Types of Strain

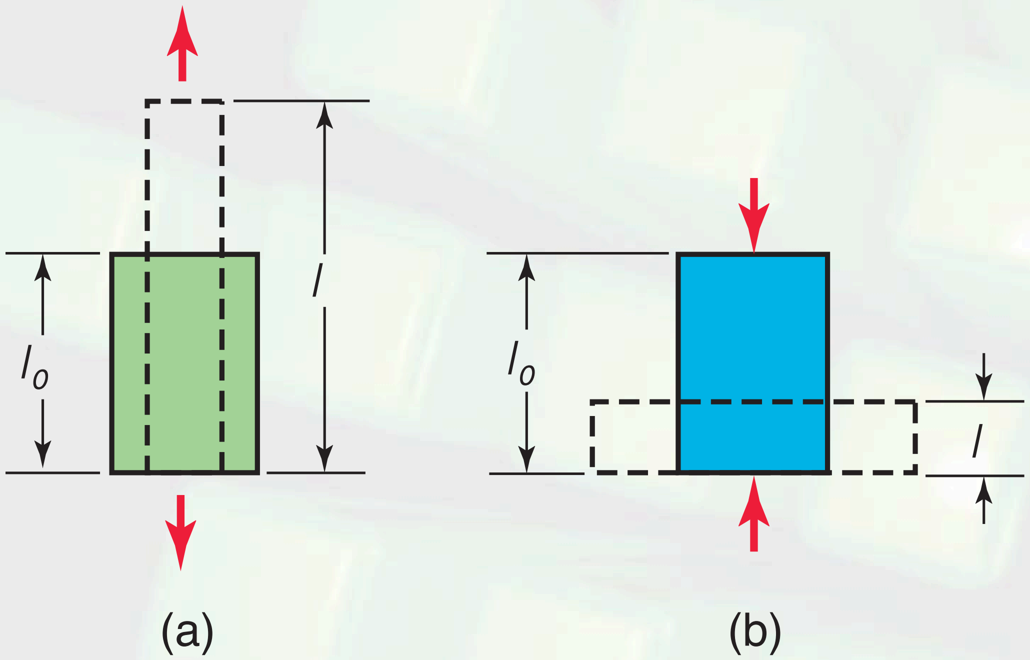

FIGURE 2.1 Types of strain. (a) Tensile. (b) Compressive. (c) Shear. All deformation processes in manufacturing involve strains of these types. Tensile strains are involved in stretching sheet metal to make car bodies, compressive strains in forging metals to make turbine disks, and shear strains in making holes by punching.

Engineering Strain:

$$e = \frac{l - l_o}{l_o}$$

Shear Strain:

$$\gamma = \frac{a}{b}$$

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$$\varepsilon_{ij} = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{yx} & \varepsilon_{zx} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{zy} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{vmatrix}.$$

Elastic stress strain relations are expressed by Hooke's laws as

$$e_x = (1/E)[\sigma_x - \nu(\sigma_y + \sigma_z)],$$
$$e_y = (1/E)[\sigma_y - \nu(\sigma_z + \sigma_x)],$$
$$e_z = (1/E)[\sigma_z - \nu(\sigma_x + \sigma_y)],$$

$$\gamma_{yz} = (1/G)\tau_{yz},$$
$$\gamma_{zx} = (1/G)\tau_{zx},$$
$$\gamma_{xy} = (1/G)\tau_{xy},$$

$$E = 2G(1 + \nu), \text{ or}$$
$$G = E/[2(1 + \nu)].$$

For an aluminum sheet under plane stress loading εx = 0.003 and εy = 0.001.
Assuming that E = 68 GPa and ν = 0.30, find εz .

A piece of steel is elastically loaded under principal stresses σ1 = 300MPa, σ2 = 250MPa and, σ3 = −200MPa. Assuming that E=205 GPa and v = 0.29 find the stored elastic energy per volume.

A slab of metal is subjected to plane-strain deformation (e2 = 0) such that σ1 = 40 ksi and σ3 = 0. Assume that the loading is elastic and that E = 205 GPa and v = 0.29 (note the mixed units).
a) Find the three normal strains.
b) Find the strain energy per volume.

True Stress and True Strain

True stress

$$\sigma = \frac{P}{A}$$

True strain

$$\varepsilon = \ln\left(\frac{l}{l_o}\right) = \ln\left(\frac{A_o}{A}\right) = \ln\left(\frac{D_o}{D}\right)^2 = 2\ln\left(\frac{D_o}{D}\right)$$

TABLE 2.2 Comparison of engineering and true strains in tension

e	0.01	0.05	0.1	0.2	0.5	1	2	5	10
ϵ	0.01	0.049	0.095	0.18	0.4	0.69	1.1	1.8	2.4

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Power Law Flow Rule

Material	K (MPa)	n
Aluminum, 1100-O	180	0.20
2024-T4	690	0.16
5052-O	210	0.13
6061-O	205	0.20
6061-T6	410	0.05
7075-O	400	0.17
Brass, 7030, annealed	895	0.49
85-15, cold rolled	580	0.34
Bronze (phosphor), annealed	720	0.46
Cobalt-base alloy, heat treated	2070	0.50
Copper, annealed	315	0.54
Molybdenum, annealed	725	0.13
Steel, low carbon, annealed	530	0.26
1045 hot rolled	965	0.14
1112 annealed	760	0.19
1112 cold rolled	760	0.08
4135 annealed	1015	0.17
4135 cold rolled	1100	0.14
4340 annealed	640	0.15
17-4 P-H, annealed	1200	0.05
52100, annealed	1450	0.07
304 stainless, annealed	1275	0.45
410 stainless, annealed	960	0.10

Note: 100 MPa = 14,500 psi.

TABLE 2.3 Typical values for K and n in Eq. 2.11 at room temperature.

Flow rule:

$$\sigma = K\varepsilon^n$$

K = Strength coefficient

n = Strain hardening exponent

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Idealized Stress-Strain Curves

