SOCIETY OF ACTUARIES

EXAM FM FINANCIAL MATHEMATICS

EXAM FM SAMPLE QUESTIONS

Interest Theory

This page indicates changes made to Study Note FM-09-05.

January 14, 2014:

Questions and solutions 58–60 were added.

June, 2014

Question 58 was moved to the Derivatives Markets set of sample questions.

Questions 61-73 were added.

December, 2014: Questions 74-76 were added.

January, 2015: Questions 77-93 were added.

May, 2015: Questions 94-133 were added.

September, 2015: Answer (D) to Question 96 was changed.

Many of the questions were re-worded to conform to the current style of question writing. The substance was not changed.

Some of the questions in this study note are taken from past SOA examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Financial Mathematics (FM) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

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1.

Bruce deposits 100 into a bank account. His account is credited interest at an annual nominal rate of interest of 4% convertible semiannually.

At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at an annual force of interest of δ.

After 7.25 years, the value of each account is the same.

Calculate δ.

(A) 0.0388

(B) 0.0392

(C) 0.0396

(D) 0.0404

(E) 0.0414

2.

Kathryn deposits 100 into an account at the beginning of each 4-year period for 40 years. The account credits interest at an annual effective interest rate of i.

The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years.

Calculate X.

(A) 4695

(B) 5070

(C) 5445

(D) 5820

(E) 6195

3.

Eric deposits 100 into a savings account at time 0, which pays interest at an annual nominal rate of i, compounded semiannually.

Mike deposits 200 into a different savings account at time 0, which pays simple interest at an annual rate of i.

Eric and Mike earn the same amount of interest during the last 6 months of the 8^{th} year.

Calculate i.

(A) 9.06%

(B) 9.26%

(C) 9.46%

(D) 9.66%

(E) 9.86%

4.

John borrows 10,000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of 1,627.45 at the end of each year. Instead, John repays the 10,000 using a sinking fund that pays an annual effective interest rate of 14%. The deposits to the sinking fund are equal to 1,627.45 minus the interest on the loan and are made at the end of each year for 10 years.

Calculate the balance in the sinking fund immediately after repayment of the loan.

(A) 2,130

(B) 2,180

(C) 2,230

(D) 2,300

(E) 2,370

5.

An association had a fund balance of 75 on January 1 and 60 on December 31. At the end of every month during the year, the association deposited 10 from membership fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31.

Calculate the dollar-weighted (money-weighted) rate of return for the year.

(A) 9.0%

(B) 9.5%

(C) 10.0%

(D) 10.5%

(E) 11.0%

6.

A perpetuity costs 77.1 and makes end-of-year payments. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, …., n at the end of year (n+1). After year (n+1), the payments remain constant at n. The annual effective interest rate is 10.5%.

Calculate n.

(A) 17

(B) 18

(C) 19

(D) 20

(E) 21

7.

1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted.

The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%.

Calculate the accumulated value of Fund Y at the end of year 10.

(A) 1519

(B) 1819

(C) 2085

(D) 2273

(E) 2431

8. Deleted

9.

A 20-year loan of 1000 is repaid with payments at the end of each year.

Each of the first ten payments equals 150% of the amount of interest due. Each of the last ten payments is X.

The lender charges interest at an annual effective rate of 10%.

Calculate X.

(A) 32

(B) 57

(C) 70

(D) 97

(E) 117

10.

A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%.

Calculate the interest portion of the 7th coupon.

(A) 632

(B) 642

(C) 651

(D) 660

(E) 667

11.

A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25-year annuity-immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment.

The annual effective rate of interest is 8%.

Calculate X.

(A) 54

(B) 64

(C) 74

(D) 84

(E) 94

12.

Jeff deposits 10 into a fund today and 20 fifteen years later. Interest for the first 10 years is credited at a nominal discount rate of *d* compounded quarterly, and thereafter at a nominal interest rate of 6% compounded semiannually. The accumulated balance in the fund at the end of 30 years is 100.

Calculate *d*.

(A) 4.33%

(B) 4.43%

(C) 4.53%

(D) 4.63%

(E) 4.73%

13.

Ernie makes deposits of 100 at time 0, and *X* at time 3. The fund grows at a force of interest $\delta_t = \frac{t^2}{100}$, $t > 0$.

The amount of interest earned from time 3 to time 6 is also *X*.

Calculate *X*.

(A) 385

(B) 485

(C) 585

(D) 685

(E) 785

14.

Mike buys a perpetuity-immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10. Beginning in year 6, the payments start to increase. For year 6 and all future years, the payment in that year is K% larger than the payment in the year immediately preceding that year, where $K < 9.2$.

At an annual effective interest rate of 9.2%, the perpetuity has a present value of 167.50.

Calculate K.

(A) 4.0

(B) 4.2

(C) 4.4

(D) 4.6

(E) 4.8

15.

A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(i) Equal annual payments at an annual effective interest rate of 8.07%.

(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective interest rate of i.

The sum of the payments under option (i) equals the sum of the payments under option (ii).

Calculate i.

(A) 8.75%

(B) 9.00%

(C) 9.25%

(D) 9.50%

(E) 9.75%

16.

A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment.

Calculate the outstanding loan balance immediately after the 40^{th} payment is made.

(A) 6750

(B) 6890

(C) 6940

(D) 7030

(E) 7340

17.

To accumulate 8000 at the end of 3*n* years, deposits of 98 are made at the end of each of the first *n* years and 196 at the end of each of the next 2*n* years.

The annual effective rate of interest is *i*. You are given $(1+i)^n = 2.0$.

Calculate *i*.

(A) 11.25%

(B) 11.75%

(C) 12.25%

(D) 12.75%

(E) 13.25%

18.

Olga buys a 5-year increasing annuity for X.

Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2.

The annual nominal interest rate is 9% convertible quarterly.

Calculate X.

(A) 2680

(B) 2730

(C) 2780

(D) 2830

(E) 2880

19.

You are given the following information about the activity in two different investment accounts:

Account K			
Date	Fund value before activity	Activity	
		Deposit	Withdrawal
January 1, 2014	100.0		
July 1, 2014	125.0		X
October 1, 2014	110.0	2*X*	
December 31, 2014	125.0		

Account L			
Date	Fund value before activity	Activity	
		Deposit	Withdrawal
January 1, 2014	100.0		
July 1, 2014	125.0		X
December 31, 2014	105.8		

During 2014, the dollar-weighted (money-weighted) return for investment account K equals the time-weighted return for investment account L, which equals *i*.

Calculate *i*.

(A) 10%

(B) 12%

(C) 15%

(D) 18%

(E) 20%

20.

