while $i<n$ and $A[i] \neq K$
$i = i+1;$
end

worst case $\sum_{i=0}^{n-1} 1 = n-1+1 = n$

$\sum_{i=a}^{b} 1 = b-a+1$

Best case: 1

average case (K is found).

$\frac{1+2+3+\dots+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{(n+1)}{2}$

p: $K \in A$

Average:

K exists: $\frac{n+1}{2}$ p

K does not exist: $n(1-p)$

CT: $p\frac{(n+1)}{2} + (1-p)n$

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for i: 1 to $n-1$
T[Inst]
endfor

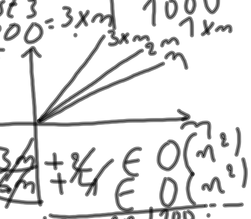
$1, 2, 3, \dots \in O(1)$.

Ex 1

for $i = 1$ to 1000
Inst 1;
Inst 2;
Inst 3
$3000 = 3 \times n$

for i: 1 to 1000
T Inst 2
1000

CT(n) =



$\cancel{3}n^2 + 3\cancel{n} + \cancel{2} \in O(n^2)$

$\cancel{5}n^2 + \cancel{6n} + \cancel{1} \in O(n^2)$

	5	10	100
n^2+1	26	101	10001
n^2	25	100	10000

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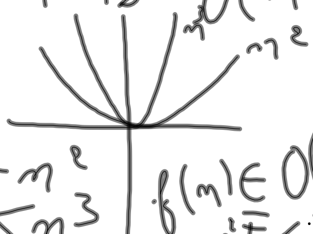
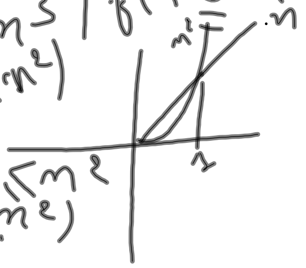
$f_1 = \cancel{5}n^3 + \cancel{2n^2} + \cancel{2} \in O(n^3)$

$f_2 = \cancel{12}n^2 + \cancel{5} \in O(n^2)$



$f(n) = n^2$
$\leq n^3$

$f(n) \in O(n^3)$



$n = O(n^2)$

$n \geq 1$, $n \leq n^2$

$n = O(n^2)$

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if $t(n) = O(g(n))$

$\Rightarrow g(n) = \Omega(t(n))$

$n = O(n^2) \Rightarrow n^2 = \Omega(n)$

Ω: $t(n) \in \Omega(g(n))$

if $\exists\, c, n_0$ such that:

$t(n) \geq c\, g(n)$ for $n \geq n_0$

Does $n^2 \in \Omega(n^3)$?

As $n^2 \leq n^3$

$n^2 \notin \Omega(n^3)$

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$t(n) \in \Theta(g(n))$

$\exists\, c_1, c_2, n_0$

$c_2 g(n) \leq t(n) \leq c_1 g(n)$
for all $n \geq n_0$

prove that:

$\frac{1}{2} n(n-1) \in \Theta(n^2)$

$(n-1) < n \Rightarrow$

$\frac{1}{2} n(n-1) < \frac{1}{2} n \times n = \frac{1}{2} n^2$

$\frac{1}{2} > \frac{1}{4}$

$\frac{1}{2} n(n-1) > \frac{1}{4} n(n-1) = \frac{1}{4} n^2 - \frac{n}{4}$

$= \; > \frac{1}{4} n^2$

$=$

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$\frac{n(n+1)}{2} \in O(n^3)$ TRUE

$\frac{n(n+1)}{2} \in \Theta(n^3)$ FALSE

$\frac{n(n+1)}{2} \in O(n^2)$ TRUE

$\frac{n(n+1)}{2} \in \Omega(n^2)$ TRUE

$\lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^3} = \lim_{n\to\infty} \frac{1}{n} = 0$