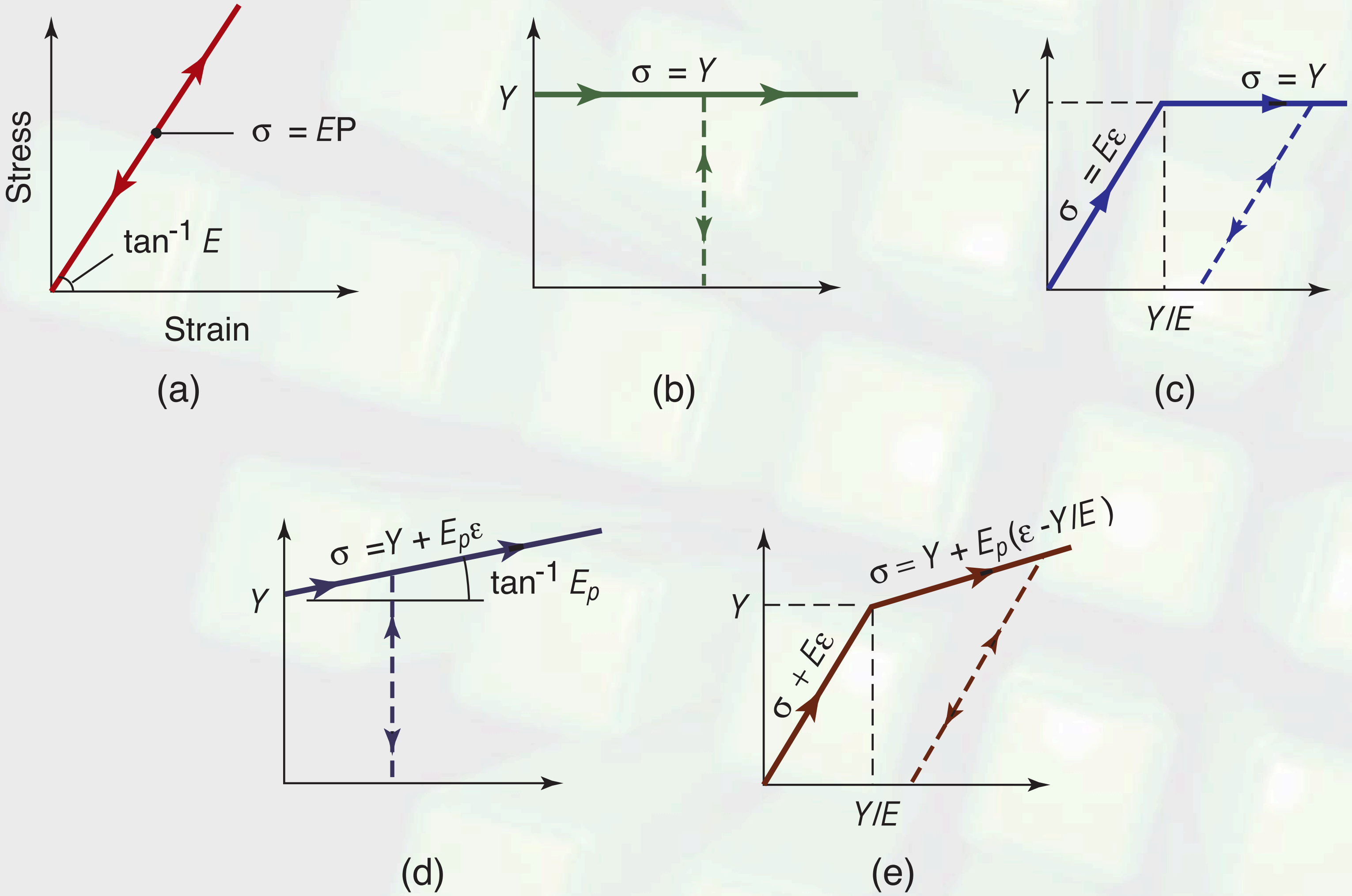

FIGURE 2.7 Schematic illustration of various types of idealized stress-strain curves. (a) Perfectly elastic. (b) Rigid, perfectly plastic. (c) Elastic, perfectly plastic. (d) Rigid, linearly strain hardening. (e) Elastic, linearly strain hardening. The broken lines and arrows indicate unloading and reloading during the test.

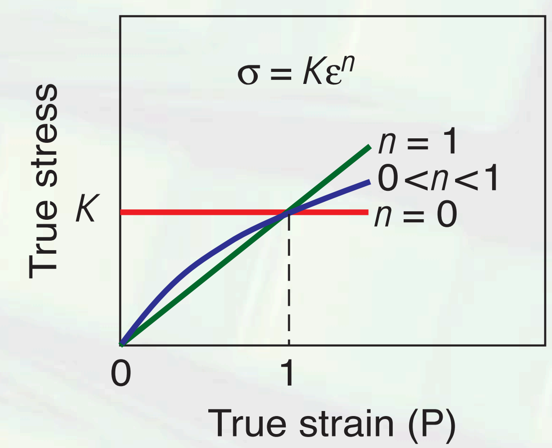

FIGURE 2.8 The effect of strain-hardening exponent n on the shape of true stress-true strain curves. When n = 1, the material is elastic, and when n = 0, it is rigid and perfectly plastic.

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Temperature and Strain Rate Effects

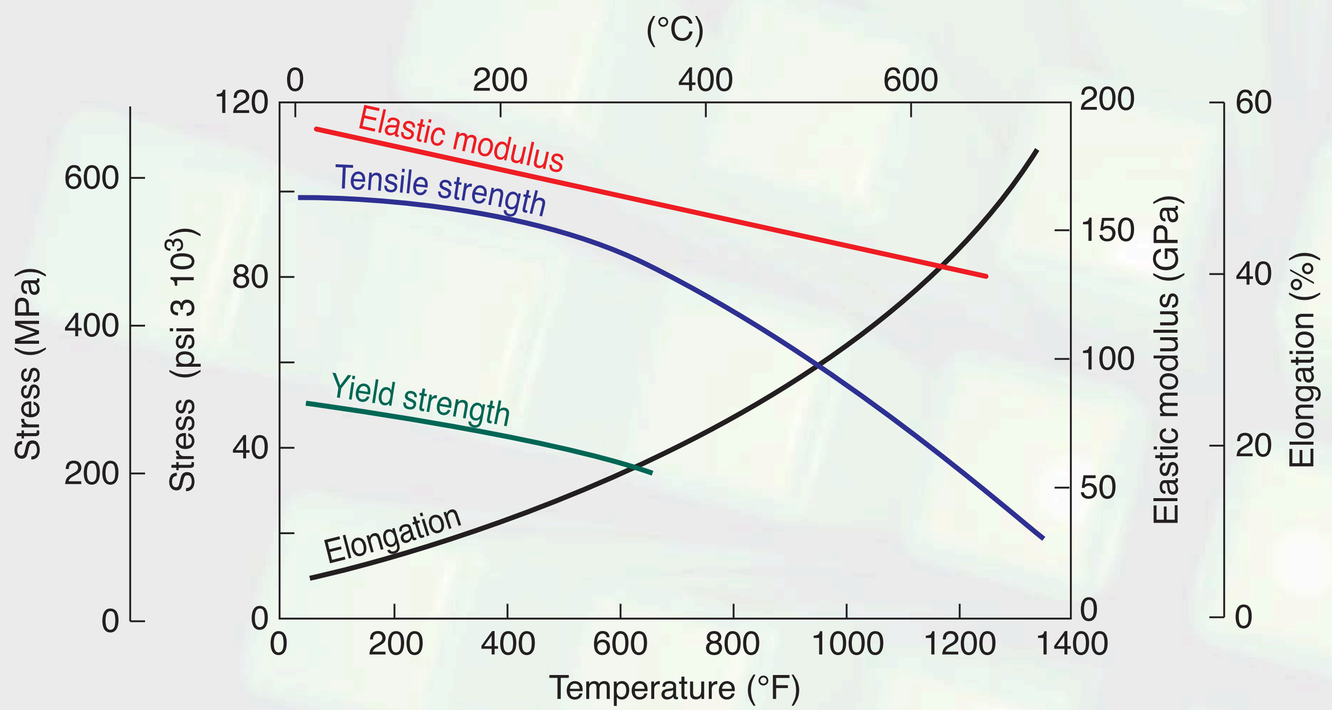

FIGURE 2.9 Effect of temperature on mechanical properties of a carbon steel. Most materials display similar temperature sensitivity for elastic modulus, yield strength, ultimate strength, and ductility.

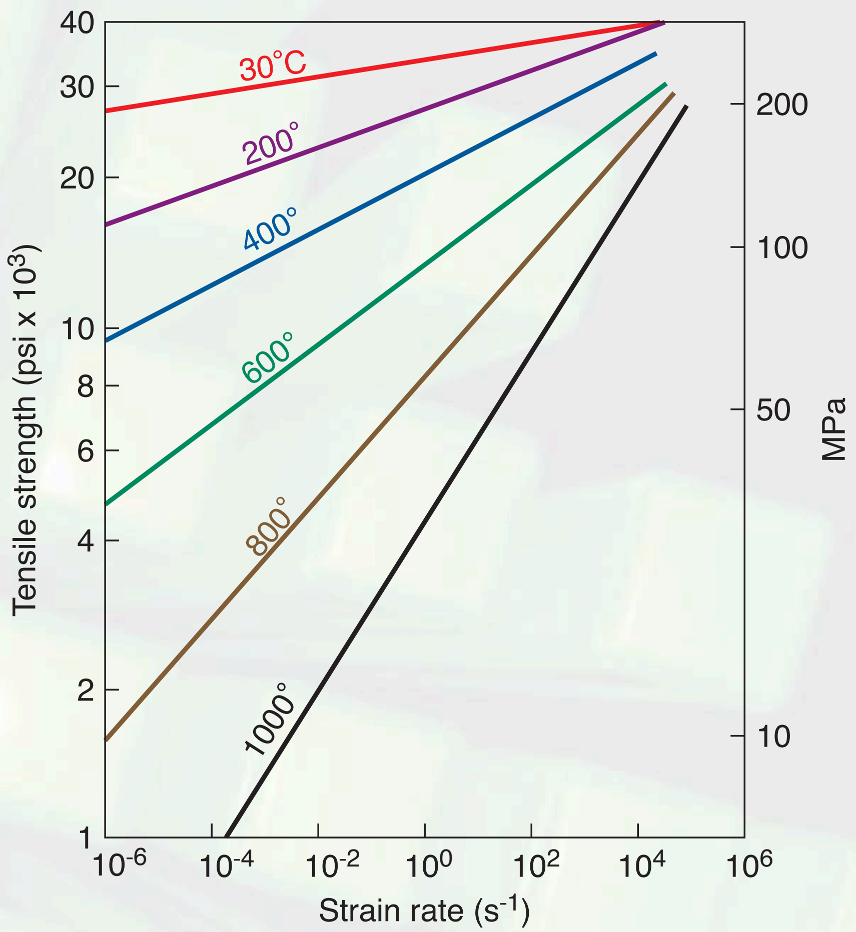

FIGURE 2.10 The effect of strain rate on the ultimate tensile strength of aluminum. Note that as temperature increases, the slope increases. Thus, tensile strength becomes more and more sensitive to strain rate as temperature increases. *Source*: After J. H. Hollomon.