David can receive one of the following two payment streams:

(i) 100 at time 0, 200 at time n years, and 300 at time $2n$ years

(ii) 600 at time 10 years

At an annual effective interest rate of i, the present values of the two streams are equal.

Given $v^n = 0.76$, calculate i.

(A) 3.5%

(B) 4.0%

(C) 4.5%

(D) 5.0%

(E) 5.5%

21.

Payments are made to an account at a continuous rate of $(8k + tk)$, where $0 \le t \le 10$.

Interest is credited at a force of interest $\delta_t = \frac{1}{8+t}$.

After time 10, the account is worth 20,000.

Calculate k.

(A) 111

(B) 116

(C) 121

(D) 126

(E) 131

22.

You have decided to invest in Bond X, an *n*-year bond with semi-annual coupons and the following characteristics:

(i) Par value is 1000.

(ii) The ratio of the semi-annual coupon rate, *r*, to the desired semi-annual yield rate, *i*, is 1.03125.

(iii) The present value of the redemption value is 381.50.

Given $(1+i)^{-n} = 0.5889$, calculate the price of bond X.

(A) 1019

(B) 1029

(C) 1050

(D) 1055

(E) 1072

23.

Project P requires an investment of 4000 today. The investment pays 2000 one year from today and 4000 two years from today.

Project Q requires an investment of *X* two years from today. The investment pays 2000 today and 4000 one year from today.

The net present values of the two projects are equal at an annual effective interest rate of 10%.

Calculate *X*.

(A) 5400

(B) 5420

(C) 5440

(D) 5460

(E) 5480

24.

A 20-year loan of 20,000 may be repaid under the following two methods:

(i) amortization method with equal annual payments at an annual effective interest rate of 6.5%

(ii) sinking fund method in which the lender receives an annual effective interest rate of 8% and the sinking fund earns an annual effective interest rate of j

Both methods require a payment of X to be made at the end of each year for 20 years.

Calculate j.

(A) 6.4%

(B) 7.6%

(C) 8.8%

(D) 11.2%

(E) 14.2%

25.

A perpetuity-immediate pays X per year. Brian receives the first n payments, Colleen receives the next n payments, and a charity receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and the charity's share is K.

Calculate K.

(A) 24%

(B) 28%

(C) 32%

(D) 36%

(E) 40%

26.

Seth, Janice, and Lori each borrow 5000 for five years at an annual nominal interest rate of 12%, compounded semi-annually.

Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.

Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years.

Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

(A) 8718

(B) 8728

(C) 8738

(D) 8748

(E) 8758

27.

Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns the same annual effective interest rate.

The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X.

Calculate X.

(A) 28.00

(B) 31.30

(C) 34.60

(D) 36.70

(E) 38.90

28.

Ron is repaying a loan with payments of 1 at the end of each year for n years. The annual effective interest rate on the loan is i. The amount of interest paid in year t plus the amount of principal repaid in year $t + 1$ equals X.

Determine which of the following is equal to X.

(A) $1+\frac{v^{n-t}}{i}$

(B) $1+\frac{v^{n-t}}{d}$

(C) $1+v^{n-t}i$

(D) $1+v^{n-t}d$

(E) $1+v^{n-t}$

29.

At an annual effective interest rate of i, $i > 0\%$, the present value of a perpetuity paying 10 at the end of each 3-year period, with the first payment at the end of year 3, is 32.

At the same annual effective rate of i, the present value of a perpetuity paying 1 at the end of each 4-month period, with first payment at the end of 4 months, is X.

Calculate X.

(A) 31.6

(B) 32.6

(C) 33.6

(D) 34.6

(E) 35.6

30.

As of 12/31/2013, an insurance company has a known obligation to pay 1,000,000 on 12/31/2017. To fund this liability, the company immediately purchases 4-year 5% annual coupon bonds totaling 822,703 of par value. The company anticipates reinvestment interest rates to remain constant at 5% through 12/31/2017. The maturity value of the bond equals the par value.

Consider two reinvestment interest rate movement scenarios effective 1/1/2014. Scenario A has interest rates drop by 0.5%. Scenario B has interest rates increase by 0.5%.

Determine which of the following best describes the insurance company's profit or (loss) as of 12/31/2017 after the liability is paid.

(A) Scenario A – 6,610, Scenario B – 11,150

(B) Scenario A – (14,760), Scenario B – 14,420

(C) Scenario A – (18,910), Scenario B – 19,190

(D) Scenario A – (1,310), Scenario B – 1,320

(E) Scenario A – 0, Scenario B – 0

31.

An insurance company has an obligation to pay the medical costs for a claimant. Annual claim costs today are 5000, and medical inflation is expected to be 7% per year. The claimant will receive 20 payments.

Claim payments are made at yearly intervals, with the first claim payment to be made one year from today.

Calculate the present value of the obligation using an annual effective interest rate of 5%.

(A) 87,900

(B) 102,500

(C) 114,600

(D) 122,600

(E) Cannot be determined

32.

An investor pays 100,000 today for a 4-year investment that returns cash flows of 60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

Using an annual effective interest rate of 5.0%, calculate the net present value of this investment today.

(A) -1398

(B) -699

(C) 699

(D) 1398

(E) 2,629

33.

You are given the following information with respect to a bond:

(i) par value: 1000

(ii) term to maturity: 3 years

(iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

Calculate the value of the bond.

(A) 906

(B) 926

(C) 930

(D) 950

(E) 1000

34.

You are given the following information with respect to a bond:

(i) par value: 1000

(ii) term to maturity: 3 years

(iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

The bond is sold at a price equal to its value.

Calculate the annual effective yield rate for the bond i.

(A) 8.1%

(B) 8.3%

(C) 8.5%

(D) 8.7%

(E) 8.9%

35.

The current price of an annual coupon bond is 100. The yield to maturity is an annual effective rate of 8%. The derivative of the price of the bond with respect to the yield to maturity is -700.

Using the bond's yield rate, calculate the Macaulay duration of the bond in years.

(A) 7.00

(B) 7.49

(C) 7.56

(D) 7.69

(E) 8.00

36.

A common stock pays a constant dividend at the end of each year into perpetuity.

Using an annual effective interest rate of 10%, calculate the Macaulay duration of the stock.

(A) 7 years
(B) 9 years
(C) 11 years
(D) 19 years
(E) 27 years

37.

A common stock pays dividends at the end of each year into perpetuity. Assume that the dividend increases by 2% each year.

Using an annual effective interest rate of 5%, calculate the Macaulay duration of the stock in years.

(A) 27
(B) 35
(C) 44
(D) 52
(E) 58

38. – 44. deleted

45.

