Manual for SOA Exam MLC.

Chapter 6. Benefit premiums
Actuarial problems.

Extract from:
"Arcones' Manual for SOA Exam MLC. Fall 2010 Edition",
available at http://www.actexmadriver.com/

(#4, Exam M, Spring 2005) For a fully discrete whole life insurance of 100,000 on (35) you are given:
(i) Percent of premium expenses are 10% per year.
(ii) Per policy expenses are 25 per year.
(iii) Per thousand expenses are 2.50 per year.
(iv) All expenses are paid at the beginning of the year.
(v) $1000P_{35} = 8.36$
Calculate the level annual expense-loaded premium using the equivalence principle.
(A) 930 (B) 1041 (C) 1142 (D) 1234 (E) 1352

(D) Let G be the expense–loaded premium. The APV of benefits is $(100000)A_{35}$. The APV of premiums is $G\ddot{a}_{35}$. The APV of expenses is

$$(0.1G + 25 + (2.5)(100))\ddot{a}_{35} = (0.1G + 275)\ddot{a}_{35}.$$

Using the equivalence principle,

$$G\ddot{a}_{35} = (100000)A_{35} + (0.1G + 275)\ddot{a}_{35}.$$

Hence,

$$G = \frac{(100000)A_{35} + (275)\ddot{a}_{35}}{(1-0.1)\ddot{a}_{35}} = \frac{(100000)P_{35} + 275}{0.9} = \frac{836 + 275}{0.9}$$

$=1234.444444.$

(#8, Exam M, Spring 2005) For a special 3–year term insurance on (30), you are given:
(i) Premiums are payable semiannually.
(ii) Premiums are payable only in the first year.
(iii) Benefits, payable at the end of the year of death, are:

k	b_{k+1}
0	1000
1	500
2	250

(iv) Mortality follows the Illustrative Life Table.
(v) Deaths are uniformly distributed within each year of age.
(vi) $i = 0.06$
Calculate the amount of each semiannual benefit premium for this insurance.
(A) 1.3 (B) 1.4 (C) 1.5 (D) 1.6 (E) 1.7

(A) Let P be the amount of each semiannual payment. We have that

$$
\begin{aligned}
& P + P(1.06)^{-1/2}{}_{0.5}p_{30} \\
=& (1000)(1.06)^{-1}q_{30} + (500)(1.06)^{-2}p_{30}q_{31} + (250)(1.06)^{-3}p_{30}p_{31}q_{32}.
\end{aligned}
$$

From the illustrative table, $(1000)q_{30} = 1.53$, $(1000)q_{31} = 1.61$ and $(1000)q_{32} = 1.70$. Hence,

$$
\begin{aligned}
& P + P(1.06)^{-1/2}{}_{0.5}p_{30} = P(1 + (1.06)^{-1/2}(1 - 0.00153/2)) \\
=& P(1.970542829), \\
& (1000)(1.06)^{-1}q_{30} + (1.06)^{-2}p_{30}q_{31} + (1.06)^{-3}p_{30}p_{31}q_{32} \\
=& (1000)(1.06)^{-1}(0.00153) + (500)(1.06)^{-2}(1 - 0.00153)(0.00161) \\
& + (250)(1.06)^{-3}(1 - 0.00153)(1 - 0.00161)(0.0017) = 2.514465799, \\
& P = \frac{2.514465799}{1.970542829} = 1.276026972.
\end{aligned}
$$

(#9, Exam M, Fall 2005) For a special fully discrete 30–payment whole life insurance on (45), you are given:
(i) The death benefit of 1000 is payable at the end of the year of death.
(ii) The benefit premium for this insurance is equal to $1000P_{45}$ for the first 15 years followed by an increased level annual premium of π for the remaining 15 years.
(iii) Mortality follows the Illustrative Life Table.
(iv) $i = 0.06$
Calculate π.
(A) 16.8 (B) 17.3 (C) 17.8 (D) 18.3 (E) 18.8

(B) We have that

$$(1000)A_{45} = 1000P_{45}\ddot{a}_{45:\overline{15}|} + \pi \cdot {}_{15}E_{45}\ddot{a}_{60:\overline{15}|}.$$

Using the illustrative mortality table, we get that $A_{45} = 0.20120$, $\ddot{a}_{45} = 14.1121$, $\ell_{45} = 9164051$, $\ell_{60} = 8188074$, $\ell_{75} = 5396081$, $\ddot{a}_{60} = 11.1454$ and $\ddot{a}_{75} = 7.2170$. So,

$$P_{45} = \frac{A_{45}}{\ddot{a}_{45}} = \frac{0.20120}{14.1121} = 0.01425726859,$$

$${}_{15}E_{45} = (1.06)^{-15}\frac{\ell_{60}}{\ell_{45}} = (1.06)^{-15}\frac{8188074}{9164051} = 0.3728260782,$$

$$\ddot{a}_{45:\overline{15}|} = \ddot{a}_{45} - {}_{15}E_{45}\ddot{a}_{60} = 14.1121 - (0.3728260782)(11.1454)$$

$$=9.956804228,$$

$${}_{15}E_{60} = (1.06)^{-15}\frac{5396081}{8188074} = 0.2749848214,$$

$$\ddot{a}_{60:\overline{15}|} = \ddot{a}_{60} - {}_{15}E_{60}\ddot{a}_{75} = 11.1454 - (0.2749848214)(7.2170)$$

$$=9.160834544.$$

Hence

$$(1000)A_{45} = 1000P_{45}\ddot{a}_{45:\overline{15}|} + \pi \cdot {}_{15}E_{45}\ddot{a}_{60:\overline{15}|}$$

becomes

$$\begin{aligned}(1000)(0.20120) = &(1000)(0.01425726859)(9.956804228)\\ &+ \pi(0.3728300028)(9.160834544).\end{aligned}$$

So,

$$\begin{aligned}\pi = &\frac{(1000)(0.20120) - (1000)(0.01425726859)(9.956804228)}{(0.3728300028)(9.160834544)}\\ =&17.34572191.\end{aligned}$$

(#14, Exam M, Spring 2005) For a special fully discrete 2–year endowment insurance of 1000 on (x), you are given:
(i) The first year benefit premium is 668.
(ii) The second year benefit premium is 258.
(iii) $d = 0.06$
Calculate the level annual premium using the equivalence principle.
(A) 469 (B) 479 (C) 489 (D) 499 (E) 509

(B) Under the non-level premiums,

$$668 + 258 v p_x = (1000) v (1 - p_x) + (1000) v^2 p_x$$

Hence,

$$p_x = \frac{668 - (1000)v}{(1000)v^2 - (1000)v - (258)v}$$
$$= \frac{668 - (1000)(0.94)}{(1000)(0.94)^2 - (1258)(0.94)} = 0.9099424595.$$

The level premium $P_{x:\overline{2}|}$ satisfies

$$668 + 258 v p_x = P_{x:\overline{2}|}(1 + v p_x).$$

So,

$$P_{x:\overline{2}|} = \frac{668 + 258 v p_x}{1 + v p_x} = \frac{668 + 258(0.94)(0.9099424595)}{1 + (0.94)(0.9099424595)} = 478.9830508$$

(#15, Exam M, Fall 2005) For a fully discrete 15-payment whole life insurance of 100,000 on (x), you are given:
(i) The expense-loaded level annual premium using the equivalence principle is 4669.95.
(ii) $100,000A_x = 51,481.97$
(iii) $\ddot{a}_{x:\overline{15}|} = 11.35$
(iv) $d = 0.02913$
(v) Expenses are incurred at the beginning of the year.
(vi) Percent of premium expenses are 10% in the first year and 2% thereafter.
(vii) Per policy expenses are K in the first year and 5 in each year thereafter until death. Calculate K.
(A) 10.0 (B) 16.5 (C) 23.0 (D) 29.5 (E) 36.5

