Chapter 5

One and Two-Sample Tests of Hypotheses

10.21 An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of **40** hours. Test the hypothesis that μ **= 800** hours against the alternative, $\mu \neq 800$ hours, if a random sample of **30** bulbs has an average life of **788** hours.

$\mu_0 = 800, \sigma = 40, n = 30, \bar{x} = 788$

σ Known

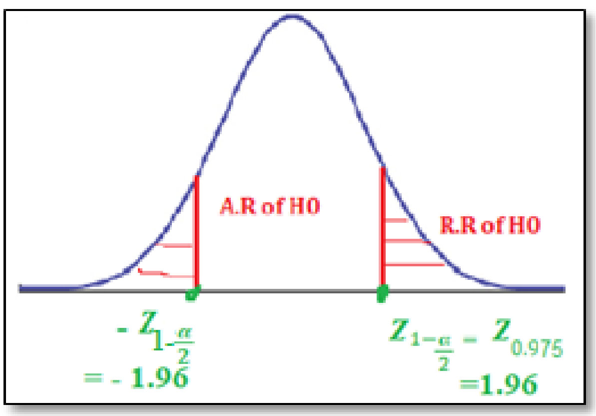
1- The hypothesis:

$H_0: \mu = 800 \quad vs \quad H_1: \mu \neq 800$

2- Test statistic (T.S.):

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{788 - 800}{\frac{40}{\sqrt{30}}} = -1.64$$

3- R.R. & A.R. of H_0: $Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}} = Z_{0.975} = 1.96$



4- Conclusion : Accept $H_0: \mu = 800$

- **p-value :** $2P(Z > |z_0|) = 2[1 - P(Z < |-1.64|)] = 2[1 - P(Z < 1.64)]$

$$= 2[1 - 0.9495] = 0.101 > \alpha = 0.05$$

10.23 Test the hypothesis that the average content of containers of a particular lubricant is 10 liters if the contents of a random sample of **10** containers are **10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3**, and **9.8** liters. Use a **0.01** level of significance and assume that the distribution of contents is normal.

$\mu_0 = 10 \quad , n = 10 , \bar{x} = 10.06 , S = 0.246$

σ Unknown and n small

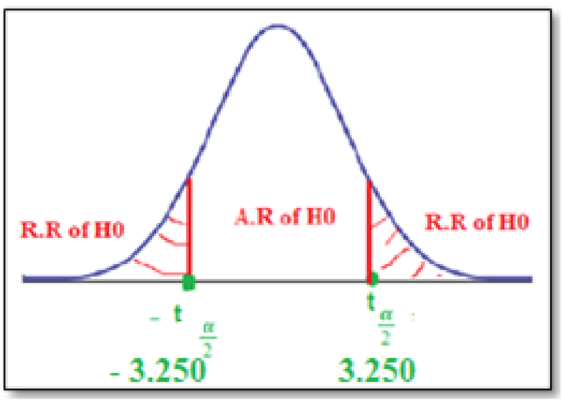
1- The hypothesis:

$$H_0: \mu = 10 \quad vs \quad H_1: \mu \neq 10$$

2- Test statistic (T.S.):

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{10.06 - 10}{\frac{0.246}{\sqrt{10}}} = 0.77$$

3- R.R. & A.R. of H_0: $t_{\frac{\alpha}{2}} = t_{\frac{0.01}{2}} = t_{0.005} = 3.250 , \ df = n - 1 \ , \alpha = 0.01$



4- Conclusion : Accept $H_0: \mu = 10$

10.25 It is claimed that automobiles are driven on average more than 20,000 kilometers per year. To test this claim, 100 randomly selected automobile owners are asked to keep a record of the kilometers they travel. Would you agree with this claim if the random sample showed an average of 23,500 kilometers **and** a standard deviation of 3900 kilometers? Use a *P*-value in your conclusion.

