End-of-chapter Questions for Practice (with Answers)

Following is a list of selected end-of-chapter questions for practice from McDonald's *Derivatives Markets*. For students who do not have a copy of the McDonald's book, be aware that a copy of the book is reserved at the main library of the University of Hong Kong for you to borrow for short period of time.

Answers provided are for your reference only. It is complied directly from the solution manual provided by the author. If you identify any error, please let me know.

Chapter 1: 1.3, 1.4, 1.11

Chapter 2: 2.5, 2.9, 2.13, 2.16

Chapter 3: 3.1, 3.3, 3.10, 3.12, 3.14, 3.15, 3.18

Chapter 4: 4.1, 4.4, 4.5, 4.15, 4.17

Chapter 5: 5.4, 5.10, 5.12, 5.15, 5.18

Chapter 7: 7.3, 7.6, 7.8, 7.9, 7.12, 7.15, 7.16

Chapter 8: 8.3, 8.7, 8.10, 8.13, 8.14, 8.15, 8.17

Chapter 9: 9.4, 9.9, 9.10, 9.12

Chapter 10: 10.1, 10.5, 10.10, 10.12, 10.14, 10.17, 10.18

Chapter 11: 11.1, 11.7, 11.16, 11.17, 11.20

Chapter 12: 12.3, 12.4, 12.5, 12.7, 12.14, 12.20

Chapter 13: 13.1, 13.3, 13.14

Chapter 14: 14.6, 14.11, 14.12

Chapter 15: 15.1, 15.3, 15.4, 15.6

Chapter 1. Introduction to Derivatives

Question 1.3.

a. Remember that the terminology bid and ask is formulated from the market makers perspective. Therefore, the price at which you can buy is called the ask price. Furthermore, you will have to pay the commission to your broker for the transaction. You pay:

$$(\$41.05 \times 100) + \$20 = \$4{,}125.00$$

b. Similarly, you can sell at the market maker's bid price. You will again have to pay a commission, and your broker will deduct the commission from the sales price of the shares. You receive:

$$(\$40.95 \times 100) - \$20 = \$4{,}075.00$$

c. Your round-trip transaction costs amount to:

$$\$4{,}125.00 - \$4{,}075.00 = \$50$$

Question 1.4.

In this problem, the brokerage fee is variable, and depends on the actual dollar amount of the sale/purchase of the shares. The concept of the transaction cost remains the same: If you buy the shares, the commission is added to the amount you owe, and if you sell the shares, the commission is deducted from the proceeds of the sale.

a.

$$\begin{aligned}(\$41.05 \times 100) + (\$41.05 \times 100) \times 0.003 &= \$4{,}117.315 \\ &= \$4{,}117.32\end{aligned}$$

b.

$$\begin{aligned}(\$40.95 \times 100) - (\$40.95 \times 100) \times 0.003 &= \$4{,}082.715 \\ &= \$4{,}082.72\end{aligned}$$

c.

$$\$4{,}117.32 - \$4{,}082.72 = \$34.6$$

The variable (or proportional) brokerage fee is advantageous to us. Our round-trip transaction fees are reduced by \$15.40.

Question 1.11.

We are interested in borrowing the asset "money." Therefore, we go to an owner (or, if you prefer, to, a collector) of the asset, called Bank. The Bank provides the \$100 of the asset money in digital form by increasing our bank account. We sell the digital money by going to the ATM and withdrawing cash. After 90 days, we buy back the digital money for \$102, by depositing cash into our bank account. The lender is repaid, and we have covered our short position.

Chapter 2. An Introduction to Forwards and Options

Question 2.5.

a. The payoff to a short forward at expiration is equal to:

$$\text{Payoff to short forward} = \text{forward price} - \text{spot price at expiration}$$

Therefore, we can construct the following table:

Price of asset in 6 months	Agreed forward price	Payoff to the short forward
40	50	10
45	50	5
50	50	0
55	50	-5
60	50	-10

b. The payoff to a purchased put option at expiration is:

$$\text{Payoff to put option} = \max[0, \text{strike price} - \text{spot price at expiration}]$$

The strike is given: It is \$50. Therefore, we can construct the following table:

Price of asset in 6 months	Strike price	Payoff to the call option
40	50	10
45	50	5
50	50	0
55	50	0
60	50	0

c. If we compare the two contracts, we see that the put option has a protection for increases in the price of the asset: If the spot price is above \$50, the buyer of the put option can walk away, and need not incur a loss. The buyer of the short forward incurs a loss and must meet her obligations. However, she has the same payoff as the buyer of the put option if the spot price is below \$50. Therefore, the put option should be more expensive. It is this attractive option to walk away if things are not as we want that we have to pay for.

