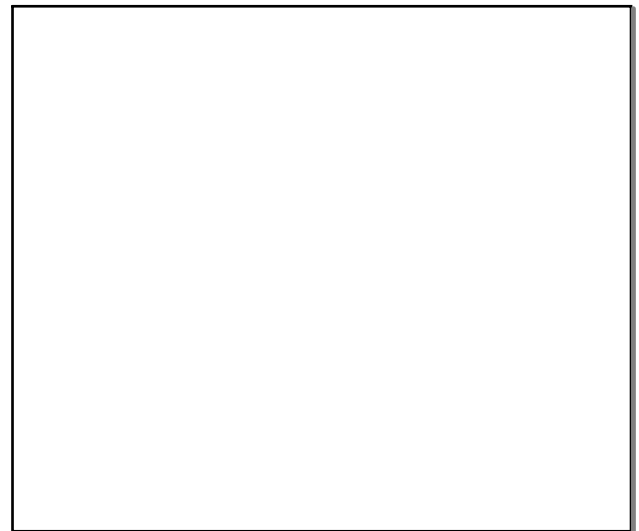
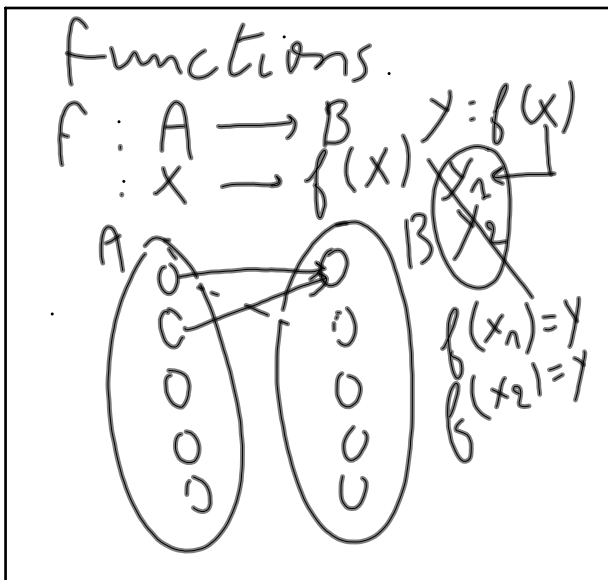


Midterm 1:
 - 2 July 9 → 11 } 20
 - Midterm 2:
 - 16 July 9 → 11 } 20
 - final → 40
 - HOMEWORK → 20
 - Quiz →

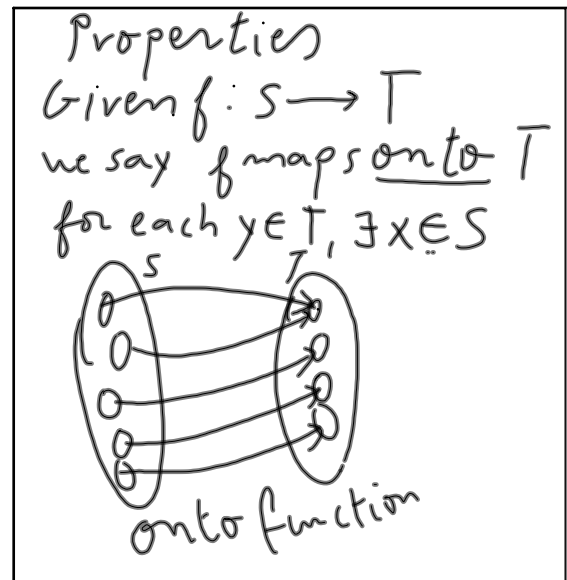
Jun 23-9:22 AM



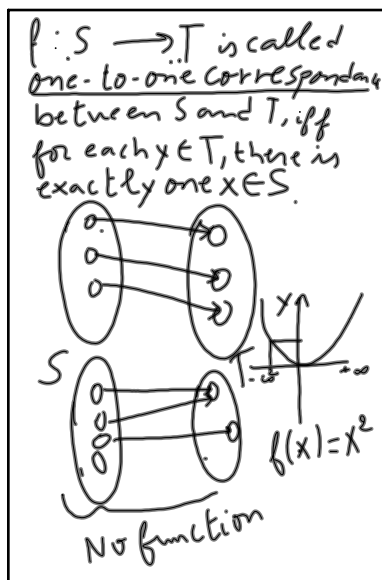
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One-to-one
 $f: S \rightarrow T$
 $x \rightarrow f(x) = y$
 $f(x_1) = f(x_2) \rightarrow x_1 = x_2$

Example:

$f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \rightarrow f(x) = 3x + 5$

$\exists x_1, x_2 \in \mathbb{R}, f(x_1) = f(x_2)$
 $f(x_1) = f(x_2)$
 $3x_1 + 5 = 3x_2 + 5$
 $x_1 = x_2$

one-to-one?