(A) We know that

$$G = 4669.95, A_x = 0.5148197, \ddot{a}_{x:\overline{15}|} = 11.35.$$

We have that

$$G\ddot{a}_{x:\overline{15}|} = (100000)A_x + 0.08G + 0.02G\ddot{a}_{x:\overline{15}|} + K - 5 + 5\ddot{a}_x.$$

We have that

$$\ddot{a}_x = \frac{1 - A_x}{d} = \frac{1 - 0.5148197}{0.02913} = 16.65569173.$$

Hence,

$$\begin{aligned} &(4669.95)(11.35) \\ =&(100000)(0.5148197) + (0.08)(4669.95) + (0.02)(4669.95)(11.35)) \\ &+ K - 5 + 5(16.65569173) \end{aligned}$$

and

$$\begin{aligned} K =&(4669.95)(11.35) - (100000)(0.5148197) - (0.08)(4669.95) \\ &+ (0.02)(4669.95)(11.35) + 5 - 5(16.65569173) = 10.00939135. \end{aligned}$$

(#29, Exam M, Fall 2005) For a fully discrete whole life insurance of 1000 on (60), you are given:
(i) The expenses, payable at the beginning of the year, are:

Expense Type	First Year	Renewal Years
% of Premium	20%	6%
Per Policy	8	2

(ii) The level expense-loaded premium is 41.20.
(iii) $i = 0.05$
Calculate the value of the expense augmented loss variable, ${}_0L_e$, if the insured dies in the third policy year.
(A) 770 (B) 790 (C) 810 (D) 830 (E) 850

(A) We have that

$$
\begin{aligned}
&{}_0L_e \\
=&(1000)v^3 + (0.20G + 8) + (0.06G + 2)v + (0.06G + 2)v^2 - G\ddot{a}_{\overline{3}|0.05} \\
=&(1000)(1.05)^{-3} + ((0.20)(41.20) + 8) + ((0.06)(41.20) + 2)(1.05)^{-1} \\
&+ ((0.06)(41.20) + 2)(1.05)^{-2} - (41.20)(2.859410431) \\
=&770.8149954.
\end{aligned}
$$

(#30, Exam M, Spring 2005) For a fully discrete whole life insurance of 1000 on (50), you are given:
(i) The annual per policy expense is 1.
(ii) There is an additional first year expense of 15.
(iii) The claim settlement expense of 50 is payable when the claim is paid.
(iv) All expenses, except the claim settlement expense, are paid at the beginning of the year.
(v) Mortality follows De Moivre's law with $\omega = 100$.
(vi) $i = 0.05$
Calculate the level expense-loaded premium using the equivalence principle.
(A) 27 (B) 28 (C) 29 (D) 30 (E) 31

(E) Let G be be the expense-loaded premium. We have that

$$G\ddot{a}_{50} = (1000)A_{50} + \ddot{a}_{50} + 15 + (50)A_{50} = (1050)A_{50} + \ddot{a}_{50} + 15.$$

We have that

$$A_{50} = \frac{1}{50}a_{\overline{50}|0.05} = 0.3651185092,$$
$$\ddot{a}_{50} = \frac{1 - A_{50}}{d} = \frac{1 - 0.3651185092}{\frac{0.05}{1.05}} = 13.33251131.$$

Hence,

$$G = \frac{(1050)A_{50} + \ddot{a}_{50} + 15}{\ddot{a}_{50}}$$
$$= \frac{(1050)(0.3651185092) + 15 + 13.33251131}{13.33251131} = 30.87992475.$$

(#35, Exam M, Spring 2005) For a special fully discrete 5–year deferred whole life insurance of 100,000 on (40), you are given:
(i) The death benefit during the 5-year deferral period is return of benefit premiums paid without interest.
(ii) Annual benefit premiums are payable only during the deferral period.
(iii) Mortality follows the Illustrative Life Table.
(iv) $i = 0.06$
(v) $(IA)^{1}_{40:\overline{5}|} = 0.04042$.
Calculate the annual benefit premiums.
(A) 3300 (B) 3320 (C) 3340 (D) 3360 (E) 3380

(D) Let P be the annual benefit premium. We have that

$$P\ddot{a}_{40:\overline{5}|} = P\,(IA)^{1}_{40:\overline{5}|} + (100000)\,{}_5E_{40}A_{45}$$

and

$$P = \frac{(100000)\,{}_5E_{40}A_{45}}{\ddot{a}_{40:\overline{5}|} - (IA)^{1}_{40:\overline{5}|}}.$$

From the illustrative life table,

$${}_5E_{40} = 0.73529, A_{45} = 0.20120, \ddot{a}_{40} = 14.8166, \ddot{a}_{45} = 14.1121,$$
$$\ddot{a}_{40:\overline{5}|} = \ddot{a}_{40} - {}_5E_{40}\ddot{a}_{45} = 14.8166 - (0.73529)(14.1121) = 4.440113991.$$

Hence,

$$P = \frac{(100000)(0.73529)(0.20120)}{4.440113991 - 0.04042} = 3362.514491.$$

(#38, Exam M, Spring 2005) A group of 1000 lives each age 30 sets up a fund to pay 1000 at the end of the first year for each member who dies in the first year, and 500 at the end of the second year for each member who dies in the second year. Each member pays into the fund an amount equal to the single benefit premium for a special 2-year term insurance, with:
(i) Benefits:

k	b_{k+1}
0	1000
1	500

(ii) Mortality follows the Illustrative Life Table.
(iii) $i = 0.06$
The actual experience of the fund is as follows:

k	Interest Rate Earned	Number of Deaths
0	0.070	1
1	0.069	1

Calculate the difference, at the end of the second year, between the expected size of the fund as projected at time 0 and the actual fund.

(A) 840 (B) 870 (C) 900 (D) 930 (E) 960

(C) From the illustrative table

$$q_{30} = 0.00153, q_{31} = 0.00161.$$

The net single premium is

$$\begin{aligned}
&(1000)vq_{30} + (500)v^2 p_{30} q_{31} \\
=&(1000)(1.06)^{-1}(0.00153) + (500)(1.06)^{-2}(1 - 0.00153)(0.00161) \\
=&2.158747197.
\end{aligned}$$

Since there are 1000 participants, the initial balance of the fund is 2158.747197. The amount in the fund at the end of the first year is

$$(2158.747197)(1.07) - 1000 = 1309.859501.$$

The amount in the fund at the end of the second year is

$$(1309.859501)(1.069) - 500 = 900.2398066.$$

Since the expected size of the fund is zero, the difference is 900.

(#18, Exam M, Fall 2006) For a special fully discrete 20–year term insurance on (30):

(i) The death benefit is 1000 during the first ten years and 2000 during the next ten years.

(ii) The benefit premium, determined by the equivalence principle, is π for each of the first ten years and 2π for each of the next ten years.

(iii) $\ddot{a}_{30:\overline{20}|} = 15.0364$

(iv)

x	$\ddot{a}_{x:10}$	$1000A^{1}_{x:\overline{10}|}$
30	8.7201	16.66
40	8.6602	32.61

Calculate π

(A) 2.9 (B) 3.0 (C) 3.1 (D) 3.2 (E) 3.3

(B) We have that

$$-\pi\ddot{a}_{30:\overline{10}|} + 2\pi\ddot{a}_{30:\overline{20}|} = (1000)A^{1}_{30:\overline{10}|} + 2 \cdot {}_{10}E_{30} \cdot A^{1}_{40:\overline{10}|}.$$

Since

$$15.0364 = \ddot{a}_{30:\overline{20}|} = \ddot{a}_{30:\overline{10}|} + {}_{10}E_{30} \cdot \ddot{a}_{40:\overline{10}|} = 8.7201 + {}_{10}E_{30}(8.6602)$$

we get that ${}_{10}E_{30} = \frac{15.0364-8.7201}{8.6602} = 0.729348052$. Hence,

$$-\pi\ddot{a}_{30:\overline{10}|} + 2\pi\ddot{a}_{30:\overline{20}|} = \pi(-8.7201 + (2)(15.0364)) = (21.3527)\pi,$$

$$(1000)A^{1}_{30:\overline{10}|} + 2 \cdot {}_{10}E_{30} \cdot A^{1}_{40:\overline{10}|} = 16.66 + (2)(0.729348052)(32.61)$$

$$=64.22807995$$

and $\pi = \frac{64.22807995}{21.3527} = 3.007960583$.

