Chapter 28

Direct Current Circuits

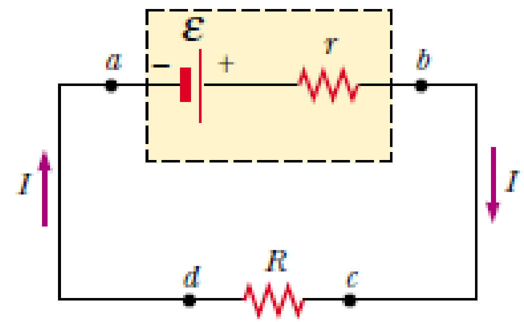
CHAPTER OUTLINE

28.1 Electromotive Force

The emf $\mathcal{E}$ of a battery is the maximum possible voltage that the battery can provide between its terminals.

$$\Delta V = \mathcal{E} - Ir$$

r is called an internal resistance.



$$\Delta V = IR$$

R is called the load resistance

$$\mathcal{E} = IR + Ir$$

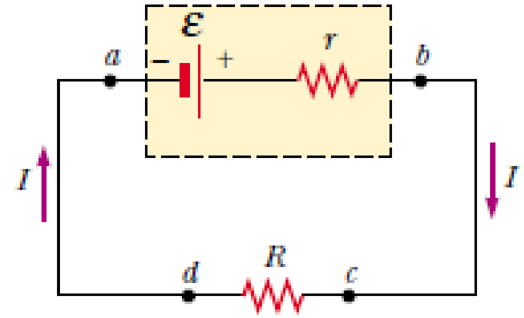
$$I = \frac{\mathcal{E}}{R + r}$$

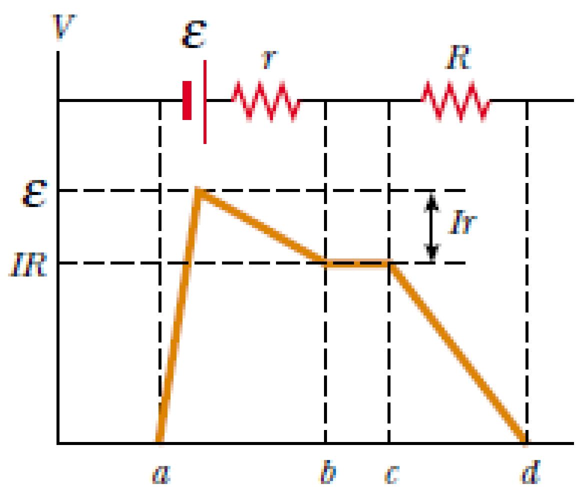
This equation shows that the current in this simple circuit depends on both the load resistance *R* external to the battery and the internal resistance *r*. If *R* is much greater than *r*, as it is in many real-world circuits, we can neglect *r*.

If we multiply Equation by the current *I*, we obtain

$$I\mathcal{E} = I^2R + I^2r$$

the total power output *I* of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r.


ε
a
−
+
r
b
I
I
d
R
c


V
ε
r
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IR
Ir
a
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d

Example 28.1 **Terminal Voltage of a Battery**

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω. Its terminals are connected to a load resistance of 3.00 Ω.

(A) Find the current in the circuit and the terminal voltage of the battery.

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \text{V}}{3.05\ \Omega} = \boxed{3.93\ \text{A}}$$

$$\Delta V = \mathcal{E} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.05\ \Omega) = \boxed{11.8\ \text{V}}$$

To check this result, we can calculate the voltage across the load resistance R:

$$\Delta V = IR = (3.93\ \text{A})(3.00\ \Omega) = 11.8\ \text{V}$$

(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

The power delivered to the load resistor is

$$\mathcal{P}_R = I^2 R = (3.93\ \mathrm{A})^2\ (3.00\ \Omega) = \boxed{46.3\ \mathrm{W}}$$

The power delivered to the internal resistance is

$$\mathcal{P}_r = I^2 r = (3.93\ \mathrm{A})^2\ (0.05\ \Omega) = \boxed{0.772\ \mathrm{W}}$$

