% (using Excel)			

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D18 =MIRR($B$2:$B$12,,C18)

	A	B	C	D
1	EOY	CF	*MARR*	*ERR*
2	0	-$500,000	0%	6.91%
3	1	$92,500	1%	7.37%
4	2	$92,500	2%	7.83%
5	3	$92,500	3%	8.31%
6	4	$92,500	4%	8.79%
7	5	$92,500	5%	9.27%
8	6	$92,500	6%	9.76%
9	7	$92,500	7%	10.26%
10	8	$92,500	8%	10.77%
11	9	$92,500	9%	11.28%
12	10	$142,500	10%	11.79%
13	*ERR* =	11.79118%	11%	12.31%
14			12%	12.84%
15	=MIRR(B2:B12,,10%)		13%	13.37%
16			14%	13.91%
17			15%	14.45%
18			16%	15.00%
19			17%	15.55%
20			18%	16.10%
21			19%	16.67%

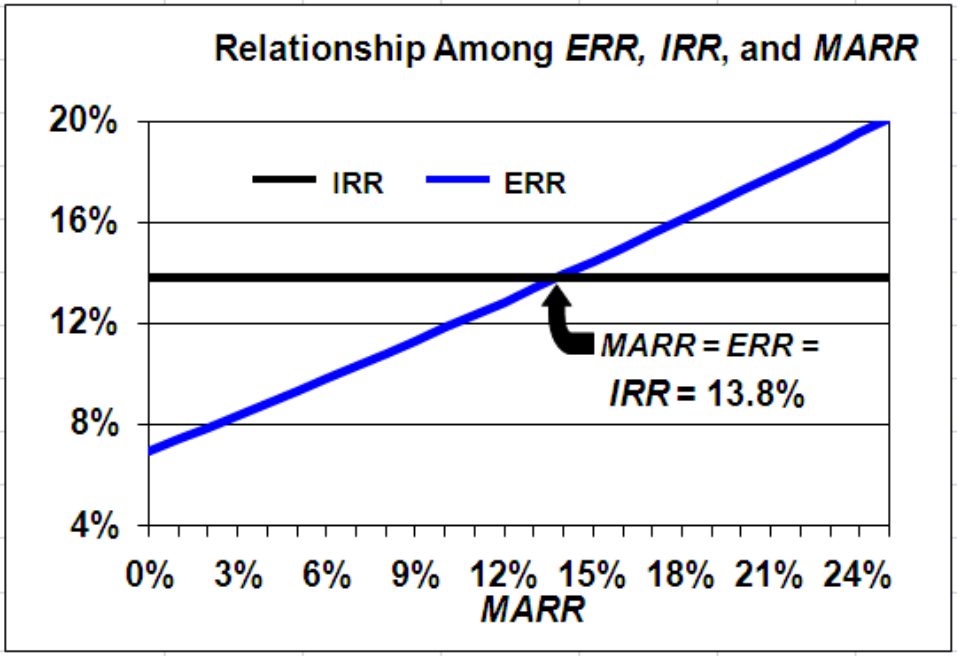


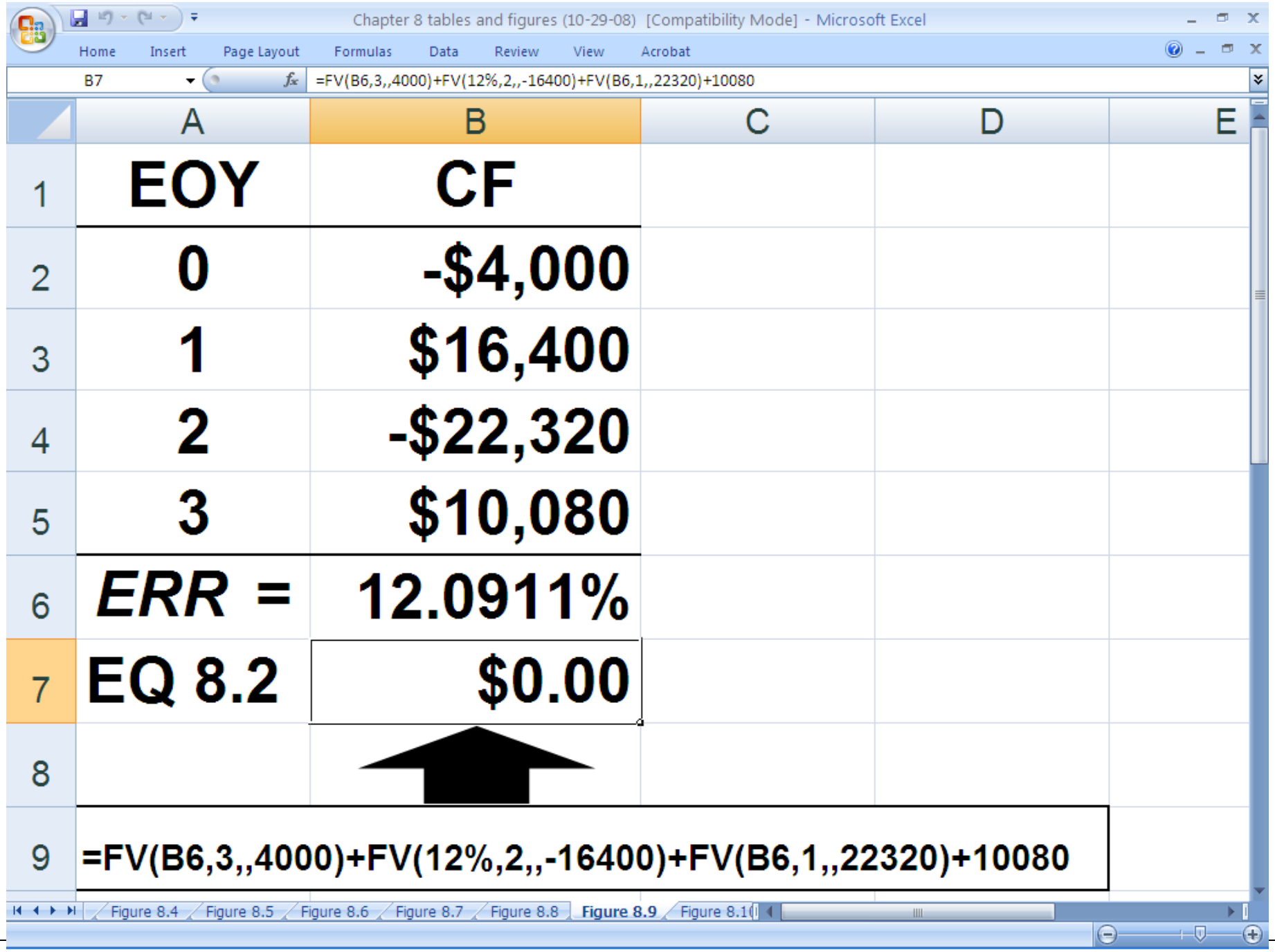
Figure 8.2 | Figure 8.3 | Figure 8.4 | Figure 8.5 | Figure 8.6 | Figure 8.7 | Figure 8.8

Example 8.9

Recall the cash flow profile, given below, that produced 3 *IRR* values: 20%, 40%, or 50%. If *MARR* = 12%, what is the *ERR*? For various values of *MARR*, what are the corresponding values of *ERR*?