You are given the following information about an investment account:

(i) The value on January 1 is 10.

(ii) The value on July 1, prior to a deposit being made, is 12.

(iii) On July 1, a deposit of X is made.

(iv) The value on December 31 is X.

Over the year, the time-weighted return is 0%, and the dollar-weighted (money-weighted) return is Y.

Calculate Y.

(A) -25%

(B) -10%

(C) 0%

(D) 10%

(E) 25%

46.

Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12.

Calculate the principal repaid in the first payment.

(A) 444

(B) 454

(C) 464

(D) 474

(E) 484

47.

Bill buys a 10-year 1000 par value bond with semi-annual coupons paid at an annual rate of 6%. The price assumes an annual nominal yield of 6%, compounded semi-annually.

As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of *i*.

At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond.

Calculate *i*.

(A) 9.50%

(B) 9.75%

(C) 10.00%

(D) 10.25%

(E) 10.50%

48.

A man turns 40 today and wishes to provide supplemental retirement income of 3000 at the beginning of each month starting on his 65th birthday. Starting today, he makes monthly contributions of *X* to a fund for 25 years. The fund earns an annual nominal interest rate of 8% compounded monthly.

On his 65^{th} birthday, each 1000 of the fund will provide 9.65 of income at the beginning of each month starting immediately and continuing as long as he survives.

Calculate *X*.

(A) 324.70

(B) 326.90

(C) 328.10

(D) 355.50

(E) 450.70

49.

Happy and financially astute parents decide at the birth of their daughter that they will need to provide 50,000 at each of their daughter's 18th, 19th, 20th and 21st birthdays to fund her college education. They plan to contribute *X* at each of their daughter's 1st through 17th birthdays to fund the four 50,000 withdrawals. They anticipate earning a constant 5% annual effective interest rate on their contributions.

Let $v = 1/1.05$.

Determine which of the following equations of value can be used to calculate *X*.

(A) $X\sum_{k=1}^{17} v^k = 50,000[v + v^2 + v^3 + v^4]$

(B) $X\sum_{k=1}^{16} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(C) $X\sum_{k=0}^{17} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(D) $X\sum_{k=1}^{17} 1.05^k = 50,000[1 + v + v^2 + v^3]$

(E) $X\sum_{k=0}^{17} v^k = 50,000[v^{18} + v^{19} + v^{20} + v^{21} + v^{22}]$

50. Deleted

51.

Joe must pay liabilities of 1,000 due 6 months from now and another 1,000 due one year from now. There are two available investments:

Bond I: A 6-month bond with face amount of 1,000, an 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually;

Bond II: A one-year bond with face amount of 1,000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually.

Calculate the amount of each bond that Joe should purchase to exactly match the liabilities.

(A) Bond I – 1, Bond II – 0.97561

(B) Bond I – 0.93809, Bond II – 1

(C) Bond I – 0.97561, Bond II – 0.94293

(D) Bond I – 0.93809, Bond II – 0.97561

(E) Bond I – 0.98345, Bond II – 0.97561

52.

Joe must pay liabilities of 2000 due one year from now and another 1000 due two years from now. He exactly matches his liabilities with the following two investments:

Mortgage I: A one year mortgage in which X is lent. It is repaid with a single payment at time one. The annual effective interest rate is 6%.

Mortgage II: A two-year mortgage in which Y is lent. It is repaid with two equal annual payments. The annual effective interest rate is 7%.

Calculate $X + Y$.

(A) 2600

(B) 2682

(C) 2751

(D) 2825

(E) 3000

53.

Joe must pay liabilities of 1,000 due one year from now and another 2,000 due three years from now. There are two available investments:

Bond I: A one-year zero-coupon bond that matures for 1000. The yield rate is 6% per year

Bond II: A two-year zero-coupon bond with face amount of 1,000. The yield rate is 7% per year.

At the present time the one-year forward rate for an investment made two years from now is 6.5%

Joe plans to buy amounts of each bond. He plans to reinvest the proceeds from Bond II in a one-year zero-coupon bond. Assuming the reinvestment earns the forward rate, calculate the total purchase price of Bond I and Bond II where the amounts are selected to exactly match the liabilities.

(A) 2584

(B) 2697

(C) 2801

(D) 2907

(E) 3000

54.

Matt purchased a 20-year par value bond with an annual nominal coupon rate of 8% payable semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The lowest yield rate that Matt can possibly receive is a nominal annual interest rate of 6% convertible semiannually.

Calculate X.

(A) 1400

(B) 1420

(C) 1440

(D) 1460

(E) 1480

55.

Toby purchased a 20-year par value bond with semiannual coupons of 40 and a redemption value of 1100. The bond can be called at 1200 on any coupon date prior to maturity, starting at the end of year 15.

Calculate the maximum price of the bond to guarantee that Toby will earn an annual nominal interest rate of at least 6% convertible semiannually.

(A) 1251

(B) 1262

(C) 1278

(D) 1286

(E) 1295

56.

Sue purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at par value X on any coupon date starting at the end of year 5. The lowest yield rate that Sue can possibly receive is an annual nominal rate of 6% convertible semiannually.

Calculate X.

(A) 1120

(B) 1140

(C) 1160

(D) 1180

(E) 1200

57.

Mary purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at 100 over the par value of 1100 on any coupon date starting at the end of year 5 and ending six months prior to maturity.

Calculate the minimum yield that Mary could receive, expressed as an annual nominal rate of interest convertible semiannually.

(A) 4.7%

(B) 4.9%

(C) 5.1%

(D) 5.3%

(E) 5.5%

58. Moved to Derivatives Section

59.

A liability consists of a series of 15 annual payments of 35,000 with the first payment to be made one year from now.

The assets available to immunize this liability are five-year and ten-year zero-coupon bonds.

The annual effective interest rate used to value the assets and the liability is 6.2%. The liability has the same present value and duration as the asset portfolio.

Calculate the amount invested in the five-year zero-coupon bonds.

(A) 127,000

(B) 167,800

(C) 208,600

(D) 247,900

(E) 292,800

60.

You are given the following information about a loan of L that is to be repaid with a series of 16 annual payments:

(i) The first payment of 2000 is due one year from now.

(ii) The next seven payments are each 3% larger than the preceding payment.

(iii) From the 9^{th} to the 16^{th} payment, each payment will be 3% less than the preceding payment.

(iv) The loan has an annual effective interest rate of 7%.

Calculate L.

(A) 20,689

(B) 20,716

(C) 20,775

(D) 21,147

(E) 22,137

61.

The annual force of interest credited to a savings account is defined by

$$\delta_t = \frac{\frac{t^2}{100}}{3 + \frac{t^3}{150}}$$

with t in years. Austin deposits 500 into this account at time 0.