Hence, the power delivered by the battery is the sum of these quantities, or 47.1 W

28.2 Resistors in Series and Parallel

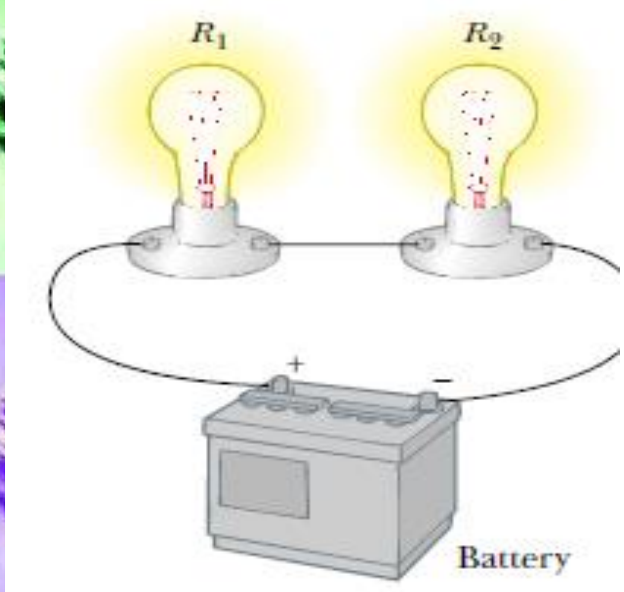
Resistors in Series

for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 in the same time interval.

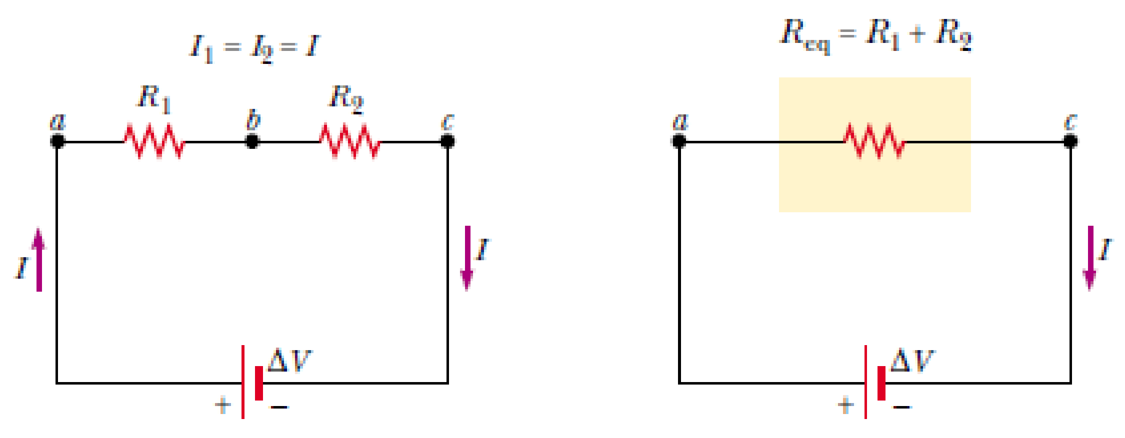
$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

$$\Delta V = IR_{eq} = I(R_1 + R_2) \longrightarrow R_{eq} = R_1 + R_2$$



(a)



(b) (c)

The equivalent resistance of three or more resistors connected in series is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

This relationship indicates that the equivalent resistance of a series connection of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.

