Multirate Digital Signal Processing

Deals with changing the sampling rate.

Many applications of DSP such as communications, speech and audio processing require changing in sampling rate.

Downsampling - 1

Input: x(n). Output: $y(m) = x(mM)$.

Taking one sample from every M samples in the input.

Example: $x(n): 8\ 7\ 4\ 8\ 9\ 6\ 4\ 2\ -2\ -5\ -7\ -7\ -6\ -4\ldots$

Downsample by a factor of 3

$y(m): 8\quad 8\quad 4\quad -5\quad -6\ldots$

Let, sampling period of x(n) is 0.1 second [10 samples per second]

Sampling period of y(m) is 3×0.1 second [3.3 samples per second]

$T \rightarrow MT$

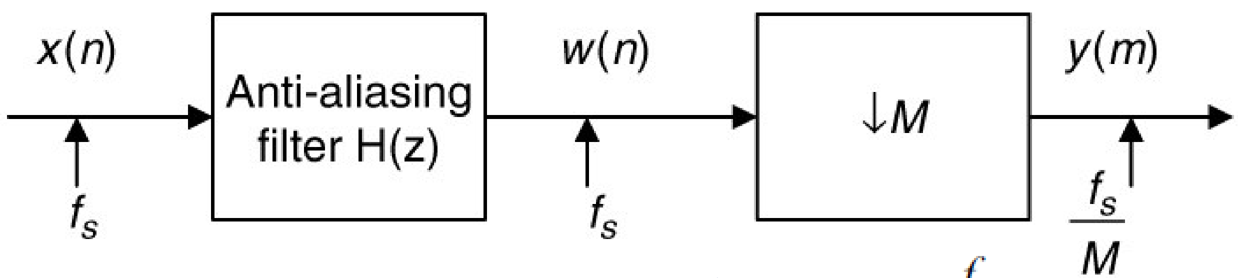
$f_s \rightarrow f_s/M$

Downsampling - 2

Folding frequency after downsampling: $f_{sM}/2 = \frac{f_s}{2M}$

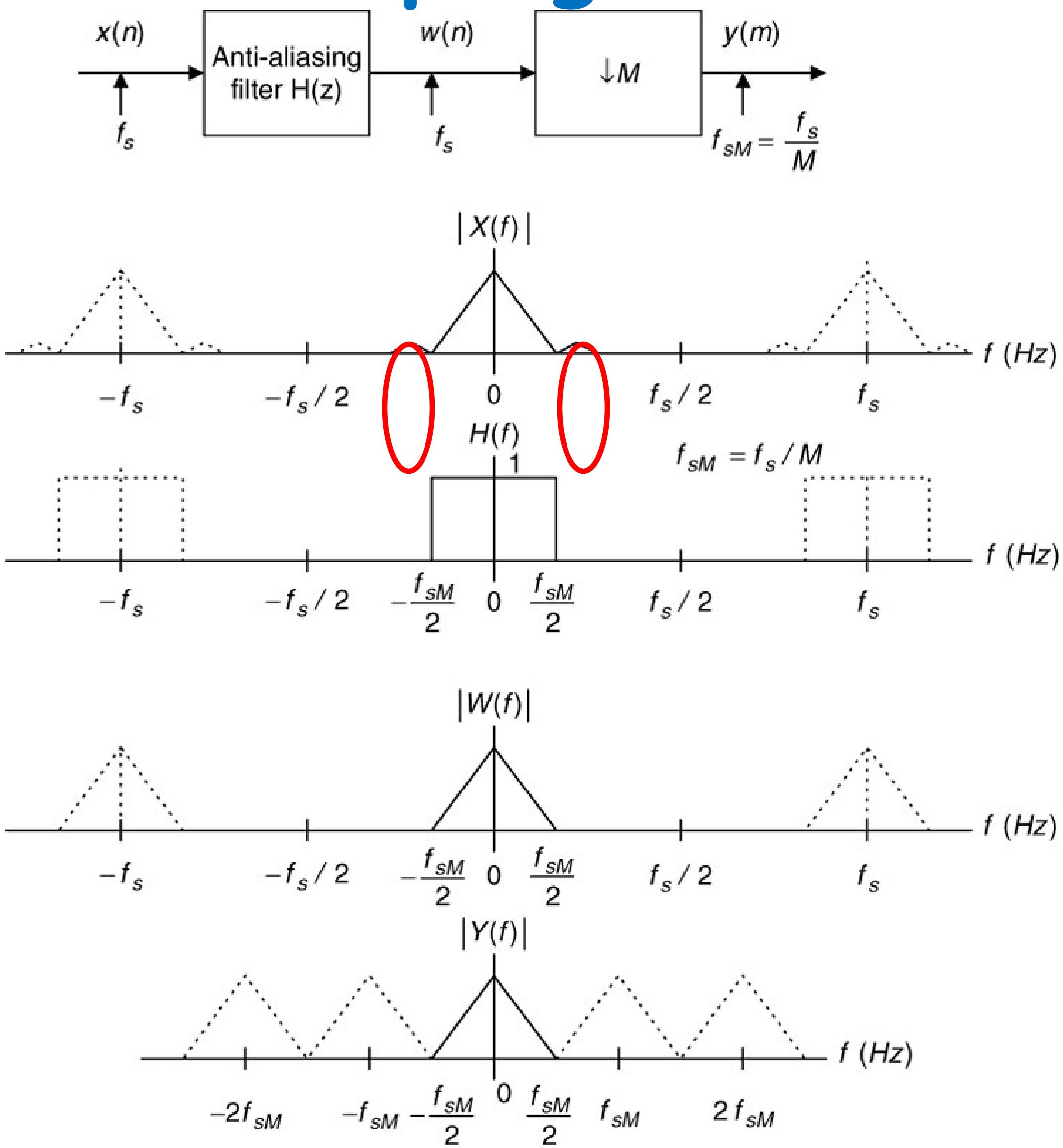
If the signal to be downsampled has frequency components larger than the folding frequency, aliasing noise will be introduced into the downsampled data.