(#28, Exam M, Fall 2006) For a fully continuous whole life insurance of 1 on (x):
(i) $\overline{A}_x = 1/3$
(ii) $\delta = 0.10$
(iii) L is the loss at issue random variable using the premium based on the equivalence principle.
(iv) $\mathrm{Var}[L] = 1/5$
(v) L' is the loss at issue random variable using the premium π.
(vi) $\mathrm{Var}[L'] = 16/45$.
Calculate π.
(A) 0.05 (B) 0.08 (C) 0.10 (D) 0.12 (E) 0.15

(C) The premium based on the equivalence principle is

$$\overline{P}(\overline{A}_x) = \frac{\overline{A}_x}{\overline{a}_x} = \frac{\overline{A}_x}{\frac{1-\overline{A}_x}{\delta}} = \frac{\frac{1}{3}}{\frac{1-\frac{1}{3}}{0.1}} = 0.05$$

We have that $L = \overline{Z}_x \left(1 + \frac{\overline{P}(\overline{A}_x)}{\delta}\right) - \frac{\overline{P}(\overline{A}_x)}{\delta}$ and

$$\frac{1}{5} = \mathrm{Var}(L) = \mathrm{Var}(\overline{Z}_x) \left(1 + \frac{\overline{P}(\overline{A}_x)}{\delta}\right)^2.$$

We also have that $L' = \overline{Z}_x \left(1 + \frac{\pi}{\delta}\right) - \frac{\pi}{\delta}$ and

$$\frac{16}{45} = \mathrm{Var}(L') = \mathrm{Var}(\overline{Z}_x) \left(1 + \frac{\pi}{\delta}\right)^2.$$

Hence,

$$\frac{9}{16} = \frac{\frac{1}{5}}{\frac{16}{45}} = \frac{\mathrm{Var}(L)}{\mathrm{Var}(L')} = \left(\frac{\delta + \overline{P}(\overline{A}_x)}{\delta + \pi}\right)^2 = \left(\frac{0.1 + 0.05}{0.1 + \pi}\right)^2 = \left(\frac{0.15}{0.1 + \pi}\right)^2$$

So, $\frac{3}{4} = \frac{0.15}{0.1+\pi}$ and $\pi = 0.1$.

(#4, Exam MLC, Fall 2007) For a fully discrete whole life insurance of 150,000 on (x), you are given:
(i) ${}^2A_x = 0.0143$
(ii) $A_x = 0.0653$
(iii) The annual premium is determined using the equivalence principle.
(iv) L is the loss–at–issue random variable.
Calculate the standard deviation of L.
(A) 14,000 (B) 14,500 (C) 15,100 (D) 15,600 (E) 16,100

(E) We have that

$$\mathrm{Var}(L) = (150000)^2 \frac{{}^2A_x - (A_x)^2}{(1 - A_x)^2} = (150000)^2 \frac{0.0143 - (0.0653)^2}{(1 - 0.0653)^2}.$$

and the standard deviation of L is

$$\frac{(150000)\sqrt{(0.0143 - (0.0653)^2}}{1 - 0.0653} = 16076.72.$$

(#11, Exam MLC, Fall 2007) For a 10–payment, fully discrete, 20–year term insurance of 1000 on (40), you are given:
(i) $i = 0.06$
(ii) Mortality follows the Illustrative Life Table.
(iii) The following expenses, which are incurred at the beginning of each policy year:

	Year 1		Years 2+	
	% of premium	Constant	% of premium	Constant
Taxes	4%	0	4%	0
Sales Commission	25%	0	5%	0
Policy maintenance	0%	10	0%	5

Calculate the expense–loaded premium using the equivalence principle.
(A) 18.21 (B) 18.35 (C) 18.53 (D) 18.71 (E) 18.95

(A) Let G be the expense–loaded premium. We have that

$$
\begin{aligned}
&G\ddot{a}_{40:\overline{10}|} = (1000)A^{1}_{40:\overline{20}|} + (0.04)G\ddot{a}_{40:\overline{10}|} + (0.20)G \\
&+ (0.05)G\ddot{a}_{40:\overline{10}|} + 5 + 5\ddot{a}_{40:\overline{20}|} \\
=&(1000)A^{1}_{40:\overline{20}|} + (0.09)G\ddot{a}_{40:\overline{10}|} + (0.20)G + 5 + 5\ddot{a}_{40:\overline{20}|}.
\end{aligned}
$$

From the illustrative table,

$$\ddot{a}_{40:\overline{10}|} = \ddot{a}_{40} - {}_{10}E_{40} \cdot \ddot{a}_{50} = 14.8166 - (0.53667)(13.2668) = 7.696706,$$

$$\ddot{a}_{40:\overline{20}|} = \ddot{a}_{40} - {}_{20}E_{40} \cdot \ddot{a}_{60} = 14.8166 - (0.27414)(11.1454) = 11.7612,$$

$$A^{1}_{40:\overline{20}|} = A_{40} - {}_{20}E_{40}A_{60} = 0.16132 - (0.27414)(0.36913) = 0.0601267.$$

Hence,

$$
\begin{aligned}
&G(7.696706) = (1000)(60.1267) + (0.09)G(7.696706) \\
&+ (0.20)G + 5 + 5(11.7612)
\end{aligned}
$$

and

$$G = \frac{(1000)(0.0601267) + 5 + 5(11.7612)}{7.696706 - (0.09)(7.696706) - 0.20} = 18.21468.$$

(#6, MLC–09–08) For a special fully discrete whole life insurance of 1000 on (40):
(i) The level benefit premium for each of the first 20 years is π.
(ii) The benefit premium payable thereafter at age x is $1000vq_x$, $x = 60, 61, 62, \ldots$
(iii) Mortality follows the Illustrative Life Table.
(iv) $i = 0.06$
Calculate π.
(A) 4.79 (B) 5.11 (C) 5.34 (D) 5.75 (E) 6.07

(B) The APV of benefit premiums is

$$\pi \ddot{a}_{40:\overline{20}|} + \sum_{j=20}^{\infty} {}_j p_{40} \cdot v^j (1000) v q_{40+j} = \pi \ddot{a}_{40:\overline{20}|} + \sum_{j=20}^{\infty} (1000) \cdot {}_{j|} q_{40} v^{j+1}$$

$$= \pi \ddot{a}_{40:\overline{20}|} + \sum_{j=21}^{\infty} (1000) \cdot {}_{j-1|} q_{40} v^j = \pi \ddot{a}_{40:\overline{20}|} + (1000)_{20|} A_{40}$$

$$= \pi \ddot{a}_{40:\overline{20}|} + (1000)_{20} E_{40} A_{60}.$$

Hence, $(1000) A_{40} = \pi \ddot{a}_{40:\overline{20}|} + (1000)_{20} E_{40} A_{60}$ and

$$\pi = \frac{(1000) A_{40} - (1000)_{20} E_{40} A_{60}}{\ddot{a}_{40:\overline{20}|}} = \frac{(1000) A_{40} - (1000)_{20} E_{40} A_{60}}{\ddot{a}_{40} - {}_{20} E_{40} \ddot{a}_{60}}$$

$$= \frac{161.32 - (369.13)(0.27414)}{14.8166 - (0.27414)(11.1454)} = 5.112293097.$$

(#14, MLC–09–08) For a fully continuous whole life insurance of 1 on (x), you are given:
(i) The forces of mortality and interest are constant.
(ii) ${}^{2}\overline{A}_x = 0.20$
(iii) $\overline{P}(\overline{A}_x) = 0.03$
(iv) ${}_0L$ is the loss-at-issue random variable based on the benefit premium.
Calculate $\mathrm{Var}({}_0L)$.
(A) 0.20 (B) 0.21 (C) 0.22 (D) 0.23 (E) 0.24

Solution 1: We know that $0.20 = \frac{\mu}{\mu+2\delta}$ and $0.03 = \mu$. Hence, $\delta = 0.06$. We have that

$$\overline{A}_x = \frac{\mu}{\mu+\delta} = \frac{1}{3},$$
$$\mathrm{Var}({}_0L) = \frac{{}^2\overline{A}_x - \overline{A}_x}{(1-\overline{A}_x)^2} = \frac{0.20-(1/3)^2}{(1-(1/3))^2} = 0.2.$$

Solution 2: Using a previous problem, $\mathrm{Var}({}_0L) = {}^2\overline{A}_x = 0.2$.

