KING SAUD UNIVERSITY **COLEGE OF SCIENCE**

M - 104

DEPARTMENT OF MATHEMATICS **TIME: 90min**

(SEMESTER 1, 1435-1436)

FIRST MID-TERM

Instruction: ***Calculators are not permitted***

1- Let $A=\begin{pmatrix} 2 & 4 & 0 \\ 1 & 3 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 1 \end{pmatrix}$. Compute (if possible): AB and BA. **(6Marks)**

2- Compute the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix}$. **(4Marks)**

3- Find all the elements of the conic section: $x^2 = 2y + 2x$ and sketch it. **(5Marks)**

4- Find the standard equation of the ellipse with vertex $(1,2)$ and with foci $(2,2)$ and $(10,2)$ and then sketch it. **(5Marks)**

5- Solve by Cramer's Rule the linear system $\begin{array}{c} 2x-3y=3 \\ x+y=4 \end{array}$. **(5Marks)**

Q.1

$$A\cdot B=\begin{pmatrix}2&4&0\\1&3&1\end{pmatrix}\cdot\begin{pmatrix}1&2\\3&1\\0&1\end{pmatrix}=\begin{pmatrix}2+12+0&4+4+0\\1++9+0&2+3+1\end{pmatrix}$$

$$=\begin{pmatrix}14&8\\10&6\end{pmatrix}$$

$$B\cdot A=\begin{pmatrix}1&2\\3&1\\0&1\end{pmatrix}\cdot\begin{pmatrix}2&4&0\\1&3&1\end{pmatrix}=\begin{pmatrix}2+2&4+6&0+2\\6+1&12+3&0+1\\0+1&0+3&0+1\end{pmatrix}$$

$$=\begin{pmatrix}4&10&2\\7&15&1\\1&3&1\end{pmatrix}$$

Q.2

$$\begin{array}{c}\begin{matrix}+&-&+\end{matrix}\\ \begin{vmatrix}1&2&3\\3&2&1\\2&1&3\end{vmatrix}\end{array}=(+1)\begin{vmatrix}2&1\\1&3\end{vmatrix}+(-2)\begin{vmatrix}3&1\\2&3\end{vmatrix}+(+3)\begin{vmatrix}3&2\\2&1\end{vmatrix}$$

$$=(6-1)-2(9-2)+3(3-4)$$

$$=5-14-3=-12.$$

Q.3 $x^2=2y+2x$, the conic is a parabola.

$x^2-2x=2y$, completing the square for x we have

$x^2-2x+1=2y+1$

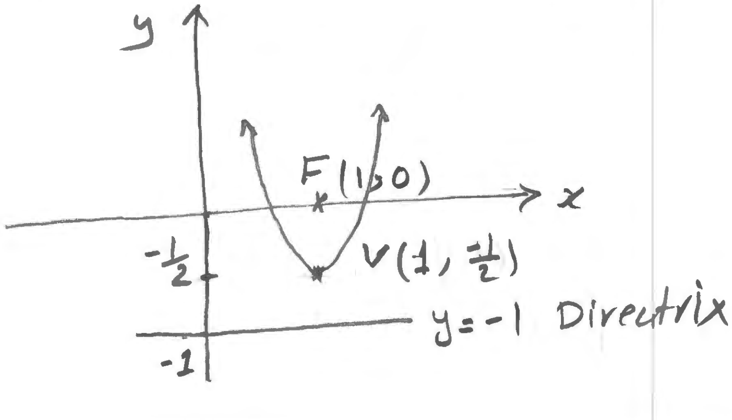
$\Longrightarrow (x-1)^2=2(y+\frac{1}{2})$, on the form $(x-h)^2=4p(y-k)$.

Hence, $h=1$, $k=-\frac{1}{2}$, $4p=2\Longrightarrow p=\frac{1}{2}$

Vertex: $(h,k)=(1,-\frac{1}{2})$, Focus: $(h,k+p)=(1,0)$

Directrix : $y=k-p=\frac{-1}{2}-\frac{1}{2}=-1$

Axis of symmetry: $x-1=0$, or $x=1$



Q.4. The Foci are (2,2) and (10,2)

$\Rightarrow$ center is $\left(\frac{2+10}{2}, \frac{2+2}{2}\right) = (6,2) = (h,k) \Rightarrow h=6, k=2$

and $2c = 10-2 = 8 \Rightarrow c=4$

a = distance between the center and the vertex $\Rightarrow a = 6-1 = 5$

But $c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2 = 25-16 = 9$,

and the major axis is horizontal, hence the equation is on the form:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{or} \quad \frac{(x-6)^2}{25} + \frac{(y-2)^2}{9} = 1$$

Q.5 $|A| = \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 2+3 = 5$

$|A_x| = \begin{vmatrix} 3 & -3 \\ 4 & 1 \end{vmatrix} = 3+12 = 15$

$|A_y| = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 8-3 = 5$

$\therefore X = \frac{|A_x|}{|A|} = \frac{15}{5} = 3$

$y = \frac{|A_y|}{|A|} = \frac{5}{5} = 1$

$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.