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for $i = 0$ to $m-2$
for $j = i+1$ to $m-2$
Instruction$_1$

$$\sum_{j=i+1}^{m-1} 1 = (m-1) - (i+1) + 1 = m - i - 1$$

$$\sum_{i=0}^{m-2}\left[\sum_{j=i+1}^{m-1} 1\right] = \sum_{i=0}^{m-2}(m-i-1)$$

$$\underbrace{\sum_{i=0}^{m-2} m}_{(m-1)m} - \sum_{i=0}^{m-2} i - \sum_{i=0}^{m-2} 1$$

$$1+2+3+\dots+m = \frac{m(m+1)}{2}$$

$$\sum_{i=0}^{m-2} i = 0 + 1 + \dots + (m-2) = \frac{(m-2)(1+m-2)}{2} = \frac{(m-2)(m-1)}{2}$$

$$\sum_{i=0}^{m-2} 1 = (m-1)$$

$$(m-1)m - \frac{(m-2)}{2}(m-1) - (m-1)$$

$$(m-1)\left[m - \frac{(m-2)}{2} - 1\right]$$

$$(m-1)\left(\frac{2m-m+2-2}{2}\right)$$

$$\frac{(m-1)m}{2} = \Theta(m^2)$$
$$O(m^2)$$

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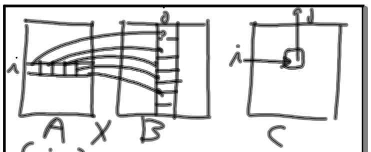

(i,j)

for $i = 0$ to $m-1$
for $j = 0$ to $m-1$
for $k = 0$ to $m-1$
Inst

$$\sum_{i=0}^{m-1}\sum_{j=0}^{m-1}\sum_{k=0}^{m-1} 1 = \sum_{i=0}^{m-1}\sum_{j=0}^{m-1} m = \sum_{i=0}^{m-1} m^2 = m^3$$

$$O(m^3)$$

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$$\sum_{i=0}^{m-2}\sum_{j=i+1}^{m-1}\sum_{k=i}^{m} 1 :$$

$$\sum_{i=0}^{m-2}\sum_{j=i+1}^{m-1}(m-i+1)$$

$$\sum_{j=i+1}^{m-1}(m-i+1) = \underbrace{\sum_{j=i+1}^{m-1} m}_{(m-i-1)m} - \sum_{j=i+1}^{m-1} i + \underbrace{\sum_{j=i+1}^{m-1} 1}_{(m-i-1)}$$

$$\sum_{j=i+1}^{m-1} i = (m-i-1)\,i$$

$$\sum_{i=0}^{m-2}(m-i-1)(m-i+1)$$

$$(x-1)(x+1) = x^2 - 1$$

$$\sum_{i=0}^{m-2}(m-i)^2 - 1$$

$$\sum (m^2 - 2mi + i^2) - 1$$

$$(m-1)m^2 - \underbrace{2m\sum_{i=0}^{m-2} i}_{m(m^2)} + \underbrace{\sum_{i=0}^{m-2}(i^2-1)}_{m^3}$$

$$= O(m^3)$$

$$\sum_{i=1}^{m} i^2 =$$

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$$\lim_{m\to\infty}\frac{5m^2+5m+3}{3m^2+6} : \frac{5m^2}{3m^2} = \frac{5}{3}.$$

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$$\lim_{m\to\infty}\frac{T(m)}{g(m)}$$

$$\begin{cases} 0 & T(m) = O(g(m)) \\ c>0 & T(m) = \Theta(g(m)) \quad T(m) = \Omega\, g(m) \\ & \text{OR } T(m) = O(g(m)) \\ \infty & T(m) = \Omega(g(m)) \end{cases}$$

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$t(m) \in O(g(m))$

if $\exists\, m_0, c$ such that:

$t(m) \leq c\, g(m)$ for all $m > m_0$

$t(n) = n$ | $t(m) \in O(g(m))$
$g(m) = m^2$ | $m \in O(m^2)$

$n \leq n^2$ for all $n \geq m_0$

$c = 1, m_0 = 1$

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prove that:

$100n+5 \in O(n^2)$

$t(n) \leq c\,g(n) \quad n \geq n_0$

$t(n) = 100n+5, \quad 5 \leq 5n \quad n \geq 1$

$\leq 100n+5n$

$\leq 105n \quad , n \leq n^2 \quad n \geq 1$

$105n \leq 105n^2$

$t(n) \leq 105n^2 \quad c = 105 \quad n_0 = 1$

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