Chapter 8

One-Way Analysis of Variance (ANOVA)

13.1 Six different machines are being considered for use in manufacturing rubber seals. The machines are being compared with respect to tensile strength of the product. A random sample of four seals from each machine is used to determine whether the mean tensile strength varies from machine to machine. The following are the tensile-strength measurements in kilograms per square centimeter $x10^{-1}$:

Machine					
1	2	3	4	5	6
17.5	16.4	20.3	14.6	17.5	18.3
16.9	19.2	15.7	16.7	19.2	16.2
15.8	17.7	17.8	20.8	16.5	17.5
18.6	15.4	18.9	18.9	20.5	20.1

Perform the analysis of variance at the 0.05 level of significance and indicate whether or not the mean tensile strengths differ significantly for the six machines.

$$\alpha = 0.05 \quad , \quad k = 6 \quad , \quad n = 4 \quad 6 \text{ rubber machines} \quad , \quad N = 24$$

Hypothesis :

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_6. \quad V.S \quad H_1: At\ least\ one\ \mu_i\ is\ different.$$

Test statistic :

i	1	2	3	4	5	6	
Total $y_i.$	68.8 $\mathbf{T_1.}$	68.7 $\mathbf{T_2.}$	72.7 $\mathbf{T_3.}$	71 $\mathbf{T_4.}$	73.7 $\mathbf{T_5.}$	72.1 $\mathbf{T_6.}$	**427** $\boldsymbol{T_{..}}$

$$\sum\sum y_{ij}^2 = 17.5^2 + 16.9^2 + \cdots \ldots\ldots\ldots\ldots\ldots\ldots + 17.5^2 + 20.1^2 = \mathbf{7665.02}$$

$$\mathbf{SST} = \sum\sum y_{ij}^2 - \frac{(T_{..})^2}{N} = 7665.02 - \frac{(427)^2}{24} = \mathbf{67.9783}$$

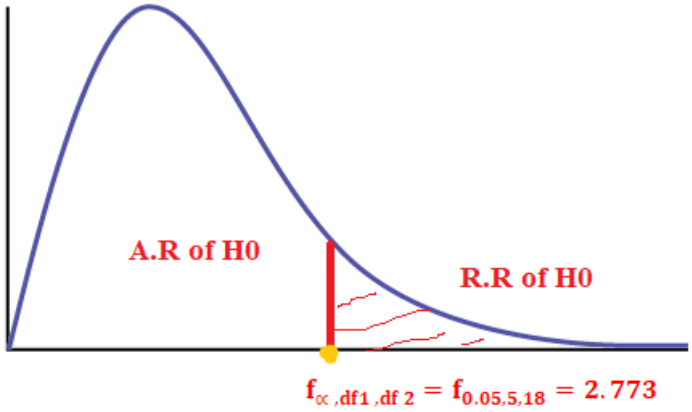
$$\mathbf{SSt_r} = \sum \frac{\boldsymbol{T_{i.}}^2}{\boldsymbol{n_{i.}}} - \frac{(T_{..})^2}{N} = \left[\frac{68.8^2}{4} + \frac{68.7^2}{4} + \frac{72.7^2}{4} + \frac{71^2}{4} + \frac{73.7^2}{4} + \frac{72.1^2}{4}\right] - \frac{(427)^2}{24} = \mathbf{5.33833}$$

$$\mathbf{SSE} = \mathbf{SST} - \mathbf{SSt_r} = 67.9783 - 5.33833 = \mathbf{62.639967}$$

ANOVA Table

source	df	SS	MS	F
Treatments	k-1= 5	SStr=5.338	1.068	$\frac{1.068}{3.48} = 0.307$
Error	N-k =18	SSE=62.64	3.48	
Total	N-1 =23	SST=67.978		

R.R & A.R of H0:



Decision:

Reject H_0 *if* $F > f_{\propto ,df1 ,df 2}$

Since $F = 0.307 \not> 2.773$, ***we cannot reject*** H_0.
thus, the mean tensile strengths is the same for the 6 machines

Two-Factor Analysis of Variance

14.1 An experiment was conducted to study the effects of temperature and type of oven on the life of a particular component. Four types of ovens and 3 temperature levels were used in the experiment. Twenty-four pieces were assigned randomly, two to each combination of treatments, and the following results recorded.

Temperature ($°F$)	Oven			
	O_1	O_2	O_3	O_4
500	227	214	225	260
550	187	181	232	246
600	174	198	178	206

Using a 0.05 level of significance, test the hypothesis that:
(a) different **ovens** have no effect on the life of the component;
(b) different **temperatures** have no effect on the life of the component;

Here we are interested in the 2 factors(A=ovens +B= temperatures). we have two tests:

i)Test whether the means of the factor A are all equal or, (i.e., the effects of all levels factor A are the same).

ii)Test whether the means of the factor B are all equal or, (i.e., the effects of all levels of factor B are the same).