Calculate the time in years it will take for the fund to be worth 2000.

(A) 6.7

(B) 8.8

(C) 14.2

(D) 16.5

(E) 18.9

62.

A 40-year bond is purchased at a discount. The bond pays annual coupons. The amount for accumulation of discount in the 15th coupon is 194.82. The amount for accumulation of discount in the 20th coupon is 306.69.

Calculate the amount of discount in the purchase price of this bond.

(A) 13,635

(B) 13,834

(C) 16,098

(D) 19,301

(E) 21,135

63.

Tanner takes out a loan today and repays the loan with eight level annual payments, with the first payment one year from today. The payments are calculated based on an annual effective interest rate of 4.75%. The principal portion of the fifth payment is 699.68.

Calculate the total amount of interest paid on this loan.

(A) 1239

(B) 1647

(C) 1820

(D) 2319

(E) 2924

64.

Turner buys a new car and finances it with a loan of 22,000. He will make n monthly payments of 450.30 starting in one month. He will make one larger payment in n+1 months to pay off the loan. Payments are calculated using an annual nominal interest rate of 8.4%, convertible monthly. Immediately after the 18th payment he refinances the loan to pay off the remaining balance with 24 monthly payments starting one month later. This refinanced loan uses an annual nominal interest rate of 4.8%, convertible monthly.

Calculate the amount of the new monthly payment.

(A) 668

(B) 693

(C) 702

(D) 715

(E) 742

65.

Kylie bought a 7-year, 5000 par value bond with an annual coupon rate of 7.6% paid semiannually. She bought the bond with no premium or discount.

Calculate the Macaulay duration of this bond with respect to the yield rate on the bond.

(A) 5.16

(B) 5.35

(C) 5.56

(D) 5.77

(E) 5.99

66.

Krishna buys an n-year 1000 bond at par. The Macaulay duration is 7.959 years using an annual effective interest rate of 7.2%.

Calculate the estimated price of the bond, using duration, if the interest rate rises to 8.0%.

(A) 940.60

(B) 942.88

(C) 944.56

(D) 947.03

(E) 948.47

67.

The prices of zero-coupon bonds are:

Maturity	Price
1	0.95420
2	0.90703
3	0.85892

Calculate the third year, one-year forward rate.

(A) 0.048

(B) 0.050

(C) 0.052

(D) 0.054

(E) 0.056

68.

Sam buys an eight-year, 5000 par bond with an annual coupon rate of 5%, paid annually. The bond sells for 5000. Let d_1 be the Macaulay duration just before the first coupon is paid. Let d_2 be the Macaulay duration just after the first coupon is paid.

Calculate $\frac{d_1}{d_2}$.

(A) 0.91

(B) 0.93

(C) 0.95

(D) 0.97

(E) 1.00

69.

An insurance company must pay liabilities of 99 at the end of one year, 102 at the end of two years and 100 at the end of three years. The only investments available to the company are the following three bonds. Bond A and Bond C are annual coupon bonds. Bond B is a zero-coupon bond.

Bond	Maturity (in years)	Yield-to-Maturity (Annualized)	Coupon Rate
A	1	6%	7%
B	2	7%	0%
C	3	9%	5%

All three bonds have a par value of 100 and will be redeemed at par.

Calculate the number of units of Bond A that must be purchased to match the liabilities exactly.

(A) 0.8807

(B) 0.8901

(C) 0.8975

(D) 0.9524

(E) 0.9724

70.

Determine which of the following statements is false with respect to Redington immunization.

(A) Modified duration may change at different rates for each of the assets and liabilities as time goes by.

(B) Redington immunization requires infrequent rebalancing to keep modified duration of assets equal to modified duration of liabilities.

(C) This technique is designed to work only for small changes in the interest rate.

(D) The yield curve is assumed to be flat.

(E) The yield curve shifts in parallel when the interest rate changes.

71.

Aakash has a liability of 6000 due in four years. This liability will be met with payments of A in two years and B in six years. Aakash is employing a full immunization strategy using an annual effective interest rate of 5%.

Calculate $|A-B|$.

(A) 0

(B) 146

(C) 293

(D) 586

(E) 881

72.

Jia Wen has a liability of 12,000 due in eight years. This liability will be met with payments of 5000 in five years and B in $8+b$ years. Jia Wen is employing a full immunization strategy using an annual effective interest rate of 3%.

Calculate $\frac{B}{b}$.

(A) 2807

(B) 2873

(C) 2902

(D) 2976

(E) 3019

73.

Trevor has assets at time 2 of A and at time 9 of B. He has a liability of 95,000 at time 5. Trevor has achieved Redington immunization in his portfolio using an annual effective interest rate of 4%.

Calculate $\frac{A}{B}$.

(A) 0.7307

(B) 0.9670

(C) 1.0000

(D) 1.0132

(E) 1.3686

74.

You are given the following information about two bonds, Bond A and Bond B:

i) Each bond is a 10-year bond with semiannual coupons redeemable at its par value of 10,000, and is bought to yield an annual nominal interest rate of i, convertible semiannually.

ii) Bond A has an annual coupon rate of $(i + 0.04)$, paid semiannually.

iii) Bond B has an annual coupon rate of $(i - 0.04)$, paid semiannually.

iv) The price of Bond A is 5,341.12 greater than the price of Bond B.

Calculate i.

(A) 0.042

(B) 0.043

(C) 0.081

(D) 0.084

(E) 0.086

75.

A borrower takes out a 15-year loan for 400,000, with level end-of-month payments, at an annual nominal interest rate of 9% convertible monthly.

Immediately after the 36th payment, the borrower decides to refinance the loan at an annual nominal interest rate of j, convertible monthly. The remaining term of the loan is kept at twelve years, and level payments continue to be made at the end of the month. However, each payment is now 409.88 lower than each payment from the original loan.

Calculate j.

(A) 4.72%

(B) 5.75%

(C) 6.35 %

(D) 6.90%

(E) 9.14%

76.

Consider two 30-year bonds with the same purchase price. Each has an annual coupon rate of 5% paid semiannually and a par value of 1000.

The first bond has an annual nominal yield rate of 5% compounded semiannually, and a redemption value of 1200.

The second bond has an annual nominal yield rate of j compounded semiannually, and a redemption value of 800.

Calculate j.

(A) 2.20%

(B) 2.34%

(C) 3.53%

(D) 4.40%

(E) 4.69%

77.

Lucas opens a bank account with 1000 and lets it accumulate at an annual nominal interest rate of 6% convertible semiannually. Danielle also opens a bank account with 1000 at the same time as Lucas, but it grows at an annual nominal interest rate of 3% convertible monthly.