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Typical Strain Rates in Metalworking

Process	True Strain	Deformation Speed (m/s)	Strain Rate (s^{-1})
Cold Working			
Forging, rolling	0.1-0.5	0.1-100	$1 - 10^3$
Wire and tube drawing	0.05-0.5	0.1-100	$1 - 10^4$
Explosive forming	0.05-0.2	10-100	$10 - 10^5$
Hot working and warm working			
Forging, rolling	0.1-0.5	0.1-30	$1 - 10^3$
Extrusion	2-5	0.1-1	$10^{-1} - 10^2$
Machining	1-10	0.1-100	$10^3 - 10^6$
Sheet-metal forming	0.1-0.5	0.05-2	$1 - 10^2$
Superplastic forming	0.2-3	$10^{-4} - 10^{-2}$	$10^{-4} - 10^{-2}$

TABLE 2.4 Typical ranges of strain, deformation speed, and strain rates in metalworking processes.

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Strain Rate Effects

Material	Temperature, °C	C: psi × 10^3	C: MPa	m
Aluminum	200-500	12-2	82-14	0.07-0.23
Aluminum alloys	200-500	45-5	310-35	0-0.20
Copper	300-900	35-3	240-20	0.06-0.17
Copper alloys (brasses)	200-800	60-2	415-14	0.02-0.3
Lead	100-300	1.6-0.3	11-2	0.1-0.2
Magnesium	200-400	20-2	140-14	0.07-0.43
Steel				
Low carbon	900-1200	24-7	165-48	0.08-0.22
Medium carbon	900-1200	23-7	160-48	0.07-0.24
Stainless	600-1200	60-5	415-35	0.02-0.4
Titanium	200-1000	135-2	930-14	0.04-0.3
Titanium alloys	200-1000	130-5	900-35	0.02-0.3
Ti-6Al-4V*	815-930	9.5-1.6	65-11	0.50-0.80
Zirconium	200-1000	120-4	830-27	0.04-0.4

* at a strain rate of 2×10^{-4} s^{-1}.

Note: As temperature increases, C decreases and m increases. As strain increases, C increases and m may increase or decrease, or it may become negative within certain ranges of temperature and strain.

Source: After T. Altan and F.W. Boulger.

TABLE 2.5 Approximate range of values for C and m in Eq. (2.16) for various annealed metals at true strains ranging from 0.2 to 1.0.

$$\sigma = C\dot{\varepsilon}^m$$

C = Strength coefficient

m = Strain-rate sensitivity exponent

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Effect of Strain Rate Sensitivity on Elongation

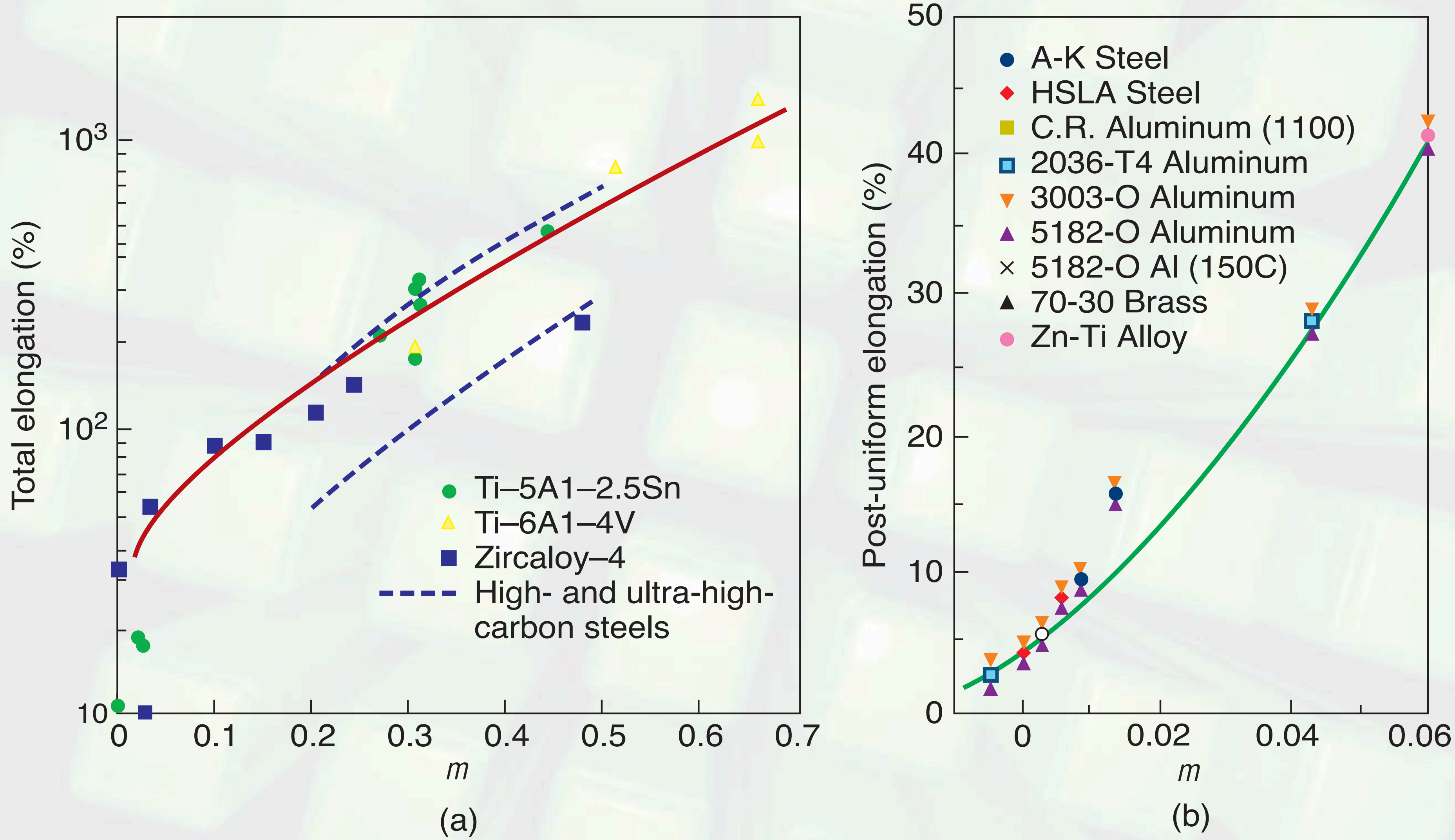

FIGURE 2.12 (a) The effect of strain-rate sensitivity exponent *m* on the total elongation for various metals. Note that elongation at high values of *m* approaches 1000%. *Source:* After D. Lee and W.A. Backofen. (b) The effect of strain-rate sensitivity exponent *m* on the post uniform (after necking) elongation for various metals. *Source:* After A.K. Ghosh.

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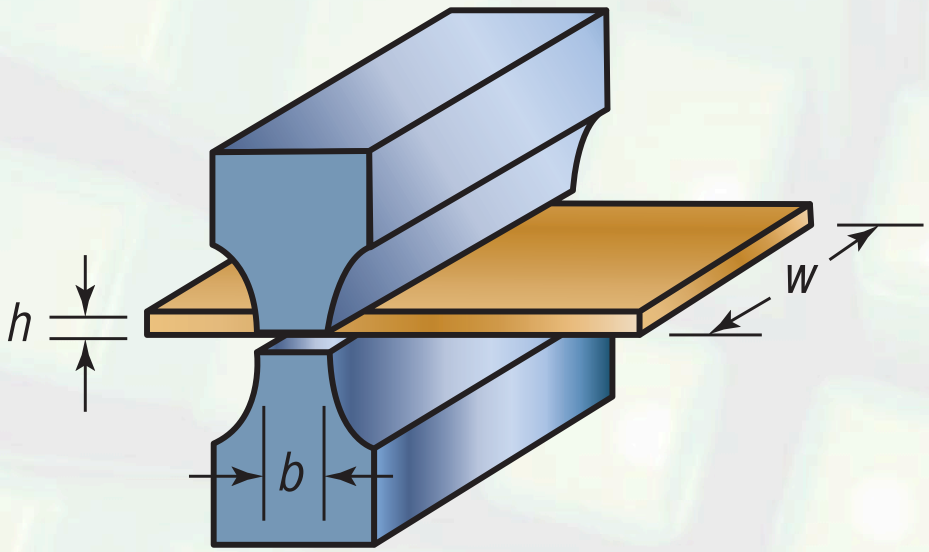
Plane-Strain Compression Test

$w > 5h$

$w > 5b$

$b > 2h$ to $4h$



Yield stress in plane strain:

$$Y' = \frac{2}{\sqrt{3}}Y = 1.15Y$$

FIGURE 2.15 Schematic illustration of the plane-strain compression test. The dimensional relationships shown should be satisfied for this test to be useful and reproducible. This test gives the yield stress of the material in plane strain, *Y'*. *Source:* After A. Nadai and H. Ford.

