Ch 10

(10.2)

Assume that you have a sample of n1=8 , with the sample mean $\bar{X}_1$= 42, and a sample standard deviation S_1=4, and you have an independent sample of n_2=15 from another population with a sample mean of $\bar{X}_2$=34 and a sample deviation S_2=5 .

$n_1 = 8$, $\bar{X}_1 = 42$, $S_1 = 4$

$n_2 = 15$, $\bar{X}_2 = 34$, $S_2 = 5$

a. What is the value of the pooled-variance t-Stat test statistic for testing $H_0: \mu_1 = \mu_2$?

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)} = \frac{(8-1) \times 4^2 + (15-1) \times 5^2}{(8-1) + (15-1)} = 22$$

$$t_{STAT} = \frac{42 - 34}{\sqrt{22\left(\frac{1}{8} + \frac{1}{15}\right)}} = \frac{8}{2.0536} = 3.8959$$

b- In finding the critical value, how many degrees of freedom are there?

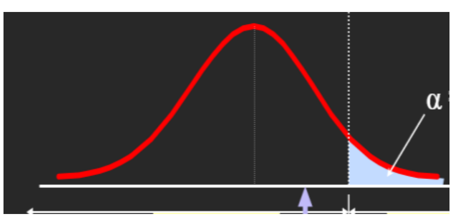
degrees of freedom (d.f) = n_1 + n_2 -2 = (8 + 15) – 2 = 21

c. Using the level of significance α = 0.01, what is the critical value for a one-tail test for the hypothesis $H_0: \mu_1 \le \mu_2$ against the alternative, $H_1: \mu_1 > \mu_2$?

$t_{0.01,21}$ = 2.5177

d. What is your statistical decision?

Reject H_0 because $t_{stat} > t_\alpha$



(10.4)

referring to problem 10.2, construct a 95% confidence interval estimate of the population mean difference between μ_1 and μ_2

$$\mu_1 - \mu_2 = (\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2}\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= (42 - 34) \pm 2.0796\sqrt{22\left(\frac{1}{8} + \frac{1}{15}\right)}$$

$$= 8 \pm 2.0796 \times 2.0536$$

$$= 8 \pm 4.2705$$

= (12.2705, 3.7295)

3.7295 $\le \mu_1$ - $\mu_2 \le$ 12.2705

(10.20) Nine experts rated two brands of coffee and its taste testing experiment. A rating on a 1 to 7-point scale [1 = extremely unpleasing, 7 = extremely pleasing] is given for each of four characteristics: taste, aroma, richness, and acidity. The accompanying data table contains the ratings accumulated over all four characteristics.

	Brand	
Expert	A	B
C.C.	26	27
S.E.	27	27
E.G.	19	21
B.L.	22	24
C.M.	22	25
C.N.	25	26
G.N.	25	24
R.M.	25	26
P.V.	21	23

a. At the 0.05 level of significance, is there evidence of a difference in the mean ratings between the two brands?

Solution:

Step 1: state the hypothesis: H_0: $\mu_D = 0$

H_1: $\mu_D \neq 0$

Step2: Select the level of significance and critical value:

d.f = n - 1 = 9-1 =8 $t_{0.025,8} = \pm\ 2.3060$

Step 3: Find the appropriate test statistic.

$$t_{stat} = \frac{\bar{D}}{S_D / \sqrt{n}} \qquad \bar{D} = \frac{\sum D_i}{n} \qquad S_D = \sqrt{\frac{\sum (D_i - \bar{D})^2}{n-1}}$$

Expert	A	B	D=A-B	D- $\bar{D}$	(D- $\bar{D}$)^2
C.C.	26	27	-1	0.2222	0.0494
S.E.	27	27	0	1.2222	1.4938
E.G.	19	21	-2	-0.7778	0.6050
B.L.	22	24	-2	-0.7778	0.6050
C.M.	22	25	-3	-1.7778	3.1606
C.N.	25	26	-1	0.2222	0.0494
G.M.	25	24	1	2.2222	4.9382
R.M.	25	26	-1	0.2222	0.0494
P.V.	21	23	-2	-0.7778	0.6050
			$\sum$ =-11		$\sum$=11.558

$$\bar{D} = \frac{\sum D_i}{n} = \frac{-11}{9} = -1.2222$$

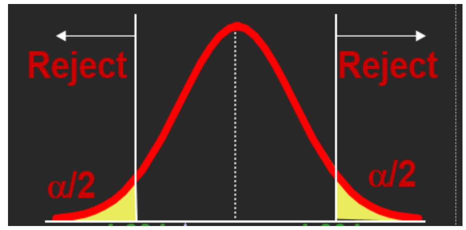
$$S_D = \sqrt{\frac{\sum(D_i - \bar{D})^2}{n-1}} = \sqrt{\frac{11.5558}{8}} = 1.2019$$

$$t_{stat} = \frac{\bar{D}}{S_D/\sqrt{n}} = \frac{-1.2222}{1.2019/3} = \frac{-1.2222}{0.4006} = -3.05$$

Step 4: State the decision rule

Reject H_0 if $t_{stat} > 2.3060$ or $t_{stat} < -2.3060$

Step 5: Decision Reject H_0 (t_{stat} is in the rejection region)



b. Construct and interpret a 95% confidence interval estimate of the difference in the mean ratings between the two brands.

$$\hat{\mu}_D = \overline{D} \pm t_{\alpha/2}\frac{S_D}{\sqrt{n}}$$
$$= -1.222 \pm 2.3060\frac{1.2019}{\sqrt{9}}$$
$$= -1.222 \pm 0.9238$$
$$-2.146 < \hat{\mu}_D < -.2984$$

<u>(10.27)</u>

Let n1=80, X1=70, and n2= 80, and X2=50.

a. at the 0.10 level of significance, is there evidence of a significant difference t between the two population proportions?