EOY	CF
0	-$4,000
1	$16,400
2	-$22,320
3	$10,080

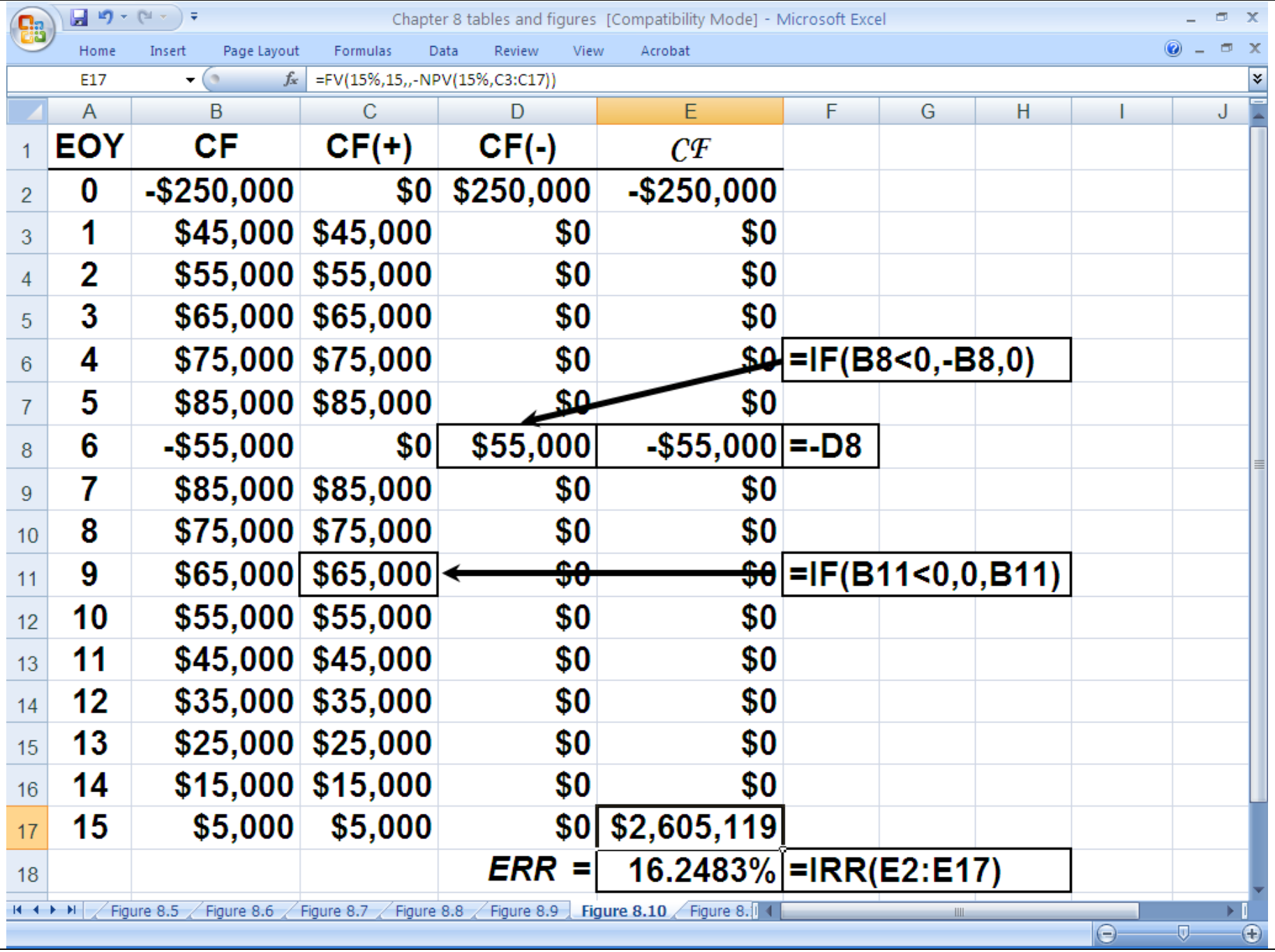
$$\$4000(F|P\ i',3) + \$22,320(F|P\ i',1) = \$16,400(F|P\ MARR,2) + \$10,080$$


Chapter 8 tables and figures (10-29-08) [Compatibility Mode] - Microsoft Excel
Home Insert Page Layout Formulas Data Review View Acrobat
B7
=FV(B6,3,,4000)+FV(12%,2,,-16400)+FV(B6,1,,22320)+10080
A B C D E
1 EOY CF
2 0 -$4,000
3 1 $16,400
4 2 -$22,320
5 3 $10,080
6 ERR = 12.0911%
7 EQ 8.2 $0.00
8
9 =FV(B6,3,,4000)+FV(12%,2,,-16400)+FV(B6,1,,22320)+10080
Figure 8.4 Figure 8.5 Figure 8.6 Figure 8.7 Figure 8.8 Figure 8.9 Figure 8.1

Example 8.10

Recall Julian Stewart's $250,000 investment in a limited partnership to drill for natural gas. The investment yielded annual returns of $45,000 the 1st yr, followed by $10,000 increases until the 6th yr, at which time an additional $150,000 had to be invested for deeper drilling. Following the 2nd drilling, the annual returns decreased by $10,000 per year, from $85,000 to $5,000. Since there were multiple negative values in the cash flow profile for the investment (EOY = 0 and EOY = 6), Excel's MIRR worksheet function cannot be used to solve for ERR.

When faced with multiple negative-valued cash flows, we construct a new CF profile that contains the negative-valued cash flows, zeroes, and the future worth of the positive-valued cash flows, with the FW based on the MARR.