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Tension & Compression; Baushinger Effect

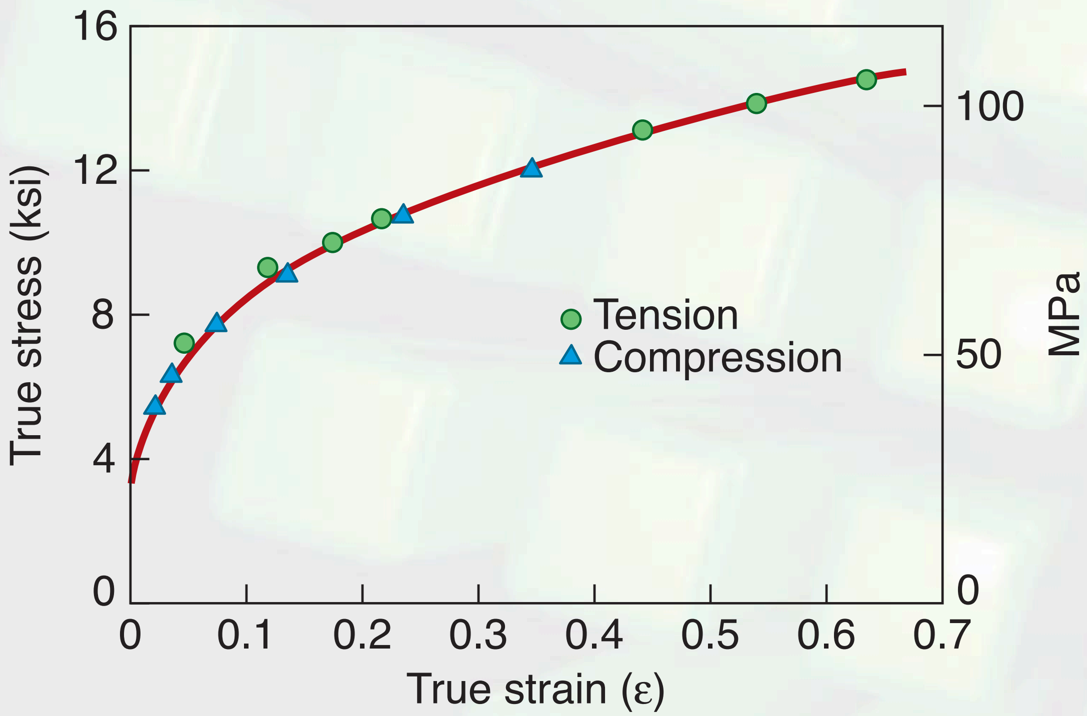

FIGURE 2.16 True stress-true strain curve in tension and compression for aluminum. For ductile metals, the curves for tension and compression are identical. *Source:* After A.H. Cottrell.

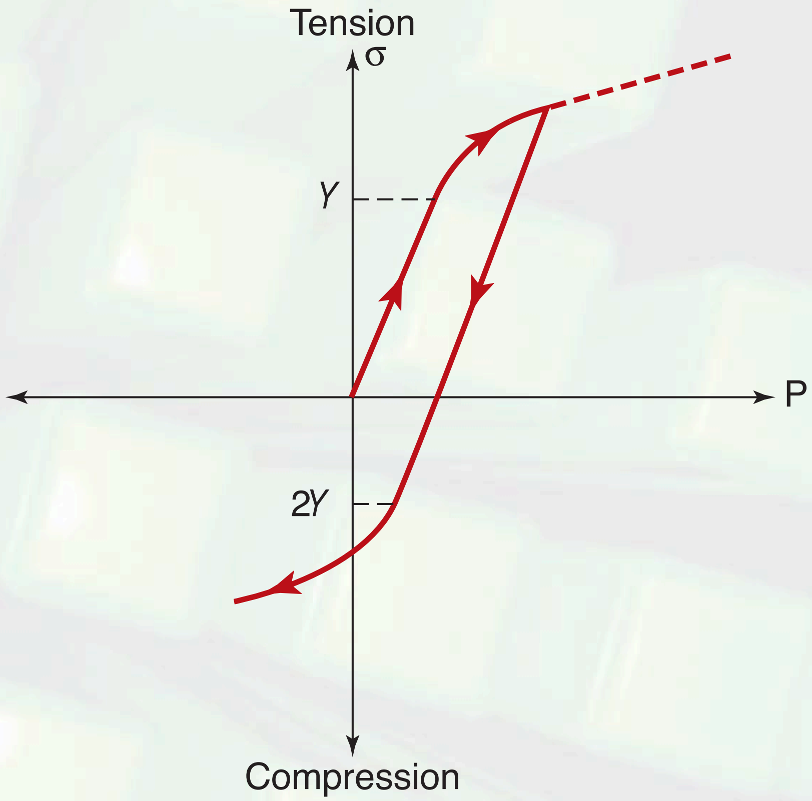

FIGURE 2.17 Schematic illustration of the Bauschinger effect. Arrows show loading and unloading paths. Note the decrease in the yield stress in compression after the specimen has been subjected to tension. The same result is obtained if compression is applied first, followed by tension, whereby the yield stress in tension decreases.

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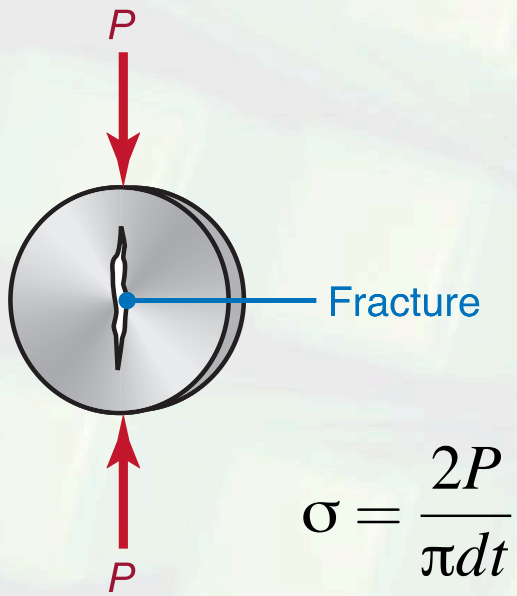
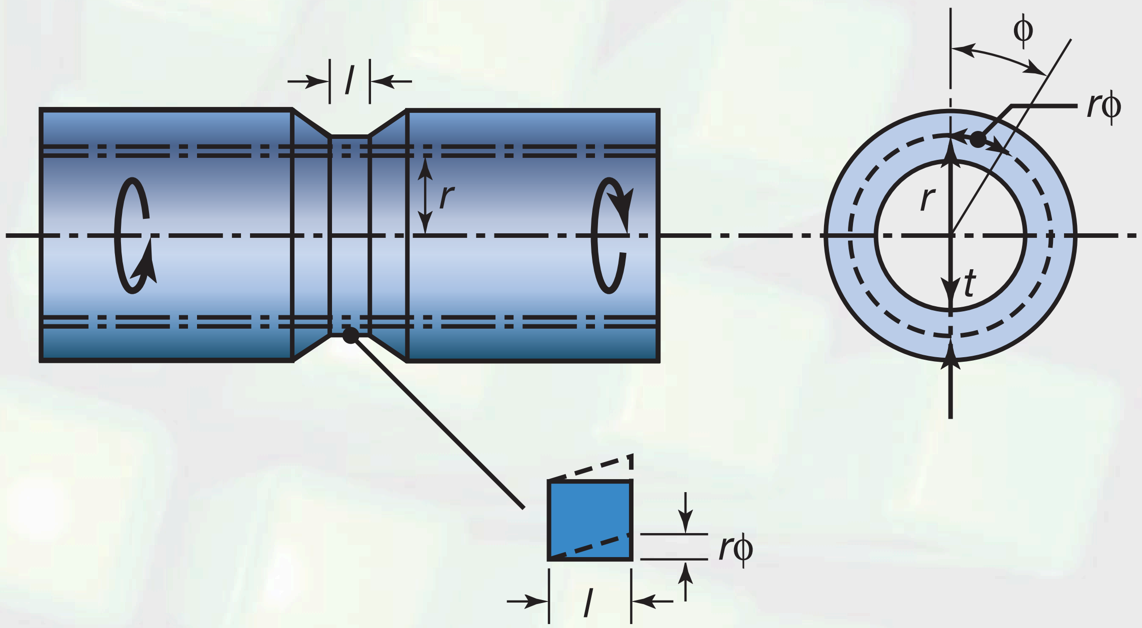
Disk & Torsion Tests