1- **The hypothesis:**

$$H_0: \mu \leq 20000 \quad vs \quad H_1: \mu > 20000$$

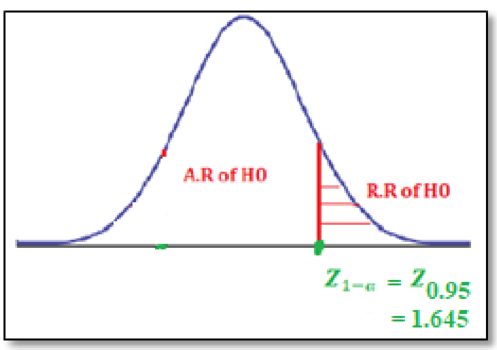
$n = 100\,, \bar{x} = 23500\,, s = 3900$

σ unknown and n ≥30

2- Test statistic (T.S.):

$$Z_c = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{23500 - 20000}{3900/\sqrt{100}} = 8.97$$

3- R.R. & A.R. of H_0: $Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$



4- Conclusion : Reject $H_0: \mu \leq 20000$

Yes $\mu > 20000$ **kilometers**

P-value : $P(Z > z_c) = [1 - P(Z < 8.97)]$

$$= [1 - 1] = 0 < \alpha = 0.05$$

Two Samples: Tests on Two Means

10.30 A random sample of size $n_1 = 25$, taken from a normal population with a standard deviation $\sigma_1 = 5.2$, has a mean $\bar{x}_1 = 81$. A second random sample of size $n_2 = 36$, taken from a different normal population with a standard deviation $\sigma_2 = 3.4$, has a mean $\bar{x}_2 = 76$. Test the hypothesis that $\mu_1 = \mu_2$ against the alternative, $\mu_1 = \mu_2$. use a P-value in your conclusion.

1- **The hypothesis:**

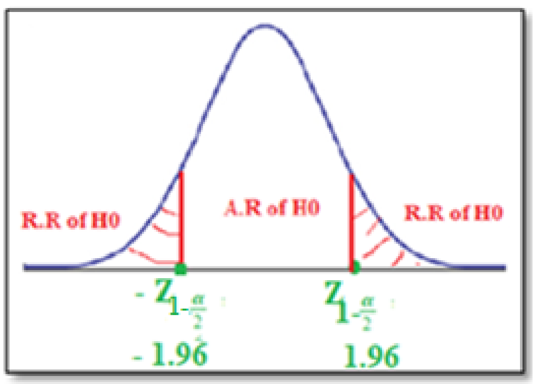
$$H_0: \mu_1 = \mu_2 \quad vs \quad H_1: \mu_1 \neq \mu_2$$

Sample 1	Sample 2
$\bar{x}_1 = 81$	$\bar{x}_2 = 76$
$n_1 = 25$	$n_2 = 36$
$\sigma_1 = 5.2$	$\sigma_2 = 3.4$

2- Test statistic (T.S.):

$$Z_c = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{81 - 76}{\sqrt{\frac{5.2^2}{25} + \frac{3.4^2}{36}}} = 4.22$$

3- R.R. & A.R. of H_0: $Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96 \quad , \alpha = 0.05$



4- Conclusion : Reject $H_0: \mu_1 = \mu_2$

We reject H_0 if $Z_c > Z_{1-\frac{\alpha}{2}}$ or $Z_c < -Z_{1-\frac{\alpha}{2}}$

Since $Z_c = 4.22 > Z_{1-\frac{\alpha}{2}} = 1.96$, we reject H_0 at $\alpha = 0.05$. Thus, $\mu_1 \neq \mu_2$

P-value : $2P(Z > |z_c|) = 2[1 - P(Z < |4.22|)]$

$$= 2[1 - 0.9927] = 0.0146 < \alpha = 0.05$$

10.36 Engineers at a large automobile manufacturing company are trying to decide whether to purchase brand A or brand B tires for the company's new models. To help them arrive at a decision, an experiment is conducted using 12 of each brand. The tires are run until they wear out. The results are as follows:

$$\text{Brand } A: \quad \bar{x}_1 = 37{,}900 \text{ kilometers}, \quad s_1 = 5100 \text{ kilometers}.$$
$$\text{Brand } B: \quad \bar{x}_1 = 39{,}800 \text{ kilometers}, \quad s_2 = 5900 \text{ kilometers}.$$

Test the hypothesis that there is no difference in the average wear of the two brands of tires. Assume the populations to be approximately normally distributed with equal variances.