<u>Solution:</u>

Step 1: State the null and alternate hypotheses.

H_0: π1 – π2 = 0

H_1: π1 – π2 ≠ 0

Step 2: State the level of significance and critical value (α= 0.10).

$$\pm Z_{\frac{\alpha}{2}} = \pm Z_{\frac{0.10}{2}} = \pm Z_{0.05} = \pm 1.645$$

Step 3: $$z_{\text{stat}} = \frac{(p_1-p_2)-(\pi_1-\pi_2)}{\sqrt{\bar{p}\,(1-\bar{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

<u>The sample proportions are:</u>

p1=70/80=0.875

p2=50/80=0.625

The pooled estimate for the overall proportion is:

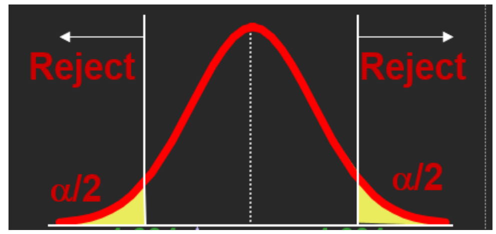
$$\overline{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{70 + 50}{80 + 80} = 0.75$$

$$z_{\text{stat}} = \frac{(0.875 - 0.625) - (0)}{\sqrt{0.75(1 - 0.75)\left(\frac{1}{80} + \frac{1}{80}\right)}} = 3.6496$$

Step 4: State the decision rule: Reject H_0 if

$z_{\text{stat}} > Z_{\frac{\alpha}{2}}$ Or $z_{\text{stat}} > -Z_{\frac{\alpha}{2}}$

Decision: $z_{\text{stat}} > Z_{\frac{\alpha}{2}}$ Reject H_0



b. Construct a 90% confidence interval estimate of the difference between the two population proportions.

$$(p_1 - p_2) \pm Z_{\alpha/2}\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

The 90% confidence interval for π1 – π2 is:

$$= (0.875 - 0.625) \pm 1.645\sqrt{\frac{0.875(1\text{-}0.875)}{80} + \frac{0.625(1\text{-}0.625)}{80}}$$

= (0.1422, 0.3578)

(10.39) The following information is available for two samples selected from independent normally distributed Populations

n1=25 s_1^2 =161.9 n2= 25 s_2^2 = 133.7

What is the value of F_{STAT} If you are testing the null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$?

$$F_{STAT} = \frac{s_1^2}{s_2^2} = \frac{161.9}{133.7} = 1.2109$$

(10.40) How many degrees of freedom are there in the numerator and denominator of the F_{STAT} ?

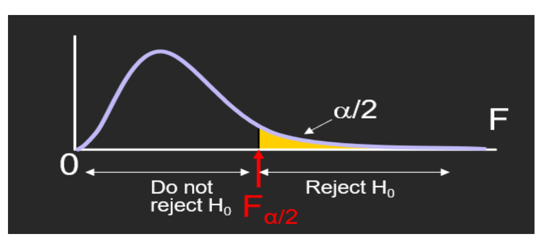
d.f1 (numerator) = n1 – 1 = 25-1=24 d.f2 (denominator) = n2 -1 = 25-1=24

(10.41) What is the upper-tail critical value of F if the level of significance, α = 0.05 and the alternative hypothesis is $H_1: \sigma_1^2 \neq \sigma_2^2$

$$F_{\frac{\alpha}{2},\text{d.f1,d.f2}} = F_{\frac{0.05}{2},24,24} = 2.27$$

(10.42) What is your statistical decision ?

Decision : $F_{STAT} < F_{\frac{\alpha}{2}}$, Don't reject



(10.51) An experiment has a single factor with three groups and six values in each group.

a) How many degrees of freedom are there in determining the among- group variation?

C – 1 = 3 – 1 =2

b) How many degrees of freedom are there in determining the within-group variation?

n – c = 18 – 3 = 15

c) How many degrees of freedom are there in determining the total variation?

n – 1 = 18 – 1 = 17

(10.52) If you are working with the same experiment as in problem 10.51,

a) If SSA = 60, and SST = 210. What is the SSW?

SST= SSA + SSW SSW = SST – SSA = 210 – 60 = 150

b) What is the MSA?

MSA = $\frac{SSA}{C-1} = \frac{60}{2}$ = 30

c) What is the MSW?

MSW = $\frac{SSW}{n-C} = \frac{150}{15} = 10$

d) What is the value of F_{stat}? $F_{stat} = \frac{MSA}{MSW} = \frac{30}{10} = 3$

(10.53)

a- Construct the ANOVA summary table and fill in all values in the table.

Source	d.f	Sum of Square	Mean Square	F
Among Groups	c-1 = 2	SSA = 60	MSA = 30	3
Within Groups	n-c = 15	SSW = 150	MSW = 10	
Total	n-1 = 17	SST = 210		

b) At the 0.05 level of significance, what is the upper -tail critical value from F distribution?

$F_{0.05\,,c-1,n-1=}$ 3.68

c) What is your statistical decision?

Don't reject (F_{stat} < $F_{\frac{\alpha}{2}}$) .