$$\sigma = \frac{2P}{\pi d t}$$

FIGURE 2.18 Disk test on a brittle material, showing the direction of loading and the fracture path. This test is useful for brittle materials, such as ceramics and carbides.



$$\tau = \frac{T}{2\pi r^2 t} \qquad \gamma = \frac{r\phi}{l}$$

FIGURE 2.19 A typical torsion-test specimen. It is mounted between the two heads of a machine and is twisted. Note the shear deformation of an element in the reduced section.

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Simple vs. Pure Shear

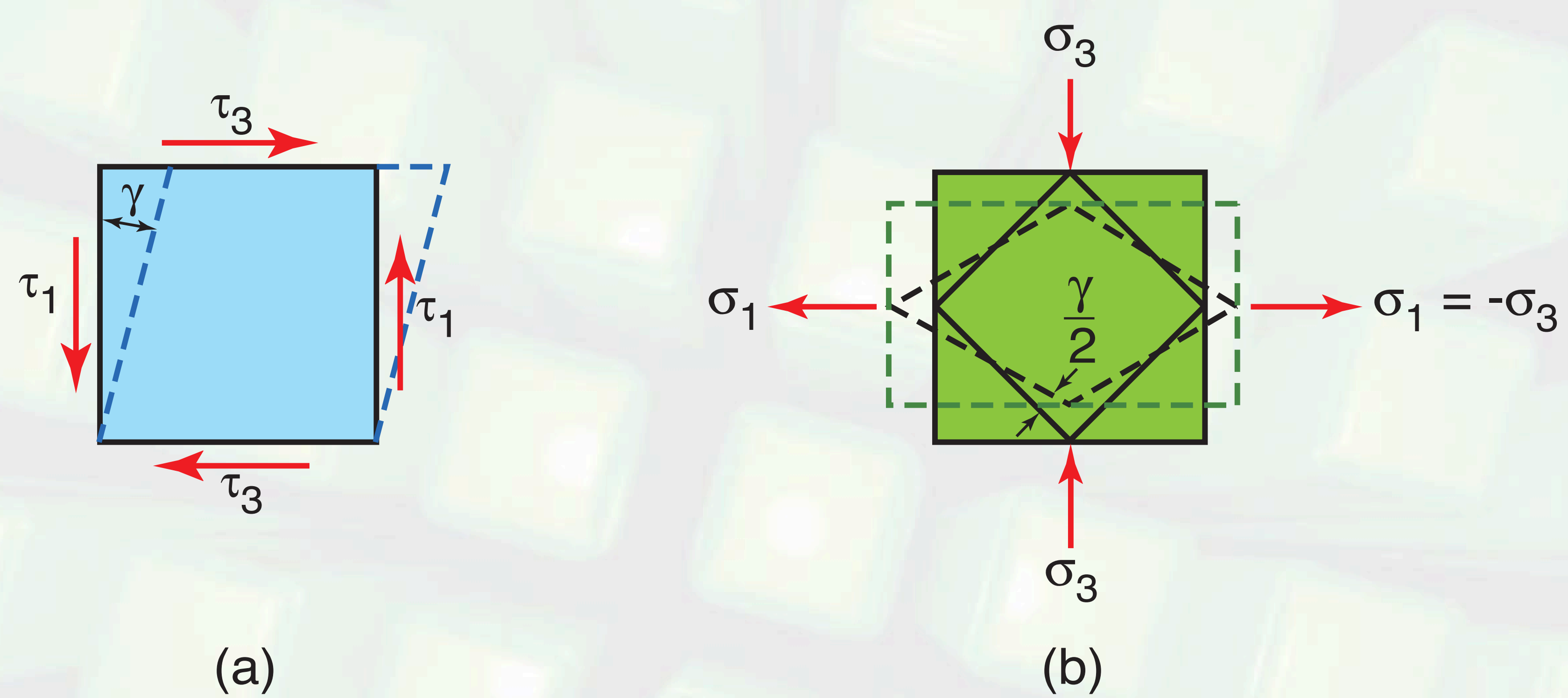

FIGURE 2.20 Comparison of (a) simple shear and (b) pure shear. Note that simple shear is equivalent to pure shear plus a rotation.

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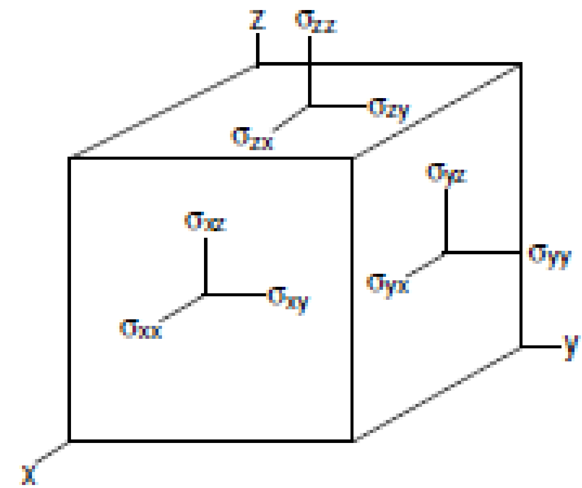
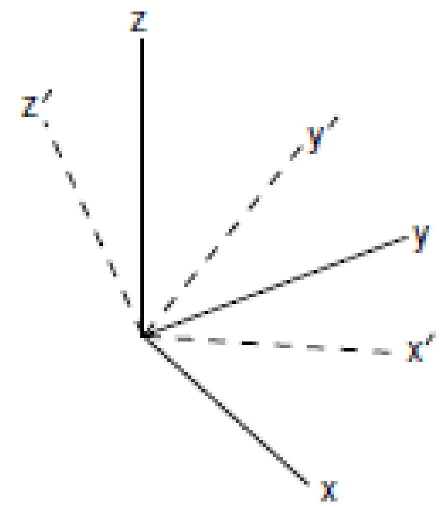
Stress Componenets

$$\sigma_{ij} = \begin{vmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix},$$



Nine components of stress acting on an infinitesimal element.

