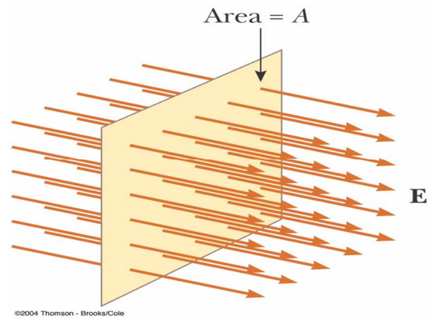
Chapter 24

Gauss's Law

24.1 Electric Flux



Consider an electric field that is uniform in both magnitude and direction, the field lines penetrate a rectangular surface of area *A*, which is perpendicular to the field.

The number of lines per unit area (in other words, the *line density*) is proportional to the magnitude of the electric field. Therefore, the total number of lines penetrating the surface is proportional to the product ***EA***. This product of the magnitude of the electric field ***E*** and surface area *A* perpendicular to the field is called the **electric flux** $\boldsymbol{\Phi_E}$ (uppercase Greek phi):

$$\Phi_E = E\,A \qquad (\mathrm{N.m^2/C})$$

Electric flux is proportional to the number of electric field lines penetrating some surface.

Example:

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of 1.00 μC at its center?

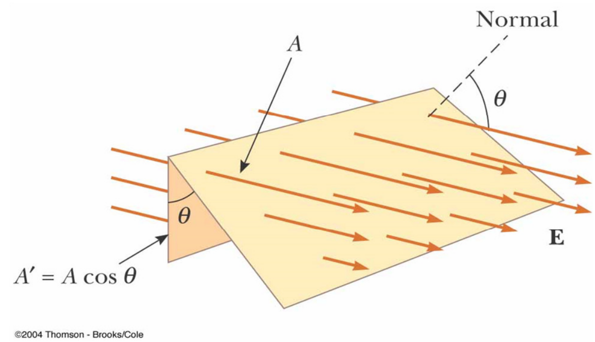
As we have studied in chapter 23, the E is given as following:

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})\,\frac{1.00 \times 10^{-6}\,\mathrm{C}}{(1.00\,\mathrm{m})^2}$$
$$= 8.99 \times 10^3\,\mathrm{N/C}$$

Since the field points radically outward they everywhere perpendicular to the surface of the sphere. The flux through the sphere (whose surface area is $A = 4\pi r^2 = 12.6\ \mathrm{m}^2$)thus

$$\Phi_E = EA = (8.99 \times 10^3\,\mathrm{N/C})(12.6\,\mathrm{m^2})$$
$$= 1.13 \times 10^5\,\mathrm{N \cdot m^2/C}$$

Field lines representing a uniform electric field penetrating an area A that is at an angle θ to the field. Because the number of lines that go through the area A' is the same as the number that go through A, the flux through A' is equal to the flux through A and is given by



$$\Phi_E = EA \cos\theta$$

$$\Phi_E = EA' = EA \cos\theta$$

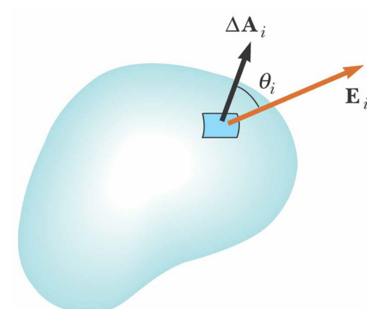
Note that:

the flux through a surface of fixed area A has a maximum value EA when the surface is perpendicular to the field (in other words, when the normal to the surface is parallel to the field, that is θ = 0.

the flux is zero when the surface is parallel to the field (in other words, when the normal to the surface is perpendicular to the field, that is, θ = 90.

We assumed a uniform electric field in the preceding discussion. In more general situations, the electric field may vary over a surface. Therefore, our definition of flux has meaning only over a small element of area.

A small element of surface area ΔA_i. The electric field makes an angle θ with the vector ΔA_i, defined as being normal to the surface element, and the flux through the element is equal to $E_i \Delta A_i \cos\theta$.



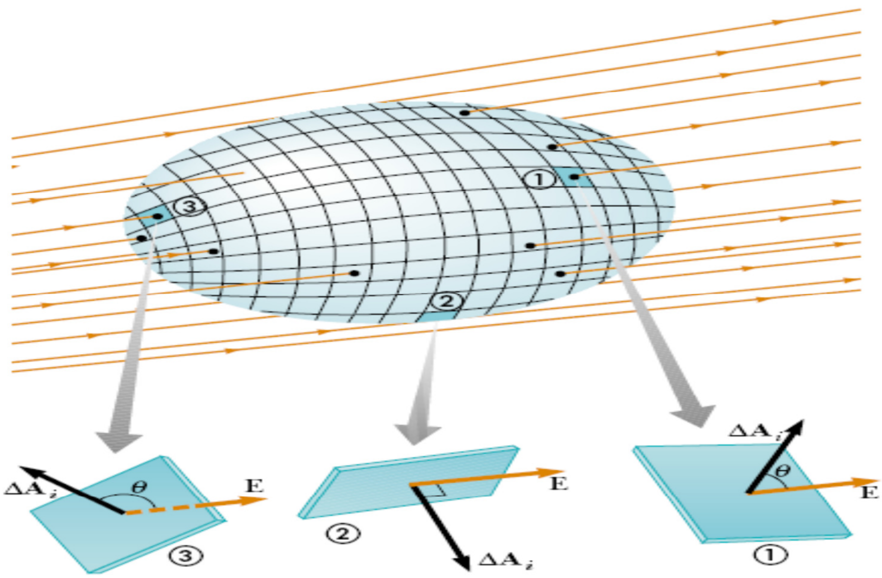
$$\Delta\phi = E_i \Delta A_i \cos\theta_i = \vec{E}_i . \Delta\vec{A}_i$$

$$\phi = \lim_{\Delta A \to 0} \sum \vec{E}_i . \Delta\vec{A}_i = \int \vec{E}.d\vec{A}$$

$$= \int E\, dA \cos\theta$$

A closed surface in an electric field

The area vectors ΔA_i are, by convention, normal to the surface and point outward. The flux through an area element can be positive (element 1), zero (element2), or negative (element 3).



The *net* flux through the surface is proportional to the net number of lines leaving the surface, where the net number means *the number leaving the surface minus the number entering the surface.*

- If more lines are leaving than entering, the net flux is **positive**.
- If more lines are entering than leaving, the net flux is **negative**.

$$\phi_c = \oint \vec{E} \cdot d\vec{A} = \oint E_n \, dA = \oint E \, dA \cos\theta$$

$$\theta < 90^0 \quad \phi_c \Rightarrow +ve \qquad \& \quad \theta > 90^0 \quad \phi_c \Rightarrow -ve$$

Example 24.2