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$$f: \text{maps } \mathbb{R} \text{ onto } \mathbb{R}$$

$$y = f(x) = 3x + 5$$

$$y = 3x + 5 \Rightarrow 3x = y - 5$$

$$\Rightarrow x = \frac{y-5}{3}$$

$\in \mathbb{R}$

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Inverse function

$$f: S \rightarrow T$$

$$x \rightarrow f(x)$$

$$f^{-1}: T \rightarrow S$$

$$y \rightarrow f^{-1}(y)$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \rightarrow f(x) = 3x + 5$$

$$f^{-1}: \mathbb{R} \rightarrow \mathbb{R}$$

$$y \rightarrow f^{-1}(y)$$

$$f(x) = 3x + 5 = y$$

$$x = \frac{y-5}{3}$$

$$f^{-1}(y) = \frac{y-5}{3}$$

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Exapb:

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$x \rightarrow f(x) = x^2$$

$$y: \mathbb{R}^+ \rightarrow \mathbb{R}^+ \quad y = x^2$$

$$y \rightarrow \sqrt{y} \quad x = \begin{cases} \sqrt{y} \\ -\sqrt{y} \end{cases}$$

$$f(x) = e^x$$

$$y = e^x \Rightarrow x = \ln y$$

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Composition

$$f: S \rightarrow T$$

$$g: T \rightarrow U$$

$$g \circ f: S \rightarrow U$$

$$x \rightarrow g(f(x))$$

Example:

$$f(x) = x^3 + 2x$$

$$g(x) = x^2$$

$$(g \circ f)(x) = g(f(x))$$

$$= g(x^3 + 2x)$$

$$= (x^3 + 2x)^2$$

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$$f(x) = \sqrt{x}$$

$$g(x) = \frac{1}{x} \quad x \in]0, \infty[$$

g o f ?

f o g ?

$$(g \circ f)(x) = g(f(x))$$

$$= g(\sqrt{x})$$

$$= \frac{1}{\sqrt{x}}$$

$$(f \circ g)(x) = f(g(x))$$

$$= f\left(\frac{1}{x}\right) = \sqrt{\frac{1}{x}} = \frac{1}{\sqrt{x}}$$

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$$f(x) = x^4$$

$$g(y) = \sqrt{y^2 + 1}$$

$$h(z) = z^2 + 7z$$

h o (g o f) h o (g(f(x)))

(h o g) o f

$$h \circ (g \circ f) = h \circ g \circ f(x)$$

$$h \circ [g(x^4)]$$

$$h \circ \left[\sqrt{x^8 + 1} \right] =$$

$$h \left(\sqrt{x^8 + 1} \right)$$

$$h \left(\frac{\sqrt{x^8 + 1}}{1} \right)$$

$$= (\sqrt{x^8 + 1})^2 + 7\sqrt{x^8 + 1}$$

$$= x^8 + 7\sqrt{x^8 + 1} = h \circ g \circ f(x)$$

(h o g) o f

$$h(g(x)) \circ f = h(\sqrt{x^2 + 1}) \circ f$$

$$= (\sqrt{x^2 + 1})^2 + 7\sqrt{x^2 + 1}$$

$$= (x^2 + 7\sqrt{x^2 + 1}) \circ f$$

$$= f(x)^2 + 7\sqrt{x^2 + 1}$$

$$= x^8 + 7\sqrt{x^2 + 1}$$

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Sequences.

$a_n = f(n)$

$a_0, a_1, \dots, a_n, a_{n+1}, \dots$

$f(0), f(1), \dots, f(n), f(n+1), \dots$

$a_n = \sum_{k=1}^n k^2$

$a_n = 1^2 + 2^2 + 3^2 + \dots + n^2$

$a_n = 1^2 \cdot n$

$a_n = 1 + 2^2 = 5$

$a_n = \prod_{k=1}^n k$

$a_n = 1 \times 2 \times 3 \times \dots \times n$

$a_n = 1 \cdot 1$

$a_n = 2 \cdot 2$

$a_n = 3 \cdot 3$

Let $\Sigma = \{a, b\}$

$\Sigma^k = \{w \in \Sigma^* : \text{length}(w) = k\}$

$\Sigma^0 = \{\lambda\}$

$\Sigma^1 = \{a, b\}$

$\Sigma^2 = \{aa, bb, ab, ba\}$

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$\sum_{k=1}^m 3^k$ $m = 4, 2, 3, 4$

