Chapter 7
Energy and Energy Transfer

Energy Concept

Most important concept in science and engineering.

Fuel for transport, heating, electricity, food .

Newton's laws enables us of solving many problems, however, yet there is a wide range of problems that can not be solved easily using such laws.

When the force acting on object is not constant (hence the net acceleration).

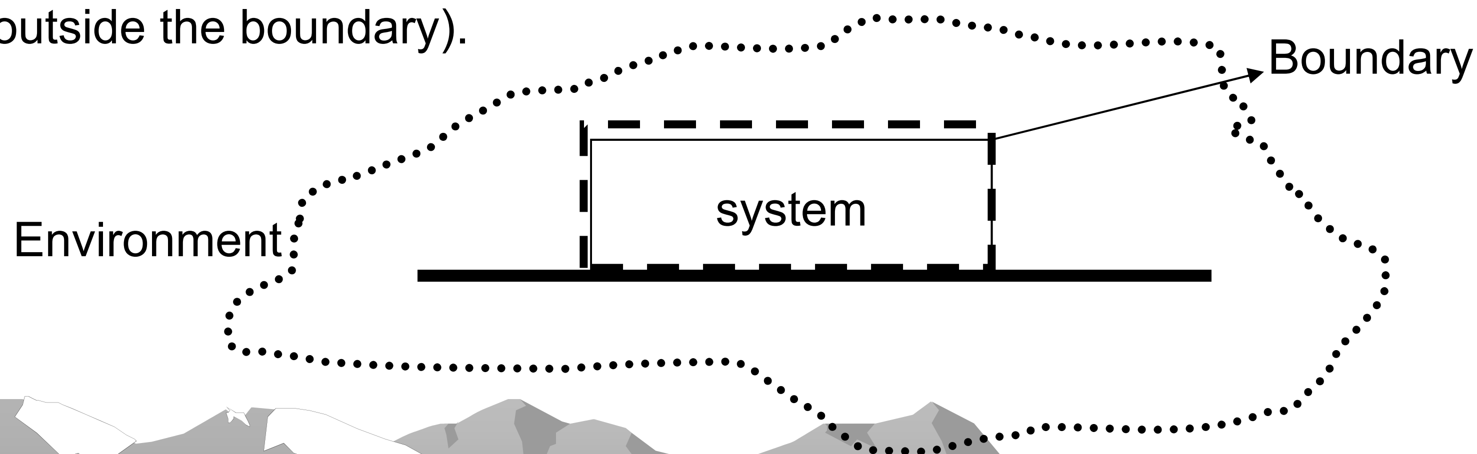
The previous approach is called particle approach, while the approach developed here is a system approach.

Systems and Environment

System is a part from the universe that satisfy the following:

- be a single object or particle
- be a collection of objects or particles
- be a region of space (such as the interior of an automobile engine combustion cylinder)
- vary in size and shape (such as a rubber ball, which deforms upon striking a wall)

Any system has an imaginary boundary that divides the universe to System (inside the boundary) and environment (outside the boundary).

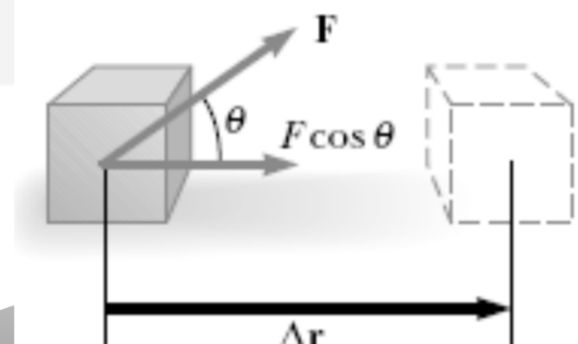


Work Done By a Constant Force

(a) (b) (c)

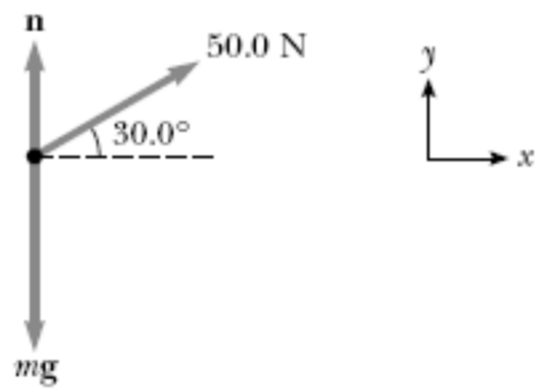
The **work** W done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and cos θ, where θ is the angle between the force and displacement vectors:

