Chapter 25

Electric Potential

CHAPTER OUTLINE

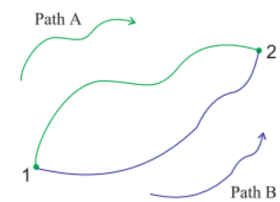
25.1 Potential Difference and Electric Potential

- When a test charge (q_0) is placed in an electric field, it experiences a force $q_0\mathbf{E}$

- The force ($q_0\mathbf{E}$) is conservative

DEFINITION: A force is **conservative** *if* the work done on a particle by the force is *independent of the path taken.*

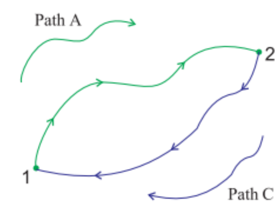
- For an infinitesimal displacement $d\mathbf{s}$ of a charge, the work done by the electric field on the charge is $\mathbf{F} \cdot d\mathbf{s} = q_0\mathbf{E} \cdot d\mathbf{s}$.



∴ For conservative forces,

$$\underbrace{\int_1^2 \vec{F}\cdot d\vec{s}}_{\text{Path A}} = \underbrace{\int_1^2 \vec{F}\cdot d\vec{s}}_{\text{Path B}}$$

- As this work is done by the field, the potential energy of the charge-field system is changed by $\Delta U = -q_o \mathbf{E} \cdot d\mathbf{s}$



Let's consider a path starting at point 1 to 2 through *Path A* and from 2 to 1 through *Path C*

$$\text{Work done} = \underbrace{\int_1^2 \vec{F}\cdot d\vec{s}}_{\text{Path A}} + \underbrace{\int_2^1 \vec{F}\cdot d\vec{s}}_{\text{Path C}}$$
$$= \underbrace{\int_1^2 \vec{F}\cdot d\vec{s}}_{\text{Path A}} - \underbrace{\int_1^2 \vec{F}\cdot d\vec{s}}_{\textbf{Path B}}$$

- For a finite displacement of the charge from A to B, the change in potential energy is

$$\Delta U = U_B - U_A = -q_0 \int_A^B \mathbf{E}\cdot d\mathbf{s}$$

this line integral does not depend on the path taken from *A* to *B*.

25.1 Potential Difference and Electric Potential

- **Electric potential** (or simply the **potential**) ***V*** *is* potential energy per unit charge U/q_0

$$V = \frac{U}{q_0}$$

- **potential** has a value at every point in an electric field

- potential energy is a scalar quantity means that electric potential also is a scalar quantity

- The **potential difference** $\Delta V = V_B - V_A$ between two points *A* and *B*

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The *potential is characteristic of the field only (source charge distribution)*, independent of a charged test particle that may be placed in the field. *Potential energy is characteristic of the charge–field system* due to an interaction between the field and a charged particle placed in the field

- **The potential difference (ΔV)** between *A* and *B* ***depends only on the source charge distribution*** (consider points *A* and *B* *without* the presence of the test charge), while the difference in **potential energy exists** only if a ***test charge is moved between the points***.

25.1 Potential Difference and Electric Potential

- ❖ Assume a charge moves in an electric field without any change in its kinetic energy
- ❖ The work performed on the charge is changes the potential energy of the system

$$\boxed{W = \Delta U = q\,\Delta V}$$

Electric potential unit is **volt** (V) $1\ \mathrm{V} \equiv 1\ \frac{\mathrm{J}}{\mathrm{C}}$

1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

Electron volt (eV) is a unit of energy and ***defines the energy a charge–field system gains or loses when a charge of magnitude e (that is, an electron or a proton) is moved through a potential difference of 1 V.***

$$\boxed{1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{C \cdot V} = 1.60 \times 10^{-19}\ \mathrm{J}}$$

25.1 Potential Difference and Electric Potential

Quick Quiz 25.1 In Figure 25.1, two points A and B are located within a region in which there is an electric field. The potential difference $\Delta V = V_B - V_A$ is (a) positive (b) negative (c) zero.

Quick Quiz 25.2 In Figure 25.1, a negative charge is placed at A and then moved to B. The change in potential energy of the charge–field system for this process is (a) positive (b) negative (c) zero.