(#24, MLC–09–08) For a block of fully discrete whole life insurances of 1 on independent lives age x, you are given:
(i) $i = 0.06$
(ii) $A_x = 0.24905$
(iii) ${}^2A_x = 0.09476$
(iv) $\pi = 0.025$, where π is the contract premium for each policy.
(v) Losses are based on the contract premium.
Using the normal approximation, calculate the minimum number of policies the insurer must issue so that the probability of a positive total loss on the policies issued is less than or equal to 0.05.
(A) 25 (B) 27 (C) 29 (D) 31 (E) 33

(B) We have that

$$E[L_x] = A_x - \pi \ddot{a}_x = 0.24905 - (0.025)\frac{1-0.24905}{6/106} = -0.0826195833$$

$$E[L_{\text{Aggreg}}] = -0.08261958333n,$$

$$\mathrm{Var}(L_x) = \left(1 + \frac{0.025}{d}\right)^2 ({}^2A_x - A_x^2)$$

$$= \left(1 + \frac{0.025}{6/106}\right)^2 (0.09476 - (0.24905)^2) = 0.06803463917,$$

$$\mathrm{Var}(L_{\text{Aggreg}}) = n(0.06803463917)$$

We have that
$0 = -0.08261958333n + (1.645)\sqrt{n(0.06803463917)}$. So,

$$n = \frac{(1.645)^2(0.06803463917)}{(0.08261958333)^2} = 26.97092799.$$

(#29, MLC–09–08) Two actuaries use the same mortality table to price a fully discrete 2-year endowment insurance of 1000 on (x).
(i) Kevin calculates non-level benefit premiums of 608 for the first year and 350 for the second year.
(ii) Kira calculates level annual benefit premiums of π.
(iii) $d = 0.05$
Calculate π.
(A) 482 (B) 489 (C) 497 (D) 508 (E) 517

(B) With $v = 0.95$, we have that

$$1000vq_x + 1000v^2 p_x = 608 + 350vp_x = \pi + \pi vp_x,$$

Hence,

$$p_x = \frac{1000v - 608}{1350v - 1000v^2} = 0.9,$$
$$\pi = \frac{608 + 350vp_x}{1 + vp_x} = \frac{608 + 350(0.95)(0.9)}{1 + (0.95)(0.9)} = 489.083558.$$

(#37, MLC–09–08) For a fully continuous whole life insurance of 1 on (x), you are given:
(i) $\delta = 0.04$
(ii) $\bar{a}_x = 12$
(iii) $\mathrm{Var}(v^T) = 0.10$
(iv) ${}_0L_e = {}_0L + E$, is the expense-augmented loss variable, where

$$
\begin{aligned}
{}_0L &= v^T - \overline{P}(\overline{A}_x)\bar{a}_{\overline{T}|}, \\
E &= c_0 + (g - e)\bar{a}_{\overline{T}|} \\
c_0 &= \text{ initial expenses} \\
g &= 0.0030, \text{is the annual rate of continuous maintenance expense;} \\
e &= 0.0066, \text{is the annual expense loading in the premium.}
\end{aligned}
$$

Calculate $\mathrm{Var}({}_0L_e)$.
(A) 0.208 (B) 0.217 (C) 0.308 (D) 0.434 (E) 0.472

(E) We have that

$$
\begin{aligned}
{}_0L_e &= {}_0L + E = v^T - \overline{P}(\overline{A}_x)\overline{a}_{\overline{T}|} + c_0 + (g-e)\overline{a}_{\overline{T}|} \\
&= v^T - \overline{P}(\overline{A}_x)\frac{1-v^T}{\delta} + c_0 + (g-e)\frac{1-v^T}{\delta} \\
&= v^T\left(1 + \frac{\overline{P}(\overline{A}_x)}{\delta} - \frac{(g-e)}{\delta}\right) - \frac{\overline{P}(\overline{A}_x)}{\delta} + c_0 + \frac{(g-e)}{\delta}
\end{aligned}
$$

Hence,

$$
\overline{P}(\overline{A}_x) = \frac{1-\delta\overline{a}_x}{\overline{a}_x} = \frac{1-(0.04)(12)}{12} = \frac{0.52}{12},
$$

$$
\begin{aligned}
\mathrm{Var}({}_0L_e) &= \mathrm{Var}(v^T)\left(1 + \frac{\overline{P}(\overline{A}_x)}{\delta} - \frac{(g-e)}{\delta}\right)^2 \\
&= (0.1)\left(1 + \frac{0.52}{(12)(0.04)} - \frac{(0.0030-0.0066)}{0.04}\right)^2 \\
&= (0.1)\left(1 + \frac{0.52}{0.48} + \frac{0.0036}{0.04}\right)^2 = 0.4723377778.
\end{aligned}
$$

(#40, MLC–09–08) For a fully discrete whole life insurance of 1000 on (60), the annual benefit premium was calculated using the following:
(i) $i = 0.06$
(ii) $q_{60} = 0.01376$
(iii) $1000A_{60} = 369.33$
(iv) $1000A_{61} = 383.00$
A particular insured is expected to experience a first-year mortality rate ten times the rate used to calculate the annual benefit premium. The expected mortality rates for all other years are the ones originally used.
Calculate the expected loss at issue for this insured, based on the original benefit premium.
(A) 72 (B) 86 (C) 100 (D) 114 (E) 128

(D) Let A^*_{60} and $\ddot{a}^*_{60}$ be the APV's values for this particular insured. We have that

$$1000P_{60} = 1000\frac{A_{60}}{\ddot{a}_{60}} = \frac{(369.33)(6/106)}{1-0.36933} = 33.14803574,$$

$$1000A^*_{60} = v(10q_{60})(1000) + v(1-10q_{60})(1000)A_{61}$$

$$=(1.06)^{-1}(10)(0.01376)(1000) + (1.06)^{-1}(1-(10)(0.01376))(383.00)$$

$$=441.4143396,$$

$$\ddot{a}^*_{60} = \frac{1-A^*_{60}}{d} = \frac{1-0.44141433962}{6/106} = 9.868346667,$$

$$E[L^*] = 1000A^*_{60} - 1000P_{60}\ddot{a}^*_{60}$$

$$=441.4143396 - (33.14803574)(9.868346667) = 114.2980316.$$

(#47, MLC–09–08) For a special whole life insurance on (35), you are given:

(i) The annual benefit premium is payable at the beginning of each year.

(ii) The death benefit is equal to 1000 plus the return of all benefit premiums paid in the past without interest.

(iii) The death benefit is paid at the end of the year of death.

(iv) $A_{35} = 0.42898$

(v) $(IA)_{35} = 6.16761$

(vi) $i = 0.05$

Calculate the annual benefit premium for this insurance.