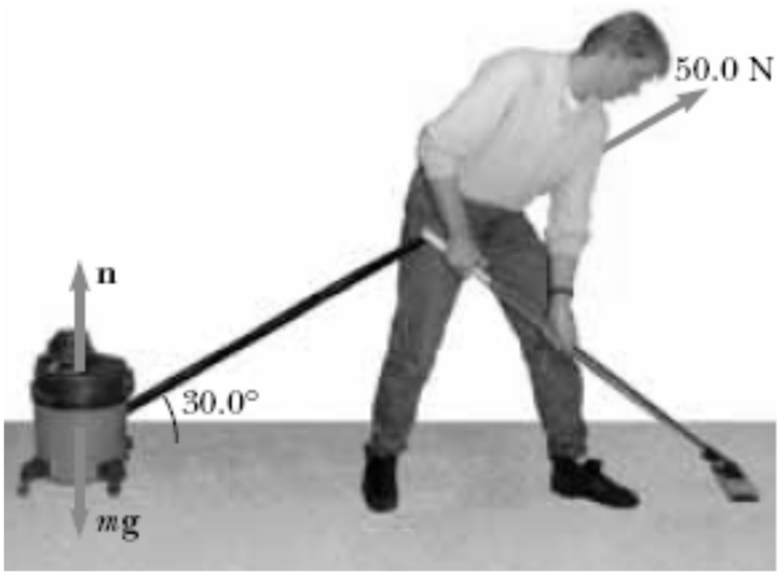
$$W \equiv F \Delta r \cos\theta$$



Example

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.5a). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.





$$W = F\Delta r\cos\theta = (50.0\ \text{N})(3.00\ \text{m})(\cos 30.0°)$$

$$= 130\ \text{N}\cdot\text{m} = 130\ \text{J}$$

Work done by a variable force

We cannot evaluate the work by using $W = F\Delta r\cos\theta$

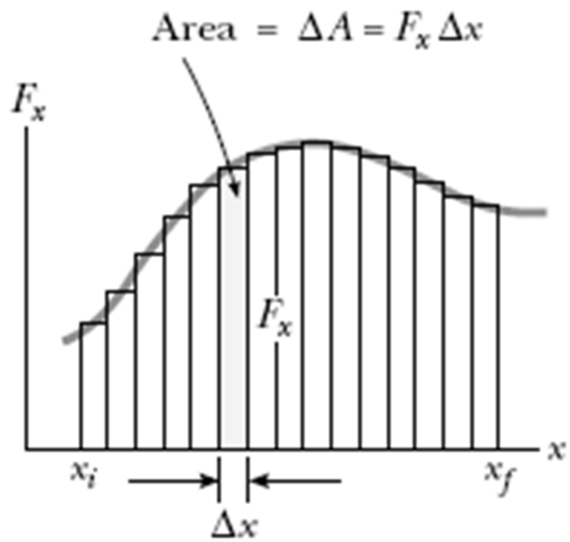
Because the force F is not constant during the displacement Δr.

Consider a very small displacement Δx, It can be assumed safely that the force can be approximated as a constant value during the small displacement, the work for this small displacement is

$$W \approx F_x \Delta x$$

we can divide xi to xf displacement to small displacements as shown in the figure, To find the total work take the summation

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x \qquad \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx \qquad W = \int_{x_i}^{x_f} F_x dx$$

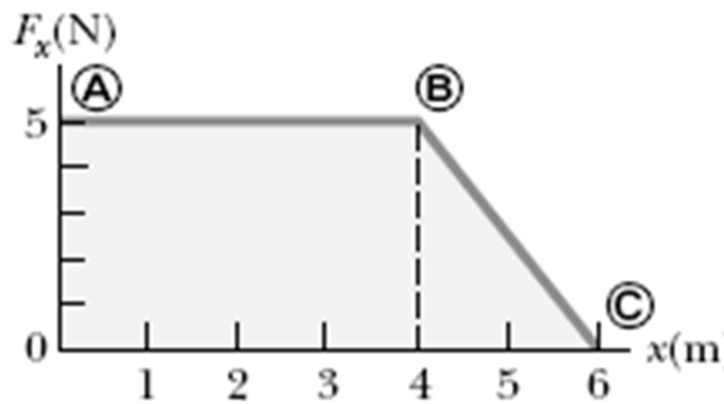


<u>Important note: the force and the displacement are in the same direction here (all in the positive x direction)</u>