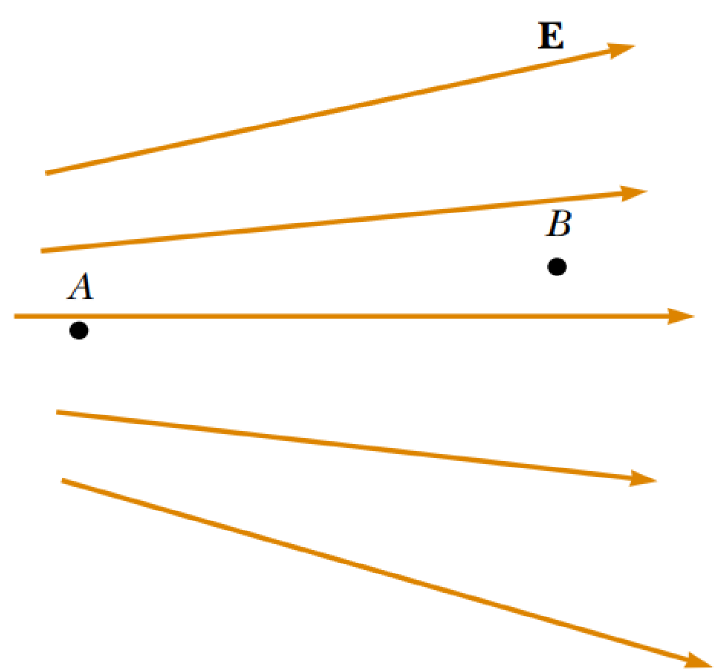
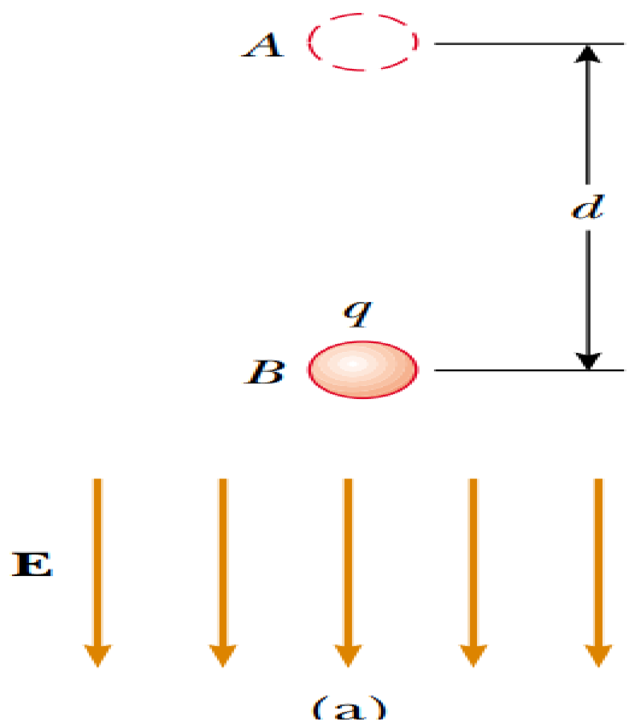


Figure 25.1 (Quick Quiz 25.1) Two points in an electric field.

25.2 Potential Differences in a Uniform Electric Field



(a) When the electric field **E** is directed downward, point *B* is at a lower electric potential than point *A*. When a positive test charge moves from point *A* to point *B*, the charge–field system loses electric potential energy.

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\int_A^B (E\cos 0°)\,ds = -\int_A^B E\,ds$$

Because E is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E\int_A^B ds = -Ed \qquad (25.6)$$

The negative sign indicates that the electric potential at point B is lower than at point A; that is, $V_B < V_A$. **Electric field lines always point in the direction of decreasing electric potential,** as shown in Figure 25.2a.

25.2 Potential Differences in a Uniform Electric Field

- The change in the potential energy $\Delta U = q_0 \, \Delta V = -q_0 E d$

- If q_0 is positive, then ΔU is negative.

We conclude that **a system consisting of a positive charge and an electric field loses electric potential energy when the charge moves in the direction of the field.**

- Electric field does work on a positive charge when the charge moves in the direction of the electric field.