$a_1 = 3, a_2 = 12$

$a_3 = 3^3 = 27$

$a_4 = 27 + 81 = 108$

$\sum_{k=1}^m k^3$ $m = 3, 4, 5$

$a_3 = 3^3 = 27$

$a_4 = 27 + 81 = 108$

$\sum_{j=1}^m j$ $m = 2, 3, 4, 5$

$a_2 = 1 + 2 = 3$

$a_3 = 1 + 2 + 3 = 6$

$a_4 = 1 + 2 + 3 + 4 = 10$

$a_5 = 1 + 2 + 3 + 4 + 5 = 15$

$a_3 = \sum_{j=1}^3 j = 6$

$a_4 = \sum_{j=1}^4 j = 10$

$a_5 = \sum_{j=1}^5 j = 15$

$a_6 = \sum_{j=1}^6 j = 21$

$a_7 = \sum_{j=1}^7 j = 28$

$a_8 = \sum_{j=1}^8 j = 36$

$a_9 = \sum_{j=1}^9 j = 45$

$a_{10} = \sum_{j=1}^{10} j = 55$

$a_{11} = \sum_{j=1}^{11} j = 66$

$a_{12} = \sum_{j=1}^{12} j = 78$

$a_{13} = \sum_{j=1}^{13} j = 91$

$a_{14} = \sum_{j=1}^{14} j = 105$

$a_{15} = \sum_{j=1}^{15} j = 120$

$a_{16} = \sum_{j=1}^{16} j = 136$

$a_{17} = \sum_{j=1}^{17} j = 153$

$a_{18} = \sum_{j=1}^{18} j = 171$

$a_{19} = \sum_{j=1}^{19} j = 190$

$a_{20} = \sum_{j=1}^{20} j = 210$

$a_{21} = \sum_{j=1}^{21} j = 231$

$a_{22} = \sum_{j=1}^{22} j = 253$

$a_{23} = \sum_{j=1}^{23} j = 276$

$a_{24} = \sum_{j=1}^{24} j = 300$

$a_{25} = \sum_{j=1}^{25} j = 325$

$a_{26} = \sum_{j=1}^{26} j = 351$

$a_{27} = \sum_{j=1}^{27} j = 378$

$a_{28} = \sum_{j=1}^{28} j = 406$

$a_{29} = \sum_{j=1}^{29} j = 435$

$a_{30} = \sum_{j=1}^{30} j = 465$

$a_{31} = \sum_{j=1}^{31} j = 496$

$a_{32} = \sum_{j=1}^{32} j = 528$

$a_{33} = \sum_{j=1}^{33} j = 561$

$a_{34} = \sum_{j=1}^{34} j = 595$

$a_{35} = \sum_{j=1}^{35} j = 630$

$a_{36} = \sum_{j=1}^{36} j = 666$

$a_{37} = \sum_{j=1}^{37} j = 703$

$a_{38} = \sum_{j=1}^{38} j = 741$

$a_{39} = \sum_{j=1}^{39} j = 780$

$a_{40} = \sum_{j=1}^{40} j = 820$

$a_{41} = \sum_{j=1}^{41} j = 861$

$a_{42} = \sum_{j=1}^{42} j = 903$

$a_{43} = \sum_{j=1}^{43} j = 946$

$a_{44} = \sum_{j=1}^{44} j = 990$

$a_{45} = \sum_{j=1}^{45} j = 1035$

$a_{46} = \sum_{j=1}^{46} j = 1081$

$a_{47} = \sum_{j=1}^{47} j = 1128$

$a_{48} = \sum_{j=1}^{48} j = 1176$

$a_{49} = \sum_{j=1}^{49} j = 1225$

$a_{50} = \sum_{j=1}^{50} j = 1275$

$a_{51} = \sum_{j=1}^{51} j = 1326$

$a_{52} = \sum_{j=1}^{52} j = 1378$

$a_{53} = \sum_{j=1}^{53} j = 1431$

$a_{54} = \sum_{j=1}^{54} j = 1485$

$a_{55} = \sum_{j=1}^{55} j = 1540$

$a_{56} = \sum_{j=1}^{56} j = 1596$

$a_{57} = \sum_{j=1}^{57} j = 1653$

$a_{58} = \sum_{j=1}^{58} j = 1711$

$a_{59} = \sum_{j=1}^{59} j = 1770$

$a_{60} = \sum_{j=1}^{60} j = 1830$

$a_{61} = \sum_{j=1}^{61} j = 1891$

$a_{62} = \sum_{j=1}^{62} j = 1953$

$a_{63} = \sum_{j=1}^{63} j = 2016$

$a_{64} = \sum_{j=1}^{64} j = 2080$

$a_{65} = \sum_{j=1}^{65} j = 2145$

$a_{66} = \sum_{j=1}^{66} j = 2211$

$a_{67} = \sum_{j=1}^{67} j = 2278$

$a_{68} = \sum_{j=1}^{68} j = 2346$

$a_{69} = \sum_{j=1}^{69} j = 2415$

$a_{70} = \sum_{j=1}^{70} j = 2485$