Question 2.9.

a. If the forward price is \$1,100, then the buyer of the one-year forward contract receives at expiration after one year a profit of: $\$S_T - \$1{,}100$, where S_T is the (unknown) value of the S&R index at expiration of the forward contract in one year. Remember that it costs nothing to enter the forward contract.

Let us again follow our strategy of borrowing money to finance the purchase of the index today, so that we do not need any initial cash. If we borrow \$1,000 today to buy the S&R index (that costs \$1,000), we have to repay in one year: $\$1{,}000 \times (1 + 0.10) = \$1{,}100$. Our total profit in one year from borrowing to buy the S&R index is therefore: $\$S_T - \$1{,}100$. The profits from the two strategies are identical.

b. The forward price of \$1,200 is worse for us if we want to buy a forward contract. To understand this, suppose the index after one year is \$1,150. While we have already made money in part a) with a forward price of \$1,100, we are still losing \$50 with the new price of \$1,200. As there was no advantage in buying either stock or forward at a price of \$1,100, we now need to be "bribed" to enter into the forward contract. We somehow need to find an equation that makes the two strategies comparable again. Suppose that we lend some money initially together with entering into the forward contract so that we will receive \$100 after one year. Then, the payoff from our modified forward strategy is: $\$S_T - \$1{,}200 + \$100 = \$S_T - \$1{,}100$, which equals the payoff of the "borrow to buy index" strategy. We have found the future value of the premium somebody needs us to pay. We still need to find out what the premium we will receive in one year is worth today.

We need to discount it: $\$100/(1 + 0.10) = \$90.91$.

c. Similarly, the forward price of \$1,000 is advantageous for us. As there was no advantage in buying either stock or forward at a price of \$1,100, we now need to "bribe" someone to sell this advantageous forward contract to us. We somehow need to find an equation that makes the two strategies comparable again. Suppose that we borrow some money initially together with entering into the forward contract so that we will have to pay back \$100 after one year. Then, the payoff from our modified forward strategy is: $\$S_T - \$1{,}000 - \$100 = \$S_T - \$1{,}100$, which equals the payoff of the "borrow to buy index" strategy. We have found the future value of the premium we need to pay. We still need to find out what this premium we have to pay in one year is worth today.

We simply need to discount it: $\$100/(1 + 0.10) = \$90.91$. We should be willing to pay \$90.91 to enter into the one year forward contract with a forward price of \$1,000.

Question 2.13.

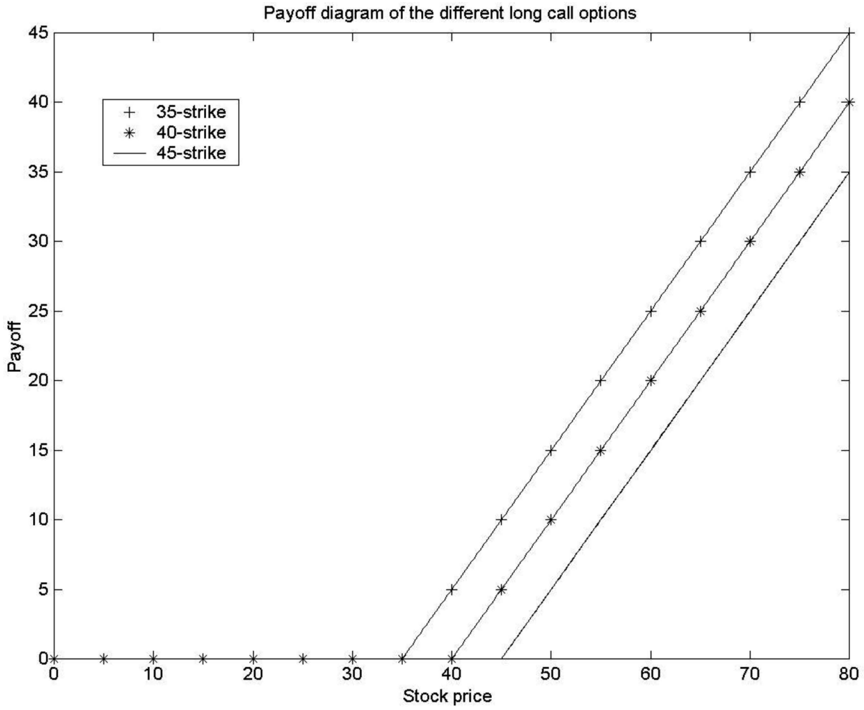
a. In order to be able to draw profit diagrams, we need to find the future values of the call premia. They are:

i) 35-strike call: $\$9.12 \times (1 + 0.08) = \$9.8496$

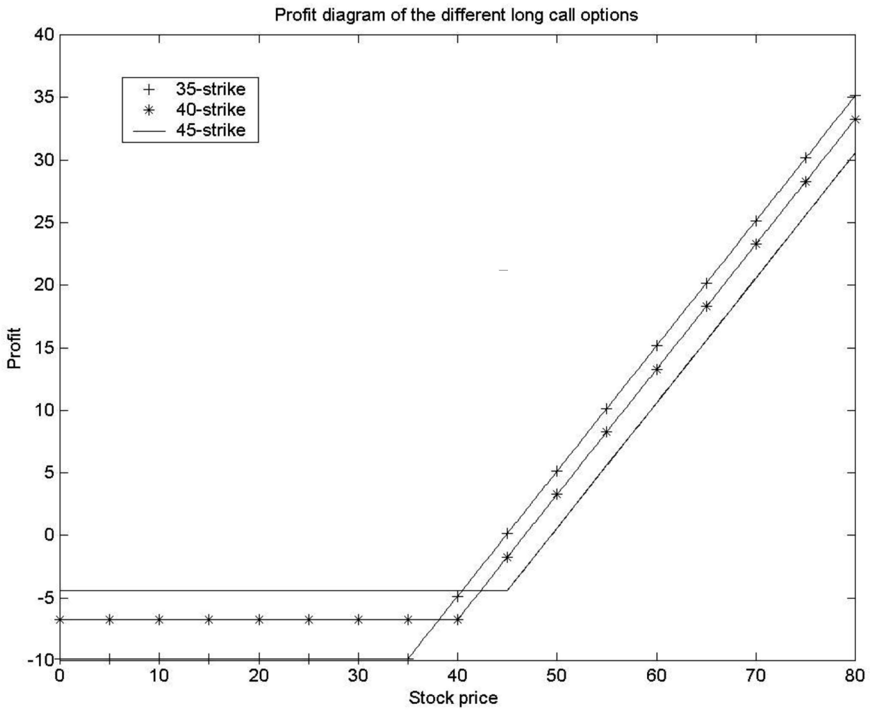
ii) 40-strike call: $\$6.22 \times (1 + 0.08) = \$6.7176$

iii) 45-strike call: $\$4.08 \times (1 + 0.08) = \$4.4064$

We can now graph the payoff and profit diagrams for the call options. The payoff diagram looks as follows:



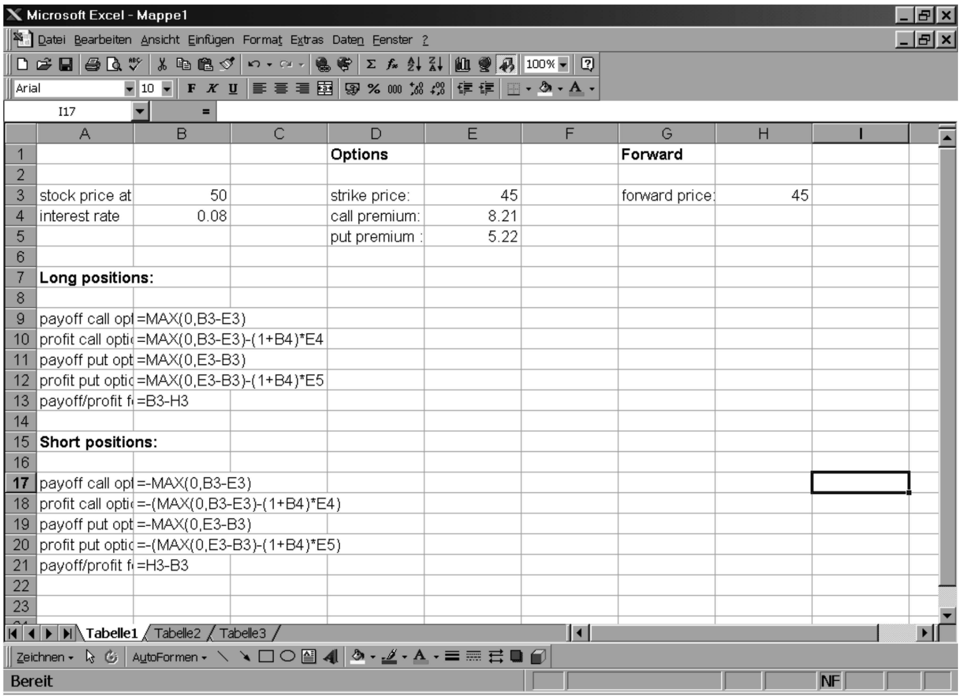
We get the profit diagram by deducting the option premia from the payoff graphs. The profit diagram looks as follows:



b. Intuitively, whenever the 45-strike option pays off (i.e., has a payoff bigger than zero), the 40-strike and the 35-strike options pay off. However, there are some instances in which the 40-strike option pays off and the 45-strike options does not. Similarly, there are some instances in which the 35-strike option pays off, and neither the 40-strike nor the 45-strike pay off. Therefore, the 35-strike offers more potential than the 40- and 45-strike, and the 40-strike offers more potential than the 45-strike. We pay for these additional payoff possibilities by initially paying a higher premium.

Question 2.16.

The following is a copy of an Excel spreadsheet that solves the problem:



	A	B	C	D	E	F	G	H
1				**Options**			**Forward**	
2								
3	stock price at	50		strike price:	45		forward price:	45
4	interest rate	0.08		call premium:	8.21			
5				put premium :	5.22			
6								
7	**Long positions:**							
8								
9	payoff call opt	=MAX(0,B3-E3)						
10	profit call optio	=MAX(0,B3-E3)-(1+B4)*E4						
11	payoff put opt	=MAX(0,E3-B3)						
12	profit put optio	=MAX(0,E3-B3)-(1+B4)*E5						
13	payoff/profit f	=B3-H3						
14								
15	**Short positions:**							
16								
17	payoff call opt	=-MAX(0,B3-E3)						
18	profit call optio	=-(MAX(0,B3-E3)-(1+B4)*E4)						
19	payoff put opt	=-MAX(0,E3-B3)						
20	profit put optio	=-(MAX(0,E3-B3)-(1+B4)*E5)						
21	payoff/profit f	=H3-B3						

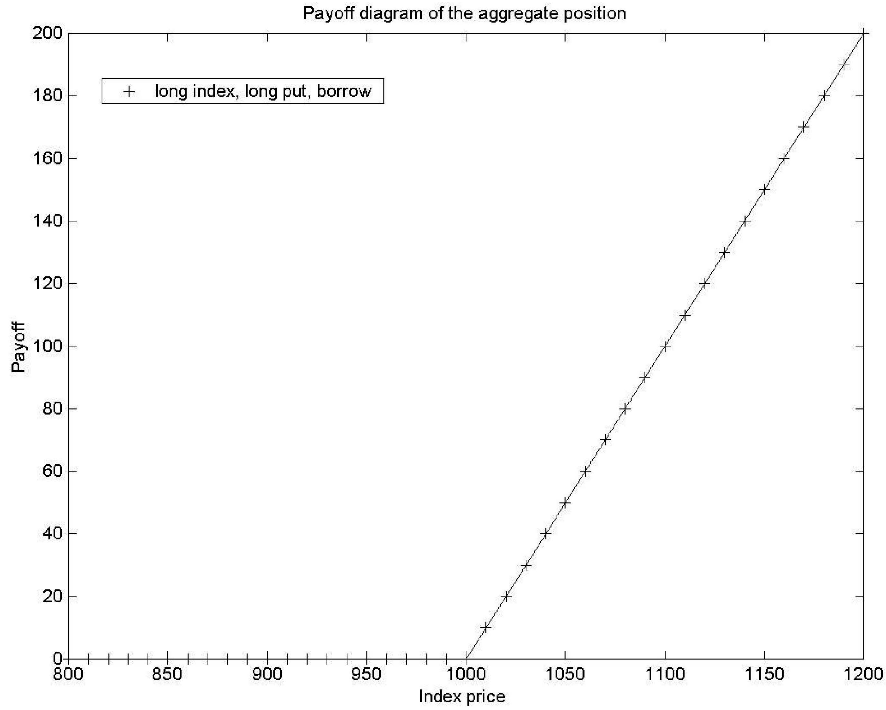
Chapter 3. Insurance, Collars, and Other Strategies