Chapter 8 tables and figures [Compatibility Mode] - Microsoft Excel
Home Insert Page Layout Formulas Data Review View Acrobat
E17
=FV(15%,15,,-NPV(15%,C3:C17))
EOY CF CF(+) CF(-) CF
0 -$250,000 $0 $250,000 -$250,000
1 $45,000 $45,000 $0 $0
2 $55,000 $55,000 $0 $0
3 $65,000 $65,000 $0 $0
4 $75,000 $75,000 $0 $0
5 $85,000 $85,000 $0 $0
6 -$55,000 $0 $55,000 -$55,000
7 $85,000 $85,000 $0 $0
8 $75,000 $75,000 $0 $0
9 $65,000 $65,000 $0 $0
10 $55,000 $55,000 $0 $0
11 $45,000 $45,000 $0 $0
12 $35,000 $35,000 $0 $0
13 $25,000 $25,000 $0 $0
14 $15,000 $15,000 $0 $0
15 $5,000 $5,000 $0 $2,605,119
ERR = 16.2483%
=IF(B8<0,-B8,0)
=-D8
=IF(B11<0,0,B11)
=IRR(E2:E17)
Figure 8.5 Figure 8.6 Figure 8.7 Figure 8.8 Figure 8.9 Figure 8.10 Figure 8.

External Rate of Return Analysis

Multiple Alternatives

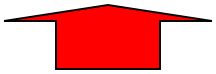
Example 8.11

Recall the 5 equal-lived, mutually exclusive investment alternatives which guaranteed your original investment back at any time you wished to end the investment. With $70,000 to invest and an 18% *MARR*, you chose investment 4 using an *IRR* analysis.

Using the external rate of return method, which would you choose? What is its external rate of return? (We use a 10-year planning horizon.)

Solution to Example 8.11

Investment	1	2	3	4	5
Initial Investment	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
Annual Return	$3,750.00	$5,000.00	$9,250.00	$11,250.00	$14,250.00
Salvage Value	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
ERR	21.27%	19.02%	20.47%	20.19%	19.19%



Investment	1	2 - 1	3 - 1	4 - 3	5 - 4
Δ Investment	$15,000.00	$10,000.00	$25,000.00	$10,000.00	$20,000.00
Δ Annual Return	$3,750.00	$1,250.00	$5,500.00	$2,000.00	$3,000.00
Δ Salvage Value	$15,000.00	$10,000.00	$25,000.00	$10,000.00	$20,000.00
Δ *ERR*	21.27%	14.70%	19.97%	19.02%	16.30%
$\geq$ *MARR*?	Yes	No	Yes	Yes	No
Defender	1	1	3	4	4

Choose 4

Solution to Example 8.11

Investment	1	2	3	4	5
Initial Investment	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
Annual Return	$3,750.00	$5,000.00	$9,250.00	$11,250.00	$14,250.00
Salvage Value	$15,000.00	$25,000.00	$40,000.00	$50,000.00	$70,000.00
ERR	21.27%	19.02%	20.47%	20.19%	19.19%

=RATE(10,,-50000,FV(18%,10,-11250)+50000)

Investment	1	2 - 1	3 - 1	4 - 3	5 - 4
Δ Investment	$15,000.00	$10,000.00	$25,000.00	$10,000.00	$20,000.00
Δ Annual Return	$3,750.00	$1,250.00	$5,500.00	$2,000.00	$3,000.00
Δ Salvage Value	$15,000.00	$10,000.00	$25,000.00	$10,000.00	$20,000.00
Δ *ERR*	21.27%	14.70%	19.97%	19.02%	16.30%
≥ *MARR*?	Yes	No	Yes	Yes	No
Defender	1	1	3	4	4

Choose 4

Example 8.12

Recall the example involving two designs (A & B) for a new ride at a theme park in Florida: A costs $300,000, has $55,000/yr revenue, and has a negligible salvage value at the end of the 10-year planning horizon; B costs $450,000, has $80,000/yr revenue, and has a negligible salvage value. Based on an *ERR* analysis and a 10% *MARR*, which is preferred?

$ERR_A(10\%)$ =RATE(10,,-300000,FV(10%,10,-55000))

= 11.31814% > *MARR* = 10% (A is acceptable)

$ERR_{B-A}(10\%)$ =RATE(10,,-150000,FV(10%,10,-25000))

= 10.26219% > *MARR* = 10% (B is preferred)