For each account, interest is credited only at the end of each interest conversion period.

Calculate the number of months required for the amount in Lucas's account to be at least double the amount in Danielle's account.

(A) 276

(B) 282

(C) 285

(D) 286

(E) 288

78.

On January 1, an investment fund was opened with an initial balance of 5000. Just after the balance grew to 5200 on July 1, an additional 2600 was deposited.

The annual effective yield rate for this fund was 9.00% over the calendar year.

Calculate the time-weighted rate of return for the year.

(A) 7.43%

(B) 8.86%

(C) 9.00%

(D) 9.17%

(E) 10.45%

79.

Bill and Joe each put 10 into separate accounts at time $t = 0$, where t is measured in years.

Bill's account earns interest at a constant annual effective interest rate of $K/25$, $K > 0$.

Joe's account earns interest at a force of interest, $\delta_t = \frac{1}{K + 0.25t}$.

At the end of four years, the amount in each account is X.

Calculate X.

(A) 20.7

(B) 21.7

(C) 22.7

(D) 23.7

(E) 24.7

80.

A student takes out a five-year loan of 1000. Interest on the loan is at an annual effective interest rate of i.

At the end of each year, the student pays the interest due on the loan and makes a deposit of twice the amount of that interest payment into a sinking fund. The sinking fund credits interest at an annual effective rate of $0.8i$. The sinking fund will accumulate the amount needed to pay off the loan at the end of five years.

Calculate i.

(A) 7.2%

(B) 8.4%

(C) 8.7%

(D) 10.6%

(E) 12.1%

81.

A borrower takes out a 50-year loan, to be repaid with payments at the end of each year. The loan payment is 2500 for each of the first 26 years. Thereafter, the payments decrease by 100 per year. Interest on the loan is charged at an annual effective rate of i ($0\% < i < 10\%$).

The principal repaid in year 26 is X.

Determine the amount of interest paid in the first year.

(A) Xv^{25}

(B) $2500v^{25} - Xv^{25}$

(C) $2500 - X$

(D) $2500 - XV^{25}$

(E) $25Xi$

82.

You are given:

i) F = the amount in a fund at the beginning of the year

ii) There are n cash flows during the year, where each cash flow, $C_k, k = 1, 2, \ldots, n$ is made at time $t_k, 0 < t \le 1$.

iii) I = the amount of interest earned during the year.

Consider the formula

$$i = \frac{I}{F + \sum_{k=1}^{n} C_k \left(1 - t_k\right)}.$$

Determine which of the following conditions produces i closest to the annual effective yield rate.

(A) When each cash flow is small relative to F

(B) When each cash flow is large relative to F

(C) When each cash flow is large relative to I

(D) When the cash flows all have the same sign

(E) When the timing of cash flows is uniform throughout the year

83.

On January 1, a fund is worth 100,000. On May 1, the value has increased to 120,000 and then 30,000 of new principal is deposited. On November 1, the value has declined to 130,000 and then 50,000 is withdrawn. On January 1 of the following year, the fund is again worth 100,000.

Calculate the time-weighted rate of return.

(A) 0.00%

(B) 17.91%

(C) 25.00%

(D) 29.27%

(E) 30.00%

84.

John made a deposit of 1000 into a fund at the beginning of each year for 20 years.

At the end of 20 years, he began making semiannual withdrawals of 3000 at the beginning of each six months, with a smaller final withdrawal to exhaust the fund. The fund earned an annual effective interest rate of 8.16%.

Calculate the amount of the final withdrawal.

(A) 561

(B) 1226

(C) 1430

(D) 1488

(E) 2240

85.

The present value of a perpetuity paying 1 every two years with first payment due immediately is 7.21 at an annual effective rate of i.

Another perpetuity paying R every three years with the first payment due at the beginning of year two has the same present value at an annual effective rate of $i + 0.01$.

Calculate R.

(A) 1.23

(B) 1.56

(C) 1.60

(D) 1.74

(E) 1.94

86.

A loan of 10,000 is repaid with a payment made at the end of each year for 20 years. The payments are 100, 200, 300, 400, and 500 in years 1 through 5, respectively. In the subsequent 15 years, equal annual payments of X are made.

The annual effective interest rate is 5%.

Calculate X.

(A) 842

(B) 977

(C) 1017

(D) 1029

(E) 1075

87.

An investor wishes to accumulate 5000 in a fund at the end of 15 years. To accomplish this, she plans to make equal deposits of X at the end of each year for the first ten years. The fund earns an annual effective rate of 6% during the first ten years and 5% for the next five years.

Calculate X.

(A) 224

(B) 284

(C) 297

(D) 312

(E) 379

88.

A borrower takes out a 15-year loan for 65,000, with level end-of-month payments. The annual nominal interest rate of the loan is 8%, convertible monthly.

Immediately after the 12th payment is made, the remaining loan balance is reamortized. The maturity date of the loan remains unchanged, but the annual nominal interest rate of the loan is changed to 6%, convertible monthly.

Calculate the new end-of-month payment.

(A) 528

(B) 534

(C) 540

(D) 546

(E) 552

89.

College tuition is 6000 for the current school year, payable in full at the beginning of the school year. College tuition will grow at an annual rate of 5%. A parent sets up a college savings fund earning interest at an annual effective rate of 7%. The parent deposits 750 at the beginning of each school year for 18 years, with the first deposit made at the beginning of the current school year. Immediately following the 18th deposit, the parent pays tuition for the 18th school year from the fund.

The amount of money needed, in addition to the balance in the fund, to pay tuition at the beginning of the 19^{th} school year is X.

Calculate X.

(A) 1439

(B) 1545

(C) 1664

(D) 1785

(E) 1870

90.

A 1000 par value 20-year bond sells for P and yields a nominal interest rate of 10% convertible semiannually. The bond has 9% coupons payable semiannually and a redemption value of 1200.

Calculate P.

(A) 914

(B) 943

(C) 1013

(D) 1034

(E) 1097

91.

An investor purchases a 10-year callable bond with face amount of 1000 for price P. The bond has an annual nominal coupon rate of 10% paid semi-annually.

The bond may be called at par by the issuer on every other coupon payment date, beginning with the second coupon payment date.

The investor earns at least an annual nominal yield of 12% compounded semi-annually regardless of when the bond is redeemed.

Calculate the largest possible value of P.

(A) 885

(B) 892

(C) 926

(D) 965

(E) 982

92.

You are given the following term structure of interest rates:

Length of investment in years	Spot rate
1	7.50%
2	8.00%
3	8.50%
4	9.00%
5	9.50%
6	10.00%

Calculate the one-year forward rate for the fifth year implied by this term structure.