Question 3.1.

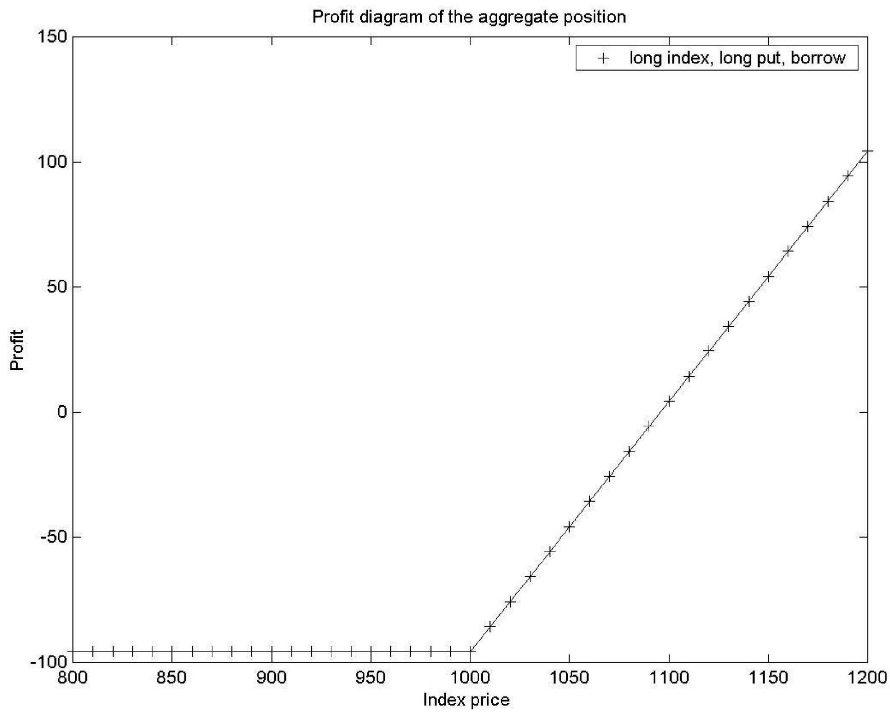
This question is a direct application of the Put-Call-Parity (equation (3.1)) of the textbook. Mimicking Table 3.1., we have:

S&R Index	S&R Put	Loan	Payoff	−(Cost + Interest)	Profit
900.00	100.00	−1000.00	0.00	−95.68	−95.68
950.00	50.00	−1000.00	0.00	−95.68	−95.68
1000.00	0.00	−1000.00	0.00	−95.68	−95.68
1050.00	0.00	−1000.00	50.00	−95.68	−45.68
1100.00	0.00	−1000.00	100.00	−95.68	4.32
1150.00	0.00	−1000.00	150.00	−95.68	54.32
1200.00	0.00	−1000.00	200.00	−95.68	104.32

The payoff diagram looks as follows:



We can see from the table and from the payoff diagram that we have in fact reproduced a call with the instruments given in the exercise. The profit diagram on the next page confirms this hypothesis.



