

## Ch 11

### Chi-Square Tests

#### 11.4

Use the following contingency table:

	A	B	Total
1	20	30	50
2	30	20	50
Total	50	50	100

a) Compute the expected frequency for each cell?

$$f_e = \frac{\text{rowtotal} \times \text{columntotal}}{n}$$

$$f_{11} = \frac{50 \times 50}{100} = 25$$

$$f_{12} = \frac{50 \times 50}{100} = 25$$

$$f_{21} = \frac{50 \times 50}{100} = 25$$

$$f_{22} = \frac{50 \times 50}{100} = 25$$

Totals for the observed and expected frequency are the same:

$$\sum f_o = 20 + 30 + 30 + 20 = 100$$

$$\sum f_e = 25 + 25 + 25 + 25 = 100$$

b) Compute  $\chi^2_{stat}$  – Is it significant at  $\alpha = 0.05$ ?

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

$$\chi^2_{stat} = \frac{(20-25)^2}{25} + \frac{(30-25)^2}{25} + \frac{(30-25)^2}{25} + \frac{(20-25)^2}{25}$$

$$\chi^2_{stat} = 1 + 1 + 1 + 1 = 4$$

\*\*\*\* the critical value:

$$\chi^2_{\alpha, (r-1)(c-1)} = 3.841$$

**Decision:** Since  $\chi^2_{stat} = 4$  is greater than the critical value of 3.841, it is significant at the 5% level of significance.

## 11.5

An online survey of 1,000 adults asked, “What do you buy from a mobile device?” The results indicated that 61% of the females said clothes as compared to 39% of the males. The results were shown in the following table:

GENDER			
	Male	Female	Total
Yes	195 <i>(250)</i>	305 <i>(250)</i>	500
No	305 <i>(250)</i>	195 <i>(250)</i>	500
Total	500	500	1000

Is there evidence of a significant difference between the portion of males and females who say they buy clothing from their mobile device at the 0.01 level of significance?

Solution:

**Step 1:** state the hypothesis:

$H_0$ : There is no different between the proportion of male and females.

$H_{01}$ : There is different between the proportion of male and females.

**Step2:** The critical value at ( $\alpha = 0.01$ ):

$$\chi^2_{\alpha, (r-1)(c-1)} = \chi^2_{0.01, (2-1)(2-1)} = 6.635$$

**Step 3:** Find the test statistic.

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

$$\chi^2_{stat} = \frac{(195 - 250)^2}{250} + \frac{(305 - 250)^2}{250} + \frac{(305 - 250)^2}{250} + \frac{(195 - 250)^2}{250}$$

$$\chi^2_{stat} = 48.4$$

**Step 4:** State the decision rule

$$\text{Reject } H_0 \text{ if } \chi^2_{stat} > \chi^2_{\alpha}$$

**Step 5:** Decision

Since  $\chi^2_{stat} = 48.4$  is larger than the upper critical bound of 6.635, reject  $H_0$ . There is enough evidence to conclude that there is significant difference between the proportions of males and females who buy clothing from their mobile devices at the 0.01 level of significance.

## 11.12

Use the following contingency table:

	A	B	C	Total
1	10	30	50	90
2	40	45	50	135
Total	50	75	100	225

a. Compute the expected frequency for each cell.

$$f_e = \frac{\text{rowtotal} \times \text{columntotal}}{n}$$

$$f_{11} = \frac{90 \times 50}{225} = 20 \quad f_{12} = \frac{90 \times 75}{225} = 30 \quad f_{13} = \frac{90 \times 100}{225} = 40$$
$$f_{21} = \frac{135 \times 50}{225} = 30 \quad f_{22} = \frac{135 \times 75}{225} = 45 \quad f_{23} = \frac{135 \times 100}{225} = 60$$

Totals for the observed and expected frequency are the same:

$$\sum f_o = 10 + 30 + 50 + 40 + 45 + 50 = 225$$

$$\sum f_e = 20 + 30 + 40 + 30 + 45 + 60 = 225$$

b. Compute  $\chi^2_{stat}$  - Is it significant at  $\alpha = 0.05$ ?

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

$$\chi^2_{stat} = \frac{(10 - 20)^2}{20} + \frac{(30 - 30)^2}{30} + \frac{(50 - 40)^2}{40} + \frac{(40 - 30)^2}{30} \\ + \frac{(45 - 45)^2}{45} + \frac{(50 - 60)^2}{60}$$

$$\chi^2_{stat} = 12.5$$

\*\*\*The critical value at ( $\alpha = 0.05$ ):

$$\chi^2_{\alpha, (r-1)(c-1)} = \chi^2_{0.05, (2-1)(3-1)} = 5.991$$

$$\chi^2_{stat} > \chi^2_{\alpha}$$

Then the result is significant at ( $\alpha = 0.05$ )

## 11.25

Where people look for news is different for various age groups. A study indicated where different age groups primarily get their news:

	AGE GROUP			
MEDIA	Under 36	36-50	50+	Total
Local TV	109	118	138	365
National TV	73	105	125	303
Radio	77	98	111	286
Local newspaper	52	78	101	231
Internet	93	87	75	255
Total	404	486	550	1440

At the 0.05 level of significance, is there evidence of a significant relationship between the age group and where people primarily get their news?

**Step 1:** state the hypothesis:

$H_0$ : The Age Group and Media are independent .

$H_{01}$ : The Age Group and Media are dependent.

**Step2:** The critical value at ( $\alpha = 0.05$ ):

$$\chi^2_{\alpha, (r-1)(c-1)} = \chi^2_{0.05, (5-1)(3-1)} = 15.507$$

Degree of freedom = (2)(4) = 8

**Step 3:** Compute the expected frequency for each cell.

$$f_e = \frac{\text{rowtotal} \times \text{columntotal}}{n}$$

$$\begin{aligned} f_{11} &= \frac{365 \times 404}{1440} = 102.40 & f_{12} &= \frac{365 \times 486}{1440} = 123.19 & f_{13} &= \frac{365 \times 550}{1440} = 139.41 \\ f_{21} &= \frac{303 \times 404}{1440} = 85.01 & f_{22} &= \frac{303 \times 486}{1440} = 102.26 & f_{23} &= \frac{303 \times 550}{1440} = 115.73 \\ f_{31} &= \frac{286 \times 404}{1440} = 80.24 & f_{32} &= \frac{286 \times 486}{1440} = 96.53 & f_{33} &= \frac{286 \times 550}{1440} = 109.24 \\ f_{41} &= \frac{231 \times 404}{1440} = 64.81 & f_{42} &= \frac{231 \times 486}{1440} = 77.96 & f_{43} &= \frac{231 \times 550}{1440} = 88.23 \\ f_{51} &= \frac{255 \times 404}{1440} = 71.57 & f_{52} &= \frac{255 \times 486}{1440} = 86.06 & f_{53} &= \frac{255 \times 550}{1440} = 97.39 \end{aligned}$$

Totals for the observed and expected frequency are the same:

$$\sum f_o = 1440 \quad \sum f_e = 1440$$

Find The test statistic.

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

$$\chi^2_{stat} = 19.34$$

**Step 4:** State the decision rule

$$\text{Reject } H_0 \text{ if } \chi^2_{stat} > \chi^2_{\alpha}$$

**Step 5:** Decision

$$\chi^2 = 19.34 > \chi^2_{\alpha} = 15.507$$

Reject  $H_0$ , That means that *The Age Group and Media are dependent*