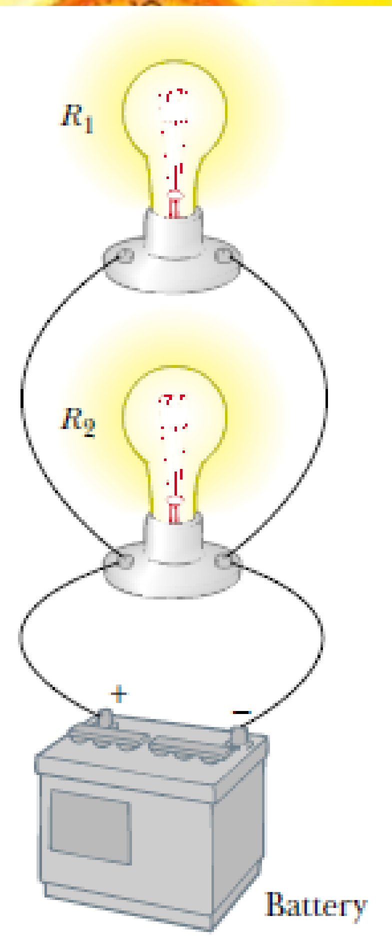
Resistors in Parallel

when resistors are connected in parallel, the potential differences across the resistors is the same.

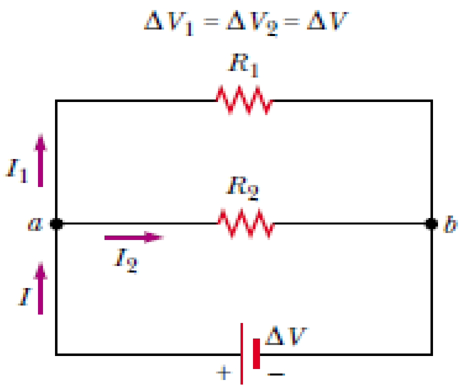
$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

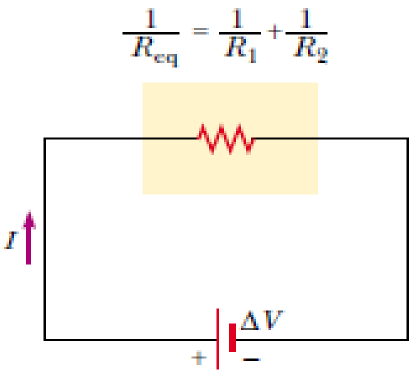
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$



(a)



(b)



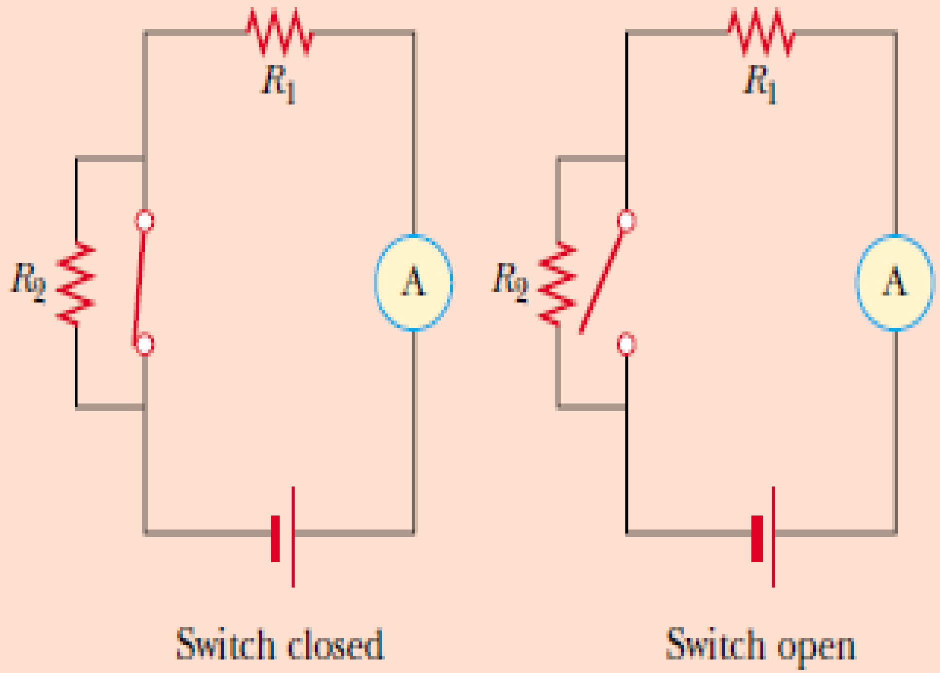
(c)