Consider transforming stresses from the *x*, *y*, *z* axis system to the *x*, *y*, *z* system shown in Figure



$$\sigma_{x'} = \ell_{x'x}^2\sigma_x + \ell_{x'y}^2\sigma_y + \ell_{x'z}^2\sigma_z + 2\ell_{x'y}\ell_{x'z}\tau_{yz} + 2\ell_{x'z}\ell_{x'x}\tau_{zx} + 2\ell_{x'x}\ell_{x'y}\tau_{xy}$$

and

$$\tau_{x'y'} = \ell_{x'x}\ell_{y'x}\sigma_x + \ell_{x'y}\ell_{y'y}\sigma_y + \ell_{x'z}\ell_{y'z}\sigma_z + (\ell_{x'y}\ell_{y'z} + \ell_{x'z}\ell_{y'y})\tau_{yz} + (\ell_{x'z}\ell_{y'x} + \ell_{x'x}\ell_{y'z})\tau_{zx} + (\ell_{x'x}\ell_{y'y} + \ell_{x'y}\ell_{y'x})\tau_{xy}.$$

Here, the term ℓ_{im} is the cosine of the angle between the *i* and *m* axes and the term ℓ_{jn} is the cosine of the angle between the *j* and *n* axes. This is often written as

PRINCIPAL STRESSES

It is always possible to find a set of axes along which the shear stress terms vanish. In this case $\sigma 1$, $\sigma 2$, and $\sigma 3$ are called the principal stresses. The magnitudes of the principal stresses, σ p, are the roots of

$$\sigma_p^3 - I_1\sigma_p^2 - I_2\sigma_p - I_3 = 0,$$

where I_1, I_2, and I_3 are called the *invariants* of the stress tensor. They are

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz},$$
$$I_2 = \sigma_{yz}^2 + \sigma_{zx}^2 + \sigma_{xy}^2 - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} - \sigma_{xx}\sigma_{yy}, \quad \text{and}$$
$$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{yz}\sigma_{zx}\sigma_{xy} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{zx}^2 - \sigma_{zz}\sigma_{xy}^2.$$

and can be expressed in terms of the principal stresses, they are

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3,$$
$$I_2 = -\sigma_2\sigma_3 - \sigma_3\sigma_1 - \sigma_1\sigma_2, \quad \text{and}$$
$$I_3 = \sigma_1\sigma_2\sigma_3.$$

EXAMPLE : Consider a stress state with $\sigma_x = 70$MPa, $\sigma_y = 35$MPa, $\tau_{xy} = 20$, $\sigma_z = \tau_{zx} = \tau_{yz} = 0$. Find the principal stresses.

SOLUTION: Using, $I_1 = 105$ MPa, $I_2 = –2050$ MPa, $I_3 = 0$. From The principal stresses are the roots $\sigma_1 = 79.1$MPa, $\sigma_2 = 25.9$MPa, and $\sigma_3 = \sigma_z = 0$.

$$\sigma_p^3 - 105\sigma_p^2 + 2050\sigma_p + 0 = 0,$$

$$\sigma_p^2 - 105\sigma_p + 2{,}050 = 0.$$

The principal stresses are the roots $\sigma_1 = 79.1$MPa, $\sigma_2 = 25.9$MPa, and $\sigma_3 = \sigma_z = 0$.

Determine the principal stresses for the stress state

$$\sigma_{ij} = \begin{vmatrix} 10 & -3 & 4 \\ -3 & 5 & 2 \\ 4 & 2 & 7 \end{vmatrix} .$$

A 5-cm diameter solid shaft is simultaneously subjected to an axial load of 80 kN and a torque of 400 Nm.

a) Determine the principal stresses at the surface assuming elastic behavior.

b) Find the largest shear stress.

State of Stress in Metalworking

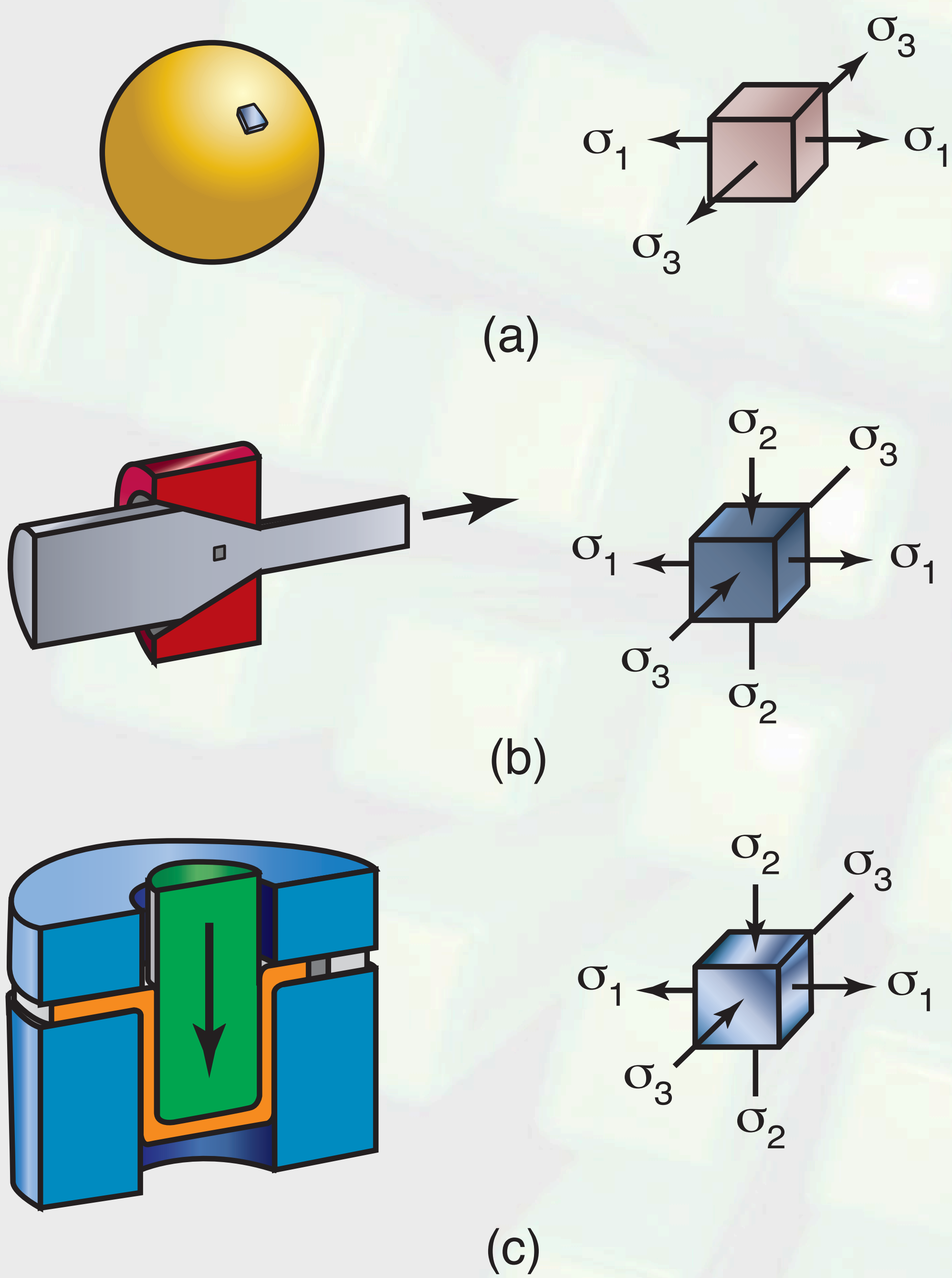


FIGURE 2.33 The state of stress in various metalworking operations. (a) Expansion of a thin-walled spherical shell under internal pressure. (b) Drawing of round rod or wire through a conical die to reduce its diameter; see Section 6.5 (c) Deep drawing of sheet metal with a punch and die to make a cup; see Section 7.6.