Example

A force acting on a particle varies with x, as shown in Figure 7.8. Calculate the work done by the force as the particle moves from $x = 0$ to $x = 6.0$ m.

$$W = \int_{x_i}^{x_f} F_x \, dx$$



Or the work = area under the Fx graph
= area of the rectangle and triangle
= 5 x 4 + 0.5 x 5 x 2
= 25 J

Work done by a spring

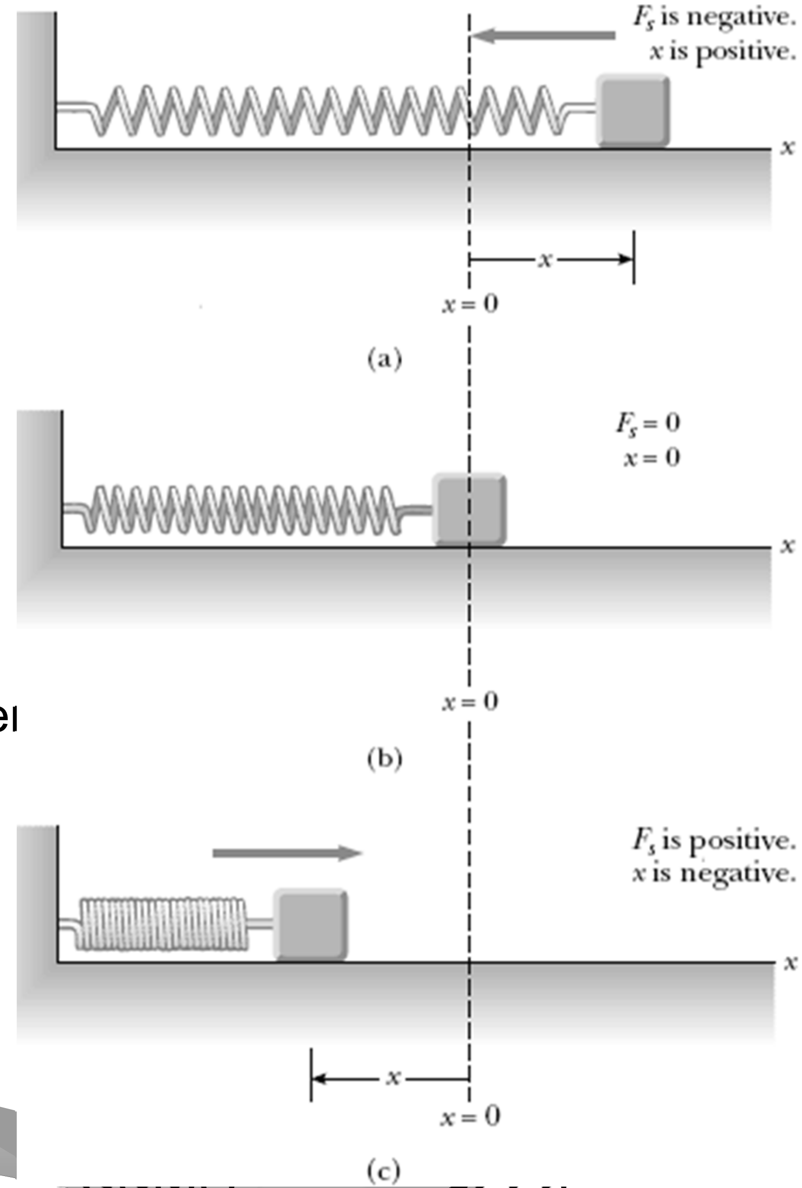
Common example for a force varies with position

$$F_s = -kx$$

Consider the work done by a spring from

$x_i = -x_{max}$ to $x_f = 0$

$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{-x_{max}}^{0} (-kx)\,dx = \frac{1}{2}kx_{max}^2$$