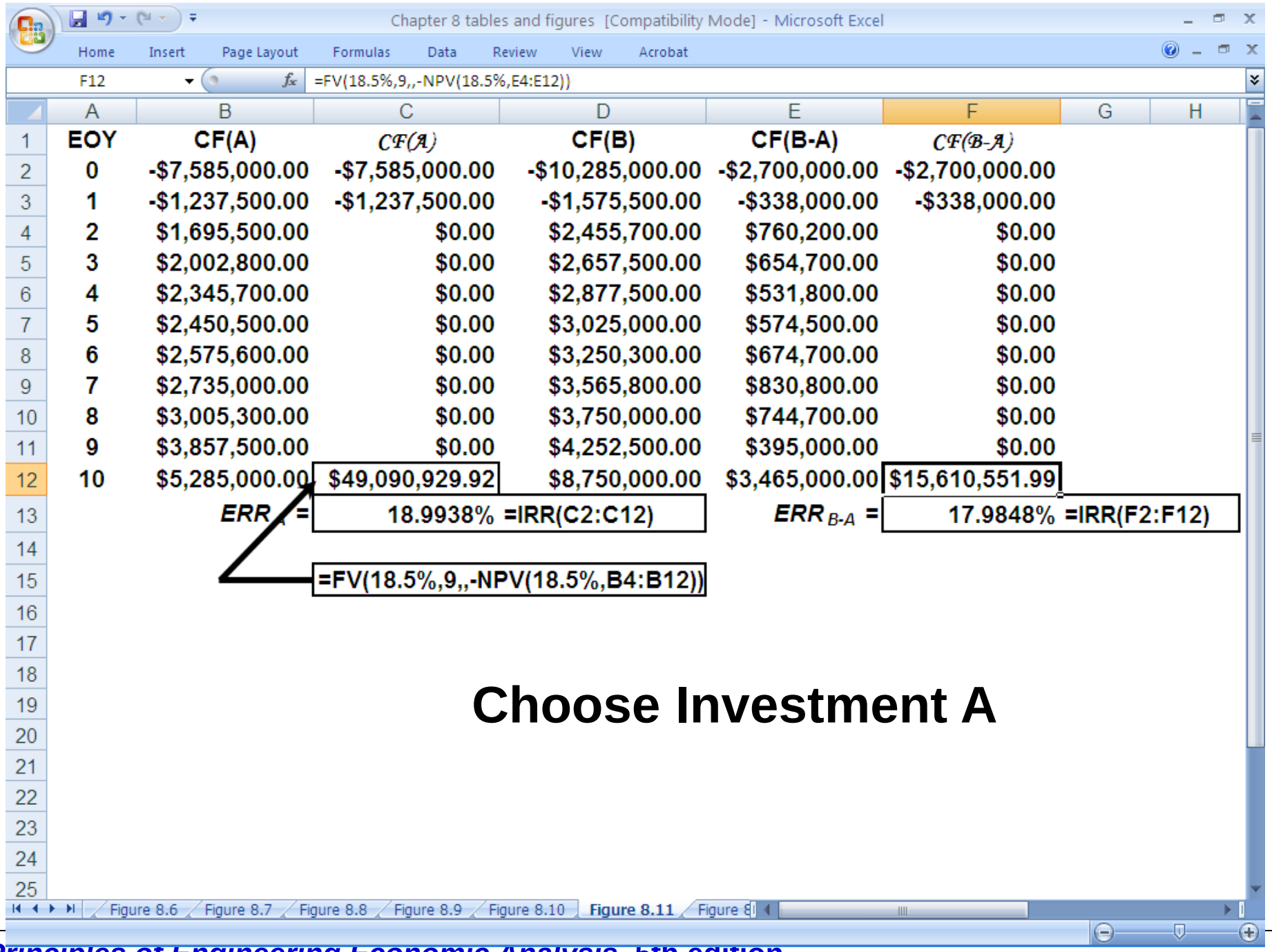
$ERR_B(10\%)$ =RATE(10,,-450000,FV(10%,10,-80000))

= 10.97611%

Example 8.13

Recall the batch chemical processing company that is considering two centrifuges for possible acquisition. The estimated cash flows are given below. With an 18.5% *MARR*, which should be chosen using an *ERR* analysis?