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States of Stress

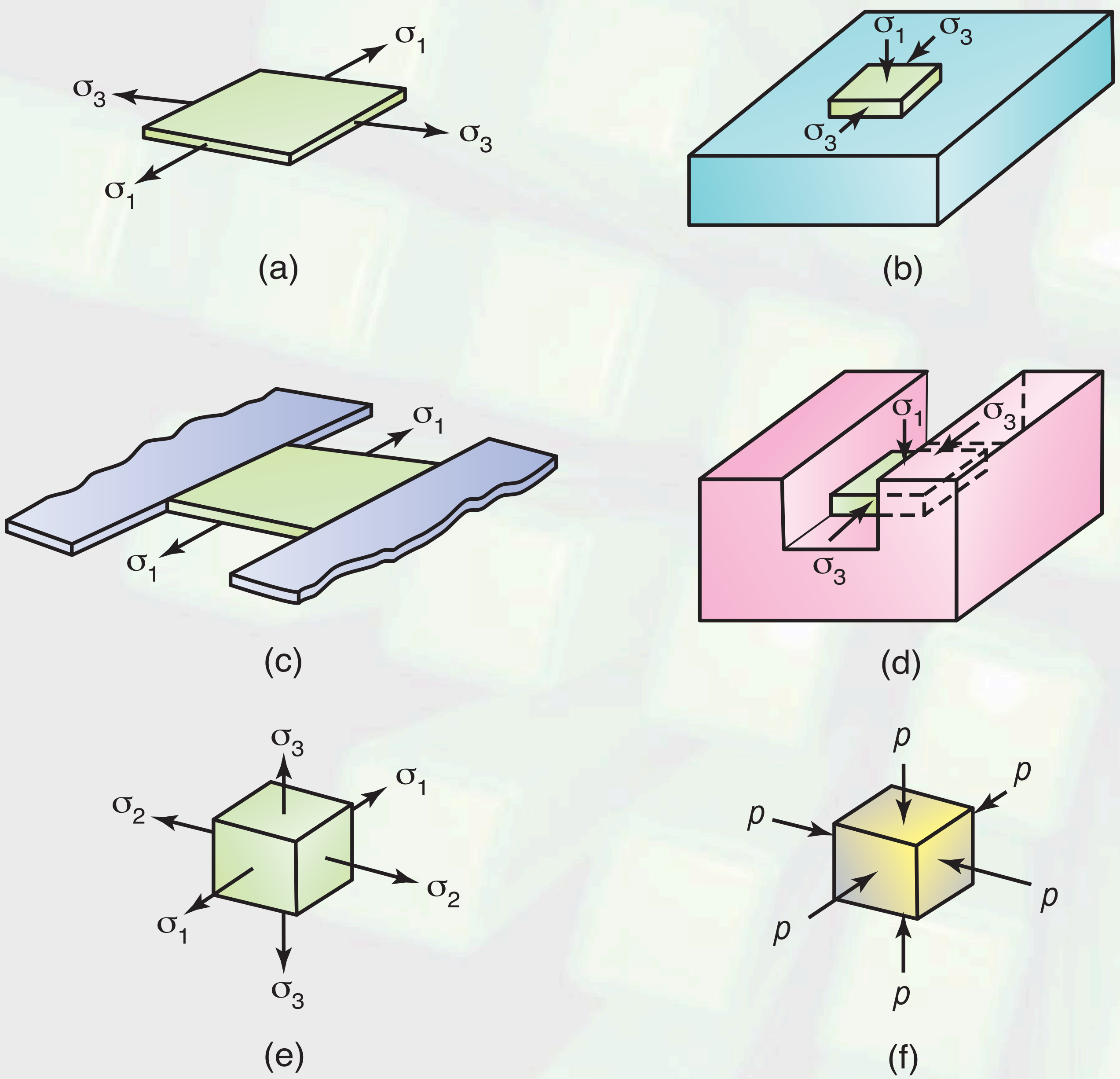

FIGURE 2.35 Examples of states of stress: (a) plane stress in sheet stretching; there are no stresses acting on the surfaces of the sheet. (b) plane stress in compression; there are no stresses acting on the sides of the specimen being compressed. (c) plane strain in tension; the width of the sheet remains constant while being stretched. (d) plane strain in compression (see also Fig. 2.15); the width of the specimen remains constant due to the restraint by the groove. (e) Triaxial tensile stresses acting on an element. (f) Hydrostatic compression of an element. Note also that an element on the cylindrical portion of a thin-walled tube in torsion is in the condition of both plane stress and plane strain (see also Section 2.11.7).

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Yield Criteria

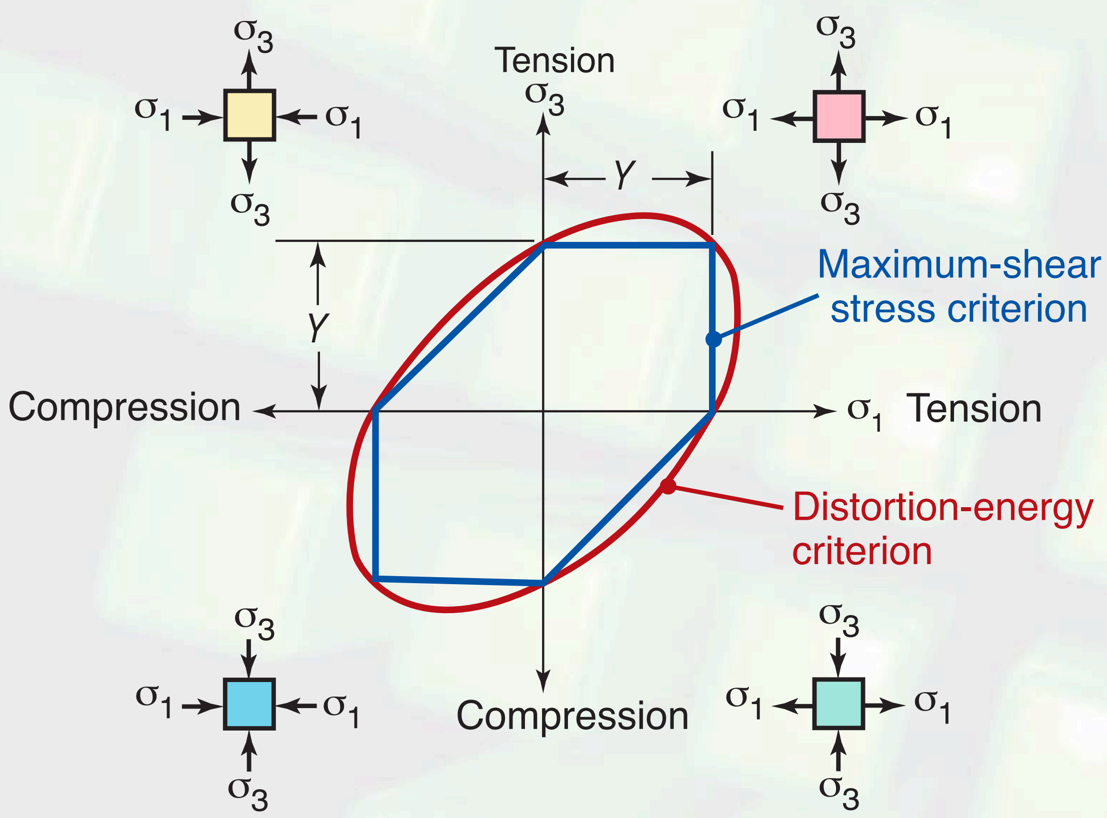

FIGURE 2.36 Plane-stress diagrams for maximum -shear-stress and distortion-energy criteria. Note that $\sigma_2 = 0$.

Maximum-shear-stress criterion:

$$\sigma_{max} - \sigma_{min} = Y$$

Distortion-energy criterion:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2$$

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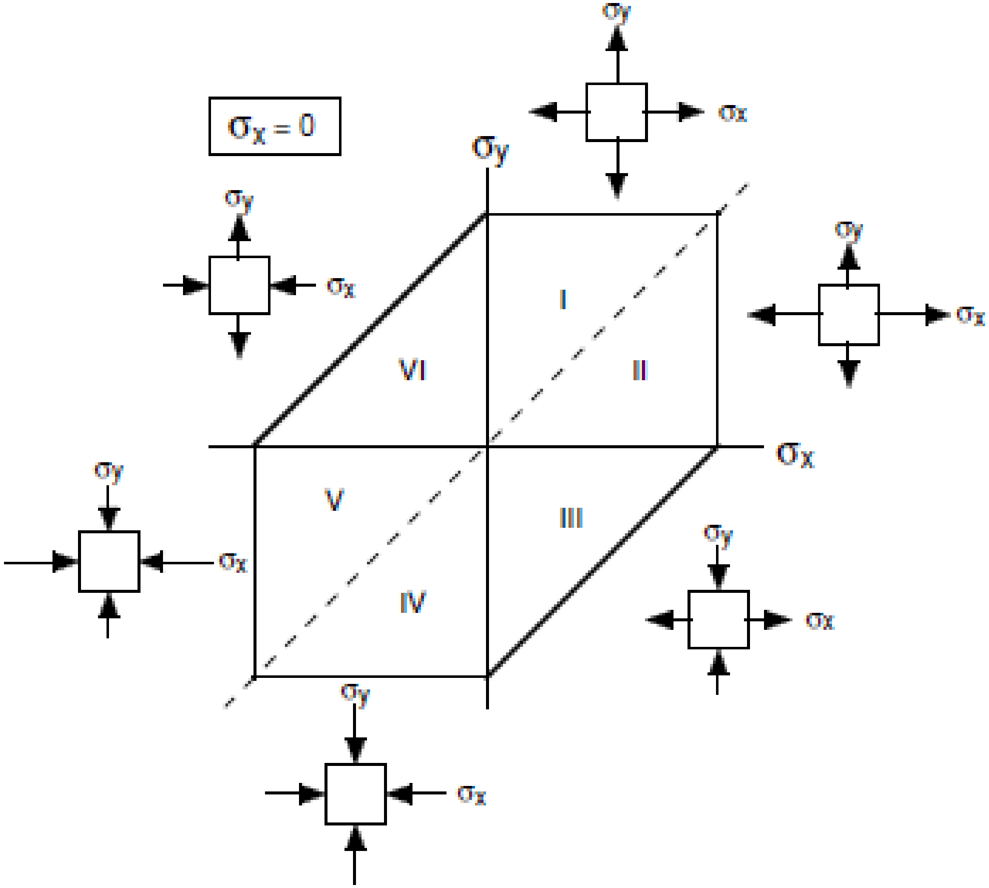