An extension of this analysis to three or more resistors in parallel gives

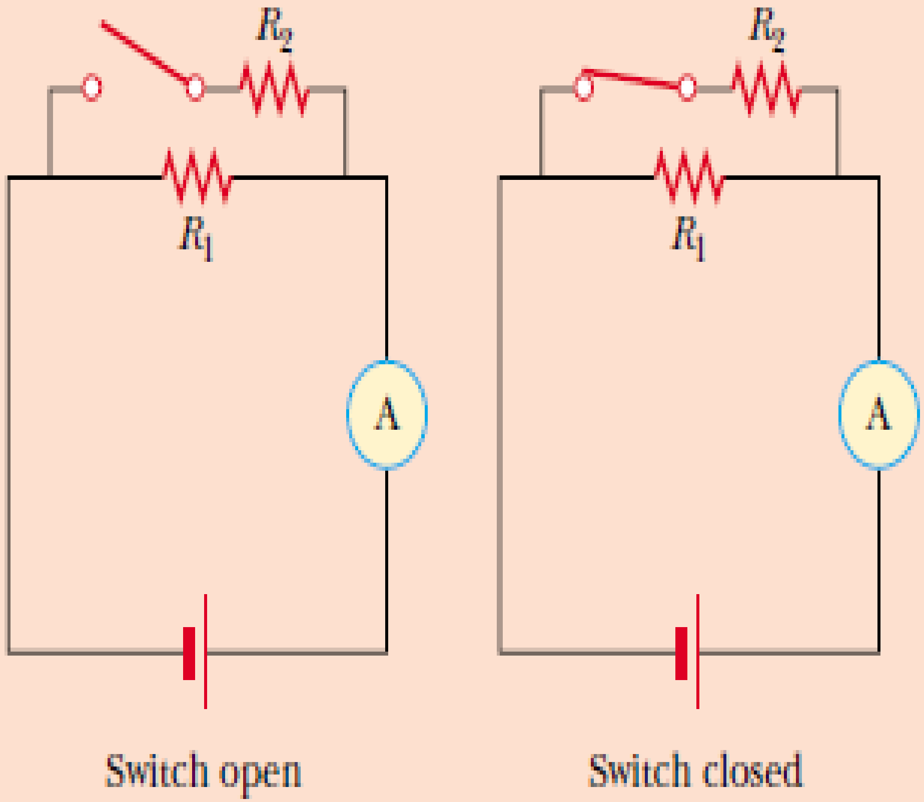
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

We can see from this expression that the inverse of the equivalent resistance of two or more resistors connected in parallel is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.

Quick Quiz 28.4 With the switch in the circuit of Figure 28.5 closed (left), there is no current in R_2, because the current has an alternate zero-resistance path through the switch. There is current in R_1 and this current is measured with the ammeter (a device for measuring current) at the right side of the circuit. If the switch is opened (Fig. 28.5, right), there is current in R_2. What happens to the reading on the ammeter when the switch is opened? (a) the reading goes up; (b) the reading goes down; (c) the reading does not change.