1- The hypothesis:

$$H_0: \mu_1 = \mu_2 \quad vs \quad H_1: \mu_1 \neq \mu_2$$

Sample 1	Sample 2
$\bar{x}_1 = 37900$	$\bar{x}_2 = 39800$
$n_1 = 12$	$n_2 = 12$
$s_1 = 5100$	$s_2 = 5900$

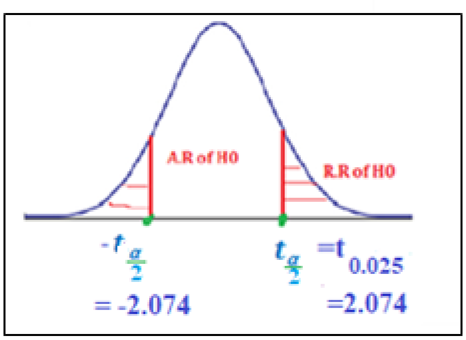
$\sigma_1 \& \sigma_2$ unknown **and** $n_1 n_2$ small

2- Test statistic (T.S.):

$$T_c = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{37900 - 39800}{5514.53\sqrt{\frac{1}{12} + \frac{1}{12}}} = -0.84$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(11 * 5100^2) + (11 * 5900^2)}{12 + 12 - 2} = 30410000$$

3- R.R. & A.R. of H_0: $t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025} = 2.074 \quad , df = n_1 + n_2 - 2 \quad , \alpha = 0.05$



4- Conclusion : Accept H0

Paired t-test

10.45 A taxi company manager is trying to decide whether the use of radial tires instead of regular belted tires improves fuel economy. Twelve cars were equipped with radial tires and driven over a prescribed test course. Without changing drivers, the same cars were then equipped with regular belted tires and driven once again over the test course. The gasoline consumption, in kilometers per liter, was recorded as follows:

	Kilometers per Liter	
Car	Radial Tires	Belted Tires
1	4.2	4.1
2	4.7	4.9
3	6.6	6.2
4	7.0	6.9
5	6.7	6.8
6	4.5	4.4
7	5.7	5.7
8	6.0	5.8
9	7.4	6.9
10	4.9	4.7
11	6.1	6.0
12	5.2	4.9

Can we conclude that cars equipped with radial tires give better fuel economy than those equipped with belted tires? Assume the populations to be normally distributed.

$\bar{d} = 0.1417 \,, S_d = 0.1975$

$d_1 = 4.2 - 4.1 = 0.1$
$d_2 = 4.7 - 4.9 = -0.2$
$d_3 = 6.6 - 6.2 = 0.4$
$d_4 = 7.0 - 6.9 = 0.1$
$d_5 = 6.7 - 6.8 = -0.1$
$d_6 = 4.5 - 4.4 = 0.1$
$d_7 = 5.7 - 5.7 = 0$
$d_8 = 6.0 - 5.8 = 0.2$
$d_9 = 7.4 - 6.9 = 0.5$
$d_{10} = 4.9 - 4.7 = 0.2$
$d_{11} = 6.1 - 6.0 = 0.1$
$d_{12} = 5.2 - 4.9 = 0.3$

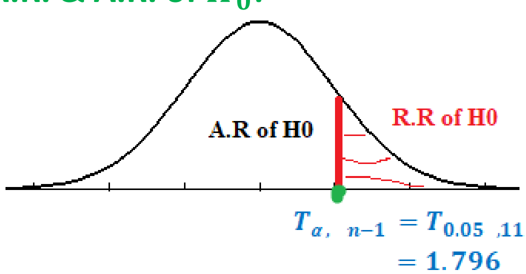
1- **The hypothesis:**

$H_0: \mu_d = 0 \quad \text{vs} \quad H_1: \mu_d > 0$

2- **Test statistic :**

$$T = \frac{\bar{d} - d_0}{\frac{S_d}{\sqrt{n}}} = \frac{0.1417}{0.1975/\sqrt{12}} = 2.485$$

3- **R.R. & A.R. of H_0:**



4- **Decision:**

Rejection Region $\left(T > T_{\alpha\,,\,n_1-1}\right)$ $T = 2.485 > 1.796 \Longrightarrow we\ \ Reject\ H_0$ **at** $\alpha = 0.05.$

i.e the cars equipped with radial tires give better fuel economy than those equipped with belted tires