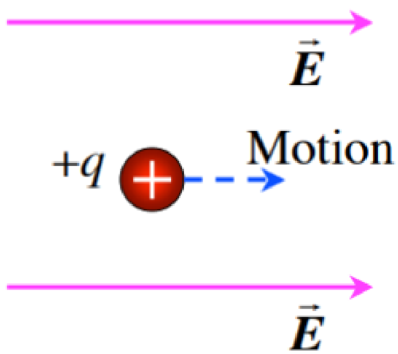
- If a positive test charge is released from rest in this electric field, it experiences an electric force $q_0\mathbf{E}$ in the direction of $\mathbf{E}$ (downward in Fig. 25.2a). Therefore, it accelerates downward, gaining kinetic energy.

  As the charged particle gains kinetic energy, the charge–field system loses an equal amount of potential energy

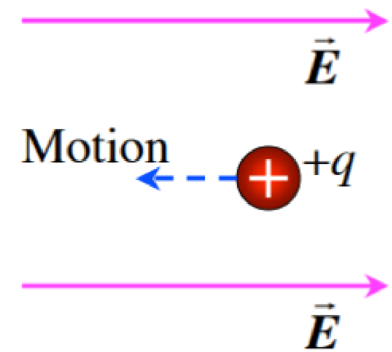
- If q_0 is negative, then ΔU is positive and the situation is reversed:

  A system consisting of a negative charge and an electric field gains electric potential energy when the charge moves in the direction of the field.
- If a negative charge is released from rest in an electric field, it accelerates in a direction opposite the direction of the field. In order for the negative charge to move in the direction of the field, an external agent must apply a force and do positive work on the charge.

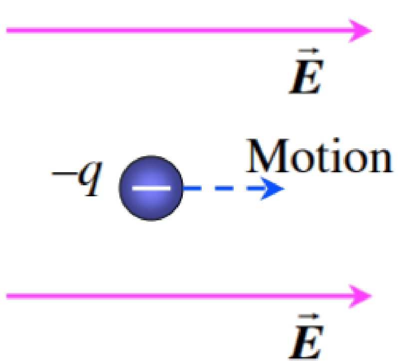
25.2 Potential Differences in a Uniform Electric Field



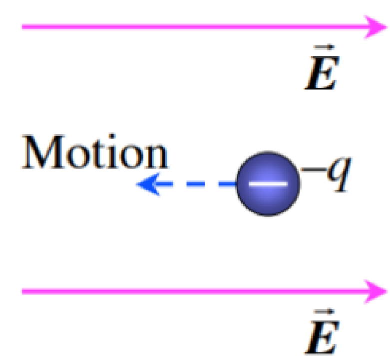
The field does positive work on it and *the potential energy decreases* ($\Delta U = -$)



The field does negative work on it and the potential energy increases($\Delta U = +$)

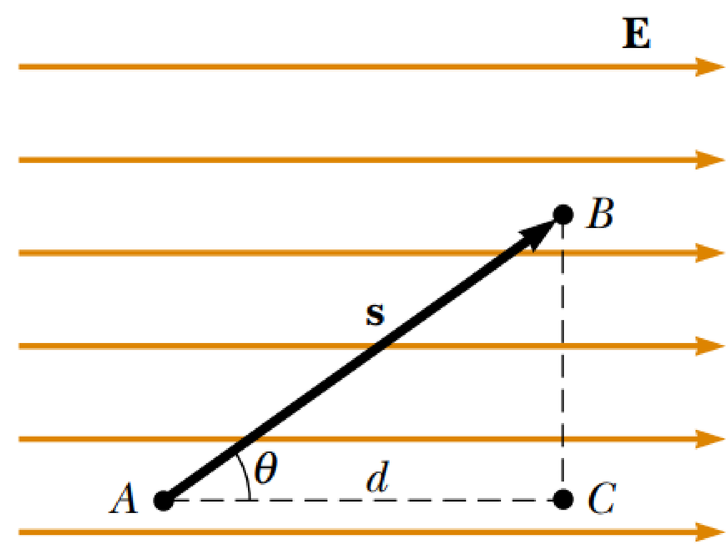


The field does negative work on it and the potential energy increases($\Delta U = +$)



The field does positive work on it and *the potential energy decreases* ($\Delta U = -$)

25.2 Potential Differences in a Uniform Electric Field



A uniform electric field directed along the positive *x* axis. Point *B* is at a lower electric potential than point *A*. Points *B* and *C* are at the *same* electric potential.

Equipotential surface

is given to any surface consisting of a continuous distribution of points having the same electric potential.