Factor	**Oven A**					
Temperature B		O1	O2	O3	O4	Total
	T=500	227	214	225	260	$T_{.1} = 926$
	T=550	187	181	232	246	$T_{2.} = 846$
	T=600	174	198	178	206	$T_{.3} = 756$
	Total	$T_{1.} = 588$	$T_{2.} = 593$	$T_{3.} = 635$	$T_{4.} = 712$	$T_{..} = 2528$

$$\sum\sum y_{ij}^2 = 227^2 + 187^2 + \cdots \ldots . + 246^2 + 206^2 = 541220$$

$$\mathbf{SST} = \sum_{i=1}^{4}\sum_{j=1}^{3} y_{ij}^2 - \frac{T_{..}^2}{N} = 541220 - \frac{(2528)^2}{12} = 8654.667$$

$$\mathbf{SSA} = \sum \frac{\boldsymbol{T_{i.}}^2}{\boldsymbol{b}} - \frac{(T_{..})^2}{N} = \left[\frac{588^2}{3} + \frac{593^2}{3} + \frac{635^2}{3} + \frac{712^2}{3}\right] - \frac{(2528)^2}{12} = 3288.667$$

$$\mathbf{SSB} = \sum \frac{\boldsymbol{T_{.j}}^2}{\boldsymbol{a}} - \frac{(T_{..})^2}{N} = \left[\frac{926^2}{4} + \frac{846^2}{4} + \frac{756^2}{4}\right] - \frac{(2528)^2}{12} = 3616.667$$

$$\mathbf{SSE} = \boldsymbol{SST - SSA - SSB} = \mathbf{8654.667 - 3288.667 - 3616.667 = 1749.33}$$

Source	df	SS	MS	F
Factor A **Oven**	a-1 **3**	SSA= **3288.667**	MSA=$\frac{SSA}{a-1}$ = 1096.222	F1=$\frac{MSA}{MSE}$=**3.7599**
Factor B **Temperature**	b-1 **2**	SSB=**3616.667**	MSB=$\frac{SSB}{b-1}$ = 1808333	F2=$\frac{MSB}{MSE}$ =**6.20236**
Error	(a-1)(b-1) **6**	SSE=**1749.333**	-	-
Total	ab-1 **11**	SST= **8654.667**	-	-

The Steps Of the test of factor A: (Oven)

Hypothesis:

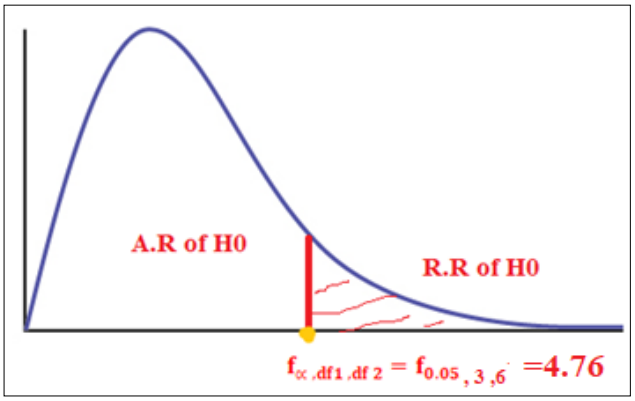
H_0: *There is no difference in the mean of the comonent life of the 4 Oven .*

H_1: *At least one mean different.*

Test statistic :

$$F1=\frac{MSA}{MSE}=3.7599$$

R.R& A.R of H0 : $F_{\alpha,(a-1),(a-1)(b-1)} = F_{0.05,3,6} = 4.76$



Decision:

Since $F = 3.7599 \not> 4.76 = f_{0.05,3,6}$, we can't Reject H_0.

The Steps Of the test of factor B : Temperature

Hypothesis:

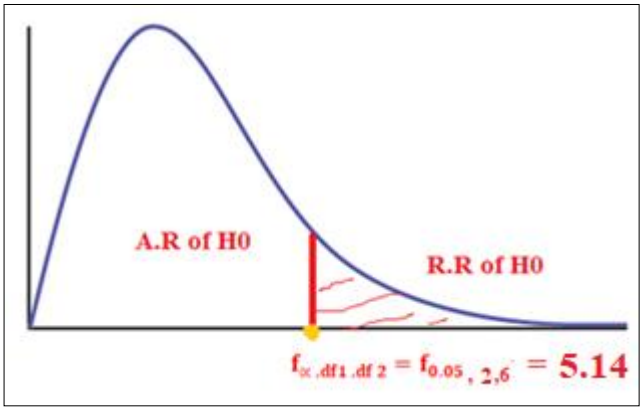
H_0: *There is no difference in the mean of the comonent life of the 3 Temperature.*

H_1: *At least one mean different.*

Test statistic :

$$F2=\frac{MSB}{MSE}=6.202$$

R.R& A.R of H0 : $F_{\alpha,\ (b-1),\ (a-1)(b-1)} = F_{0.05,2,6} = 5.14$



Decision:

Since $F = 6.202 > 5.14 = f_{0.05,3,6}$, we Reject H_0.