Anti- aliasing filter: H(z) → Low pass filter with stop frequency edge at $f_{sM}/2 = \frac{f_s}{2M}$



⇒ $f_{\max} < \frac{f_s}{2M}$

Downsampling - 3



Frequency response of anti -aliasing filter

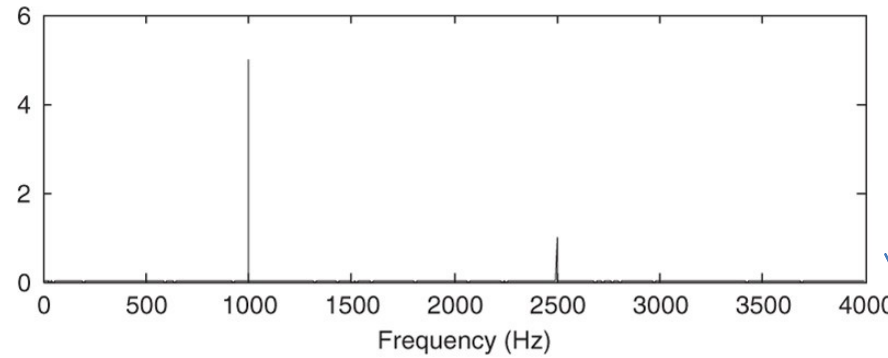
Output spectrum

Downsampling - 4

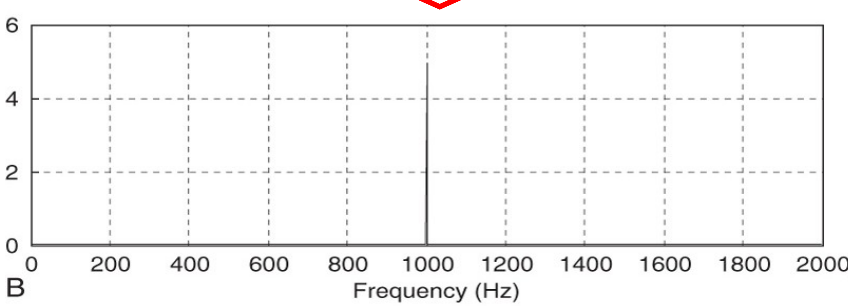
Let: $$x(n) = 5\sin\left(\frac{2\pi \times 1000n}{8000}\right) + \cos\left(\frac{2\pi \times 2500n}{8000}\right)$$

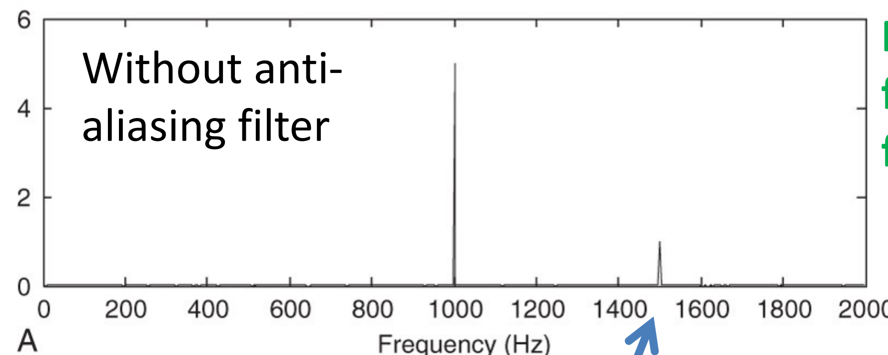
$f_s = 8{,}000\ \text{Hz}$

Original folding freq.



With anti-aliasing filter





New folding freq.

$$4\ \text{kHz} - 2.5\ \text{kHz} = 1.5\ \text{kHz}$$

Downsampling - Example

Given:

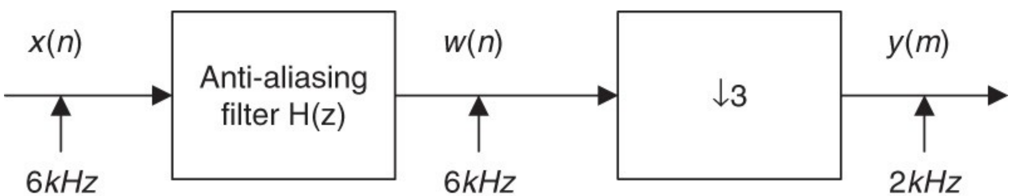
Sampling rate = 6,000 Hz
Input audio frequency range = 0–800 Hz
Passband ripple = 0.02 dB
Stopband attenuation = 50 dB
Downsample factor $M = 3$,

(Passband ripple and stopband attenuation) → Hamming window (Table 7.7)

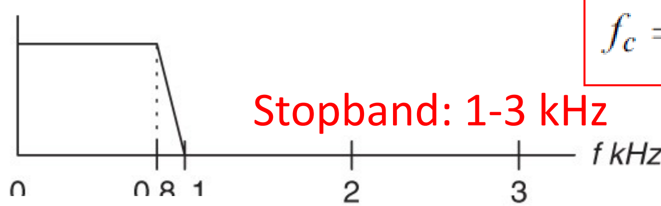
Determine:

FIR filter length, cutoff frequency, and window type.