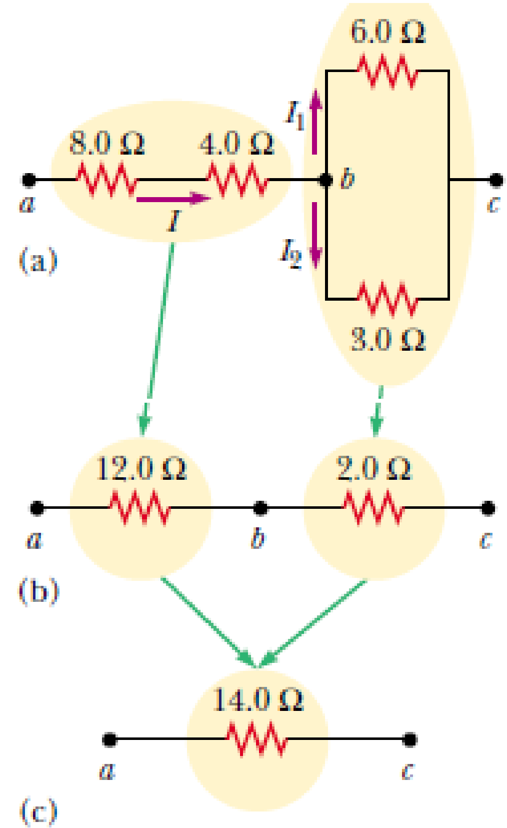
Quick Quiz 28.7 With the switch in the circuit of Figure 28.7 open (left), there is no current in R_2. There is current in R_1 and this current is measured with the ammeter at the right side of the circuit. If the switch is closed (Fig. 28.7, right), there is current in R_2. What happens to the reading on the ammeter when the switch is closed? (a) the reading goes up; (b) the reading goes down; (c) the reading does not change.



Example 28.4 **Find the Equivalent Resistance**

Four resistors are connected as shown in Figure 28.9a.

(A) Find the equivalent resistance between points *a* and *c*.



(B) What is the current in each resistor if a potential difference of 42 V is maintained between *a* and *c*?

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\ \text{V}}{14.0\ \Omega} = 3.0\ \text{A}$$

$$I_1 = 1.0\ \text{A}$$

$$I_2 = 2.0\ \text{A}$$

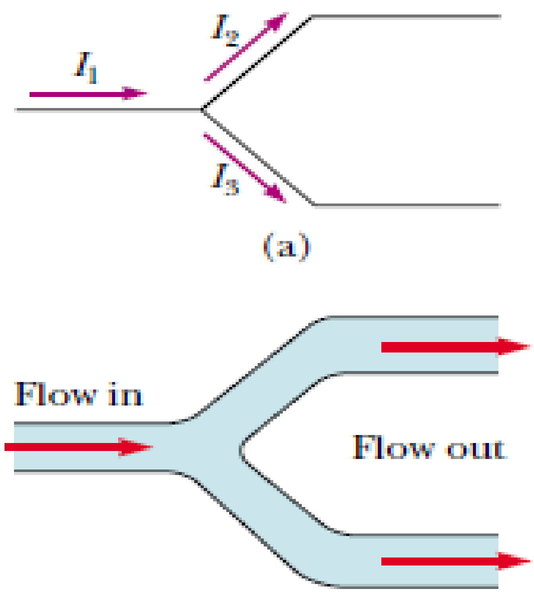
28.3 Kirchhoff's Rules

As we saw in the preceding section, simple circuits can be analyzed using the expression $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop. The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff 's rules:

1. **Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}} \tag{28.9}$$

Kirchhoff's first rule is a statement of conservation of electric charge. All charges that enter a given point in a circuit must leave that point because charge cannot build up at a point.



$$I_1 = I_2 + I_3$$

2. **Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \qquad (28.10)$$

Kirchhoff's second rule follows from the law of conservation of energy

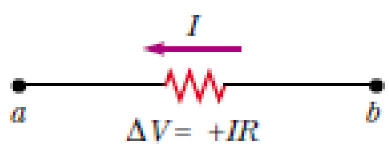
When applying Kirchhoff's second rule in practice, we imagine *traveling* around the loop and consider changes in *electric potential,* rather than the changes in *potential energy* described in the preceding paragraph. You should note the following sign conventions when using the second rule:

•Because charges move from the high-potential end of a resistor toward the low potential end, if a resistor is traversed in the direction of the current, the potential difference ΔV across the resistor is $-IR$ (Fig. 28.15a).

(a)

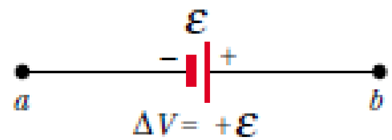
•If a resistor is traversed in the direction *opposite* the current, the potential difference ΔV across the resistor is $+IR$ (Fig. 28.15b).