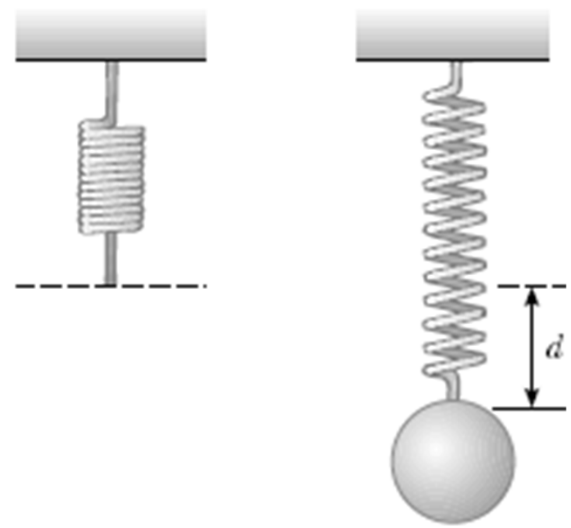


And generally the work done by the spring experie
displacement from x_i to x_f

$$W_s = \int_{x_i}^{x_f} (-kx)\,dx = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

Example

A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure 7.12. The spring is hung vertically, and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position.



(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

(B) How much work is done by the spring as it stretches through this distance?

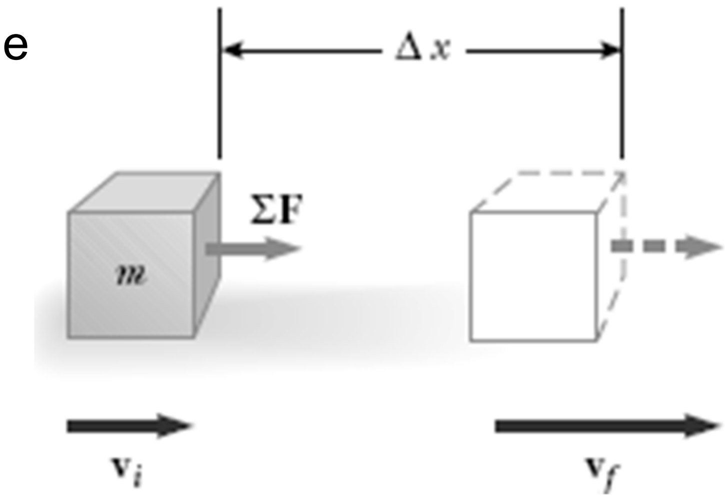
$$k = \frac{mg}{d} = \frac{(0.55\ \text{kg})(9.80\ \text{m/s}^2)}{2.0 \times 10^{-2}\ \text{m}} = \boxed{2.7 \times 10^2\ \text{N/m}}$$

$$W_s = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}(2.7 \times 10^2\ \text{N/m})(2.0 \times 10^{-2}\ \text{m})^2$$

$$= \boxed{-5.4 \times 10^{-2}\ \text{J}}$$

Kinetic Energy and Work-Kinetic Energy Theorem

When doing work, the speed of an object could change



$$\Delta \mathbf{r} = \Delta x \hat{\mathbf{i}} = (x_f - x_i)\,\hat{\mathbf{i}}$$

Net force

$$\sum W = \int_{x_i}^{x_f} \left(\sum F\right) dx$$

$$\sum W = \int_{x_i}^{x_f} ma\,dx = \int_{x_i}^{x_f} m\,\frac{dv}{dt}\,dx = \int_{x_i}^{x_f} m\,\frac{dv}{dx}\,\frac{dx}{dt}\,dx = \int_{v_i}^{v_f} mv\,dv$$