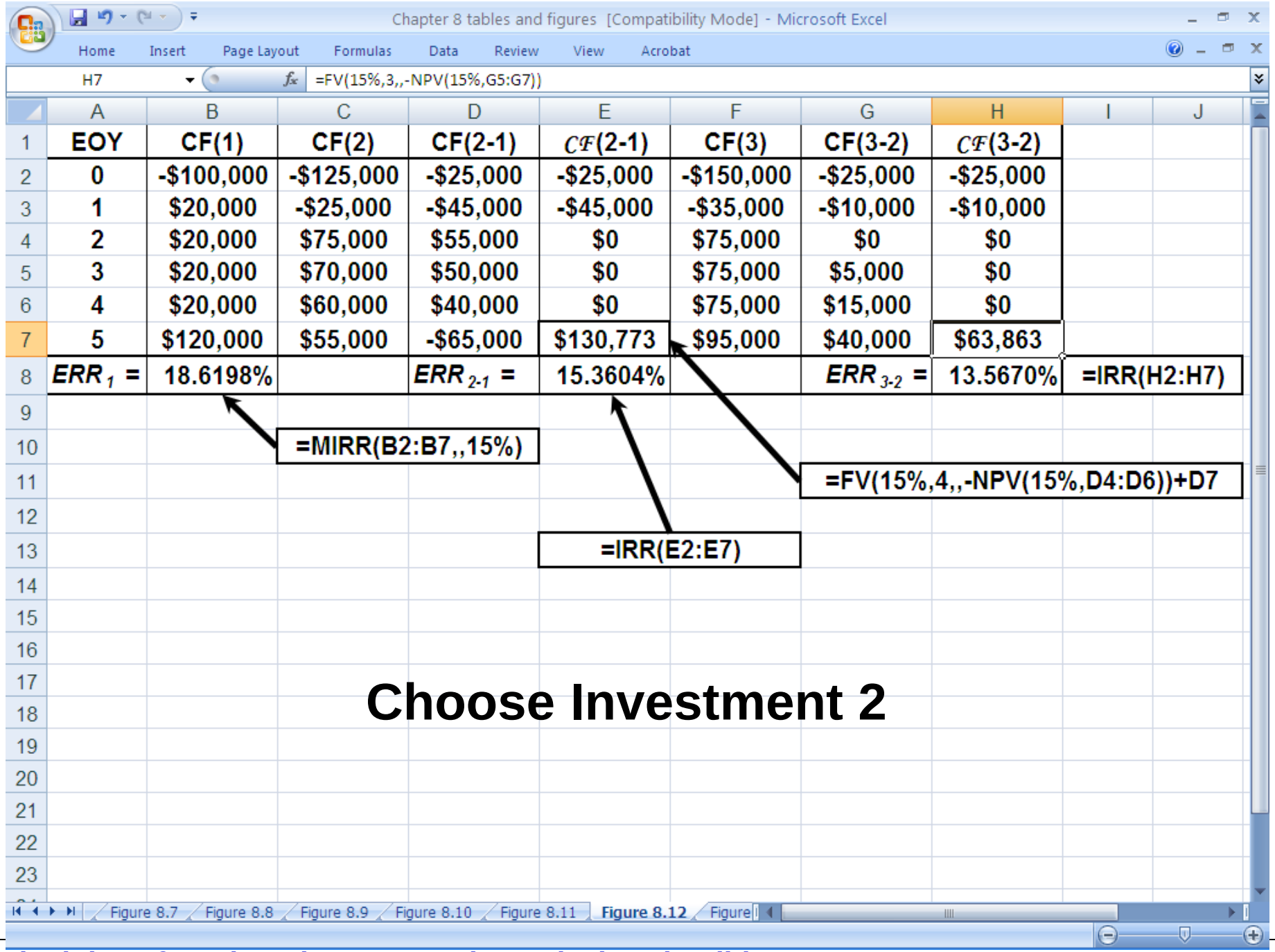
EOY	CF(A)	CF(B)
0	-$7,585,000.00	-$10,285,000.00
1	-$1,237,500.00	-$1,575,500.00
2	$1,695,500.00	$2,455,700.00
3	$2,002,800.00	$2,657,500.00
4	$2,345,700.00	$2,877,500.00
5	$2,450,500.00	$3,025,000.00
6	$2,575,600.00	$3,250,300.00
7	$2,735,000.00	$3,565,800.00
8	$3,005,300.00	$3,750,000.00
9	$3,857,500.00	$4,252,500.00
10	$5,285,000.00	$8,750,000.00


Chapter 8 tables and figures [Compatibility Mode] - Microsoft Excel
Home Insert Page Layout Formulas Data Review View Acrobat
F12
=FV(18.5%,9,,-NPV(18.5%,E4:E12))
EOY
CF(A)
CF(A)
CF(B)
CF(B-A)
CF(B-A)
0 -$7,585,000.00 -$7,585,000.00 -$10,285,000.00 -$2,700,000.00 -$2,700,000.00
1 -$1,237,500.00 -$1,237,500.00 -$1,575,500.00 -$338,000.00 -$338,000.00
2 $1,695,500.00 $0.00 $2,455,700.00 $760,200.00 $0.00
3 $2,002,800.00 $0.00 $2,657,500.00 $654,700.00 $0.00
4 $2,345,700.00 $0.00 $2,877,500.00 $531,800.00 $0.00
5 $2,450,500.00 $0.00 $3,025,000.00 $574,500.00 $0.00
6 $2,575,600.00 $0.00 $3,250,300.00 $674,700.00 $0.00
7 $2,735,000.00 $0.00 $3,565,800.00 $830,800.00 $0.00
8 $3,005,300.00 $0.00 $3,750,000.00 $744,700.00 $0.00
9 $3,857,500.00 $0.00 $4,252,500.00 $395,000.00 $0.00
10 $5,285,000.00 $49,090,929.92 $8,750,000.00 $3,465,000.00 $15,610,551.99
ERR A = 18.9938% =IRR(C2:C12)
ERR B-A = 17.9848% =IRR(F2:F12)
=FV(18.5%,9,,-NPV(18.5%,B4:B12))
Choose Investment A
Figure 8.6 Figure 8.7 Figure 8.8 Figure 8.9 Figure 8.10 Figure 8.11 Figure 8