The potential difference $V_B - V_A$ is equal to the potential difference $V_C - V_A$

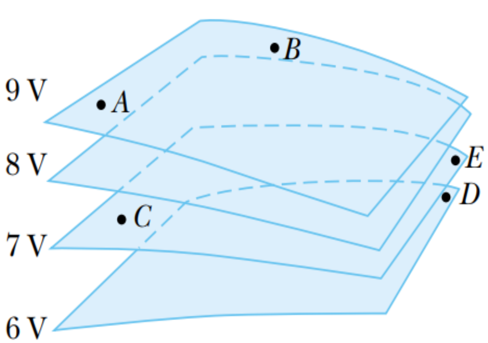


Figure 25.4 (Quick Quiz 25.3) Four equipotential surfaces.

Quick Quiz 25.3 The labeled points in Figure 25.4 are on a series of equipotential surfaces associated with an electric field. Rank (from greatest to least) the work done by the electric field on a positively charged particle that moves from *A* to *B*; from *B* to *C*; from *C* to *D*; from *D* to *E*.

Quick Quiz 25.4 For the equipotential surfaces in Figure 25.4, what is the *approximate* direction of the electric field? (a) Out of the page (b) Into the page (c) Toward the right edge of the page (d) Toward the left edge of the page (e) Toward the top of the page (f) Toward the bottom of the page.

Example 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge

A battery produces a specified potential difference ΔV between conductors attached to the battery terminals. A 12-V battery is connected between two parallel plates, as shown in Figure 25.5. The separation between the plates is $d = 0.30$ cm, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and if we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.

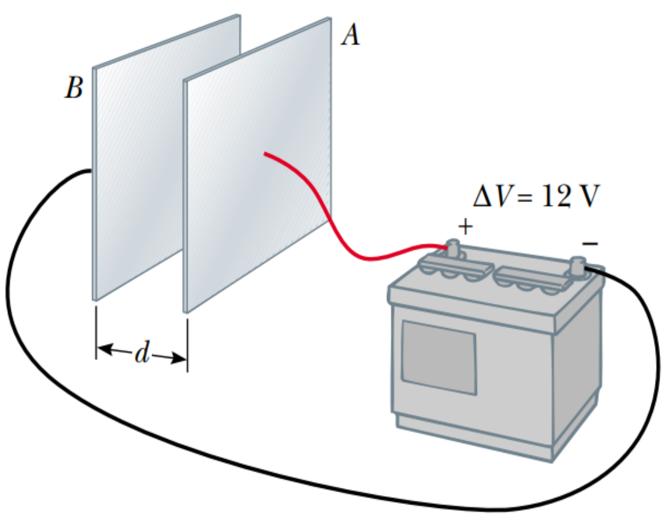


Figure 25.5 (Example 25.1) A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference ΔV divided by the plate separation d.

Solution The electric field is directed from the positive plate (A) to the negative one (B), and the positive plate is at a higher electric potential than the negative plate is. The potential difference between the plates must equal the potential difference between the battery terminals. We can understand this by noting that all points on a conductor in equilibrium are at the same electric potential[1]; no potential difference exists between a terminal and any portion of the plate to which it is connected. Therefore, the magnitude of the electric field between the plates is, from Equation 25.6,

$$E = \frac{|V_B - V_A|}{d} = \frac{12\text{ V}}{0.30 \times 10^{-2}\text{ m}} = \boxed{4.0 \times 10^3\text{ V/m}}$$

The configuration of plates in Figure 25.5 is called a *parallel-plate capacitor*, and is examined in greater detail in Chapter 26.

Example 25.2 Motion of a Proton in a Uniform Electric Field

Interactive

A proton is released from rest in a uniform electric field that has a magnitude of 8.0×10^4 V/m (Fig. 25.6). The proton undergoes a displacement of 0.50 m in the direction of **E**.

(A) Find the change in electric potential between points A and B.

Solution Because the positively charged proton moves in the direction of the field, we expect it to move to a position of lower electric potential. From Equation 25.6, we have

$$\Delta V = -Ed = -(8.0 \times 10^4 \text{ V/m})(0.50 \text{ m}) = \boxed{-4.0 \times 10^4 \text{ V}}$$

(B) Find the change in potential energy of the proton–field system for this displacement.