(b)



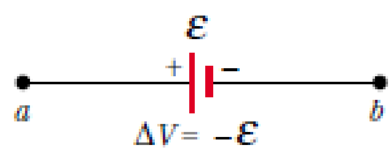
•If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from - to +), the potential difference ΔV is + (Fig. 28.15c). The emf of the battery increases the electric potential as we move through it in this direction.

(c)



- If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from + to −), the potential difference ΔV is $-\mathcal{E}$ (Fig. 28.15d). In this case the emf of the battery reduces the electric potential as we move through it.

(d)



PROBLEM-SOLVING HINTS

Kirchhoff's Rules

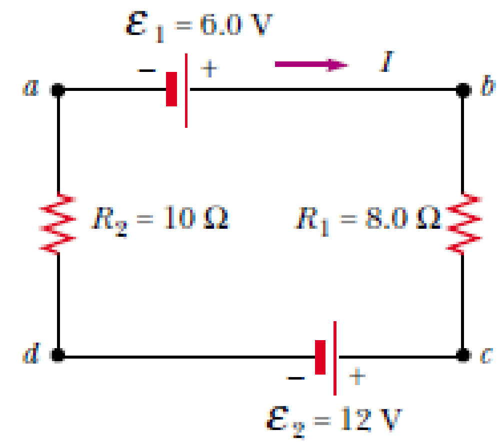
- Draw a circuit diagram, and label all the known and unknown quantities. You must assign a *direction* to the current in each branch of the circuit. Although the assignment of current directions is arbitrary, you must adhere rigorously to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junctions in the circuit that provide new relationships among the various currents.
- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the potential difference as you imagine crossing each element while traversing the closed loop (either clockwise or counterclockwise). Watch out for errors in sign!
- Solve the equations simultaneously for the unknown quantities. Do not be alarmed if a current turns out to be negative; *its magnitude will be correct and the direction is opposite to that which you assigned.*

A single-loop circuit contains two resistors and two batteries, as shown in Figure 28.16. (Neglect the internal resistances of the batteries.)

(A) Find the current in the circuit.

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$



$$(1) \qquad I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\ \text{V} - 12\ \text{V}}{8.0\ \Omega + 10\ \Omega} = -0.33\ \text{A}$$

(B) What power is delivered to each resistor? What power is delivered by the 12-V battery?

$$\mathcal{P}_1 = I^2R_1 = (0.33\ \text{A})^2(8.0\ \Omega) = \boxed{0.87\ \text{W}}$$

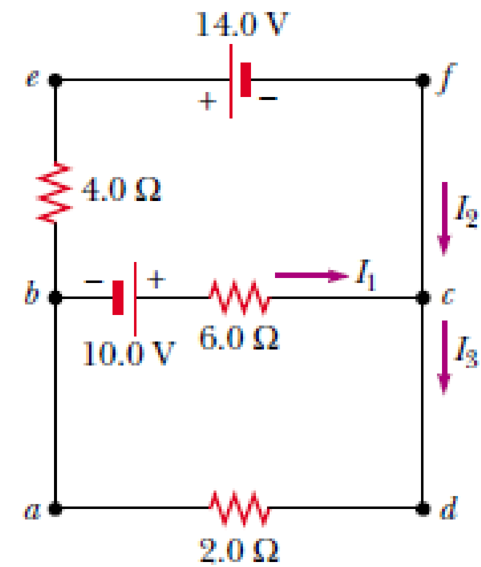
$$\mathcal{P}_2 = I^2R_2 = (0.33\ \text{A})^2(10\ \Omega) = \boxed{1.1\ \text{W}}$$

The 12-V battery delivers power $I\mathcal{E}_2 = 4.0$

Example 28.9 **Applying Kirchhoff's Rules**

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 28.17.

(1) $I_1 + I_2 = I_3$



(2) *abcda* $10.0\ \text{V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)I_3 = 0$

(3) *befcb* $-14.0\ \text{V} + (6.0\ \Omega)I_1 - 10.0\ \text{V} - (4.0\ \Omega)\ I_2 = 0$

Expressions (1), (2), and (3) represent three independent equations with three unknowns. Substituting Equation (1) into Equation (2) gives

$$10.0\ \text{V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)\ (I_1 + I_2) = 0$$

$$(4) \qquad 10.0\ \text{V} = (8.0\ \Omega)I_1 + (2.0\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \qquad -12.0\ \text{V} = -(3.0\ \Omega)I_1 + (2.0\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22.0\ \text{V} = (11.0\ \Omega)I_1$$

$$I_1 = \boxed{2.0\ \text{A}}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

$$(2.0\ \Omega)\,I_2 = (3.0\ \Omega)\,I_1 - 12.0\ \text{V}$$
$$= (3.0\ \Omega)(2.0\ \text{A}) - 12.0\ \text{V} = -6.0\ \text{V}$$

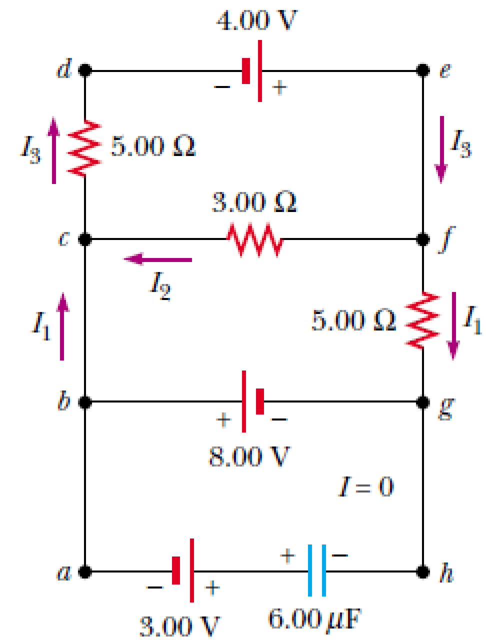
$$I_2 = \boxed{-3.0\ \text{A}}$$

Finally,

$$I_3 = I_1 + I_2 = \boxed{-1.0\ \text{A}}$$

Example 28.10 A Multiloop Circuit

(A) Under steady-state conditions, find the unknown currents I_1, I_2, and I_3 in the multiloop circuit shown in Figure 28.18.



$$(1) \qquad I_1 + I_2 = I_3$$

applied to loops *defcd* and *cfgbc*, traversed clockwise, gives

$$(2) \qquad defcd \quad 4.00\text{ V} - (3.00\ \Omega)I_2 - (5.00\ \Omega)I_3 = 0$$

$$(3) \qquad cfgbc \quad (3.00\ \Omega)I_2 - (5.00\ \Omega)I_1 + 8.00\text{ V} = 0$$

From Equation (1) we see that $I_1 = I_3 - I_2$, which, when substituted into Equation (3), gives

$$(4) \qquad (8.00\ \Omega)I_2 - (5.00\ \Omega)I_3 + 8.00\text{ V} = 0$$

Subtracting Equation (4) from Equation (2), we eliminate I_3 and find that

$$I_2 = -\frac{4.00\ \text{V}}{11.0\ \Omega} = \boxed{-0.364\ \text{A}}$$

Using $I_2 = -0.364$ A in Equations (3) and (1) gives

$$I_1 = \boxed{1.38\ \text{A}} \qquad I_3 = \boxed{1.02\ \text{A}}$$

(B) What is the charge on the capacitor?

$$-8.00\ \text{V} + \Delta V_{\text{cap}} - 3.00\ \text{V} = 0$$

$$\Delta V_{\text{cap}} = 11.0\ \text{V}$$

$$Q = (6.00\ \mu\text{F})(11.0\ \text{V}) = \boxed{66.0\ \mu\text{C}}$$