Example 8.14

Recall, the three mutually exclusive investment alternatives having the cash flow profiles shown below. Based on a 15% *MARR* and *ERR* analysis, which should be chosen?

EOY	CF(1)	CF(2)	CF(3)
0	-$100,000	-$125,000	-$150,000
1	$20,000	-$25,000	-$35,000
2	$20,000	$75,000	$75,000
3	$20,000	$70,000	$75,000
4	$20,000	$60,000	$75,000
5	$120,000	$55,000	$95,000


Chapter 8 tables and figures [Compatibility Mode] - Microsoft Excel
Home Insert Page Layout Formulas Data Review View Acrobat
H7
=FV(15%,3,,-NPV(15%,G5:G7))
EOY CF(1) CF(2) CF(2-1) CF(2-1) CF(3) CF(3-2) CF(3-2)
0 -$100,000 -$125,000 -$25,000 -$25,000 -$150,000 -$25,000 -$25,000
1 $20,000 -$25,000 -$45,000 -$45,000 -$35,000 -$10,000 -$10,000
2 $20,000 $75,000 $55,000 $0 $75,000 $0 $0
3 $20,000 $70,000 $50,000 $0 $75,000 $5,000 $0
4 $20,000 $60,000 $40,000 $0 $75,000 $15,000 $0
5 $120,000 $55,000 -$65,000 $130,773 $95,000 $40,000 $63,863
ERR 1 = 18.6198% ERR 2-1 = 15.3604% ERR 3-2 = 13.5670% =IRR(H2:H7)
=MIRR(B2:B7,,15%)
=FV(15%,4,,-NPV(15%,D4:D6))+D7
=IRR(E2:E7)
Choose Investment 2
Figure 8.7 Figure 8.8 Figure 8.9 Figure 8.10 Figure 8.11 Figure 8.12 Figure

Analyzing Alternatives with No Positive Cash Flows

Example 8.15

A company must purchase a new incinerator to meet air quality standards. Three alternatives have been identified, with cash flow profiles given below. Based on a 7-year planning horizon and a 12% *MARR*, which should be purchased? Perform *PW, AW, IRR,* and *ERR* analyses.

Alt	Initial Investment	Annual Operating Cost	Annual Maintenance Cost
A	$250,000	$105,000	$42,000
B	$385,000	$78,000	$28,000
C	$475,000	$65,000	$18,000

Solution to Example 8.15

$PW_A(12\%) = \$250{,}000 + \$147{,}000(P|A\ 12\%,7)$

$= \$250{,}000 + \$147{,}000(4.56376)$

$= \$920{,}872.72$

=PV(12%,7,-147000)+250000

$= \$920{,}872.21$

$PW_B(12\%) = \$385{,}000 + \$106{,}000(P|A\ 12\%,7)$

$= \$385{,}000 + \$106{,}000(4.56376)$

$= \$868{,}758.56$

=PV(12%,7,-106000)+385000

$= \$868{,}758.19$

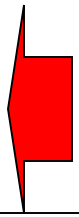
$PW_C(12\%) = \$475{,}000 + \$83{,}000(P|A\ 12\%,7)$

$= \$475{,}000 + \$83{,}000(4.56376)$

$= \$853{,}792.08$

=PV(12%,7,-83000)+475000

$= \$853{,}791.79$



Choose C

Solution to Example 8.15 (Continued)

$EUAC_A(12\%) = \$250{,}000(A|P\ 12\%,7) + \$147{,}000$

$= \$250{,}000(0.21912) + \$147{,}000$

$= \$201{,}780.00$

=PMT(12%,7,-250000)+147000

= $201,779.43

$EUAC_B(12\%) = \$385{,}000(A|P\ 12\%,7) + \$106{,}000$

$= \$385{,}000(0.21912) + \$106{,}000$

$= \$190{,}361.20$

=PMT(12%,7,-385000)+106000

= $190,360.33

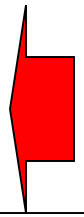
$EUAC_C(12\%) = \$475{,}000 + \$83{,}000(P|A\ 12\%,7)$