Solution Using Equation 25.3,

$$\Delta U = q_0 \,\Delta V = e\,\Delta V$$
$$= (1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V})$$
$$= \boxed{-6.4 \times 10^{-15} \text{ J}}$$

The negative sign means the potential energy of the system decreases as the proton moves in the direction of the electric field. As the proton accelerates in the direction of the field, it gains kinetic energy and at the same time the system loses electric potential energy.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

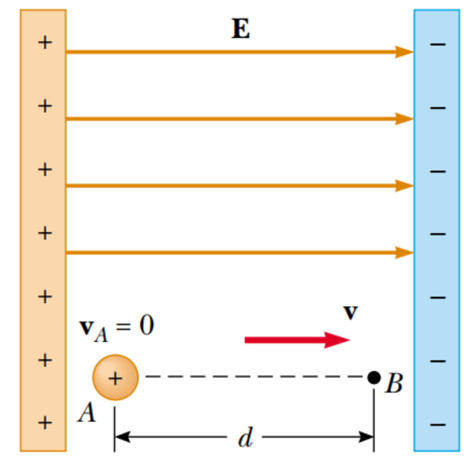


Figure 25.6 (Example 25.2) A proton accelerates from A to B in the direction of the electric field.

Solution The charge–field system is isolated, so the mechanical energy of the system is conserved:

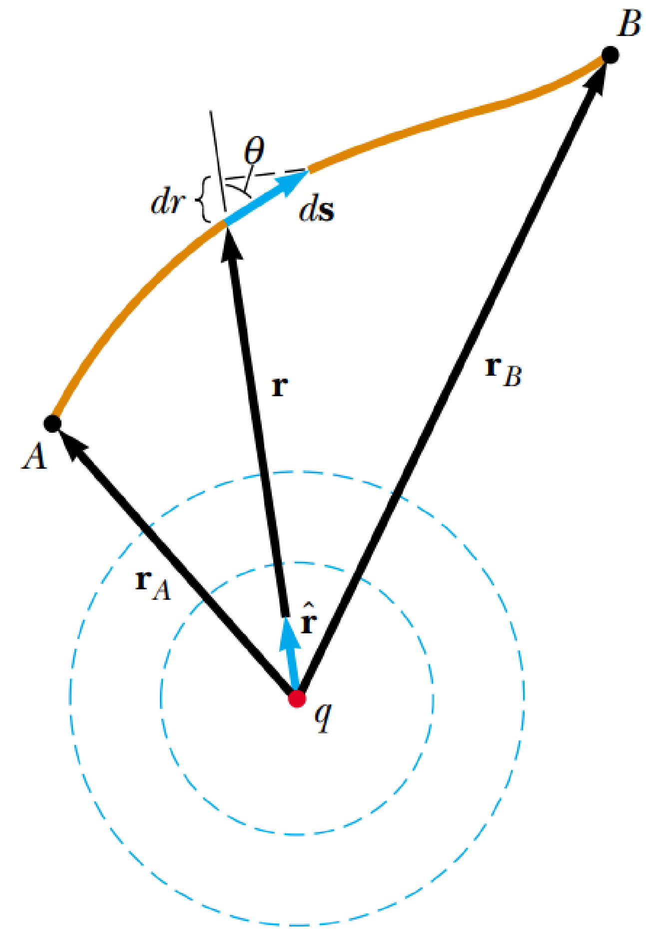
$$\Delta K + \Delta U = 0$$
$$(\tfrac{1}{2}mv^2 - 0) + e\,\Delta V = 0$$
$$v = \sqrt{\frac{-(2e\,\Delta V)}{m}}$$
$$= \sqrt{\frac{-2(1.6 \times 10^{-19} \text{ C})(-4.0 \times 10^4 \text{ V})}{1.67 \times 10^{-27} \text{ kg}}}$$
$$= \boxed{2.8 \times 10^6 \text{ m/s}}$$

What If? **What if the situation is exactly the same as that shown in Figure 25.6, but no proton is present? Could both parts (A) and (B) of this example still be answered?**

25.3 Electric Potential and Potential Energy Due to Point Charges

- Isolated positive point charge q produces an electric field that is directed radially outward from the charge.

A and *B* are the two arbitrary points



- The electric potential difference at a point located a distance r from the charge

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

- The electric field due to the point charge

$$\mathbf{E} = k_e q \hat{\mathbf{r}} / r^2$$

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{s}$$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\Big]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

The potential difference between points *A* and *B* due to a point charge *q* depends *only* on the initial and final radial coordinates r_A and r_B.