Solution:



$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{800 + 1000}{2} = 900 \text{ Hz}$$



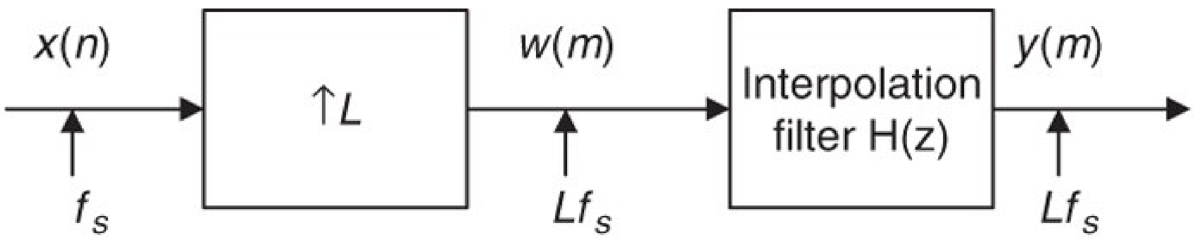
Stopband: 1-3 kHz

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_s} = \frac{1000 - 800}{6000} = 0.033 \Rightarrow N = \frac{3.3}{\Delta f} = \frac{3.3}{0.033} = 100 \Rightarrow N = 101$$

Upsampling - 1

Increase the number of samples by a factor of L, in a given period.

$$y(m) = \begin{cases} x\left(\frac{m}{L}\right) & m = nL \\ 0 & otherwise \end{cases}$$



$x(n)$: 8 8 4 −5 −6....

Upsample by a factor of 3

$w(m)$: 8 0 0 8 0 0 4 0 0 −5 0 0 −6 0 0....

$T \rightarrow T/L$

$f_s \rightarrow Lf_s$

Upsampling - 2

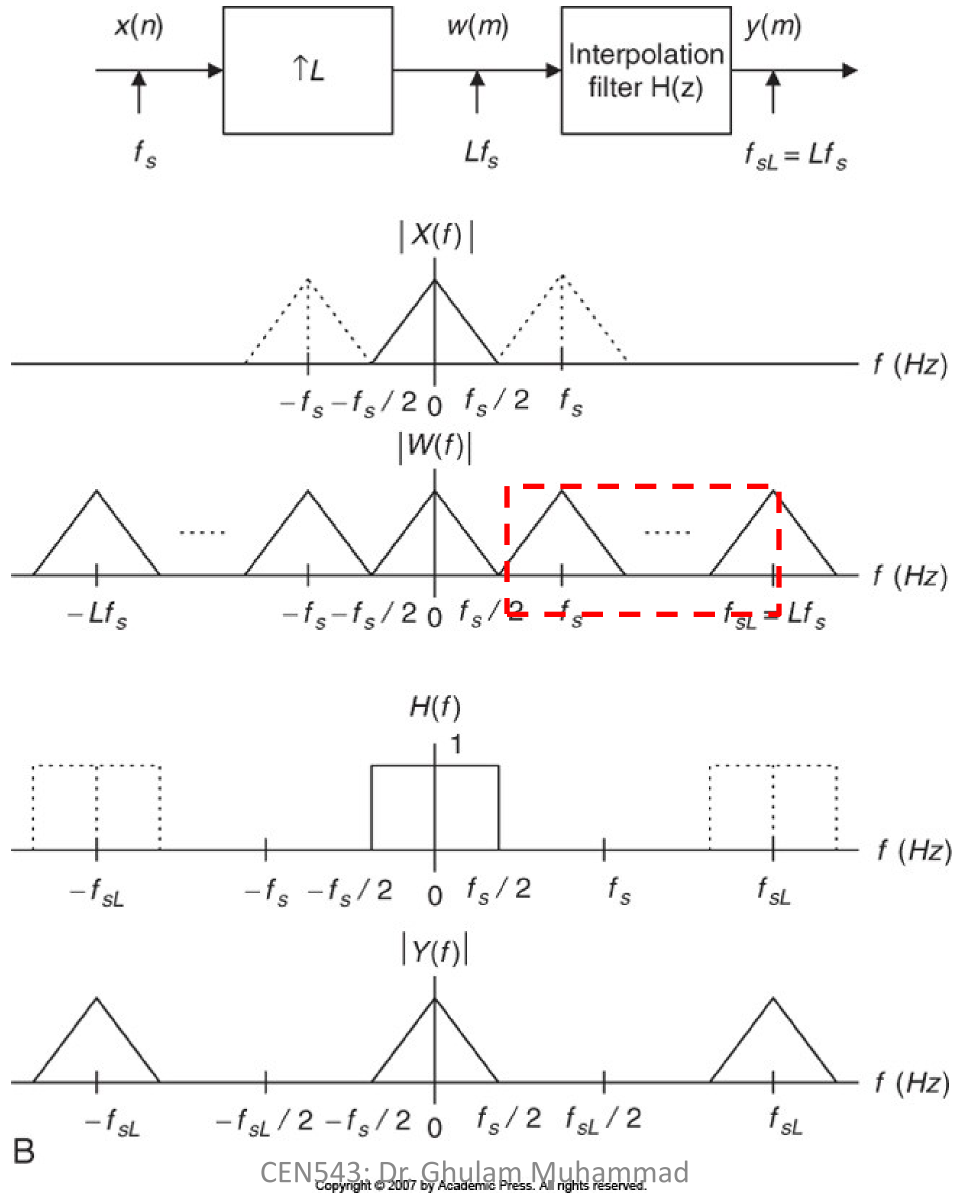
Folding frequency after upsampling: $f_{sL} = Lf_s$