$a_{71} = \sum_{j=1}^{71} j = 2556$

$a_{72} = \sum_{j=1}^{72} j = 2628$

$a_{73} = \sum_{j=1}^{73} j = 2701$

$a_{74} = \sum_{j=1}^{74} j = 2775$

$a_{75} = \sum_{j=1}^{75} j = 2850$

$a_{76} = \sum_{j=1}^{76} j = 2926$

$a_{77} = \sum_{j=1}^{77} j = 3003$

$a_{78} = \sum_{j=1}^{78} j = 3081$

$a_{79} = \sum_{j=1}^{79} j = 3160$

$a_{80} = \sum_{j=1}^{80} j = 3240$

$a_{81} = \sum_{j=1}^{81} j = 3321$

$a_{82} = \sum_{j=1}^{82} j = 3403$

$a_{83} = \sum_{j=1}^{83} j = 3486$

$a_{84} = \sum_{j=1}^{84} j = 3570$

$a_{85} = \sum_{j=1}^{85} j = 3655$

$a_{86} = \sum_{j=1}^{86} j = 3741$

$a_{87} = \sum_{j=1}^{87} j = 3828$

$a_{88} = \sum_{j=1}^{88} j = 3916$

$a_{89} = \sum_{j=1}^{89} j = 4005$

$a_{90} = \sum_{j=1}^{90} j = 4095$

$a_{91} = \sum_{j=1}^{91} j = 4186$

$a_{92} = \sum_{j=1}^{92} j = 4278$

$a_{93} = \sum_{j=1}^{93} j = 4371$

$a_{94} = \sum_{j=1}^{94} j = 4465$

$a_{95} = \sum_{j=1}^{95} j = 4560$

$a_{96} = \sum_{j=1}^{96} j = 4656$

$a_{97} = \sum_{j=1}^{97} j = 4753$

$a_{98} = \sum_{j=1}^{98} j = 4851$

$a_{99} = \sum_{j=1}^{99} j = 4950$

$a_{100} = \sum_{j=1}^{100} j = 5050$

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$\sum_{i=1}^{10} (-1)^i = (-1)^1 + (-1)^2 + (-1)^3 + (-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8 + (-1)^9 + (-1)^{10} = 0$

$\prod_{m=1}^5 (2m+1) = 3 \times 5 \times 7 \times 9 \times 11$

$\prod_{j=4}^8 (j-1) = 3 \times 4 \times 5 \times 6 \times 7$

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| | | | |
|---|---|-------|---------|
| P | q | P ∨ q | ¬P ∨ ¬q |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

$P \vee q \Leftrightarrow \neg(P \wedge \neg q)$

- show that $\neg P \Leftrightarrow P \vee \neg P$

- show that $P \vee q \Leftrightarrow (P \vee P) \vee (q \vee q)$

- find a proposition equivalent to $P \wedge q$ using \neg .

Jun 23-11:06 AM

$\neg P \Leftrightarrow P \vee \neg P$

$\neg P \Leftrightarrow \neg(P \wedge \neg P)$

$P \wedge q \quad (P \wedge \neg P) \Leftrightarrow \neg P$

$\neg P \quad \neg P \Leftrightarrow \neg P$

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| | | | | | | |
|---|---|-------|---------|---|---|-------|
| P | q | P ∨ q | ¬P ∨ ¬q | A | B | A ∨ B |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 |

$P \vee q \Leftrightarrow (P \vee P) \vee (q \vee q)$

$P \vee P \Leftrightarrow P$

$q \vee q \Leftrightarrow q$

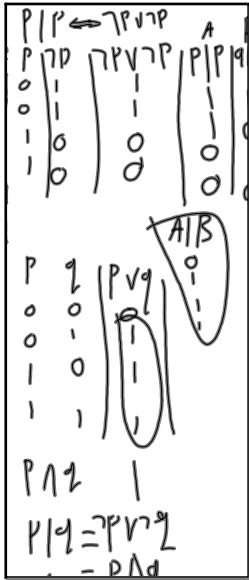
$A \vee B \Leftrightarrow (A \vee A) \vee (B \vee B)$

$\Leftrightarrow P \vee q$

$\neg(P \vee q) \Leftrightarrow \neg(P \vee P) \vee \neg(q \vee q)$

$\Leftrightarrow \neg P \vee \neg q$

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