I $\sigma_y > \sigma_x > 0$, so $\sigma_y = Y$

II $\sigma_x > \sigma_y > 0$, so $\sigma_x = Y$

III $\sigma_x > 0 > \sigma_y$, so $\sigma_x - \sigma_y = Y$

IV $0 > \sigma_x > \sigma_y$, so $\sigma_y = -Y$

V $0 > \sigma_y > \sigma_x$, so $\sigma_x = -Y$

VI $\sigma_y > 0 > \sigma_x$, so $\sigma_y - \sigma_x = Y$

A thin-wall tube with closed ends is subjected to a maximum internal pressure of 35 MPa in service. The mean radius of the tube is 30 cm.
(a) If the tensile yield strength is 700 MPa,what minimum thickness must be specified
to prevent yielding?
(b) If the material has a yield strength in shear of k = 280 MPa, what minimum thickness must be specified to prevent yielding?
SOLUTION:
(a) Hoop stress, $\sigma 1 = Pr/t = 35(30\ cm)/t = \sigma max$, longitudinal stress = $\sigma 2 = Pr/(2t) = (35MPa)(30\ cm)/(2t)$, σmax = thickness stress, $\sigma 3 \approx 0$. Yielding occurs when $\sigma 1 = 700$, or $t = (35MPa)(30\ cm)/(700MPa) = 1.5\ cm$.
(b) $\sigma 1 - \sigma 3 = 2k = 560$ MPa at yielding, so yielding occurs when $t = (35MPa)(30\ cm)/(560MPa) = 1.875\ cm$.

FLOW RULES

$$d\varepsilon_1 = d\lambda[\sigma_1 - (1/2)(\sigma_2 + \sigma_3)]$$

$$d\varepsilon_2 = d\lambda[\sigma_2 - (1/2)(\sigma_3 + \sigma_1)]$$

$$d\varepsilon_3 = d\lambda[\sigma_3 - (1/2)(\sigma_1 + \sigma_2)].$$

NORMALITY PRINCIPLE

$$\frac{d\varepsilon_2}{d\varepsilon_1} = -\frac{\partial\sigma_1}{\partial\sigma_2}.$$

Consider the stress states

$$\begin{vmatrix} 15 & 3 & 0 \\ 3 & 10 & 0 \\ 0 & 0 & 5 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} 10 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 0 \end{vmatrix}.$$

a) Find σ_m for each.
b) Find the deviatoric stress in the normal directions for each
c) What is the sum of the deviatoric stresses for each?

A thin wall tube with closed ends is made from steel with a yield strength of 250 MPa. The tube is 2 m long with a wall thickness of 2 mm and a diameter of 8 cm. In service it will experience an axial load of 8 kN and a torque of 2.7 Nm. What is the maximum internal pressure it can withstand without yielding according to (a) the Tresca criterion and (b) the von Mises criterion?

When a brass tensile specimen, initially 0.505 in in diameter, is tested, the maximum load of 120 ksi was recorded at an elongation of 40%. What would the load be on an identical tensile specimen when the elongation is 20%?

During a tension test the tensile strength was found to be 340 MPa. This was recorded at an elongation of 30%. Determine n and K if the approximation $\bar{\sigma} = K\bar{\varepsilon}^n$ applies.

Flow Stress and Work of Deformation

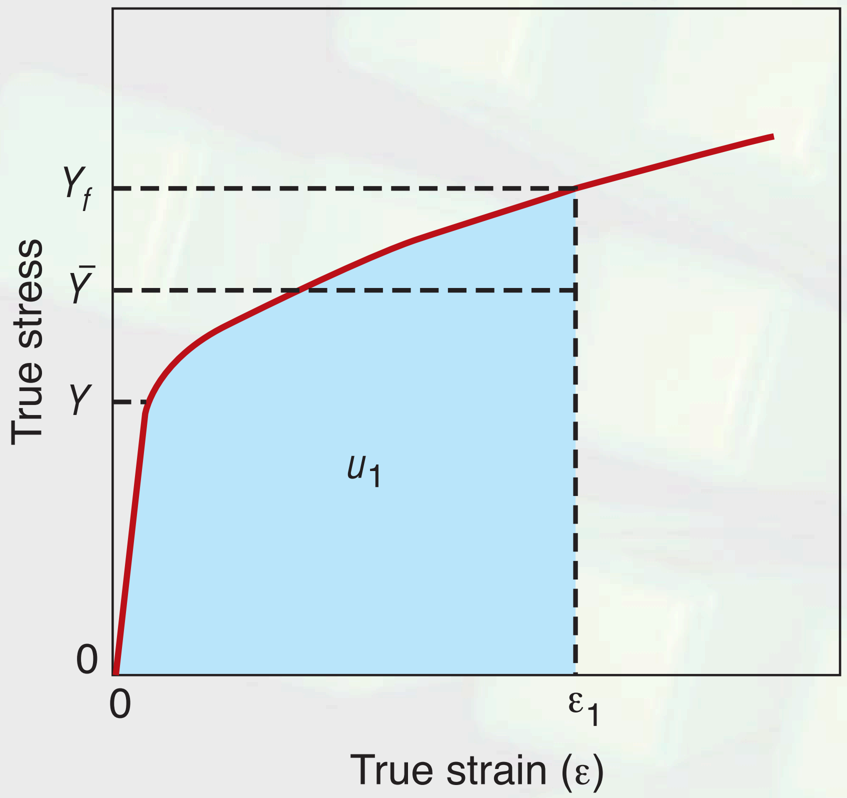

FIGURE 2.37 Schematic illustration of true stress-true strain curve showing yield stress Y, average flow stress, specific energy u_1 and flow stress Y_f.

Flow stress:

$$\bar{Y} = \frac{K\varepsilon_1^n}{n+1}$$

Specific energy

$$u = \int_0^{\varepsilon} \bar{\sigma}\, d\bar{\varepsilon}$$

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Ideal & Redundant Work

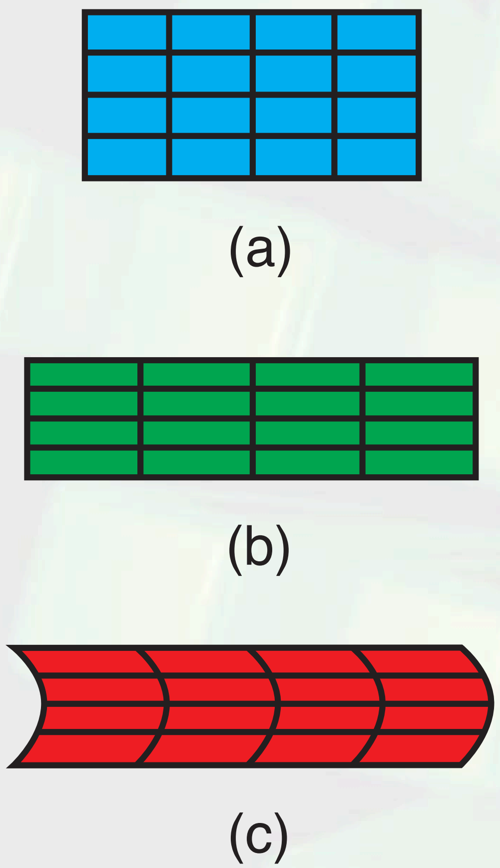

FIGURE 2.38 Deformation of grid patterns in a workpiece: (a) original pattern; (b) after ideal deformation; (c) after inhomogeneous deformation, requiring redundant work of deformation. Note that (c) is basically (b) with additional shearing, especially at the outer layers. Thus (c) requires greater work of deformation than (b). See also Figs. 6.3 and 6.49.

Total specific energy:

$$u_{total} = u_{ideal} + u_{friction} + u_{redundant}$$

Efficiency:

$$\eta = \frac{u_{ideal}}{u_{total}}$$

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