After upsampling, the spectral replicas originally centered at $\pm f_s$, $\pm 2f_s$,… are included in the frequency range from 0 Hz to $Lf_s/2$ Hz.

To remove these unwanted replicas, interpolation filter is used.

Normalized stop frequency edge: $\Omega_{stop} = 2\pi\left(\frac{f_s}{2}\right) \times \left(\frac{T}{L}\right) = \frac{\pi}{L}$ radians

Upsampling - 3

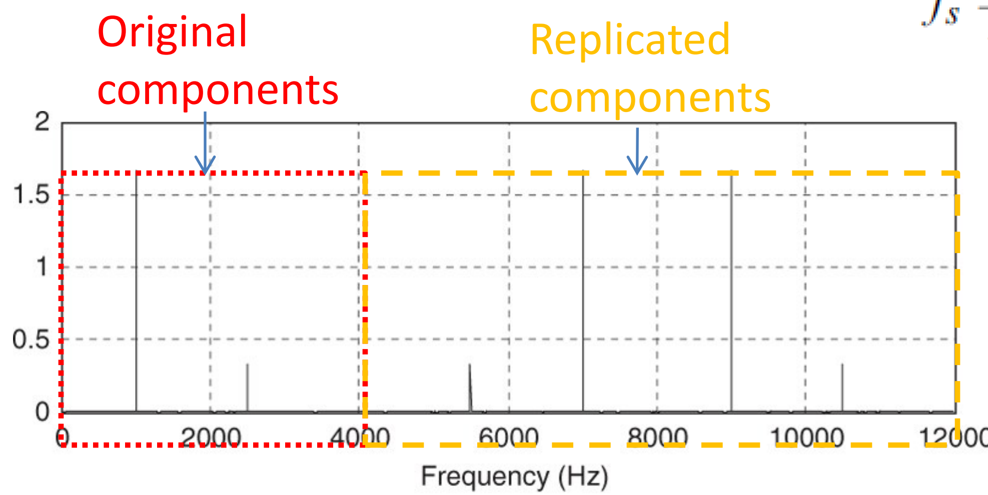


Interpolation filter frequency response

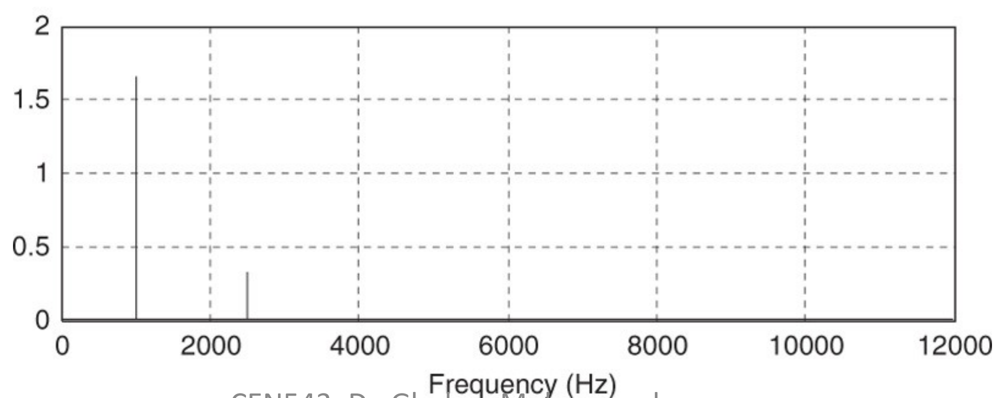
Upsampling - 4

$$x(n) = 5\sin\left(\frac{2\pi \times 1000n}{8000}\right) + \cos\left(\frac{2\pi \times 2500n}{8000}\right)$$

$f_s = 8{,}000\ \text{Hz}$



After applying interpolation filter



Upsampling - Example

Given:

Sampling rate = 6,000 Hz
Input audio frequency range = 0–800 Hz
Passband ripple = 0.02 dB
Stopband attenuation = 50 dB
Upsample factor $L = 3$,

Determine:

FIR filter length, cutoff frequency, and window type.

