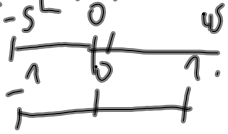
How many numbers are there between 1 an 10? [1, 10]

General: How many numbers are there between m and n such that m < n ? [m, n]

(5, 45)

Answer: n - m + 1

[-5, 45] 51



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How many even integer numbers between m and n ?

2, 4, 6, 8 ....

1, 2, 3, 4, ... , 100, 101

2, 4, 6, 8, . . . , 100 = 50 . .

$\frac{(n-m+1)}{2}$ $\left\lfloor \frac{n-m+1}{2} \right\rfloor = 50$

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factors and multiples.

- If n and m are integers we say n is a multiple of m if $n = Km$ and K is an integer. m divides n, and we write $m | n$ m divisor of n.

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3 | 6 , 4 | 20 , 15 | 15

4 ∤ 7 , 11 ∤ 12 , 0 | 0

Proposition: if m and n are positive integers such that $m | n$ then $m \leq n$

$n = Km$, K integers positive > 0, $K = 1, 2, \ldots$

$K = 1, n = m \Rightarrow m \leq n$

$K > 1, n > m$

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Theorem: Every positive integer n can be written as a product of primes

$48 = 1 \times 2 \times 2 \times 2 \times 2 \times 3$

$36 = 1 \times 2 \times 2 \times 3 \times 3$

GCD(48, 36) : 12

Jun 9-10:00 AM

Least common multiple (m, n)

$48 = 1 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

$36 = 1 \times 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$lcm(48, 36) = 2^4 \times 3^2 = 16 \times 9$

n and m.

$GCD(n, m) \times lcm(m, n) = n \times m$

if $m | n$ and $n | q \Rightarrow m | q$

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sets

$\mathbb{N} = \{0, 1, \ldots,\}$

$\mathbb{Z} = \{\ldots, -1, 0, 1, \ldots.\}$

$\mathbb{Q}$

$\mathbb{R}$

$\{2, 4, 6\}$

$\{x : x \in \mathbb{R}\}$

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$\{x : x \in \mathbb{R} \text{ and } 1 < x < 3\}$

$\{n : n \in \mathbb{N} \text{ and } n \text{ is even}\}$

$\{(-1)^n : n \in \mathbb{N}\} = \{1, -1\}$

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$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$

$(a, b) =]a, b[= \{x \in \mathbb{R} : a < x < b\}$

$[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$

$[a, \infty) = \{x \in \mathbb{R} : x \geq a\}$

$\{n \in \mathbb{N} : 2 < n < 3\} = \emptyset$ empty set

Jun 9-10:38 AM

$S, T, S = T$

$S \subseteq T$

power set of S: is all the subsets of S, $\mathcal{P}(S)$

$S = \{a\}, S = \{a, b\}$

$|\mathcal{P}(S)| = 2^{|S|}$

$\mathcal{P}(S) = \{\emptyset, \{a\}\}$

$\mathcal{P}(S) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Jun 9-10:44 AM

$\Sigma = \{a, b\}$ $\{A, \ldots, Z\}$ $\{a, \ldots, z, 1 \ldots 9\}$

$\Sigma^* = \{a, b, ab, aa, aaa \ldots$ $+$ {if, then, integer.

$aba a, \ldots\}$

$\Sigma = \{0, 1\}$

$\Sigma^* = \{0, 1, 010, 11, \ldots\}$

empty word $= \lambda$.

Jun 9-10:48 AM

Set operation

- $A \cup B = \{x : x \in A \text{ or } x \in B \text{ or } x \in (A \cap B)\}$
- $A \cap B = \{x : x \in A \text{ and } x \in B\}$

if $(A \cap B = \emptyset)$ A and B disjoint sets.

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Let $\Sigma = \{a, b\}$ $\Sigma^* = \{a, b, aa, ab, ba, bb, aaa, \ldots\}$

$A = \{\lambda, a, aa, aaa\}$

$C = \{w \in \Sigma^* : \text{length}(w) = 2\}$

$A \cup C$ and $A \cap C$.

$C = \{aa, ab, ba, bb\}$

$A \cup C = \{\lambda, a, aa, aaa, ab, ba, bb\}$

$A \cap C = \{aa\}$

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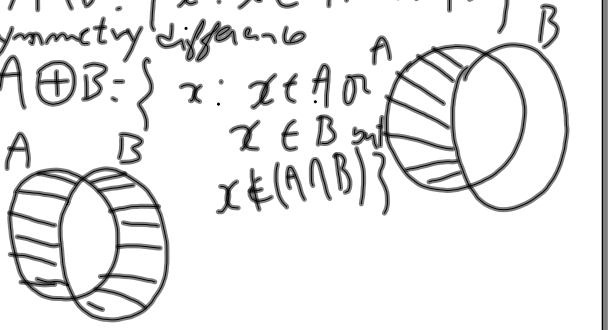
$A \setminus B = \{x : x \in A \text{ and } x \notin B\}$

symmetry difference

$A \oplus B = \{x : x \in A \text{ or } x \in B \text{ but } x \notin (A \cap B)\}$



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Let $\Sigma = \{a, b\}$, $C = \{\lambda, a, b, aa, bb, ab, ba\}$

$A = \{\lambda, a, aa, aaa\}$

$B = \{\lambda, b, bb, bbb\}$

$C = \{w \in \Sigma^* : \text{length}(w) \leq 2\}$

$A \setminus B = \{a, aa, aaa\}$

$B \setminus A = \{b, bb, bbb\}$

$C \setminus A = \{b, bb, ab, ba\}$

$A \setminus \Sigma = \{\lambda, aa, aaa\}$

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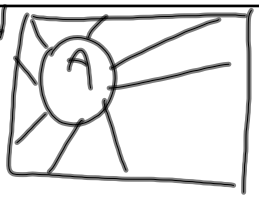
$U = \mathbb{R}$

$A \subseteq U$

$A^c = \{x : x \in U \text{ and } x \notin A\}$



$A = \{n \in \mathbb{N} : n < 12\}$

$A^c = \{m \in \mathbb{N} : n \geq 12\}$

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De Morgan Law

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

$(A^c)^c = A$. $A \cup A^c = U$, $A \cap A^c = \emptyset$

$(U^c) = \emptyset$, $\emptyset^c = U$

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Let S, T two sets

$S \times T = \{(s, t) : s \in S \text{ and } t \in T\}$

$|S| \times |T|$

$S = \{1, 2\}$ $T = \{a, b, c\}$

$S \times T = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$

$S \times T = \{$

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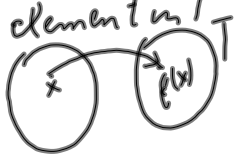
function:
f: assigns to each element $x \in S$ a unique element in T

$f: S \to T$

$x \to f(x)$



$x \in Dom(f)$

$Im(f) := \{ f(x) : x \in Dom(f) \}$

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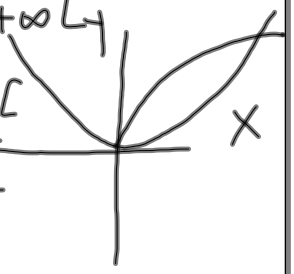
$f_1(x) := x^2$, $f_2(x) := \sqrt{x}$

$Dom(f_2) :=]-\infty, +\infty[= \mathbb{R}$

$Dom(f_2) := [0, +\infty[$

$Im(f_2) := [0, +\infty[$

$Im(f_2) := [0, +\infty[$



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