One Sample: Test on a Single Proportion

10.55 A marketing expert for a pasta-making company believes that 40% of pasta lovers prefer lasagna. If 9 out of 20 pasta lovers choose lasagna over other pastas, what can be concluded about the expert's claim? Use a 0.05 level of significance.

n=20 , X=9 , $\alpha = 0.05$, $\widehat{P} = \frac{x}{n} = \frac{\text{جزء}}{\text{كل}} = \frac{9}{20} = 0.45$

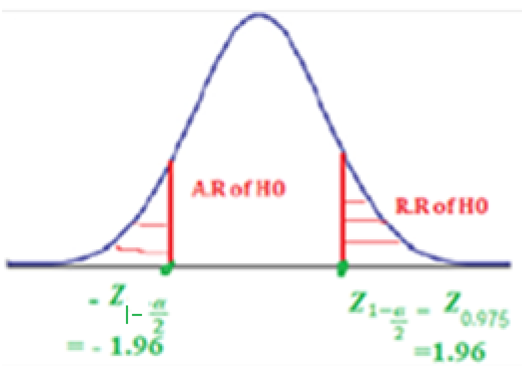
1- The hypothesis:

$H_0: P = 0.4$ vs $H_1: P \neq 0.4$

2- Test statistic :

$$Z_C = \frac{\widehat{P} - P_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.45 - 0.4}{\sqrt{\frac{0.4(0.6)}{20}}} = 0.46$$

3- R.R. & A.R. of H_0: $Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$, $\alpha = 0.05$



4- Decision:

We reject H_0 if $Z_c > Z_{1-\frac{\alpha}{2}}$ or $Z_c < -Z_{1-\frac{\alpha}{2}}$

Since $Z_c = 0.46 \ngtr Z_{1-\frac{\alpha}{2}} = 1.96$, we can't reject H_0 at $\alpha = 0.05$.

Thus, his claim is correct.

Two Samples: Tests on Two Proportions

10.63 In a study to estimate the proportion of residents in a certain city and its suburbs who favor the construction of a nuclear power plant, it is found that 63 of 100 urban residents favor the construction while only 59 of 125 suburban residents are in favor.

Is there a significant difference between the proportions of urban and suburban residents who favor construction of the nuclear plant?

$$n_1 = 100,\ x_1 = 63, \quad \rightarrow \hat{P}_1 = \frac{63}{100} = 0.63$$

$$n_2 = 125,\ X_2 = 59, \quad \rightarrow \hat{P}_2 = \frac{59}{125} = 0.472, \quad \alpha = 0.05$$

1- The hypothesis:

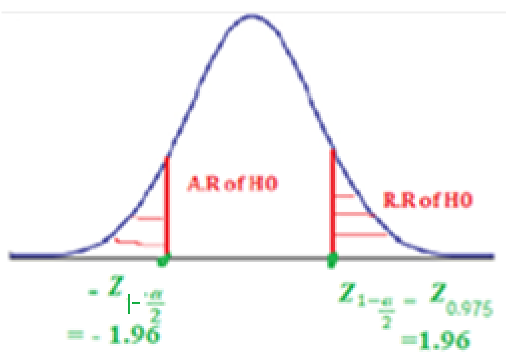
$H_0: p_1 = p_2$ vs $H_1: p_1 \neq p_2$

2- Test statistic :

$$Z_C = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{p}\hat{q}(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{(0.63) - 0.472}{\sqrt{(0.542)(0.458)\frac{1}{100} + \frac{1}{125}}} = 2.36$$

Where, $\hat{p} = \frac{X_1+X_2}{n_1+n_2} = \frac{63+59}{100+125} = 0.542$

3- R.R. & A.R. of H_0: $Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96 \quad , \alpha = 0.05$



4- Decision:

We reject H_0 $if\ Z_c > Z_{1-\frac{\alpha}{2}}$ $or\ Z_c < -Z_{1-\frac{\alpha}{2}}$

Since $Z_c = 2.36 > Z_{1-\frac{\alpha}{2}} = 1.96$, we reject H_0 at $\alpha = 0.05$.

One- and Two-Sample Tests Concerning Variances

10.67 The content of containers of a particular lubricant is known to be normally distributed with a variance of 0.03 liter.