Solution:

Interpolation filter operating at the sampling rate = 18,000 Hz

Stopband frequency range = 3–9 kHz

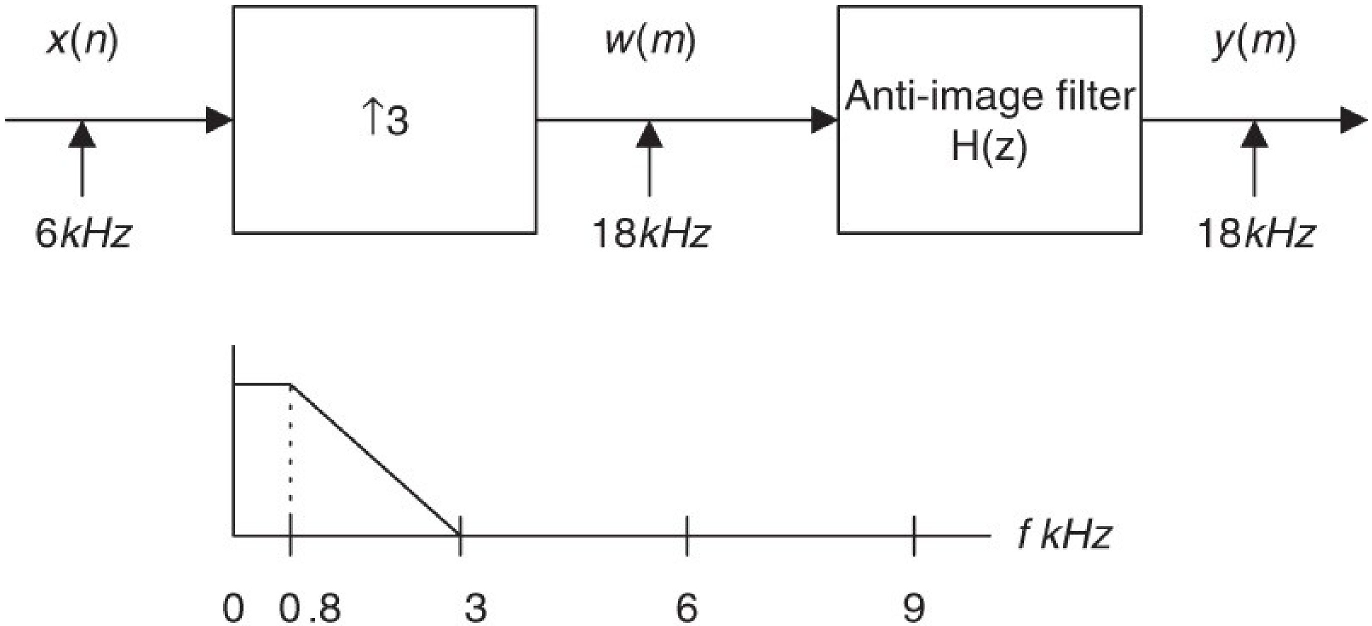
Normalized transition band: $\Delta f = \frac{f_{stop} - f_{pass}}{f_{sL}} = \frac{3000 - 800}{18000} = 0.1222$

Filter length, $N = \frac{3.3}{\Delta f} = \frac{3.3}{0.1222} = 27$

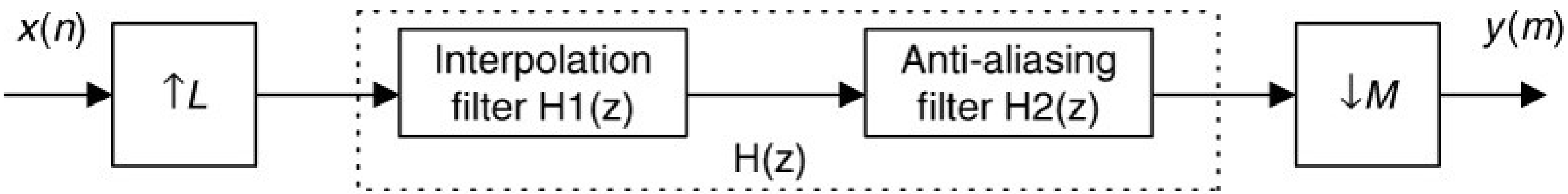
Hamming window

Upsampling - Example (contd.)

Cutoff frequency: $f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{3000 + 800}{2} = 1900 \text{ Hz}$



Changing Sampling Rate by a Non-Integer Factor L/M -1



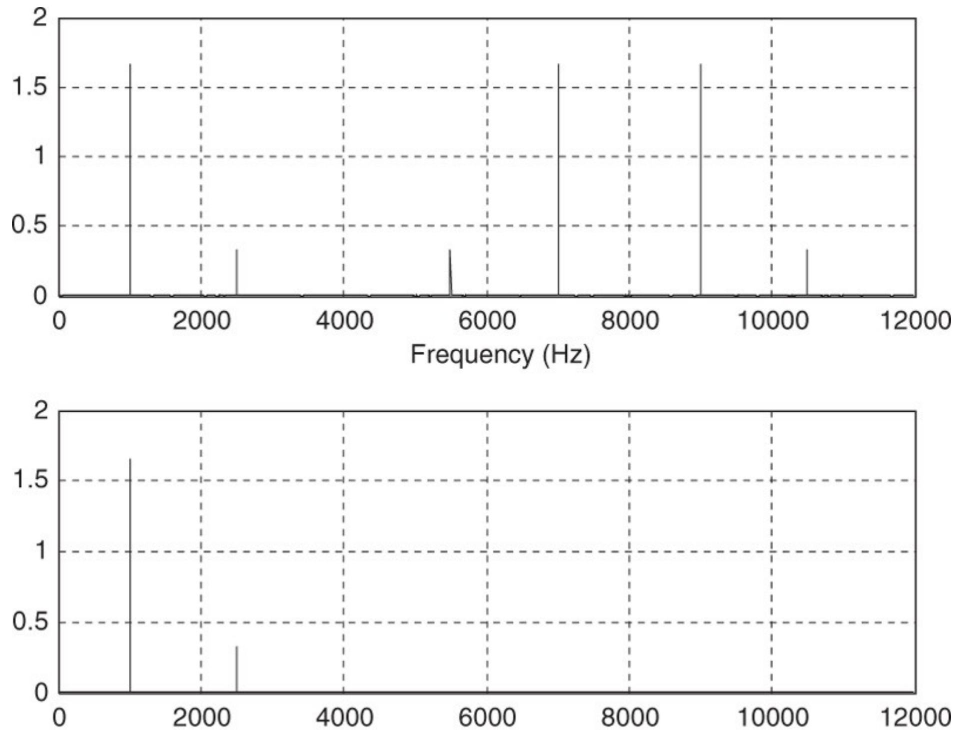
Let: $$x(n) = 5\sin\left(\frac{2\pi \times 1000n}{8000}\right) + \cos\left(\frac{2\pi \times 2500n}{8000}\right)$$ $$f_s = 8{,}000\,\text{Hz}$$