$$\sum W = \tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2}$$

The kinetic energy (K) is defined as $K \equiv \frac{1}{2}mv^2$

$$\sum W = K_f - K_i = \Delta K$$

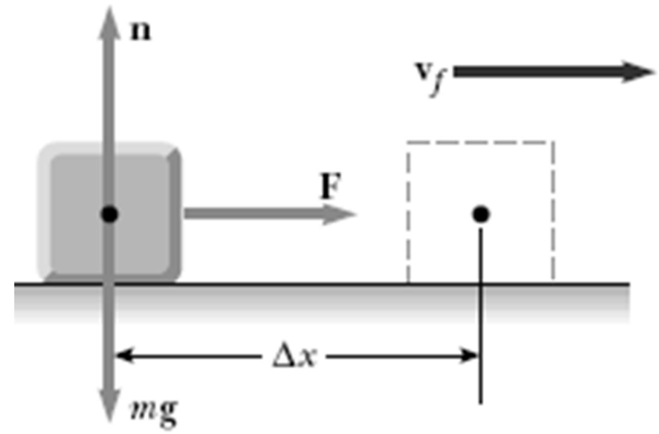
Example

A 6.0-kg block initially at rest is pulled to the right along a horizontal, frictionless surface by a constant horizontal force of 12 N. Find the speed of the block after it has moved 3.0 m.

$$W = F\Delta x = (12\,\text{N})(3.0\,\text{m}) = 36\,\text{J}$$

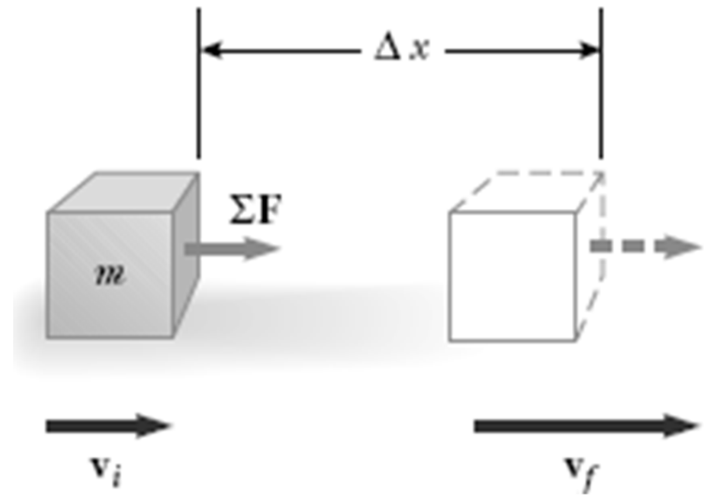
$$W = K_f - K_i = \tfrac{1}{2}mv_f^2 - 0$$

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(36\,\text{J})}{6.0\,\text{kg}}} = 3.5\,\text{m/s}$$



Power

Power is the time rate of energy transferred (work).



The average power is $$\overline{\mathcal{P}} \equiv \frac{W}{\Delta t}$$

And the instantaneous power is

$$\mathcal{P} \equiv \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

$$\mathcal{P} = \frac{dW}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}$$

The unit of power is Watt, sometimes horsepower (hp) is used   1 hp = 746 W

kWh is a unit of energy = 1000 x 3600 J = 3.6 x 10^6 J

Example

An elevator car has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion upward, as shown in Figure 7.19a.

(A) What power delivered by the motor is required to lift the elevator car at a constant speed of 3.00 m/s?

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of 1.00 m/s^2?

$$\sum F_y = T - f - Mg = 0$$

$$\begin{aligned} T &= f + Mg \\ &= 4.00 \times 10^3\,\text{N} + (1.80 \times 10^3\,\text{kg})(9.80\,\text{m/s}^2) \\ &= 2.16 \times 10^4\,\text{N} \end{aligned}$$

$$\mathcal{P} = \mathbf{T}\cdot\mathbf{v} = Tv$$

$$= (2.16 \times 10^4\,\text{N})(3.00\,\text{m/s}) = 6.48 \times 10^4\,\text{W}$$

$$\sum F_y = T - f - Mg = Ma$$

$$\begin{aligned} T &= M(a + g) + f \\ &= (1.80 \times 10^3\,\text{kg})(1.00\,\text{m/s}^2 + 9.80\,\text{m/s}^2) \\ &\quad + 4.00 \times 10^3\,\text{N} \\ &= 2.34 \times 10^4\,\text{N} \end{aligned}$$

$$\mathcal{P} = Tv = (2.34 \times 10^4\,\text{N})v$$

$$\mathcal{P} = (2.34 \times 10^4\,\text{N})(3.00\,\text{m/s}) = 7.02 \times 10^4\,\text{W}$$