Potential difference between any two points *A* and *B* in a field created by a point charge depends only on the radial coordinates r_A and r_B

25.3 Electric Potential and Potential Energy Due to Point Charges

❖ To choose the reference of electric potential for a point charge

$$V = 0 \text{ at } r_A = \infty.$$

$$V = k_e \frac{q}{r}$$

Electric potential due to several point charges

applying the superposition principle

$$V = k_e \sum_i \frac{q_i}{r_i}$$

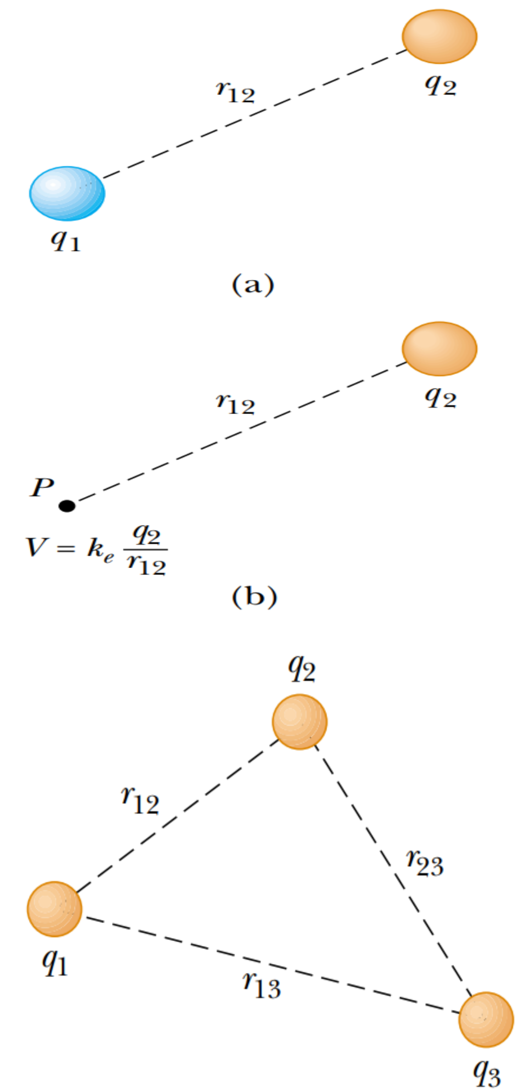
Easier to evaluate *V* than to evaluate **E**.

➢ Potential energy of a system of two charged particle

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

➢ total potential energy of the system of three charges

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$



Example 25.3 The Electric Potential Due to Two Point Charges Interactive

A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure 25.12a.

(A) Find the total electric potential due to these charges at the point P, whose coordinates are (4.00, 0) m.

Solution For two charges, the sum in Equation 25.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{2.00 \times 10^{-6}\ \text{C}}{4.00\ \text{m}} - \frac{6.00 \times 10^{-6}\ \text{C}}{5.00\ \text{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \text{V}}$$

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P (Fig. 25.12b).

Solution When the charge q_3 is at infinity, let us define $U_i = 0$ for the system, and when the charge is at P, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \text{C})(-6.29 \times 10^3\ \text{V})$$

$$= \boxed{-1.89 \times 10^{-2}\ \text{J}}$$

Therefore, because the potential energy of the system has decreased, positive work would have to be done by an external agent to remove the charge from point P back to infinity.

What If? **You are working through this example with a classmate and she says, "Wait a minute! In part (B), we ignored the potential energy associated with the pair of charges q_1 and q_2!" How would you respond?**

Answer Given the statement of the problem, it is not necessary to include this potential energy, because part (B) asks for the *change* in potential energy of the system as q_3 is brought in from infinity. Because the configuration of charges q_1 and q_2 does not change in the process, there is no ΔU associated with these charges. However, if part (B) had asked to find the change in potential energy when *all three* charges start out infinitely far apart and are then brought to the positions in Figure 25.12b, we would need to calculate the change as follows, using Equation 25.14:

$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) \times \left(\frac{(2.00 \times 10^{-6}\ \text{C})(-6.00 \times 10^{-6}\ \text{C})}{3.00\ \text{m}} + \frac{(2.00 \times 10^{-6}\ \text{C})(3.00 \times 10^{-6}\ \text{C})}{4.00\ \text{m}} + \frac{(3.00 \times 10^{-6}\ \text{C})(-6.00 \times 10^{-6}\ \text{C})}{5.00\ \text{m}}\right)$$

$$= -5.48 \times 10^{-2}\ \text{J}$$

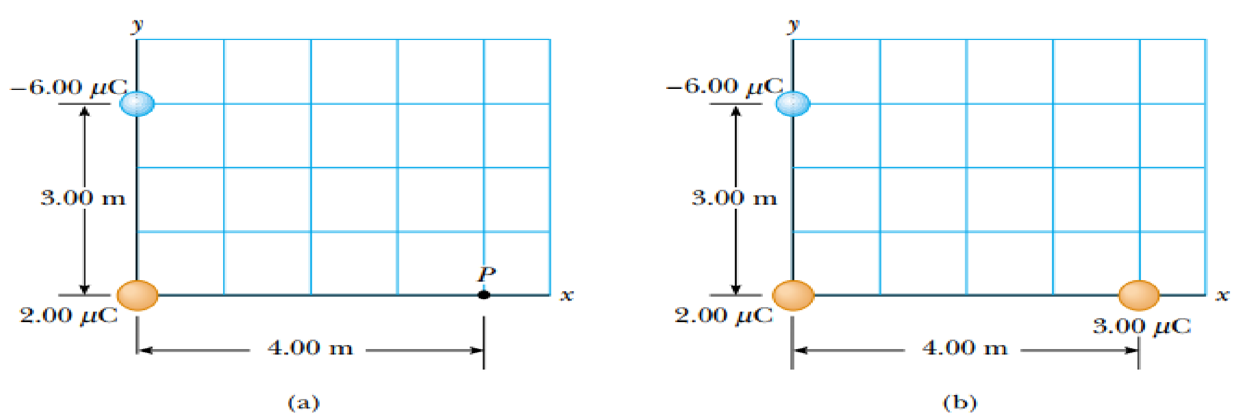
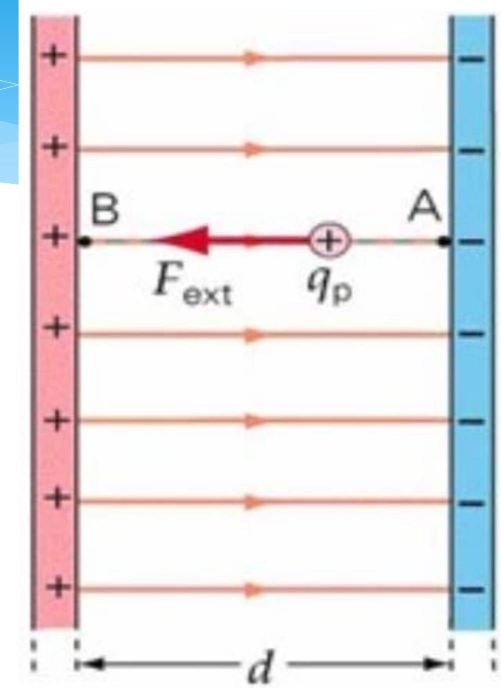


Figure 25.12 (Example 25.3) (a) The electric potential at P due to the two charges q_1 and q_2 is the algebraic sum of the potentials due to the individual charges. (b) A third charge $q_3 = 3.00\ \mu C$ is brought from infinity to a position near the other charges.

Selected Solved Problems (Chapter # 25)



Q1: A proton is moved from the negative plate to the positive plate of a parallel-plate arrangement. The plates are 1.5cm apart, and the electric field is uniform with a magnitude of 1500N/C.

A)What is the potential difference between the plates?

B) How much work would be required to move a proton from the negative to the positive plate?

C) If the proton is released from rest at the positive plate, what speed will it have just before it hits the negative plate?

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The difference in potential between the accelerating plates in the electron gun of a TV picture tube is about 25 000 V. If the distance between these plates is 1.50 cm, what is the magnitude of the uniform electric field in this region?

16. Given two 2.00-μC charges, as shown in Figure P25.16, and a positive test charge $q = 1.28 \times 10^{-18}$ C at the origin, (a) what is the net force exerted by the two 2.00-μC charges on the test charge q? (b) What is the electric field at the origin due to the two 2.00-μC charges? (c) What is the electric potential at the origin due to the two 2.00-μC charges?