We want sampling rate = 3000 Hz. ⇨ $$\left(\frac{L}{M}\right) = 0.375 = \frac{3}{8}$$

Upsample first by a factor of 3.

Then apply FIR LPF (interpolation filter) with N = 53 and cutoff freq. = 3250 Hz at sampling rate = 24,000 Hz.

Changing Sampling Rate by a Non-Integer Factor L/M -2



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After upsampling and interpolation filter

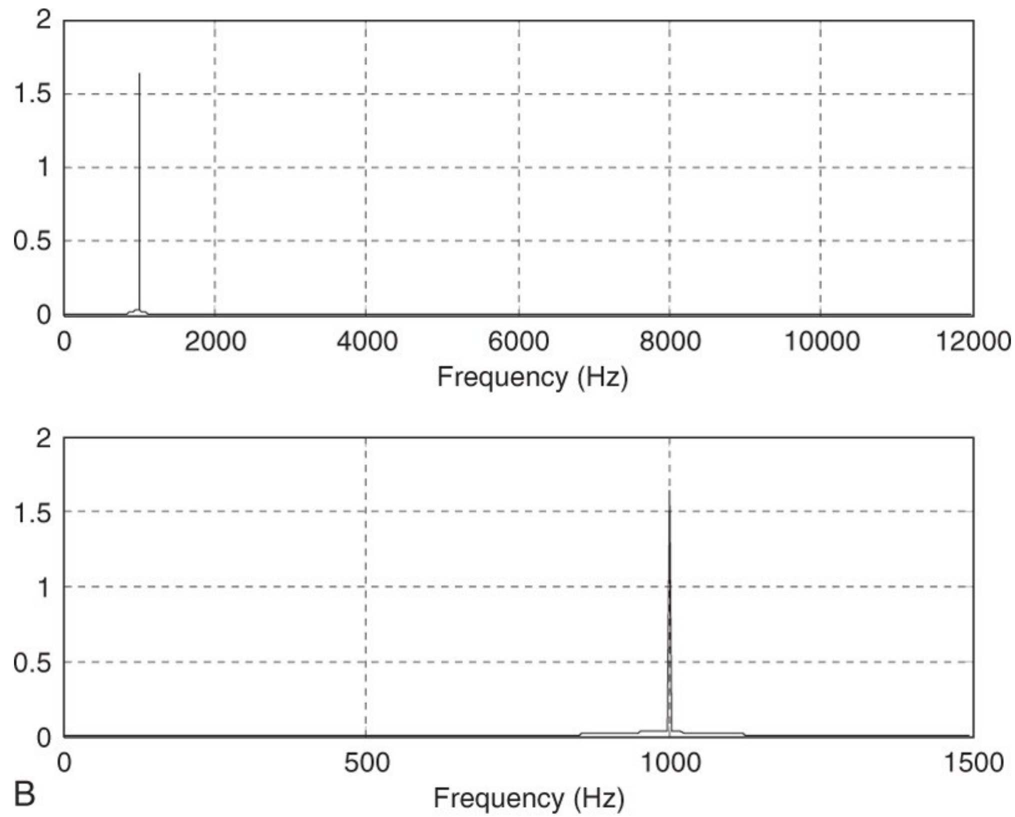
To downsample by a factor of M=8:

Filter length = 159
Cutoff freq. = 1250 Hz
Nyquist limit = 1500 Hz

Changing Sampling Rate by a Non-Integer Factor L/M -3

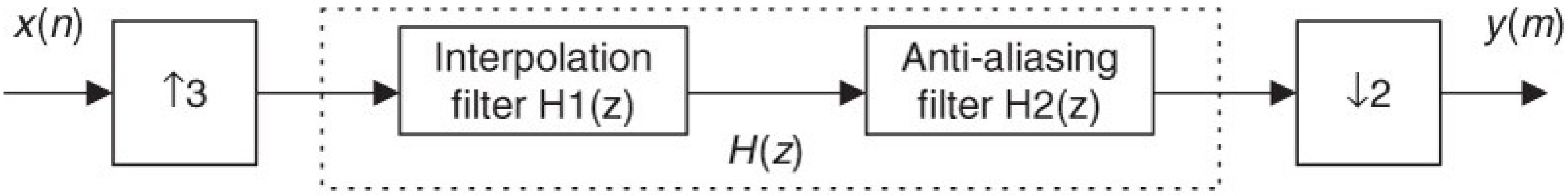
After anti-aliasing filter

After downsampling



To use only one filter, we choose N = 159 and cutoff freq. = 1250 Hz (lower bandwidth).

