1.4 WHAT IS DISCRETE-EVENT SIMULATION?

Figure 1.5 depicts a single server system consisting of a machine and a buffer in a factory. The dynamics of the system may be described as follows: (1) a job arrives at the system with an inter-arrival time of t_a, and the job is loaded on the machine if it is idle; otherwise, the job is put into the buffer; (2) the loaded job is processed for a service time of t_s and unloaded; (3) when a job is unloaded, the next job is loaded if the buffer is not empty. In Fig. 1.5, the state variables of the system are q and m, where q is the number of jobs in the buffer

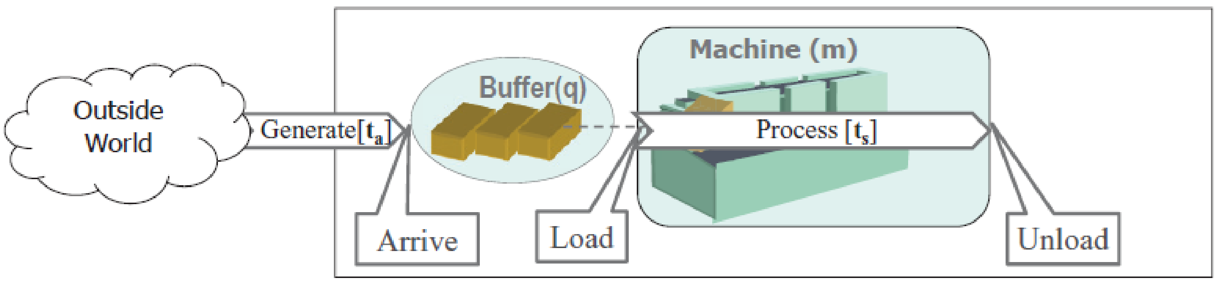

Fig. 1.5. A single server system model.

and m denotes the status (Idle or Busy) of the machine, and the events are Arrive, Load, and Unload.

1.4.1 Description of System Dynamics

Using the state variables and events, the system dynamics of the single server system may be described more rigorously as follows: (1) when an Arrive event occurs, q is increased by one, the next Arrive event is scheduled to occur after t_a time units, and a Load event is scheduled to occur immediately if $m \equiv$ Idle(=0); (2) when a Load event occurs, q is decreased by one, m is set to Busy(=1), and an Unload event is scheduled to occur after t_s time units; (3) when an Unload event occurs, m is set to Idle and a Load event is scheduled to occur immediately if $q > 0$. The dynamics of the single server system may be described as a graph as given in Fig. 1.6, which is called an *event graph*.

1.4.2 Simulation Model Trajectory

An executable model of a system is called a *simulation model*, and the trajectory of the state variables of the model is called the *simulation model trajectory*. Let $\{a_k\}$ and $\{s_k\}$ denote the sequences of inter-arrival times (t_a) and service times (t_s), respectively. Then, the simulation model trajectory of the single server system would look like Fig. 1.7, where $\{t_i\}$ are event times, X(t) is input trajectory, and Q(t) = {q(t), m(t)} denotes the trajectory of the system

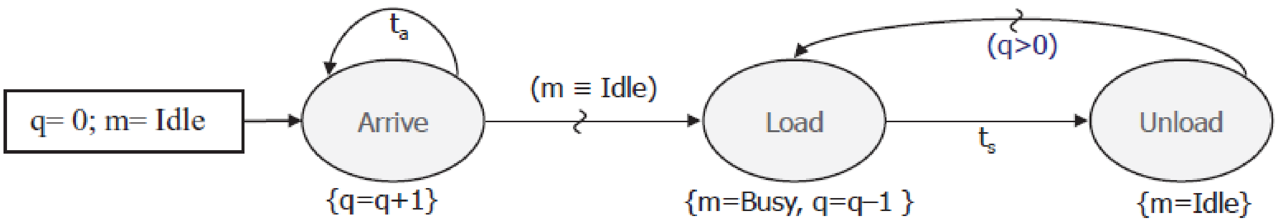

Fig. 1.6. Event graph describing the system dynamics of the single server system.

state variables. The "time" here means a simulation time, which is a logical time used by the simulation model to represent physical time of the target system to be simulated.

At time t_1 ($=a_1$), a job J_1 arrives at an empty system and is loaded on the idle machine to be processed for a time period of s_1. In the meantime, another job J_2 arrives at time t_2 ($=a_1 + a_2$), which will be put into the buffer since the machine is busy. Thus, the buffer will have one job during the time period $[t_2, t_3]$, which is denoted as a shaded bar in the buffer graph q(t) of Fig. 1.7. At t_3 ($=t_1 + s_1$), the first job J_1 is unloaded and the job J_2 in the buffer is loaded on the machine. At t_4 ($=t_3 + s_2$), J_2 is finished and unloaded, which will make the system empty again. Thus, the machine is busy during the time period $[t_1, t_4]$. At time t_5 ($=a_1 + a_2 + a_3$), another job J_3 arrives at the system and is loaded on the machine, and so on.

1.4.3 Collecting Statistics from the Model Trajectory

When simulating a service system, one may be interested in such items as (1) queue length, (2) waiting time distribution, (3) sojourn time, (4) server utilization, etc. In the case of the single server system, the following statistics can be collected from the model trajectory.

1. Queue length $q(t)$ statistics during $t \in [t_0, t_{10}]$: AQL (average queue length)
   - $AQL = \{(t_3 - t_2) + (t_7 - t_6) + 2(t_8 - t_7) + (t_9 - t_8) + 2(t_{10} - t_9)\}/t_{10}$
2. Waiting time $\{W_j\}$ statistics for the first four jobs: AWT (average waiting time)
   - $AWT = \{W_1 + W_2 + W_3 + W_4\}/4 = \{0 + (t_3 - t_2) + 0 + (t_8 - t_6)\}/4 = (t_3 - t_2 + t_8 - t_6)/4$
3. Sojourn time $\{S_j\}$ statistics for the first four jobs: AST (average sojourn time)
   - AST = AWT + Average service time = AWT + $(s_1 + s_2 + s_3 + s_4)/4$
4. Server utilization during $t \in [t_0, t_{10}]$: U (utilization)
   - $U = \{(t_4 - t_1) + (t_{10} - t_5)\}/t_{10}$