$= \$475{,}000 + \$83{,}000(4.56376)$

$= \$187{,}082.00$

=PMT(12%,7,-475000)+83000

= $187,080.92



Choose C

Solution to Example 8.15 (Continued)

IRR analysis

Incremental solution: B-A ($135,000 incremental investment produces $41,000 incremental reduction in annual costs)

$385,000 - $250,000 (A|P IRR_{B-A},7)= $147,000 - $106,000

IRR_{B-A} = 23.4% > 12% (B>>A)

Incremental solution: C-B ($90,000 incremental investment produces $23,000 incremental reduction in annual costs)

$475,000 - $385,000 (A|P IRR_{C-B},7)= $106,000 - $83,000

IRR_{C-B}(12%) = 17.082% > 12% (C>>B)

Choose C

Solution to Example 8.15 (Continued)

ERR analysis

Incremental solution: B-A ($135,000 incremental investment yields $41,000 reduction in annual costs)

$$\$135{,}000(1+ERR_{B-A})^7 = \$41{,}000(F|A\ 12\%,7)$$

$$(1+ERR_{B-A})^7 = \$41{,}000(10.08901)/\$135{,}000$$

$$ERR_{B-A} = 17.347\% > 12\%\ (B>>A)$$

Incremental solution: C-B ($90,000 incremental investment yields $23,000 reduction in annual costs)

$$\$90{,}000(1+ERR_{C-B})^7 = \$23{,}000(F|A\ 12\%,7)$$

$$(1+ERR_{C-B})^7 = \$23{,}000(10.08901)/\$90{,}000$$

$$ERR_{C-B} = 14.489\% > 12\%\ (C>>B)$$

Choose C

Pit Stop #8—Halfway Home! Miles to Go!

1. True or False: For personal investment decision making, rates of return are used more frequently than present worth.
2. True or False: Unless non-monetary considerations dictate otherwise, you should choose the mutually exclusive investment alternative having the greatest rate of return over the planning horizon.
3. True or False: If *ERR* > *MARR*, then *IRR* > *ERR* > *MARR*.
4. True or False: If *PW* > 0, then *IRR* > *MARR*.
5. True or False: If *ERR* > *MARR*, then *MIRR* > *MARR*.
6. True or False: If *IRR*(A) > *IRR*(B), then *ERR*(A) > *ERR*(B).
7. True or False: If *PW*(A) > *PW*(B), then *FW*(A) > *FW*(B), *AW*(A) > *AW*(B), *CW*(A) > *CW*(B), and *IRR*(A) > *IRR*(B).
8. True or False: Multiple roots can exist when using *IRR* and *MIRR* methods.
9. True or False: Excel's IRR worksheet function signals if multiple roots exist for a cash flow series.
10. True or False: Of all the equivalent *DCF* methods, the one that is the most difficult to use is the external rate of return method because of its requirement of a reinvestment rate for recovered capital.

Pit Stop #8—Halfway Home! Miles to Go!

1. True or False: For personal investment decision making, rates of return are used more frequently than present worth. **TRUE**
2. True or False: Unless non-monetary considerations dictate otherwise, you should choose the mutually exclusive investment alternative having the greatest rate of return over the planning horizon. **FALSE**
3. True or False: If *ERR* > *MARR*, then *IRR* > *ERR* > *MARR*. **TRUE**
4. True or False: If *PW* > 0, then *IRR* > *MARR*. **TRUE**
5. True or False: If *ERR* > *MARR*, then *MIRR* > *MARR*. **FALSE**
6. True or False: If *IRR*(A) > *IRR*(B), then *ERR*(A) > *ERR*(B). **FALSE**
7. True or False: If *PW*(A) > *PW*(B), then *FW*(A) > *FW*(B), *AW*(A) > *AW*(B), *CW*(A) > *CW*(B), and *IRR*(A) > *IRR*(B). **FALSE**
8. True or False: Multiple roots can exist when using *IRR* and *MIRR* methods. **FALSE**
9. True or False: Excel's IRR worksheet function signals if multiple roots exist for a cash flow series. **FALSE**
10. True or False: Of all the equivalent *DCF* methods, the one that is the most difficult to use is the external rate of return method because of its requirement of a reinvestment rate for recovered capital. **FALSE**