Changing Sampling Rate by a Non-Integer Factor L/M -Example



Audio input $x(n)$ is sampled at the rate of 6,000 Hz,

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Audio output $y(m)$ is operated at the rate of 9,000 Hz.

Upsample and H1(z):

Input sampling rate = 6000 Hz. Nyquist freq. = 3,000 Hz.

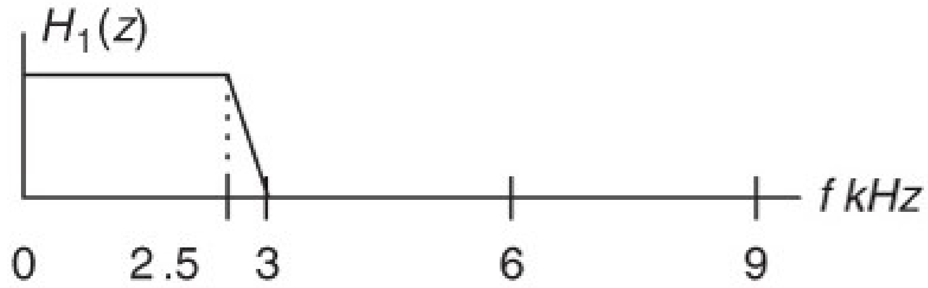
New sampling rate = 18,000 Hz. New Nyquist freq. = 9,000 Hz.

So, we have to stop frequency 3,000 – 9,000 Hz.

Changing Sampling Rate by a Non-Integer Factor L/M –Example (contd1)

Specifications for the interpolation filter $H_1(z)$:

Passband frequency range = 0–2500 Hz
Passband ripples for $H_1(z) = 0.04$ dB
Stopband frequency range = 3000–9000 Hz
Stopband attenuation = 42 dB



Input to $H_2(z)$ has sampling rate = 18000 Hz. Nyquist freq. = 9,000 Hz.

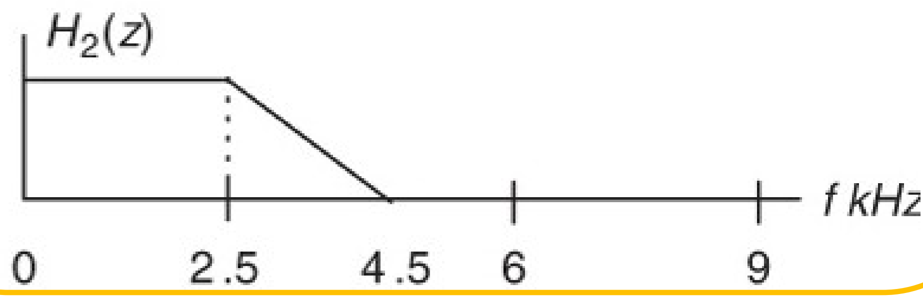
New sampling rate = 9,000 Hz. New Nyquist freq. = 4,500 Hz.

So, we have to stop frequency 4,500 – 9,000 Hz.

Changing Sampling Rate by a Non-Integer Factor L/M –Example (contd2)

Specifications for the anti-aliasing filter $H_2(z)$:

Passband frequency range = 0–2500 Hz
Passband ripples for $H_2(z)$ = 0.02 dB
Stopband frequency range = 4500–9000 Hz
Stopband attenuation = 46 dB



Combined specifications $H(z)$:

Passband frequency range = 0–2500 Hz
Passband ripples for $H(z)$ = 0.02 dB
Stopband frequency range = 3000–9000 Hz
Stopband attenuation = 46 dB.

Choose lower one

Changing Sampling Rate by a Non-Integer Factor L/M –Example (contd3)

Transition band:

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_{sL}} = \frac{3000 - 2500}{18000} = 0.0278$$

Filter length:

$$N = \frac{3.3}{\Delta f} = \frac{3.3}{0.0278} = 118.8$$

We choose $N = 119$

Cutoff frequency:

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{3000 + 2500}{2} = 2750\,\text{Hz}$$

Multistage Decimation - 1

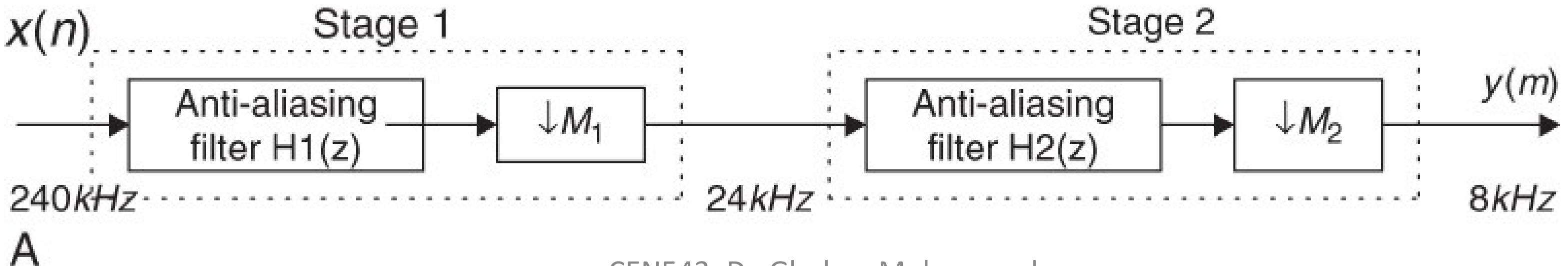
Multistage approach for downsampling rate conversion is useful to reduce filter length.