(A) 9.5%

(B) 10.0%

(C) 11.5%

(D) 12.0%

(E) 12.5%

93.

Seth has two retirement benefit options.

His first option is to receive a lump sum of 374,500 at retirement.

His second option is to receive monthly payments for 25 years starting one month after retirement. For the first year, the amount of each monthly payment is 2000. For each subsequent year, the monthly payments are 2% more than the monthly payments from the previous year.

Using an annual nominal interest rate of 6%, compounded monthly, the present value of the second option is P.

Determine which of the following is true.

(A) P is 323,440 more than the lump sum option amount.

(B) P is 107,170 more than the lump sum option amount.

(C) The lump sum option amount is equal to P.

(D) The lump sum option amount is 60 more than P.

(E) The lump sum option amount is 64,090 more than P.

94.

A couple decides to save money for their child's first year college tuition.

The parents will deposit 1700 n months from today and another 3400 $2n$ months from today.

All deposits earn interest at a nominal annual rate of 7.2%, compounded monthly.

Calculate the maximum integral value of n such that the parents will have accumulated at least 6500 five years from today.

(A) 11

(B) 12

(C) 18

(D) 24

(E) 25

95.

Let S be the accumulated value of 1000 invested for two years at a nominal annual rate of discount d convertible semiannually, which is equivalent to an annual effective interest rate of i.

Let T be the accumulated value of 1000 invested for one year at a nominal annual rate of discount d convertible quarterly.

$S / T = (39 / 38)^4$.

Calculate i.

(A) 10.0%

(B) 10.3%

(C) 10.8%

(D) 10.9%

(E) 11.1%

96,

An investor's retirement account pays an annual nominal interest rate of 4.2%, convertible monthly.

On January 1 of year y, the investor's account balance was X. The investor then deposited 100 at the end of every quarter. On May 1 of year $(y + 10)$, the account balance was $1.9X$.

Determine which of the following is an equation of value that can be used to solve for X.

(A) $$\frac{1.9X}{(1.0105)^{\frac{124}{3}}} + \sum_{k=1}^{42} \frac{100}{(1.0105)^{k-1}} = X$$

(B) $$X + \sum_{k=1}^{42} \frac{100}{(1.0035)^{3(k-1)}} = \frac{1.9X}{(1.0035)^{124}}$$

(C) $$X + \sum_{k=1}^{41} \frac{100}{(1.0035)^{3k}} = \frac{1.9X}{(1.0035)^{124}}$$

(D) $$X + \sum_{k=1}^{41} \frac{100}{(1.0105)^{k-1}} = \frac{1.9X}{(1.0105)^{\frac{124}{3}}}$$

(E) $$X + \sum_{k=1}^{42} \frac{100}{(1.0105)^{k-1}} = \frac{1.9X}{(1.0105)^{\frac{124}{3}}}$$

97.

Five deposits of 100 are made into a fund at two-year intervals with the first deposit at the beginning of the first year.

The fund earns interest at an annual effective rate of 4% during the first six years and at an annual effective rate of 5% thereafter.

Calculate the annual effective yield rate earned over the investment period ending at the end of the tenth year.

(A) 4.18%

(B) 4.40%

(C) 4.50%

(D) 4.58%

(E) 4.78%

98.

John finances his daughter's college education by making deposits into a fund earning interest at an annual effective rate of 8%. For 18 years he deposits X at the beginning of each month.

In the 16^{th} through the 19^{th} years, he makes a withdrawal of 25,000 at the beginning of each year. The final withdrawal reduces the fund balance to zero.

Calculate X.

(A) 207

(B) 223

(C) 240

(D) 245

(E) 260

99.

Jack inherited a perpetuity-due, with annual payments of 15,000. He immediately exchanged the perpetuity for a 25-year annuity-due having the same present value. The annuity-due has annual payments of X.

All the present values are based on an annual effective interest rate of 10% for the first 10 years and 8% thereafter.

Calculate X.

(A) 16,942

(B) 17,384

(C) 17,434

(D) 17,520

(E) 18,989

100.

An investor owns a bond that is redeemable for 300 in seven years. The investor has just received a coupon of 22.50 and each subsequent semiannual coupon will be X more than the preceding coupon. The present value of this bond immediately after the payment of the coupon is 1050.50 assuming an annual nominal yield rate of 6% convertible semiannually.

Calculate X.

(A) 7.54

(B) 10.04

(C) 22.37

(D) 34.49

(E) 43.98

101.

A 30-year annuity is arranged to pay off a loan taken out today at a 5% annual effective interest rate. The first payment of the annuity is due in ten years in the amount of 1,000. The subsequent payments increase by 500 each year.

Calculate the amount of the loan.

(A) 58,283

(B) 61,197

(C) 64,021

(D) 64,257

(E) 69,211

102.

A woman worked for 30 years before retiring. At the end of the first year of employment she deposited 5000 into an account for her retirement. At the end of each subsequent year of employment, she deposited 3% more than the prior year. The woman made a total of 30 deposits.

She will withdraw 50,000 at the beginning of the first year of retirement and will make annual withdrawals at the beginning of each subsequent year for a total of 30 withdrawals. Each of these subsequent withdrawals will be 3% more than the prior year. The final withdrawal depletes the account.

The account earns a constant annual effective interest rate.

Calculate the account balance after the final deposit and before the first withdrawal.

(A) 760,694

(B) 783,948

(C) 797,837

(D) 805,541

(E) 821,379

103.

An insurance company purchases a perpetuity-due providing a geometric series of quarterly payments for a price of 100,000 based on an annual effective interest rate of i. The first and second quarterly payments are 2000 and 2010, respectively.

Calculate i.

(A) 10.0%

(B) 10.2%

(C) 10.4%

(D) 10.6%

(E) 10.8%

104.

A perpetuity provides for continuous payments. The annual rate of payment at time t is

$$\begin{cases} 1, & \text{for } 0 \le t < 10, \\ (1.03)^{t-10} & \text{for } t \ge 10. \end{cases}$$

Using an annual effective interest rate of 6%, the present value at time $t = 0$ of this perpetuity is x.

Calculate x.

(A) 27.03

(B) 30.29

(C) 34.83

(D) 38.64

(E) 42.41

105.

A bank agrees to lend 10,000 now and X three years from now in exchange for a single repayment of 75,000 at the end of 10 years. The bank charges interest at an annual effective rate of 6% for the first 5 years and at a force of interest $\delta_t = \frac{1}{t+1}$ for $t \ge 5$.

Calculate X.

(A) 23,500

(B) 24,000

(C) 24,500

(D) 25,000

(E) 25,500

106.