Test the hypothesis that $\sigma^2 = 0.03$ against the alternative that $\sigma^2 \neq 0.03$ for the random sample of 10 containers in Exercise 10.23 on page 356.

10.23 Test the hypothesis that the **average** content of containers of a particular lubricant زيوت التشحيم is 10 liters if the contents of a random sample of **10** containers are **10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3**, and **9.8** liters. Use a **0.01** level of significance and assume that the distribution of contents is normal.

$\sigma_0^2 = 0.03$, $n = 10$, $S = 0.2459$ (from exercise 10.23),

$\alpha = 0.01$

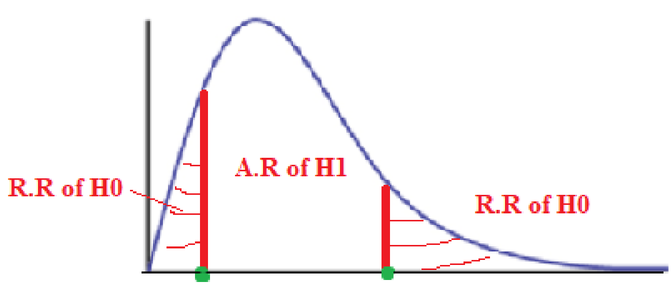
1- The hypothesis:

$H_0: \sigma^2 = 0.03$ VS $H_1: \sigma^2 \neq 0.03$

2- Test statistic :

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{(9)(0.2459)^2}{0.03} = 18.14$$

3- R.R. & A.R. of H_0



$\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{0.995,9} = 1.735$

$\chi^2_{\frac{\alpha}{2}} = \chi^2_{0.005,9} = 23.589$

4- Decision:

We reject H_0 if $\chi^2 > \chi^2_{\frac{\alpha}{2}}$ or $\chi^2 < \chi^2_{1-\frac{\alpha}{2}}$

Since $\chi^2 = 18.14 \ngtr 23.589$ & Since $\chi^2 = 18.14 \nless 1.735$, we can't reject H_0

10.73 A study is conducted to compare the lengths of time required by men and women to assemble a certain product. Past experience indicates that the distribution of times for both men and women is approximately normal but the variance of the times for women is less than that for men. A random sample of times for 11 men and 14 women produced the following data:

Men	women
$n_1 = 11$	$n_2 = 14$
$s_1 = 6.1$	$s_2 = 5.3$

Test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against the alternative that $\sigma_1^2 > \sigma_2^2$. Use $\alpha = 0.05$.

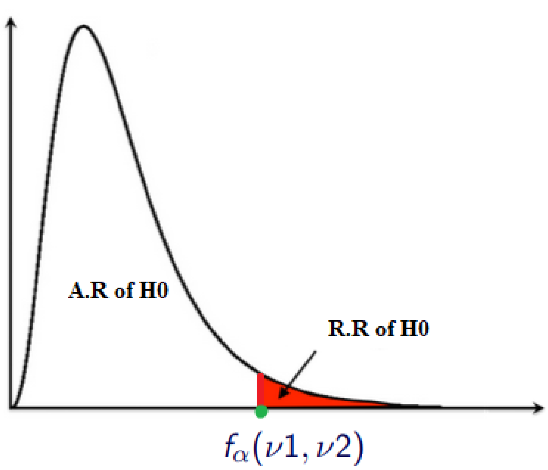
1- **The hypothesis:**

$\boldsymbol{H_0: \sigma_1^2 = \sigma_2^2}$ **vs** $\boldsymbol{H_1: \sigma_1^2 > \sigma_2^2}$

2. **Test statistic :**

$$F = \frac{S_1^2}{S_2^2} = \frac{(6.1)^2}{(5.3)^2} = 1.32$$

5- **R.R. & A.R. of H_0 ;** $\quad F_{\alpha, n_1-1, n_2-1} = f_{0.05, 10, 13} = 2.67$



$F_{\alpha, n_1-1, n_2-1} = f_{0.05, 10, 13}$
$= 2.67$

3. **Decision:**

We reject H_0 *if* $F > F_{\alpha, n_1-1, n_2-1}$; Since $F = 1.32 \not> 2.67$, we can't reject H_0.

i.e the variance for men is equal to the variance for women.