Chapter 6

Chi square tests

10.80 The grades in a statistics course for a particular semester were as follows:

Grade	A	B	C	D	F
f	14	18	32	20	16

Test the hypothesis, at the 0.05 level of significance, that the distribution of grades is uniform.

1. **Hypotheses :**

$H_0: The\ data\ \ follow\ uniform\ distribution.$

$H_1: The\ data\ doen't\ follow\ uniform\ distribution.$

We will use χ^2 test for goodness of fit.

Calculate Expected frequencies.

N=14+18+32+20+16=100

$e_i = Np_i\ ,\quad p_i = \frac{1}{5} = 0.2\ \ , i = 1, \dots, 5$

Thus, $E_i = 100(0.2) = 20\ ;\forall\ i$

Grade	A	B	C	D	F
Observed	14	18	32	20	16
Expected	20	20	20	20	20

2. **Test statistic:**

$$\chi^2 = \sum_{i=1}^{5} \frac{(O_i - e_i)^2}{e_i}$$

$$\chi^2 = \frac{(14-20)^2}{20} + \cdots + \frac{(16-20)^2}{20} = 10$$

3.Decision:

We Reject H_0 if $\chi^2 > \ \chi^2_{\alpha,k-1} = \chi^2_{0.05,4} = 9.488$
Since, $\chi^2 = 10 > 9.488$ we reject H_0 . i.e the data doesn't follow uniform distribution.

10.81 A die is tossed 180 times with the following results:

x	1	2	3	4	5	6
f	28	36	36	30	27	23

is this a balanced die? Use a 0.01 level of significance.

1. **Hypotheses :**

H_0: *The die is* balanced .

H_1: *The die is not* balanced .

We will use χ^2 test for goodness of fit.

Calculate Expected frequencies.

N=28+36+36+30+27+23=180 ;

$e_i = Np_i$, *if the die is balanced* $p_i = \frac{1}{6}$, $i = 1, \dots, 6$

$e_i = 180\left(\frac{1}{6}\right) = 30$

x	1	2	3	4	5	6
Observed	28	36	36	30	27	23
Expected	30	30	30	30	30	30

2. Test statistic:

$$\chi^2 = \sum_{i=1}^{6} \frac{(O_i - e_i)^2}{e_i}$$

$$\chi^2 = \frac{(28-30)^2}{30} + \cdots + \frac{(23-30)^2}{30} = 4.47$$

3.Decision:

We Reject H_0 if $\chi^2 > \chi^2_{\alpha,n-1} = \chi^2_{0.01,5} = 15.086$
Since, $\chi^2 = 4.47 \ngtr 15.086$ we can't reject H_0. i.e the data is balanced.

10.87 A random sample of 90 adults is classified according to gender and the number of hours of television watched during a week:

	Gender	
	Male	Female
Over 25 hours	15	29
Under 25 hours	27	19

Use a 0.01 level of significance and test the hypothesis that the time spent watching television is independent of whether the viewer is male or female.

1. **Hypotheses :**

H_0: *Time spent watching TV independent of grades.*

(The two random variable are independent)

H_1: *The two random variable are dependent .*

We will use χ^2 test Independence Test

Calculate Expected frequencies.

2. **Test statistic:**

$$\chi^2 = \sum_{i,j} \frac{(o_{ij}-e_{ij})^2}{e_{ij}}; \qquad e_{ij} = \frac{\sum_i o_{ij} \sum_j o_{ij}}{N} = \frac{n_r n_c}{N}$$

	Gender		$\sum_j O_{ij} = O_{i.} = n_r$
	Male	Female	
Over 25 hours	15	29	**44**
Under 25 hours	27	19	**46**
$\sum_i O_{ij} = O_{j.} = n_c$	**42**	**48**	**N= 90**

$$e_{11} = \frac{42(44)}{90} = 20.53; \quad e_{21} = \frac{42(46)}{90} = 21.47$$

$$e_{12} = \frac{48(49)}{90} = 23.47\,; \quad e_{22} = \frac{48(46)}{90} = 24.53$$

$$\chi^2 = \frac{(15-20.53)^2}{20.53} + \frac{(27-21.47)^2}{21.53} + \frac{(29-23.47)^2}{23.47} + \frac{(19-24.53)^2}{24.53} = 5.47$$

3. **Decision:**

We Reject H_0 if $\chi^2 > \chi^2_{\alpha,(c-1)(r-1)} = \chi^2_{0.01,1*1} = \chi^2_{0.01,1} = 6.635$
Since, $\chi^2 = 5.47 \ngtr 6.635$ we can't reject H_0. i.e The two variables are independent.

10.93 To determine current attitudes about prayer in public schools, a survey was conducted in four Virginia counties. The following table gives the attitudes of 200 parents from Craig County, 150 parents from Giles County, 100 parents from Franklin County, and 100 parents from Montgomery County:

	County				
Attitude	**Craig**	**Giles**	**Franklin**	**Mont.**	n_r
Favor	65	66	40	34	205
Oppose	42	30	33	42	147
No opinion	93	54	27	24	198
n_c	200	150	100	100	550

Test for homogeneity of attitudes among the four counties concerning prayer in the public schools.

1. **Hypotheses :**

H_0: $for\ each\ row\ i\ p_{i1} = \cdots = p_{ic}$.

H_1: $at\ least\ one\ of\ H_0\ statments\ is\ false$.

H_0: For each attitude, the proportions of craig, Giles, Franklin and Montgomery counties are the same.

H_1: $At\ least\ one\ of\ the\ counties\ proportions\ is\ different.$

We will use χ^2 test Homogeneity Test.

2. **Test statistic:**

$$\chi^2 = \sum_{i,j} \frac{(o_{ij} - e_{ij})^2}{e_{ij}}; \qquad e_{ij} = \frac{\sum_i o_{ij} \sum_j o_{ij}}{N} = \frac{n_r n_c}{N}$$

$$e_{11} = 74.55 \quad e_{12} = 55.91 \quad e_{13} = 37.27 \quad e_{14} = 37.27$$

$$e_{21} = 53.45 \quad e_{22} = 40.09 \quad e_{23} = 26.73 \quad e_{24} = 26.73$$

$$e_{31} = 72.00 \quad e_{32} = 54.00 \quad e_{33} = 36.00 \quad e_{34} = 36.00$$

$$\chi^2 = \frac{(65 - 74.55)^2}{74.55} + \cdots + \frac{(24 - 36)^2}{36} = 31.1$$

3. **Decision:**

We Reject H_0 if $\chi^2 > \chi^2_{\alpha,(c-1)(r-1)} = \chi^2_{0.05,\ 2*3} = \chi^2_{0.05,\ 6} = 12.592$
Since, $\chi^2 = 31.1 > 12.592$ we reject H_0. i.e attitudes are not homogeneous.