A company takes out a loan of 15,000,000 at an annual effective discount rate of 5.5%. You are given:

i) The loan is to be repaid with n annual payments of 1,200,000 plus a drop payment one year after the nth payment.

ii) The first payment is due three years after the loan is taken out.

Calculate the amount of the drop payment.

(A) 79,100

(B) 176,000

(C) 321,300

(D) 959,500

(E) 1,180,300

107.

Tim takes out an n-year loan with equal annual payments at the end of each year.

The interest portion of the payment at time $(n - 1)$ is equal to 0.5250 of the interest portion of the payment at time $(n - 3)$ and is also equal to 0.1427 of the interest portion of the first payment.

Calculate n.

(A) 18

(B) 20

(C) 22

(D) 24

(E) 26

108.

You are given the following information about an eleven-year loan of L to be repaid by the sinking fund method:

i) The sinking fund earns an annual effective interest rate of 4.70%.

ii) Immediately following the seventh payment and deposit, the difference between what is owed to the lender on the loan and the accumulated value of the sinking fund is 6241.

Calculate the sinking fund deposit.

(A) 1019

(B) 1055

(C) 1067

(D) 1084

(E) 1104

109.

On January 1, 2003 Mike took out a 30-year mortgage loan in the amount of 200,000 at an annual nominal interest rate of 6% compounded monthly. The loan was to be repaid by level end-of-month payments with the first payment on January 31, 2003.

Mike repaid an extra 10,000 in addition to the regular monthly payment on each December 31 in the years 2003 through 2007.

Determine the date on which Mike will make his last payment (which is a drop payment).

(A) July 31, 2013

(B) November 30, 2020

(C) December 31, 2020

(D) December 31, 2021

(E) January 31, 2022

110.

A 5-year loan of 500,000 with an annual effective discount rate of 8% is to be repaid by level end-of-year payments.

If the first four payments had been rounded up to the next multiple of 1,000, the final payment would be *X*.

Calculate *X*.

(A) 103,500

(B) 111,700

(C) 115,200

(D) 125,200

(E) 127,500

111.

A company plans to invest *X* at the beginning of each month in a zero-coupon bond in order to accumulate 100,000 at the end of six months. The price of each bond as a percentage of redemption value is given in the following chart:

Maturity (months)	1	2	3	4	5	6
Price	99%	98%	97%	96%	95%	94%

Calculate *X* given that the bond prices will not change during the six-month period.

(A) 15,667

(B) 16,078

(C) 16,245

(D) 16,667

(E) 17,271

112.

A loan of *X* is repaid with level annual payments at the end of each year for 10 years.

You are given:

i) The interest paid in the first year is 3600; and

ii) The principal repaid in the 6^{th} year is 4871.

Calculate *X*.

(A) 44,000

(B) 45,250

(C) 46,500

(D) 48,000

(E) 50,000

113.

An investor purchased a 25-year bond with semiannual coupons, redeemable at par, for a price of 10,000. The annual effective yield rate is 7.05%, and the annual coupon rate is 7%.

Calculate the redemption value of the bond.

(A) 9,918

(B) 9,942

(C) 9,981

(D) 10,059

(E) 10,083

114.

Jeff has 8000 and would like to purchase a 10,000 bond. In doing so, Jeff takes out a 10 year loan of 2000 from a bank and will make interest-only payments at the end of each month at a nominal rate of 8.0% convertible monthly. He immediately pays 10,000 for a 10-year bond with a par value of 10,000 and 9.0% coupons paid monthly.

Calculate the annual effective yield rate that Jeff will realize on his 8000 over the 10-year period.

(A) 9.30%

(B) 9.65%

(C) 10.00%

(D) 10.35%

(E) 10.70%

115.

A bank issues three annual coupon bonds redeemable at par, all with the same term, price, and annual effective yield rate.

The first bond has face value 1000 and annual coupon rate 5.28%.

The second bond has face value 1100 and annual coupon rate 4.40%.

The third bond has face value 1320 and annual coupon rate r.

Calculate r.

(A) 2.46%

(B) 2.93%

(C) 3.52%

(D) 3.67%

(E) 4.00%

116.

An investor owns a bond that is redeemable for 250 in 6 years from now. The investor has just received a coupon of c and each subsequent semiannual coupon will be 2% larger than the preceding coupon. The present value of this bond immediately after the payment of the coupon is 582.53 assuming an annual effective yield rate of 4%.

Calculate c.

(A) 32.04

(B) 32.68

(C) 40.22

(D) 48.48

(E) 49.45

117.

An n-year bond with annual coupons has the following characteristics:

i) The redemption value at maturity is 1890;

ii) The annual effective yield rate is 6%;

iii) The book value immediately after the third coupon is 1254.87; and

iv) The book value immediately after the fourth coupon is 1277.38.

Calculate n.

(A) 16

(B) 17

(C) 18

(D) 19

(E) 20

118.

An *n*-year bond with semiannual coupons has the following characteristics:

i) The par value and redemption value are 2500;

ii) The annual coupon rate is 7% payable semi-annually;

iii) The annual nominal yield to maturity is 8% convertible semiannually; and

iv) The book value immediately after the fourth coupon is 8.44 greater than the book value immediately after the third coupon.

Calculate *n*.

(A) 6.5

B) 7.0

(C) 9.5

(D) 12.0

(E) 14.0

119.

For the next four years, the one-year forward rates of interest are estimated to be:

Year	0	1	2	3	4
Forward Rate	4%	6%	8%	10%	12%

Calculate the spot rate for a zero-coupon bond maturing three years from now.

(A) 4%

(B) 5%

(C) 6%

(D) 7%

(E) 8%

120.

On January 1, an investment account is worth 50,000. On May 1, the value has increased to 52,000 and 8,000 of new principal is deposited. At time t, in years, $(4/12 < t < 1)$ the value of the fund has increased to 62,000 and 10,000 is withdrawn. On January 1 of the next year, the investment account is worth 55,000. The dollar-weighted rate of return (using the simple interest approximation) is equal to the time-weighted rate of return for the year.

Calculate t.

(A) 0.411

(B) 0.415

(C) 0.585

(D) 0.589

(E) 0.855

121.

Annuity A pays 1 at the beginning of each year for three years. Annuity B pays 1 at the beginning of each year for four years.

The Macaulay duration of Annuity A at the time of purchase is 0.93. Both annuities offer the same yield rate.

Calculate the Macaulay duration of Annuity B at the time of purchase.

(A) 1.240

(B) 1.369

(C) 1.500

(D) 1.930

(E) 1.965

122.