Example:

Original sampling rate: $f_s = 240\,\text{kHz}$
Audio frequency range: 0–3,400 Hz
Passband ripple: $\delta_p = 0.05$ (absolute)
Stopband attenuation: $\delta_s = 0.005$ (absolute)
FIR filter design using the window method
New sampling rate: $f_{sM} = 8\,\text{kHz}$

Design a two-stage decimator

$$M = \frac{240\,kHz}{8\,kHz} = 30 = 10 \times 3 \Rightarrow M_1 = 10 \text{ and } M_2 = 3$$

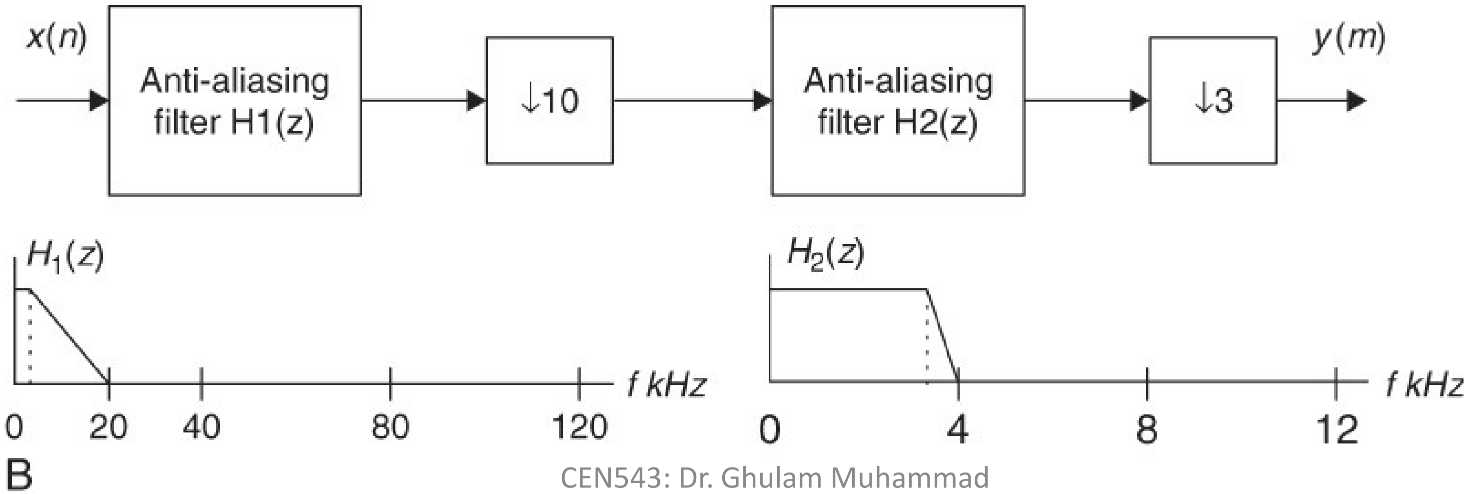


Multistage Decimation - 2

Filter specification for $H_1(z)$:

Passband frequency range: 0–3,400 Hz
Passband ripples: $0.05/2 = 0.025$ ($\delta_s\ dB = 20log_{10}(1+\delta_p) = 0.212\,\text{dB}$)
Stopband frequency range: 20,000–120,000 Hz
Stopband attenuation: 0.005, $\delta_s\ dB = -20 \times \log_{10}(\delta_s) = 46\,\text{dB}$
Filter type: FIR, Hamming window.

$$f_{stop} = \frac{f_s}{M_1} - \frac{f_s}{2 \times M} = \frac{240000}{10} - \frac{240000}{2 \times 30} = 20000\ Hz$$



Multistage Decimation - 3

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_s} = \frac{20000 - 3400}{240000} = 0.06917$$

$$N = \frac{3.3}{\Delta f} = 47.7 \Rightarrow N = 49$$

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{20000 + 3400}{2} = 11700 \text{ Hz}$$

Filter specification for $H_2(z)$:

Passband frequency range: 0–3,400 Hz
Passband ripples: $0.05/2 = 0.025$ (0.212 dB)
Stopband frequency range: 4,000–12,000 Hz
Stopband attenuation: 0.005, $\delta_s\ dB = 46$ dB
Filter type: FIR, Hamming window

If use one stage, the filter length would be 1321

$$\Delta f = \frac{f_{stop} - f_{pass}}{f_{sM1}} = \frac{4000 - 3400}{24000} = 0.025$$

$$N = \frac{3.3}{\Delta f} = 132. \rightarrow N = 133$$

$$f_c = \frac{f_{pass} + f_{stop}}{2} = \frac{4000 + 3400}{2} = 3700 \text{ Hz}$$

What is Forensics?

The use of science and technology to investigate and establish facts in criminal or civil courts of law.

[The American Heritage® Dictionary of the English Language]

***Human Witness*: (may be)biased, fading memory, etc.**

***Forensics Sciences* : unbiased…..if applied correctly.**

Types of Digital Forensics

- **Digital Image Forensics**
- **Digital Video Forensics**
- **Digital Audio Forensics**
- **Biometric Forensics**
- **Multimedia Forensics**

Example: Digital Image Forgery

Original Image

Tampered Image

Digital Audio Forensics

Classification: