324 Stat

Lecture Notes

(5) Some Continuous Probability Distributions

(Book*: Chapter 6 ,pg171)

Probability& Statistics for Engineers & Scientists

By Walpole, Myers, Myers, Ye

5.1 Normal Distribution:

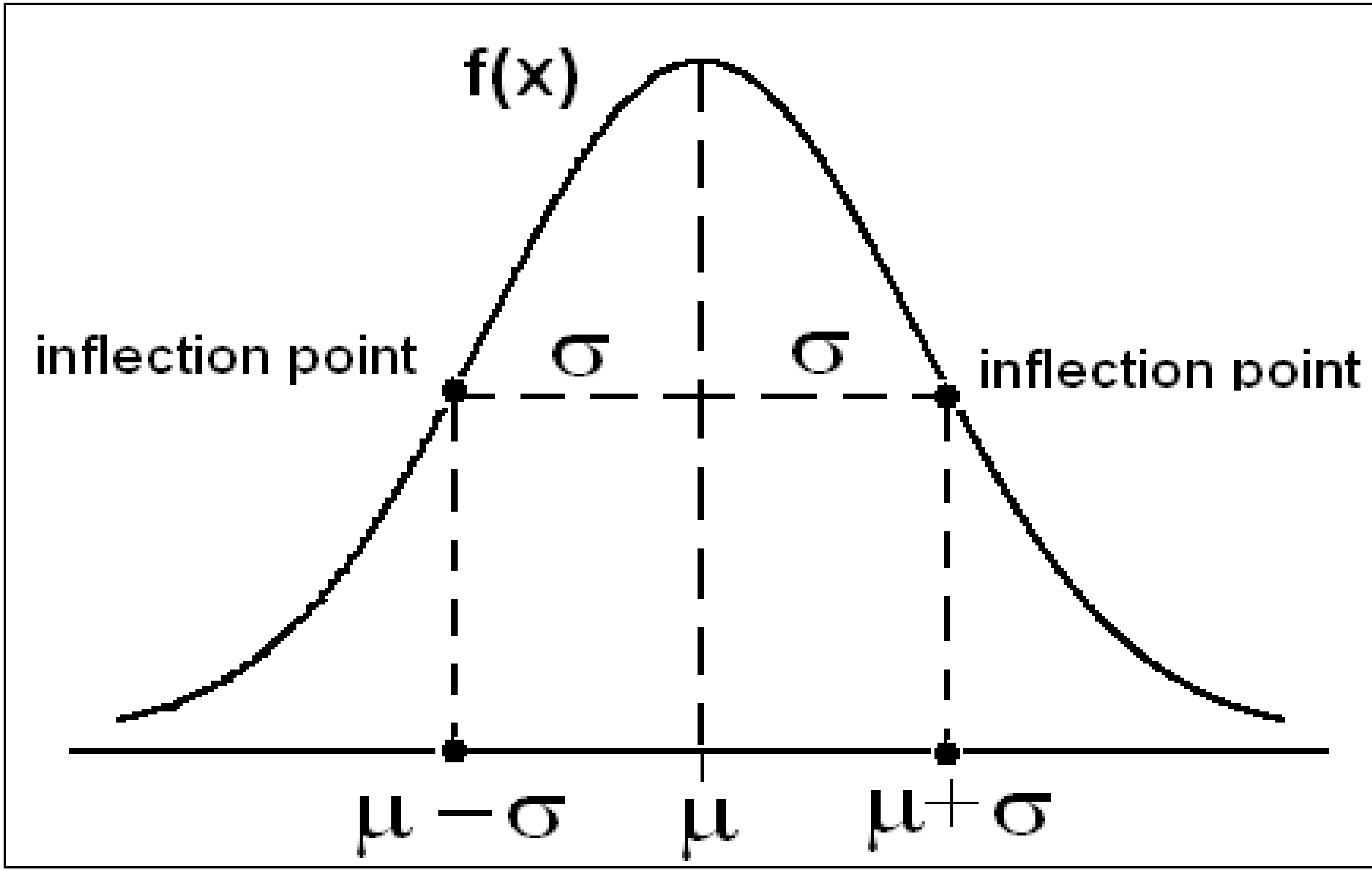
- The probability density function of the normal random variable **X,** with mean μ and variance σ² is given by:

$$f(x,\mu,\sigma^2)=\frac{1}{\sigma\sqrt{2\Pi}}\; e^{-\frac{1}{2}(\frac{X-\mu}{\sigma})^2}, \qquad -\infty < x < \infty$$

5.1.1 The Normal Curve has the Following Properties:

- The mode, which is the point on the horizontal axis where the curve is a maximum, occurs at X= μ , (Mode = Median = Mean).
- The curve is symmetric about a vertical axis through the mean μ .

- The total area under the curve and above the horizontal axis is equal to **1**.


f(x)
inflection point
σ
σ
inflection point
μ − σ
μ
μ+σ

Definition: Standard Normal Distribution:

- The distribution of a normal random variable with mean **zero** and variance **one** is called a standard normal distribution denoted by $Z \approx N(0,1)$

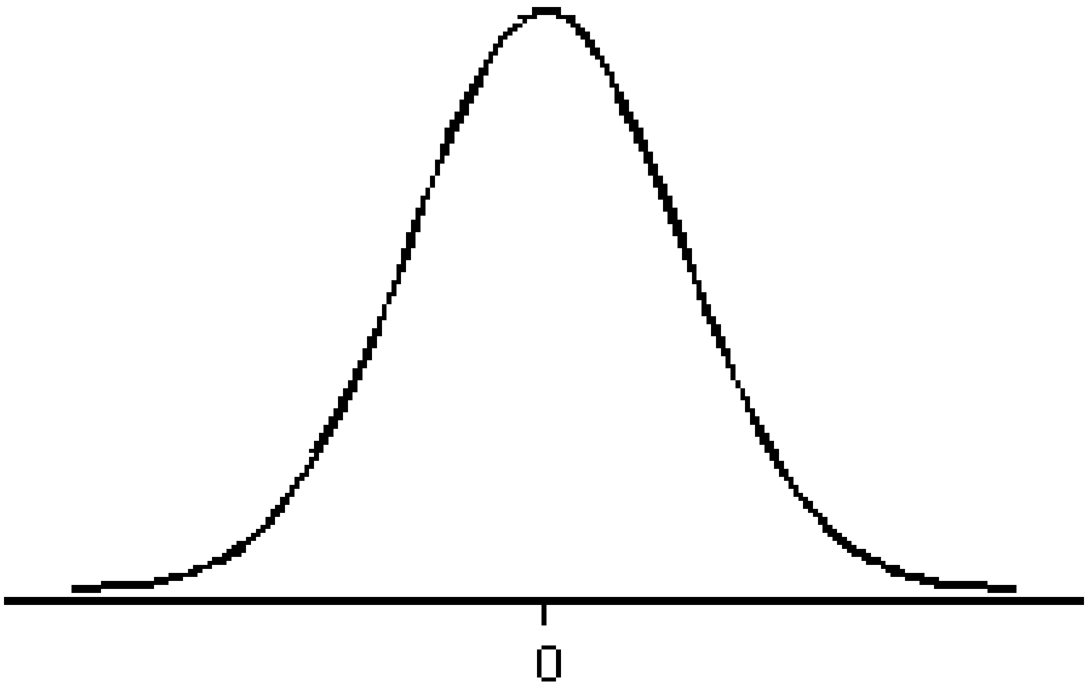
- **Areas under the Normal Curve:**

$$X \approx N(\mu, \sigma)$$

$$Z = \frac{X - \mu}{\sigma} \approx N(0,1)$$

- Using the standard normal tables to find the areas under the curve.

The pdf of Z~N(0,1) is given by:



EX (1):

Using the tables of the standard normal distribution, find:

$$(a)\, P(Z < 2.11)$$

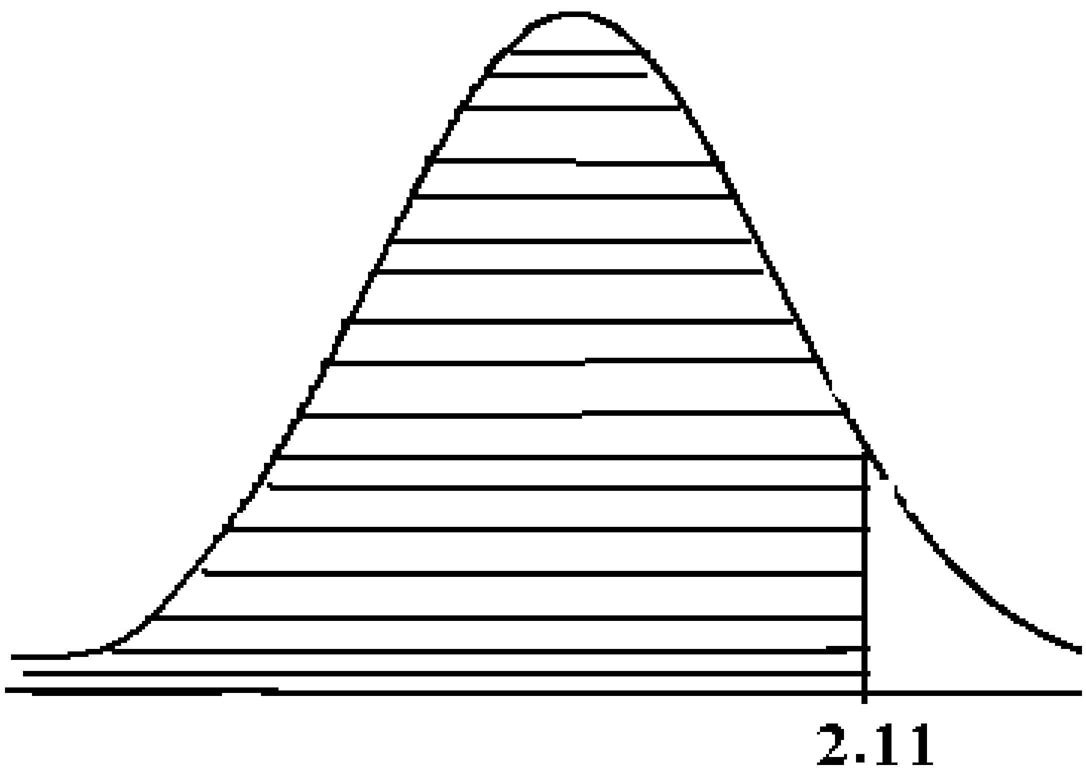
$$(b)\, P(Z > -1.33)$$

$$(c)\, P(Z = 3)$$

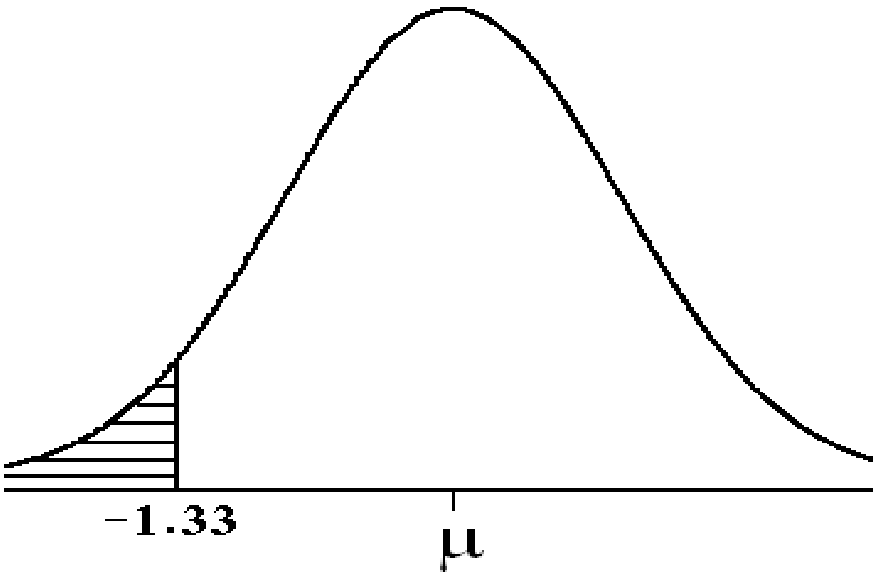
$$(d)\, P(-1.2 < Z < 2.1)$$

Solution:

$$(a)\,P(Z < 2.11) = 0.9826$$

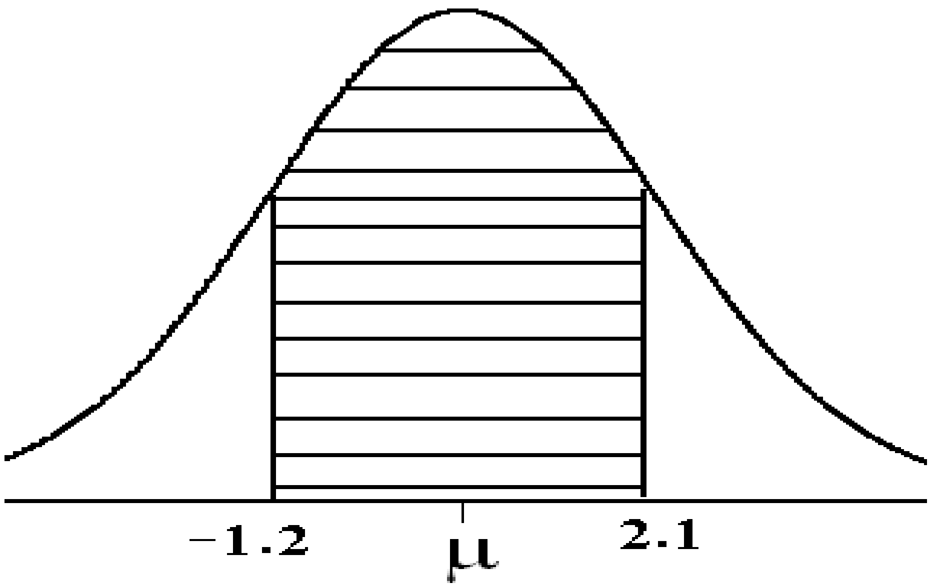


$$(b)\, P(Z > -1.33) = 1 - 0.0918 = 0.9082$$



$$(c)\ P(Z = 3) = 0$$

$$(d)\ P(-1.2 < Z < 2.1) = 0.9821 - 0.1151 = 0.867$$



See Ex: 6.2, 6.3, pg 178-179

EX (6.2 pg 178):

Using the standard normal tables, find the area under the curve that lies:

A. to the right of $Z=1.84$

B. to the left of $z=2.51$

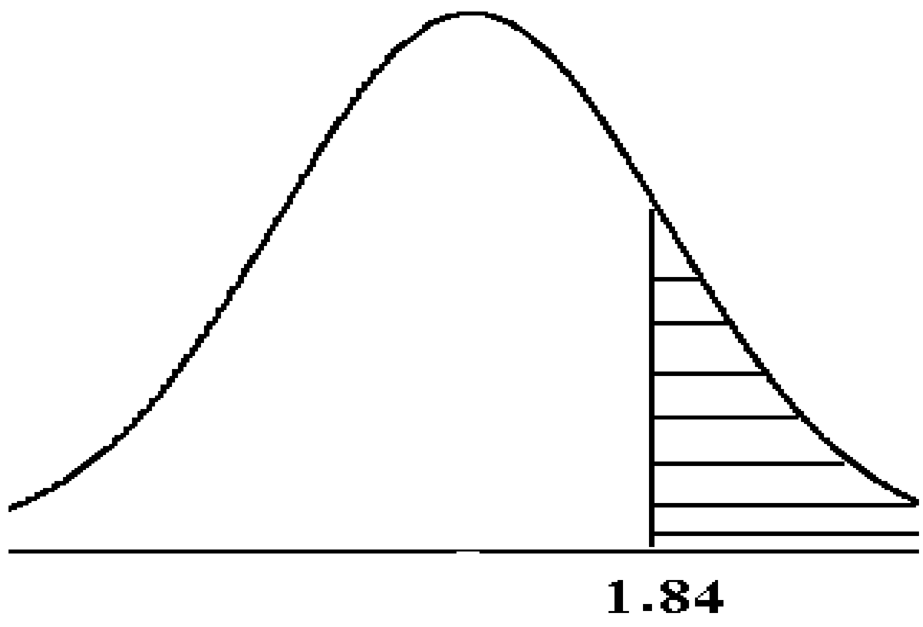
C. between $z=-1.97$ and $z=0.86$

A. at the point $z= -2.15$

Solution:

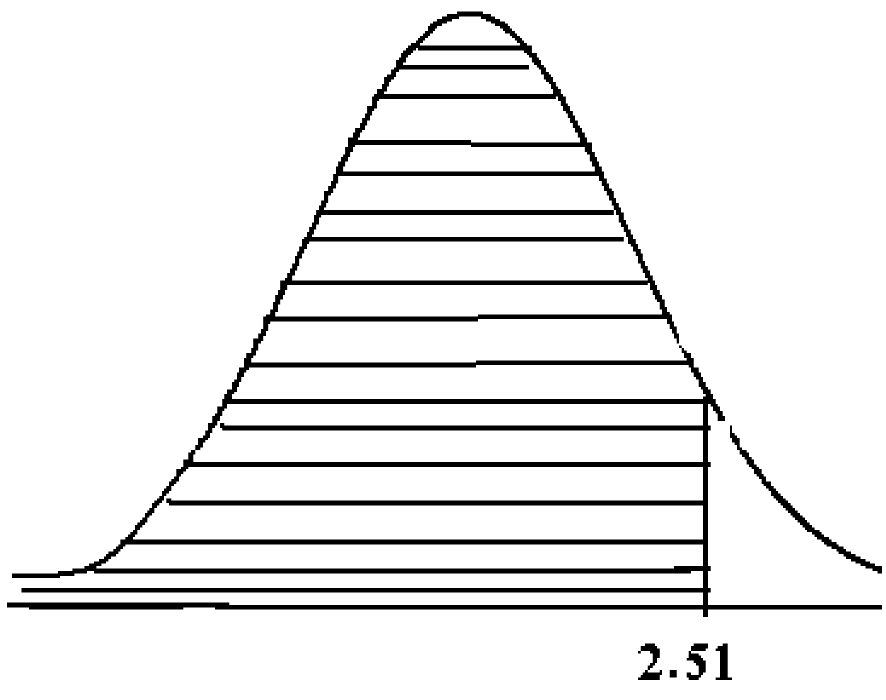
A. to the right of Z=1.84

$$P(Z > 1.84) = 1 - 0.9671 = 0.0329$$



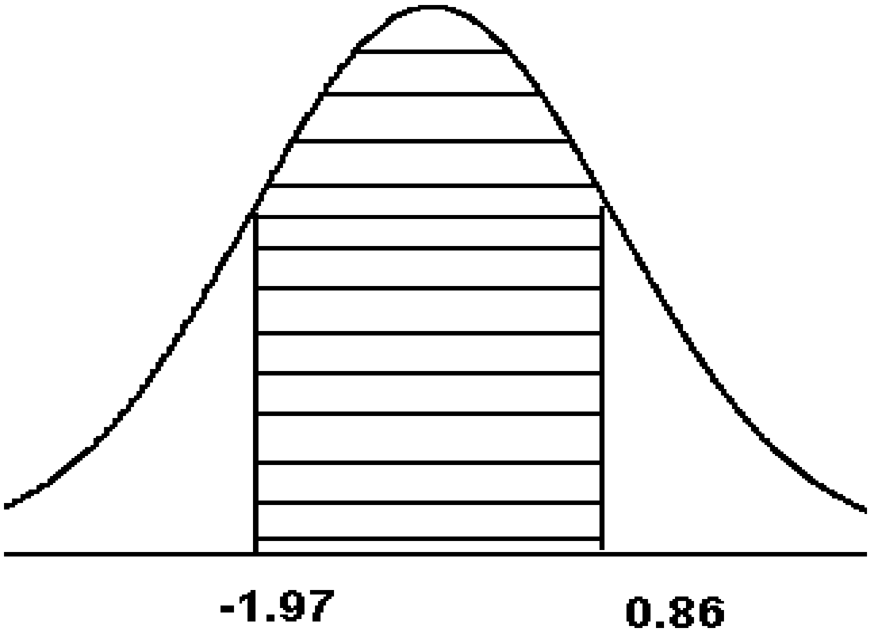
B. to the left of z=2.51

$$P(Z < 2.51) = 0.9940$$



C. between z=-1.97 and z=0.86

$$P(-1.97 < Z < 0.86) = 0.8051 - 0.0244 = 0.7807$$



D. at the point z= -2. 15

$$P(Z = -2.15) = 0$$

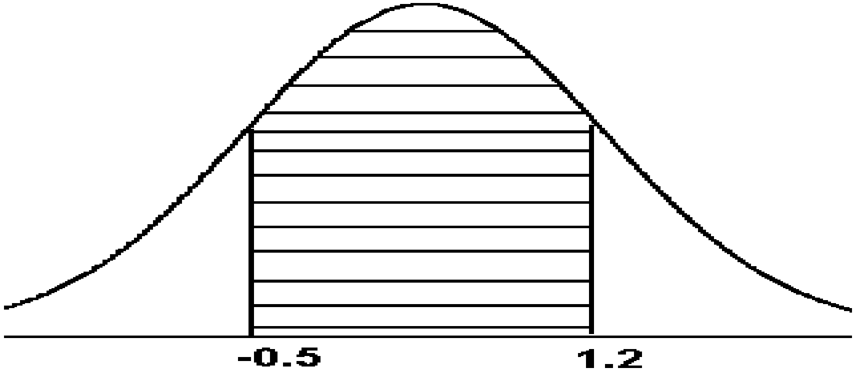
EX (6.4 pg 179):

Given a normal distribution with $\mu=50$, $\sigma=10$. Find the probability that **X** assumes a value between **45** and **62**.

Solution:

$$P(45 < X < 62) = P\left(\frac{45-50}{10} < Z < \frac{62-50}{10}\right) = P(-0.5 < Z < 1.2)$$

$$= 0.8849 - 0.3085 = 0.5764$$



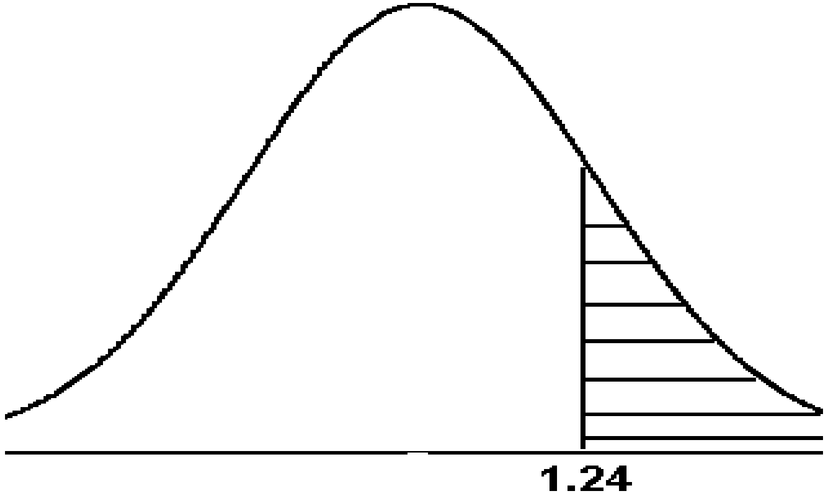
EX(6.5 pg 180) :

Given a normal distribution with $\mu=300$, $\sigma=50$, find the probability that **X** assumes a value greater than **362**.

Solution:

$$P(X > 362) = P(Z > \frac{362-300}{50}) = P(Z > 1.24)$$

$$= 1 - 0.8925 = 0.1075$$



Applications of the Normal Distribution:

EX (1):

The reaction time of a driver to visual stimulus is normally distributed with a mean of **0.4** second and a standard deviation of **0.05** second.

(a) What is the probability that a reaction requires more than **0.5** second?

(b) What is the probability that a reaction requires between **0.4** and **0.5** second?

(c) Find mean and variance.

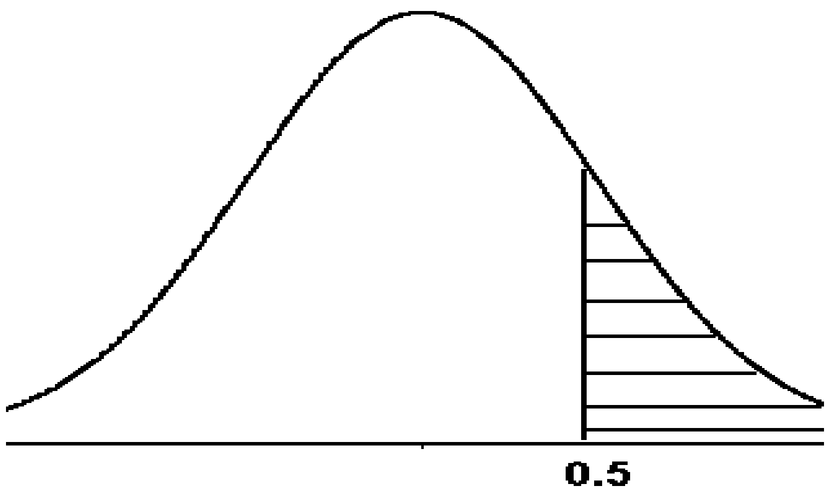
Solution:

$$\mu_X = 0.4, \sigma_X = 0.05$$

(a) What is the probability that a reaction requires more than **0.5** second?

$$(a) P(X > 0.5) = P(Z > \frac{0.5 - 0.4}{0.05}) = P(Z > 2)$$

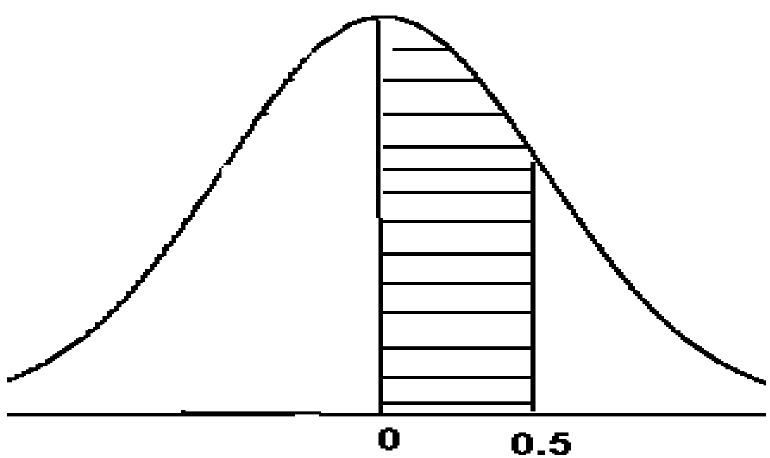
$$= 1 - 0.9772 = 0.0228$$



(b) What is the probability that a reaction requires between 0.4 and 0.5 second?

$$(b)\, P(0.4 < X < 0.5) = P\left(\frac{0.4-0.4}{0.05} < Z < \frac{0.5-0.4}{0.05}\right)$$

$$= P(0 < Z < 2) = 0.9772 - 0.5 = 0.4772$$



(c) Find mean and variance.

$$(c)\ \mu = 0.4, \sigma^2 = 0.0025$$

EX (2):

The line width of a tool used for semiconductor manufacturing is assumed to be normally distributed with a mean of 0.5 micrometer and a standard deviation of 0.05 micrometer.

(a) What is the probability that a line width is greater than 0.62 micrometer?

(b) What is the probability that a line width is between 0.47 and 0.63 micrometer?

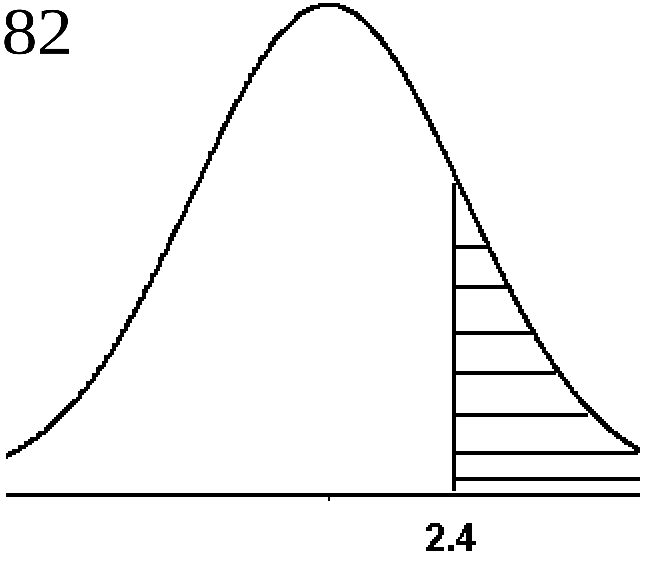
Solution:

(a) What is the probability that a line width is greater than 0.62 micrometer?

$$\mu_X = 0.5, \quad \sigma_X = 0.05$$

$$(a)\, P(x > 0.62) = P(Z > \frac{0.62 - 0.5}{0.05}) = P(Z > 2.4)$$

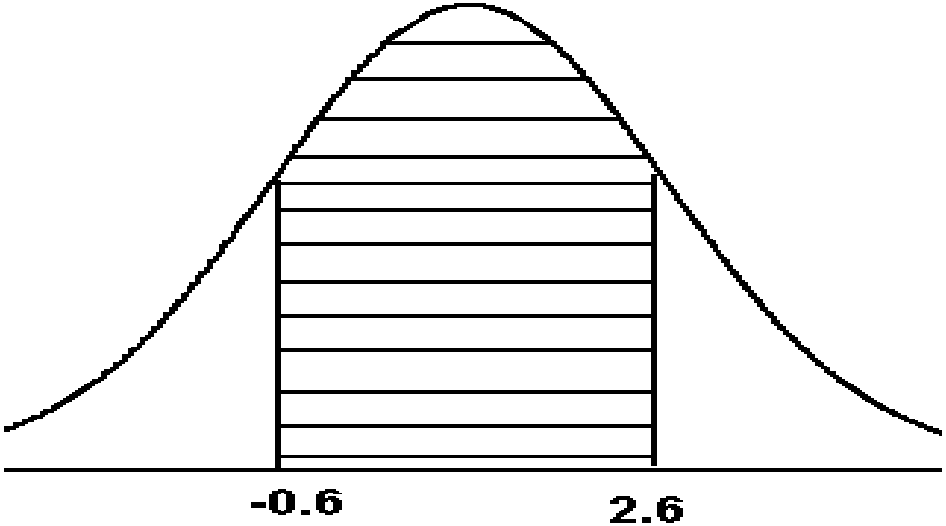
$$= 1 - 0.9918 = 0.0082$$



(b) What is the probability that a line width is between 0.47 and 0.63 micrometer?

$$(b)\,P(0.47 < X < 0.63) = P\left(\frac{0.47-0.5}{0.05} < Z < \frac{0.63-0.5}{0.05}\right)$$

$$= P(-0.6 < Z < 2.6) = 0.9953 - 0.2743 = 0.721$$



See Ex 6.7, 6.8 pg 182

Normal Approximation to the Binomial(Reading):
Theorem:

If **X** is a binomial random variable with mean $\mu = n\,p$ and variance $\sigma^2 = n\,p\,q$, then the limiting form of the distribution of

$$Z = \frac{X - n\,p}{\sqrt{n\,p\,q}} \quad as\ n \to \infty$$

is the standard normal distribution $N(0,1)$.

EX

The probability that a patient recovers from rare blood disease is **0.4**. If **100** people are known to have contracted this disease, what is the probability that less than **30** survive?

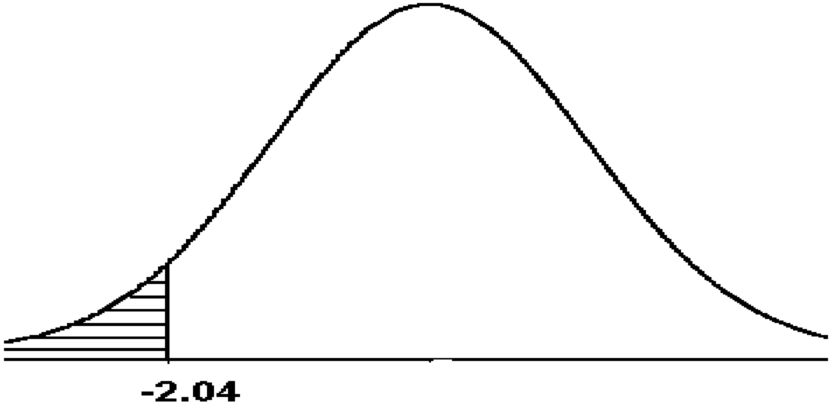
Solution:

$$n = 100 \quad , \quad p = 0.4 \ , \quad q = 0.6$$

$$\mu = np = (100)(0.4) = 40 \quad ,$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(0.4)(0.6)} = 4.899$$

$$P(X < 30) = P(Z < \frac{30-40}{4.899}) = P(Z < -2.04) = 0.0207$$



EX (6.16, pg 192)

A multiple – choice quiz has **200** questions each with **4** possible answers of which only **1** is the correct answer. What is the probability that sheer guess – work yields from **25** to **30** correct answers for **80** of the **200** problems about which the student has no knowledge?

Solution:

$$p = np = (80)(0.25) = 20 \quad ,$$

$$\sigma = \sqrt{npq} = \sqrt{(80)(0.25)(0.75)} = 3.873$$

$$P(25 < X < 30) = P(\frac{25-20}{3.873} < Z < \frac{30-20}{3.873}) = P(1.29 < Z < 2.58) = 0.9951 - 0.9015 = 0.0936$$