Cash flows are 40,000 at time 2 (in years), 25,000 at time 3, and 100,000 at time 4. The annual effective yield rate is 7.0%.

Calculate the Macaulay duration.

(A) 2.2

(B) 2.3

(C) 3.1

(D) 3.3

(E) 3.4

123.

Bond A is a 3-year bond that pays annual coupons and is priced at par. The annual coupon rate is 10%.

Bond B pays semiannual coupons and yields 10% convertible semiannually.

Bond A and Bond B have the same modified duration.

Calculate the Macaulay duration of Bond B.

(A) 2.49

(B) 2.61

(C) 2.74

(D) 2.87

(E) 3.01

124.

Rhonda purchases a perpetuity providing a payment of 1 at the beginning of each year. The perpetuity's Macaulay duration is 30 years.

Calculate the modified duration of this perpetuity.

(A) 28.97

(B) 29.00

(C) 29.03

(D) 29.07

(E) 29.10

125.

Stocks F and J are valued using the dividend discount model. The required annual effective rate of return is 8.8%. The dividend of Stock F has an annual growth rate of g and the dividend of Stock J has an annual growth rate of $-g$.

The dividends of both stocks are paid annually on the same date.

The value of Stock F is twice the value of Stock J. The next dividend on Stock F is half of the next dividend on Stock J.

Calculate g.

(A) 0.0%

(B) 0.8%

(C) 2.9%

(D) 5.3%

(E) 8.8%

126.

Which of the following statements regarding immunization are true?

I. If long-term interest rates are lower than short-term rates, the need for immunization is reduced.

II. Either Macaulay or modified duration can be used to develop an immunization strategy.

III. Both processes of matching the present values of the flows or the flows themselves will produce exact matching.

(A) I only

(B) II only

(C) III only

(D) I, II and III

(E) The correct answer is not given by (A), (B), (C), or (D).

127.

A company owes 500 and 1000 to be paid at the end of year one and year four, respectively. The company will set up an investment program to match the duration and the present value of the above obligation using an annual effective interest rate of 10%.

The investment program produces asset cash flows of X today and Y in three years.

Calculate X and determine whether the investment program satisfies the conditions for Redington immunization.

(A) $X = 75$ and the Redington immunization conditions are not satisfied.

(B) $X = 75$ and the Redington immunization conditions are satisfied.

(C) $X = 1138$ and the Redington immunization conditions are not satisfied.

(D) $X = 1138$ and the Redington immunization conditions are satisfied.

(E) $X = 1414$ and the Redington immunization conditions are satisfied.

128.

An insurance company has a known liability of 1,000,000 that is due 8 years from now. The technique of full immunization is to be employed. Asset I will provide a cash flow of 300,000 exactly 6 years from now. Asset II will provide a cash flow of X, exactly y years from now, where $y > 8$.

The annual effective interest rate is 4%.

Calculate X.

(A) 697,100

(B) 698,600

(C) 700,000

(D) 701,500

(E) 702,900

129.

A company has liabilities of 573 due at the end of year 2 and 701 due at the end of year 5.

A portfolio comprises two zero-coupon bonds, Bond A and Bond B.

Determine which portfolio produces a Redington immunization of the liabilities using an annual effective interest rate of 7.0%.

(A) Bond A: 1-year, current price 500; Bond B: 6-years, current price 500

(B) Bond A: 1-year, current price 572; Bond B: 6-years, current price 428

(C) Bond A: 3-years, current price 182; Bond B: 4-years, current price 1092

(D) Bond A: 3-years, current price 637; Bond B: 4-years, current price 637

(E) Bond A: 3.5 years, current price 1000; Bond B: Not used

130.

A company has liabilities of 402.11 due at the end of each of the next three years. The company will invest 1000 today to fund these payouts. The only investments available are one-year and three-year zero-coupon bonds, and the yield curve is flat at a 10% annual effective rate. The company wishes to match the duration of its assets to the duration of its liabilities.

Determine how much the company should invest in each bond.

(A) 366 in the one-year bond and 634 in the three-year bond.

(B) 484 in the one-year bond and 516 in the three-year bond.

(C) 500 in the one-year bond and 500 in the three-year bond.

(D) 532 in the one-year bond and 468 in the three-year bond.

(E) 634 in the one-year bond and 366 in the three-year bond.

131.

You are given the following information about a company's liabilities:

- Present value: 9697
- Macaulay duration: 15.24
- Macaulay convexity: 242.47

The company decides to create an investment portfolio by making investments into two of the following three zero-coupon bonds: 5-year, 15-year, and 20-year. The company would like its position to be Redington immunized against small changes in yield rate.

The annual effective yield rate for each of the bonds is 7.5%.

Determine which of the following portfolios the company should create.

(A) Invest 3077 for the 5-year bond and 6620 for the 20-year bond.

(B) Invest 6620 for the 5-year bond and 3077 for the 20-year bond.

(C) Invest 465 for the 15-year bond and 9232 for the 20-year bond.

(D) Invest 4156 for the 15-year bond and 5541 for the 20-year bond.

(E) Invest 9232 for the 15-year bond and 465 for the 20-year bond.

132.

A bank accepts a 20,000 deposit from a customer on which it guarantees to pay an annual effective interest rate of 10% for two years. The customer needs to withdraw half of the accumulated value at the end of the first year. The customer will withdraw the remaining value at the end of the second year.

The bank has the following investment options available, which may be purchased in any quantity:

Bond H: A one-year zero-coupon bond yielding 10% annually

Bond I: A two-year zero-coupon bond yielding 11% annually

Bond J: A two-year bond that sells at par with 12% annual coupons

Any portion of the 20,000 deposit that is not needed to be invested in bonds is retained by the bank as profit.

Determine which of the following investment strategies produces the highest profit for the bank and is guaranteed to meet the customer's withdrawal needs.

(A) 9,091 in Bond H, 8,264 in Bond I, 2,145 in Bond J

(B) 10,000 in Bond H, 10,000 in Bond I

(C) 10,000 in Bond H, 9,821 in Bond I

(D) 8,910 in Bond H, 731 in Bond I, 10,000 in Bond J

(E) 8,821 in Bond H, 10,804 in Bond J

133.

An insurance company wants to match liabilities of 25,000 payable in one year and 20,000 payable in two years with specific assets. The following assets are currently available:

i) One-year bond with an annual coupon of 6.75% at par

ii) Two-year bond with annual coupons of 4.50% at par

iii) Two-year zero-coupon bond yielding 5.00% annual effective

Calculate the smallest amount the company needs to disburse today to purchase assets that will exactly match these liabilities.

(A) 41,220

(B) 41,390

(C) 41,560

(D) 41,660

(E) 41,750