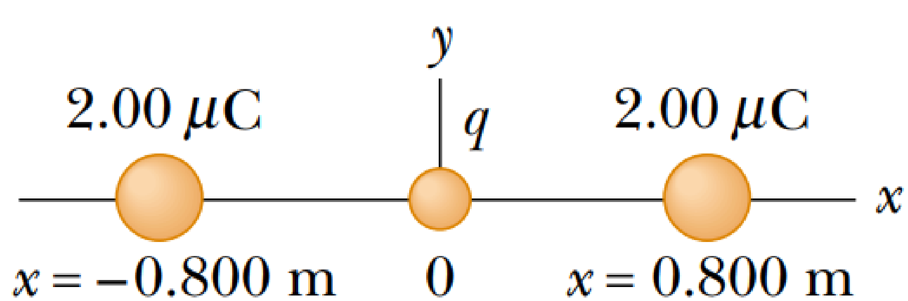

Figure P25.16

17. At a certain distance from a point charge, the magnitude of the electric field is 500 V/m and the electric potential is -3.00 kV. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

20. Two point charges, $Q_1 = +5.00$ nC and $Q_2 = -3.00$ nC, are separated by 35.0 cm. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

Selected Solved Problems (Chapter # 25)

19. The three charges in Figure P25.19 are at the vertices of an isosceles triangle. Calculate the electric potential at the midpoint of the base, taking $q = 7.00\ \mu C$.

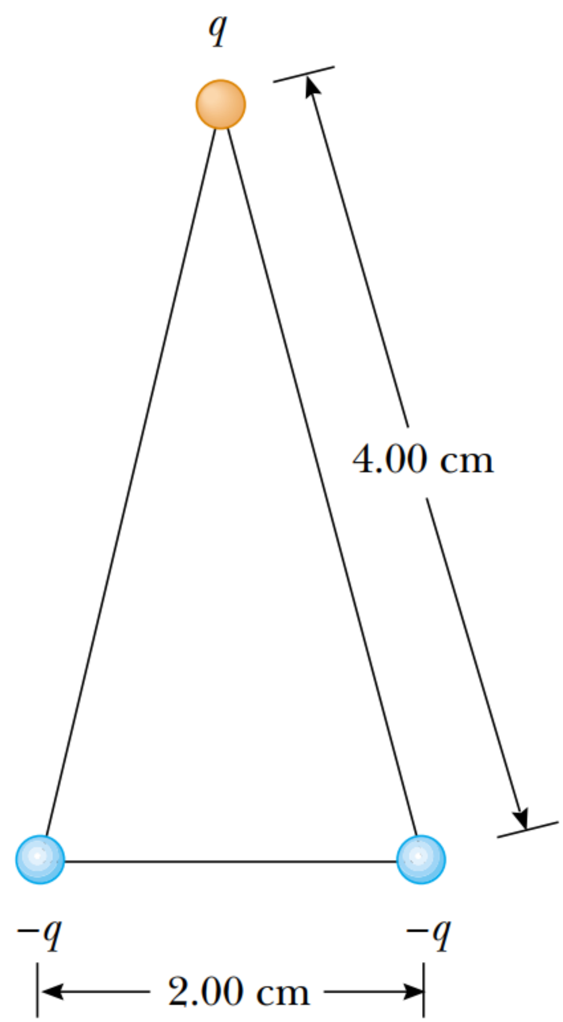

Figure P25.19

Selected Solved Problems (Chapter # 25)

7) A charge $q = -4.0\ \mu C$ is moved 0.25 m horizontally to point P in a region where an electric field is 150 V/m and directed vertically as shown. What is the change in the electric potential energy of the charge?

شحنة $q = -4.0\ \mu C$ 0.25 تحركت مسافة ٠,٢٥ متر أفقيا إلى نقطة P في المنطقة المجال الكهربائي هو ١٥٠ V/m والموجهه عموديا كما هو مبين .ما هو التغيير في مجال الطاقة الكهربائية للشحنة

a) -2.4×10^{-3} J
b) $+1.5 \times 10^{-4}$ J
c) -1.5×10^{-4} J
d) $+2.4 \times 10^{-3}$ J
e) zero joules