(A) 73.66 (B) 75.28 (C) 77.42 (D) 78.95 (E) 81.66

(A) By the equivalence principle $\pi\ddot{a}_{35} = 1000A_{35} + \pi(IA)_{35}$. Hence,

$$\ddot{a}_{35} = \frac{1 - 0.42898}{5/105} = 11.99142,$$

$$\pi = \frac{000A_{35}}{\ddot{a}_{35} - (IA)_{35}} = \frac{(1000)(0.42898)}{11.99142 - 6.16761} = 73.65968327.$$

(#51, MLC–09–08) For a fully discrete whole life insurance of 1000 on (60), you are given:
(i) $i = 0.06$
(ii) Mortality follows the Illustrative Life Table, except that there are extra mortality risks at age 60 such that $q_{60} = 0.015$.
Calculate the annual benefit premium for this insurance.
(A) 31.5 (B) 32.0 (C) 32.1 (D) 33.1 (E) 33.2

(E) We have that

$$
\begin{aligned}
&1000A_{60} = (1000)vq_{60} + (1000)vp_{60A_{61}} \\
=&v(1000)(0.015) + v(0.985)(382.79) = 369.8567453, \\
&P = \frac{1000A_{60}}{\ddot{a}_{60}} = \frac{369.8567453}{(1 - 0.3698567453)/(6/106)} = 33.22306049.
\end{aligned}
$$

(#60, MLC–09–08) For a fully discrete whole life insurance of 100,000 on each of 10,000 lives age 60, you are given:
(i) The future lifetimes are independent.
(ii) Mortality follows the Illustrative Life Table.
(iii) $i = 0.06$.
(iv) π is the premium for each insurance of 100,000.
Using the normal approximation, calculate π, such that the probability of a positive total loss is 1%.
(A) 3340 (B) 3360 (C) 3380 (D) 3390 (E) 3400

(C) We have that $L = (10^5)Z_x - \pi \ddot{Y}_x = \left(10^5 + \frac{\pi}{d}\right) Z_x - \frac{\pi}{d}$. So, $E[L] = (10^5)A_x - \pi \ddot{a}_x$ and $\mathrm{Var}(L) = \left(10^5 + \frac{\pi}{d}\right)^2 \mathrm{Var}(Z_x)$. For 10000 lives,

$$E[L_{\mathrm{Aggr}}] = (10)^4((10^5)A_x - \pi \ddot{a}_x), \mathrm{Var}(L_{\mathrm{Aggr}}) = (10)^4 \left(10^5 + \frac{\pi}{d}\right)^2 \mathrm{Var}(Z_x$$

π is the solution of

$$0 = (10)^4((10^5)A_x - \pi \ddot{a}_x) + (2.326)(10)^2 \left(10^5 + \frac{\pi}{d}\right) \sqrt{\mathrm{Var}(Z_x)}.$$

So,

$$\pi = \frac{(10^7)A_x + (2.326)10^5 \sqrt{\mathrm{Var}(Z_x)}}{100\ddot{a}_x - \frac{(2.326)\sqrt{\mathrm{Var}(Z_x)}}{d}}$$

$$= \frac{(10^7)(0.36913) + (2.326)10^5 \sqrt{0.17741 - 0.36913^2}}{100(11.1454) - \frac{(2.326)\sqrt{0.17741 - 0.36913^2}}{6/106}} = 3379.563104.$$

(#84, MLC–09–08) For a special 2-payment whole life insurance on (80):

(i) Premiums of π are paid at the beginning of years 1 and 3.

(ii) The death benefit is paid at the end of the year of death.

(iii) There is a partial refund of premium feature:

If (80) dies in either year 1 or year 3, the death benefit is $1000 + \frac{\pi}{2}$

Otherwise, the death benefit is 1000.

(iv) Mortality follows the Illustrative Life Table.

(v) $i = 0.06$

Calculate π, using the equivalence principle.

(A) 369 (B) 381 (C) 397 (D) 409 (E) 425

(C) Equating benefits and premiums, we get that

$$\pi + \pi v^2 {}_2p_{80} = 1000A_{80} + v\pi(0.5)q_{80} + v^3\pi(0.5)p_{80}p_{81}q_{82}.$$

From the Illustrative Life Table;

$$q_{80} = 0.08030,\ \ q_{81} = 0.08742,\ \ q_{82} = 0.09561, A_{80} = 665.75.$$

Hence,

$$\begin{aligned}
&1 + v^2 p_{80}p_{81} - v(0.5)q_{80} - v^{-3}\pi(0.5)p_{80}p_{81}q_{82}\\
=&1 + (1.06)^{-2}(1-0.08030)(1-0.08742) - (1.06)^{-1}(0.5)(0.08030)\\
&-(1.06)^{-3}(0.5)(1-0.08030)(1-0.08742)(0.09561) = 1.675408682,\\
&\pi = \frac{665.75}{1.675408682} = 397.3657335,
\end{aligned}$$

and

$$\pi = \frac{665.75}{1.675408682} = 397.3657335.$$

(#92, MLC–09–08) For a fully continuous whole life insurance of 1:
(i) $\mu = 0.04$
(ii) $\delta = 0.08$
(iii) L is the loss-at-issue random variable based on the benefit premium.
Calculate $\mathrm{Var}(L)$.
(A) $\frac{1}{10}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

(B) We have that

$$\overline{A}_x = \frac{\mu}{\mu+\delta} = \frac{4}{12} = \frac{1}{3},\ {}^2\overline{A}_x = \frac{\mu}{\mu+2\delta} = \frac{4}{20} = \frac{1}{5},$$

$$\mathrm{Var}(\overline{L}(\overline{A}_x)) = \frac{{}^2\overline{A}_x - (\overline{A}_x)^2}{(1-\overline{A}_x)^2} = \frac{\frac{1}{5}-\frac{1}{9}}{\left(1-\frac{1}{3}\right)^2} = \frac{\frac{4}{45}}{\frac{4}{9}} = \frac{1}{5}.$$

(#96, MLC–09–08) For a special 3-year deferred whole life annuity-due on (x):
(i) $i = 0.04$
(ii) The first annual payment is 1000.
(iii) Payments in the following years increase by 4% per year.
(iv) There is no death benefit during the three year deferral period.
(v) Level benefit premiums are payable at the beginning of each of the first three years.
(vi) $e_x = 11.05$ is the curtate expectation of life for (x).
(vii)

k	1	2	3
${}_k p_x$	0.99	0.98	0.97

Calculate the annual benefit premium.
(A) 2625 (B) 2825 (C) 3025 (D) 3225 (E) 3425

(B) The APV of benefits is

$$\sum_{j=3}^{\infty}(1000)v^j(1+i)^{j-3}\cdot {}_jp_x$$

$$=(1000)v^3\sum_{j=3}^{\infty}{}_jp_x$$

$$=(1000)v^3(e_x-p_x-{}_2p_x)=(1000)v^3(11.05-0.99-0.98)=8072.0869$$

The APV of premiums is

$$\pi+v\pi p_x+v^2\pi\cdot{}_2p_x=\pi(1+v(0.99)+v^2(0.98))=2.857988166\pi.$$

Hence, $\pi=\frac{8072.086937}{2.857988166}=2824.394808.$

(#97, MLC–09–08) For a special fully discrete 10-payment whole life insurance on (30) with level annual benefit premium π:
(i) The death benefit is equal to 1000 plus the refund, without interest, of the benefit premiums paid.
(ii) $A_{30} = 0.102$
(iii) ${}_{10|}A_{30} = 0.088$
(iv) $(IA)^1_{30:\overline{10|}} = 0.078$
(v) $\ddot{a}_{30:\overline{10|}} = 7.747$
Calculate π.
(A) 14.9 (B) 15.0 (C) 15.1 (D) 15.2 (E) 15.3

(B) We have that

$$\pi \ddot{a}_{30:\overline{10}|} = (1000)A_{30} + \pi (IA)^{1}_{30:\overline{10}|} + 10 \cdot {}_{10|}A_{30}.$$

Hence,

$$\pi = \frac{(1000)(0.102)}{7.747 - 0.078 - (10)(0.088)} = 15.02430402.$$

(#99, MLC–09–08) On January 1, 2002, Pat, age 40, purchases a 5-payment, 10-year term insurance of 100,000:
(i) Death benefits are payable at the moment of death.
(ii) Contract premiums of 4000 are payable annually at the beginning of each year for 5 years.
(iii) $i = 0.05$
(iv) L is the loss random variable at time of issue.
Calculate the value of L if Pat dies on June 30, 2004.
(A) 77,100 (B) 80,700 (C) 82,700 (D) 85,900 (E) 88,000

(A) Pat dies 2.5 years after issue of the contract. Hence,

$$_0L = (100000)v^{2.5} - 4000(1 + v + v^2) = 77079.3717.$$

(#111, MLC–09–08) For a special fully discrete 3-year term insurance on (x):

(i) $b_{k+1} = \begin{cases} 0 & \text{for } k = 0 \\ 1,000(11-k) & \text{for } k = 1,2 \end{cases}$

(ii)

k	q_{x+k}
0	0.200
1	0.100
2	0.097

(iii) $i = 0.06$

Calculate the level annual benefit premium for this insurance.

(A) 518 (B) 549 (C) 638 (D) 732 (E) 799

(A) Let B be the APV of benefits. Let P be the APV of premiums. Let π be the annual benefit premium. We have that

$$B = (10000)v^2(0.8)(0.1) + (9000)v^3(0.8)(0.9)(0.097) = 1239.748249$$
$$= P = \pi(1 + v(0.8) + v^2(0.8)(0.9)) = 2.395514418\pi.$$

So, $\pi = \frac{1239.748249}{2.395514418} = 517.5290283.$

(#119, MLC–09–08) For a special fully continuous whole life insurance on (x):
(i) The level premium is determined using the equivalence principle.
(ii) Death benefits are given by $b_t = (1+i)^t$ where i is the interest rate.
(iii) L is the loss random variable at $t = 0$ for the insurance.
(iv) T is the future lifetime random variable of (x).
Which of the following expressions is equal to L?
(A) $\frac{(v^T - \overline{A}_x)}{(1-\overline{A}_x)}$ (B) $(v^T - \overline{A}_x)(1+\overline{A}_x)$ (C) $\frac{(v^T - \overline{A}_x)}{(1+\overline{A}_x)}$ (D) $(v^T + \overline{A}_x)(1-\overline{A}_x))$ (E) $\frac{(v^T + \overline{A}_x)}{(1+\overline{A}_x)}$

(A) The actuarial present value of the death benefits is $\int_0^\infty (1+i)^t v^t f_{T(x)}(t)\, dt\, dt = 1$. Let $\overline{P}$ be the rate of benefit premiums. The actuarial present value of the benefit premiums is $\overline{P}\overline{a}_x$. Hence, $\overline{P} = \frac{1}{\overline{a}_x}$ and the loss is

$$1 - \frac{1}{\overline{a}_x}\overline{Y}_x = 1 - \frac{1}{\overline{a}_x}\frac{1-v^T}{\delta} = 1 - \frac{1-v^T}{1-\overline{A}_x} = \frac{v^T - \overline{A}_x}{1-\overline{A}_x}.$$

(#127, MLC–09–08) For a special fully discrete 35-payment whole life insurance on (30):
(i) The death benefit is 1 for the first 20 years and is 5 thereafter.
(ii) The initial benefit premium paid during the each of the first 20 years is one fifth of the benefit premium paid during each of the 15 subsequent years.
(iii) Mortality follows the Illustrative Life Table.
(iv) $i = 0.06$
(v) $A_{30:\overline{20}|} = 0.32307$.
(vi) $\ddot{a}_{30:\overline{35}|} = 14.835$
Calculate the initial annual benefit premium.
(A) 0.010 (B) 0.015 (C) 0.020 (D) 0.025 (E) 0.030

(B) Let π be the initial premium. Equating benefits and premiums,

$$5\pi\ddot{a}_{30:\overline{35}|} - 4\pi\ddot{a}_{30:\overline{20}|} = A_{30} + 4 \cdot {}_{20|}A_{30} = A_{30} + 4 \cdot {}_{20}E_{30}A_{50}.$$

From the life table $A_{30} = 0.10248$. ${}_{20}E_{30} = 0.29347$ and $A_{50} = 0.24905$. The APV of benefits is

$$A_{30}+4\cdot{}_{20}E_{30}A_{50} = 0.10248+4(0.29347)(0.24905) = 0.394834814.$$

The APV of premiums is

$$\ddot{a}_{30:\overline{35}|} = \ddot{a}_{30} - {}_{35}E_{30}\ddot{a}_{65} = \ddot{a}_{30} - {}_{20}E_{30} \cdot {}_{15}E_{50}\ddot{a}_{65}$$
$$=15.8561 - (0.29374)(0.51081)(9.8969) = 14.37111638,$$
$$\ddot{a}_{30:\overline{20}|} = \ddot{a}_{30} - {}_{20}E_{30}\ddot{a}_{50} = 15.8561 - (0.29374)(13.2668) = 11.9591101$$
$$5\pi\ddot{a}_{30:\overline{35}|} - 4\pi\ddot{a}_{30:\overline{20}|} = (5)(14.37111638)\pi - (4)(11.95911017)\pi$$
$$=26.33855932\pi$$

Hence, $\pi = \frac{0.394834814}{26.33861332} = 0.01499075212.$

(#139, MLC–09–08) For a fully discrete whole life insurance of 10,000 on (30):
(i) π denotes the annual premium and $L(\pi)$ denotes the loss-at-issue random variable for this insurance.
(ii) Mortality follows the Illustrative Life Table.
(iii) $i = 0.06$
Calculate the lowest premium, π', such that the probability is less than 0.5 that the loss $L(\pi')$ is positive.
(A) 34.6 (B) 36.6 (C) 36.8 (D) 39.0 (E) 39.1

(C) We have that $\frac{\ell_{30}}{2} = \frac{9501381}{2} = 4750690.5$ and $\ell_{77} = 4828182$ and $\ell_{78} = 4530360$. So,

$$\mathbb{P}\{K_{30} \geq 49\} = \mathbb{P}\{T_{30} \geq 48\} < 0.5 < \mathbb{P}\{T_{30} \geq 47\} = \mathbb{P}\{K_{30} \geq 48\}$$

and $\mathbb{P}\{K_{30} < 48\} < 0.5 < \mathbb{P}\{K_{30} < 49\}$. We have that $\pi' = \frac{10000}{\ddot{s}_{\overline{48|}0.06}} = 36.77032951$.

(#142, MLC–09–08) For a fully continuous whole life insurance of 1 on (x):
(i) π is the benefit premium.
(ii) L is the loss-at-issue random variable with the premium equal to π.
(iii) L^* is the loss-at-issue random variable with the premium equal to 1.25π.
(iv) $\bar{a}_x = 5.0$.
(v) $\delta = 0.08$
(vi) $\mathrm{Var}(L) = 0.5625$
Calculate the sum of the expected value and the standard deviation of L^*.
(A) 0.59 (B) 0.71 (C) 0.86 (D) 0.89 (E) 1.01

(B) We have that

$$\pi = \frac{\overline{A}_x}{\overline{a}_x} = \frac{1 - \overline{a}_x\delta}{\overline{a}_x} = \frac{1-(5)(0.08)}{5} = 0.12, \ \pi^* = (1.25)\pi = 0.15,$$

$$E[L^*] = \overline{A}_x - (1.25)\pi\overline{a}_x = -(0.25)\pi\overline{a}_x = -(0.25)(0.12)(5) = -0.15,$$

$$\mathrm{Var}(L^*) = \mathrm{Var}(\overline{Z}_x)\left(1+\frac{\pi^*}{\delta}\right)^2 = \mathrm{Var}(L)\frac{\left(1+\frac{\pi^*}{\delta}\right)^2}{\left(1+\frac{\pi}{\delta}\right)^2} = 0.5625\frac{\left(1+\frac{0.15}{0.08}\right)^2}{\left(1+\frac{0.12}{0.08}\right)^2}$$

$$\sqrt{\mathrm{Var}(L^*)} = \sqrt{0.5625}\frac{1+\frac{0.15}{0.08}}{1+\frac{0.12}{0.08}} = 0.8625,$$

$$E[L^*] + \sqrt{\mathrm{Var}(L^*)} = -0.15 + 0.7744565217 = 0.7125.$$

(#204, MLC–09–08) For a 10-payment, 20-year term insurance of 100,000 on Pat:
(i) Death benefits are payable at the moment of death.
(ii) Contract premiums of 1600 are payable annually at the beginning of each year for 10 years.
(iii) $i = 0.05$
(iv) L is the loss random variable at the time of issue.
Calculate the minimum value of L as a function of the time of death of Pat.
(A) $-21{,}000$ (B) $-17{,}000$ (C) $-13{,}000$ (D) $-12{,}400$ (E) $-12{,}000$

(C) The minimum loss is attained if Pat dies after paying all benefit premiums, but before the death benefit is made. The minimum loss is

$$-1600\ddot{a}_{\overline{10}|} = -12972.51468.$$

237–239. Use the following information for questions 237–239.
For a semicontinuous 20-year endowment insurance of 25,000 on (x), you are given:
(i) The following expenses are payable at the beginning of the year:

	Percent of Premium	Per 1000 Insurance	Per Policy
First Year	25%	2.00	15.00
Renewal	5%	0.50	3.00

(ii) Deaths are uniformly distributed over each year of age.
(iii) $\overline{A}_{x:\overline{20}|} = 0.4058$
(iv) $A_{x:\overline{20}|}^{\;\;1} = 0.3195$
(v) $\ddot{a}_{x:\overline{20}|} = 12.522$
(vi) $i = 0.05$
(vii) Premiums are determined using the equivalence principle.

(#237, MLC–09–08) Calculate the expense-loaded first-year net premium including policy fee assuming that per–policy expenses are matched separately by first-year and renewal policy fees.
(A) 884 (B) 899 (C) 904 (D) 909 (E) 924

(C) Let G be the annual expense-loaded premium excluding renewal policy fees. We have that

$$G\ddot{a}_{x:\overline{20|}} = (25000)\overline{A}_{x:\overline{20|}} + (0.2)G + (0.05)G\ddot{a}_{x:\overline{20|}} + (12.5)\ddot{a}_{x:\overline{20|}} + (25)(1.5).$$

Hence,

$$G = \frac{(25000)(0.4058) + (12.5)(12.522) + 37.5}{(12.522)(1 - 0.05) - 0.20} = 883.9871237.$$

Let F_1 be the one year policy fee. We have that $F_1 = 15 + (0.25)F_1$. So, $F_1 = \frac{15}{0.75} = 20$. The expense-loaded first-year net premium including policy fee assuming that per–policy expenses are matched separately by first-year and renewal policy fees is $883.9871237 + 20 = 903.9871237$.

(#238, MLC–09–08) Calculate the expense-loaded renewal net premiums including policy fee assuming that per–policy expenses are matched separately by first-year and renewal policy fees.
(A) 884 (B) 887 (C) 899 (D) 909 (E) 912

(B) By the previous problem the annual expense-loaded premium excluding renewal policy fees is 883.9871237. Let F_R be the renewal policy fee. We have that $(3 + 0.05F_R)a_{x:\overline{19}|} = F_R a_{x:\overline{19}|}$. Hence, $F_R = \frac{3}{1-0.05} = 3.157894737$ and the expense-loaded renewal net premiums including policy fee assuming that per–policy expenses are matched separately by first-year and renewal policy fees is $883.9871237 + 3.157894737 = 887.1450184$.

(#239, MLC–09–08) Calculate the level annual expense-loaded premium.

(A) 884 (B) 888 (C) 893 (D) 909 (E) 913

(B) We have that

$$G\ddot{a}_{x:\overline{20}|} = (25000)\overline{A}_{x:\overline{20}|} + (0.2)G + (0.05)G\ddot{a}_{x:\overline{20}|} + (12.5+3)\ddot{a}_{x:\overline{20}|} + ((25)(1.5)+12).$$

Hence,

$$G = \frac{(25000)(0.4058) + (15.5)(12.522) + 37.5 + 12}{(12.522)(1-0.05) - 0.20} = 888.225019.$$

(#240, MLC–09–08) For a 10-payment 20-year endowment insurance of 1000 on (40), you are given:
(i) The following expenses:

	First Year		Subsequent Years	
	Percent of Premium	Per Policy	Percent of Premium	Per Policy
Taxes	4%	0	4%	0
Sales Commission	25%	0	5%	0
Policy Maintenance	0	10	0	5

(ii) Expenses are paid at the beginning of each policy year.
(iii) Death benefits are payable at the moment of death.
(iv) The expense-loaded premium is determined using the equivalence principle.
Which of the following is a correct expression for the expense-loaded premium?

(A) $(1000\overline{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{9}|})/(0.96\ddot{a}_{40:\overline{10}|} - 0.25 - 0.05\ddot{a}_{40:\overline{9}|})$

(B) $(1000\overline{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{9}|})/(0.91\ddot{a}_{40:\overline{10}|} - 0.2)$

(C) $(1000\overline{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{19}|})/(0.96\ddot{a}_{40:\overline{10}|} - 0.25 - 0.05\ddot{a}_{40:\overline{9}|})$

(D) $(1000\overline{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{19}|})/(0.91\ddot{a}_{40:\overline{10}|} - 0.2)$

(E) $(1000\overline{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{9}|})/(0.95\ddot{a}_{40:\overline{10}|} - 0.2 - 0.04\ddot{a}_{40:\overline{20}|})$

(D) Equating the APV of premiums and expenses, we get that

$$G\ddot{a}_{40:\overline{10}|} = (1000)\overline{A}_{40:\overline{20}|} + G(0.09)\ddot{a}_{40:\overline{10}|} + G(0.2) + 5\ddot{a}_{40:\overline{20}|} + 5.$$

Noticing that $5\ddot{a}_{40:\overline{20}|} + 5 = 5a_{40:\overline{19}|} + 10$, we get that

$$G = \frac{(1000)\overline{A}_{40:\overline{20}|} + 5\ddot{a}_{40:\overline{19}|} + 5}{0.91\ddot{a}_{40:\overline{10}|} - 0.2} = \frac{(1000)\overline{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{19}|}}{0.91\ddot{a}_{40:\overline{10}|} - 0.2}$$

(#241, MLC–09–08) For a fully discrete whole life insurance of 100,000 on (x), you are given:
(i) Expenses, paid at the beginning of the year, are as follows:

Year	Percentage of Premium Expenses	Per 1000 Expenses	Per Policy Expenses
1	50%	2.0	150
2+	4%	0.5	25

(ii) $i = 0.04$
(iii) $\ddot{a}_x = 10.8$
(iv) Per policy expenses are matched by a level policy fee to be paid in each year.
Calculate the expense-loaded premium using the equivalence principle.
(A) 5800 (B) 5930 (C) 6010 (D) 6120 (E) 6270

(C) Equating the APV of premiums and expenses, we get that

$$G\ddot{a}_x = (100000)A_x + G(0.04)\ddot{a}_x + G(0.46) + ((100)(0.5) + 25)\ddot{a}_x$$
$$+ ((100)(1.5) + 125)$$
$$=(100000)A_x + G(0.04)\ddot{a}_x + G(0.46) + 75\ddot{a}_x + 275.$$

Hence

$$A_x = 1 - d\ddot{a}_x = 1 - (10.8)(4/104) = 0.5846153846,$$
$$G = \frac{(100000)A_x + 75\ddot{a}_x + 275}{0.96\ddot{a}_x - 0.46}$$
$$=\frac{(100000)(0.5846153846) + (75)(10.8) + 275}{(0.96)(10.8) - 0.46}$$
$$=6009.945343.$$

(#245, MLC–09–08) For a fully discrete 5-payment 10-year deferred 20-year term insurance of 1000 on (30), you are given:
(i) The following expenses:

	Year 1		Years 2–10	
	Percent of Premium	Per Policy	Percent of Premium	Per Policy
Taxes	5%	0	5%	0
Sales commission	25%	0	10%	0
Policy maintenance	0	20	0	10

(ii) Expenses are paid at the beginning of each policy year.
(iii) The expense-loaded premium is determined using the equivalence principle.
Which of the following is correct expression for the expense-loaded premium?

(A) $(1000\ {}_{10|20}A_{30} + 20 + 10a_{30:\overline{19|}})/(0.95\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10\ddot{a}_{30:\overline{4|}})$
(B) $(1000\ {}_{20|30}A_{30} + 20 + 10a_{30:\overline{19|}})/(0.85\ddot{a}_{30:\overline{5|}} - 0.15)$
(C) $(1000\ {}_{10|20}A_{30} + 20 + 10a_{30:\overline{19|}})/(0.95\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10a_{30:\overline{4|}})$
(D) $(1000\ {}_{10|20}A_{30} + 20 + 10a_{30:\overline{9|}})/(0.95\ddot{a}_{30:\overline{5|}} - 0.25 - 0.10\ddot{a}_{30:\overline{4|}})$
(E) $(1000\ {}_{10|20}A_{30} + 20 + 10a_{30:\overline{9|}})/(0.85\ddot{a}_{30:\overline{5|}} - 0.15)$

(E) Equating the APV of benefits and expenses,

$$G\ddot{a}_{30:\overline{5}|} = 1000\,{}_{10|20}A_{30} + 0.15G\ddot{a}_{30:\overline{5}|} + 0.15G + 10 + 10\ddot{a}_{30:\overline{10}|}.$$

Hence,

$$G = \frac{1000\ {}_{10|20}A_{30} + 10 + 10\ddot{a}_{30:\overline{10}|}}{0.85\ddot{a}_{30:\overline{5}|} - 0.15} = \frac{1000\ {}_{10|20}A_{30} + 20 + 10a_{30:\overline{9}|}}{0.85\ddot{a}_{30:\overline{5}|} - 0.15}.$$

(#246, MLC–09–08) For a special single premium 2-year endowment insurance on (x), you are given:
(i) Death benefits, payable at the end of the year of death, are:

$$b_1 = 3000, \quad b_2 = 2000$$

(ii) The maturity benefit is 1000.
(iii) Expenses, payable at the beginning of the year:
(a) Taxes are 2% of the expense-loaded premium.
(b) Commissions are 3% of the expense-loaded premium.
(c) Other expenses are 15 in the first year and 2 in the second year.
(iv) $i = 0.04$
(v) $p_x = 0.9 \quad p_{x+1} = 0.8$
Calculate the expense-loaded premium using the equivalence principle.
(A) 670 (B) 940 (C) 1000 (D) 1300 (E) 1370

(E) The APV of benefits is

$$(3000)(0.1)v+(2000)(0.9)(0.2)v^2+(1000)(0.9)(0.8)v^2 = 1286.982249.$$

Equating the APV of benefits and expenses,

$$G = 1286.98224915+15+2v(0.9)+0.05G = 1303.713018+0.05G.$$

So, $G = \frac{1303.713018}{0.95} = 1372.329492.$

(#247, MLC–09–08) For a fully discrete 2-payment, 3-year term insurance of 10,000 on (x), you are given:

(i) $i = 0.05$

(ii) $q_x = 0.10$, $q_{x+1} = 0.15$, $q_{x+2} = 0.20$

(iii) Death is the only decrement.

(iv) Expenses, paid at the beginning of the year, are:

Policy Year	Per policy	Per 1000 of insurance	Fraction of premium
1	25	4.50	0.20
2	10	1.50	0.10
3	10	1.50	–

(v) Settlement expenses, paid at the end of the year of death, are 20 per policy plus 1 per 1000 of insurance.

(vi) G is the expense-loaded level annual premium for this insurance.

(vii) The single benefit premium for this insurance is 3499.

Calculate G, using the equivalence principle.

(A) 1597 (B) 2296 (C) 2303 (D) 2343 (E) 2575

(C) Equating benefits and expenses,

$$G\ddot{a}_{x:\overline{2}|} = (10000)A^{1}_{x:\overline{3}|} + (30)A_{x:\overline{3}|} + 25\ddot{a}_{x:\overline{3}|} + 45 + 0.1G + 0.1G\ddot{a}_{x:\overline{2}|}$$
$$=(10030)A_{x:\overline{3}|} + 25\ddot{a}_{x:\overline{3}|} + 45 + 0.1G + 0.1G\ddot{a}_{x:\overline{2}|}.$$

We know that $(10000)A^{1}_{x:\overline{3}|} = 3499$. We have that

$$\ddot{a}_{x:\overline{2}|} = 1 + v(0.9) = 1.857142857,$$
$$\ddot{a}_{x:\overline{3}|} = 1 + v(0.9) + v^2(0.9)(0.85) = 2.551020408,$$
$$G = \frac{(10030)A_{x:\overline{3}|} + 25\ddot{a}_{x:\overline{3}|} + 45}{0.9\ddot{a}_{x:\overline{2}|} - 0.1}$$
$$=\frac{(1.003)(3499) + (25)(2.551020408) + 45}{(0.9)(1.857142857) - 0.1} = 2302.537052.$$

(#248, MLC–09–08) For a fully discrete 20-year endowment insurance of 10,000 on (50), you are given:
(i) Mortality follows the Illustrative Life Table.
(ii) $i = 0.06$
(iii) The annual contract premium is 495.
(iv) Expenses are payable at the beginning of the year.
(v) The expenses are:

	Percent of Premium	Per Policy	Per 1000 of Insurance
First Year	35%	20	15.00
Renewal	5%	5	1.50

Calculate the actuarial present value of amounts available for profit and contingencies. (A) 930 (B) 1080 (C) 1130 (D) 1180 (E) 1230

(D) From the table $A_{50} = 0.24905$, $A_{70} = 0.51495$ and ${}_{20}E_{50} = 0.23047$. Hence,

$$A_{50:\overline{20}|} = A_{50} + {}_{20}E_{50}(1 - A_{70}) = 0.24905 + (0.23047)(1 - 0.51495)$$
$$=0.3608394735,$$
$$\ddot{a}_{50:\overline{20}|} = \frac{1 - 0.3608394735}{6/106} = 11.29183597.$$

The APV of amounts available for profit and contingencies

$$\begin{aligned}
&(495)\ddot{a}_{50:\overline{20}|} - (10000)A_{50:\overline{20}|} - (0.05)(495)\ddot{a}_{50:\overline{20}|} - 20\ddot{a}_{50:\overline{20}|}\\
&\quad - 150 - (0.3)(495)\\
=&((495)(0.95) - 20)\ddot{a}_{50:\overline{10}|} - (10000)A_{50:\overline{20}|} - 150 - (0.3)(495)\\
=&((495)(0.95) - 20)(11.29183597) - (10000)(0.3608394735)\\
&\quad - 150 - (0.3)(495)\\
=&1177.25441.
\end{aligned}$$