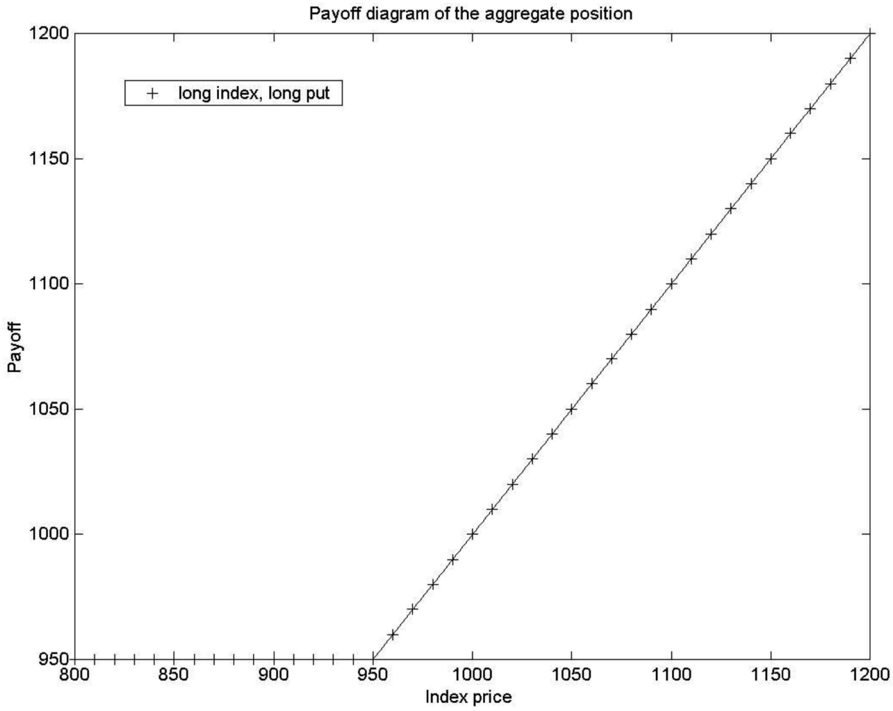
Question 3.3.

In order to be able to draw profit diagrams, we need to find the future value of the put premium, the call premium and the investment in zero-coupon bonds. We have for:

the put premium: $\$51.777 \times (1 + 0.02) = \$52.81$,
the call premium: $\$120.405 \times (1 + 0.02) = \$122.81$ and
the zero-coupon bond: $\$931.37 \times (1 + 0.02) = \$950.00$

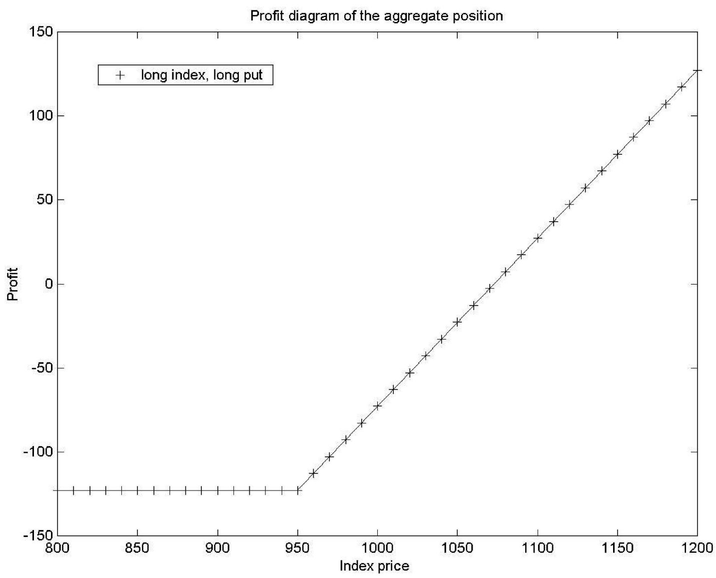
Now, we can construct the payoff and profit diagrams of the aggregate position:

Payoff diagram:



From this figure, we can already see that the combination of a long put and the long index looks exactly like a certain payoff of \$950, plus a call with a strike price of 950. But this is the alternative given to us in the question. We have thus confirmed the equivalence of the two combined positions for the payoff diagrams. The profit diagrams on the next page confirm the equivalence of the two positions (which is again an application of the Put-Call-Parity).

Profit Diagram for a long 950-strike put and a long index combined:



Question 3.10.

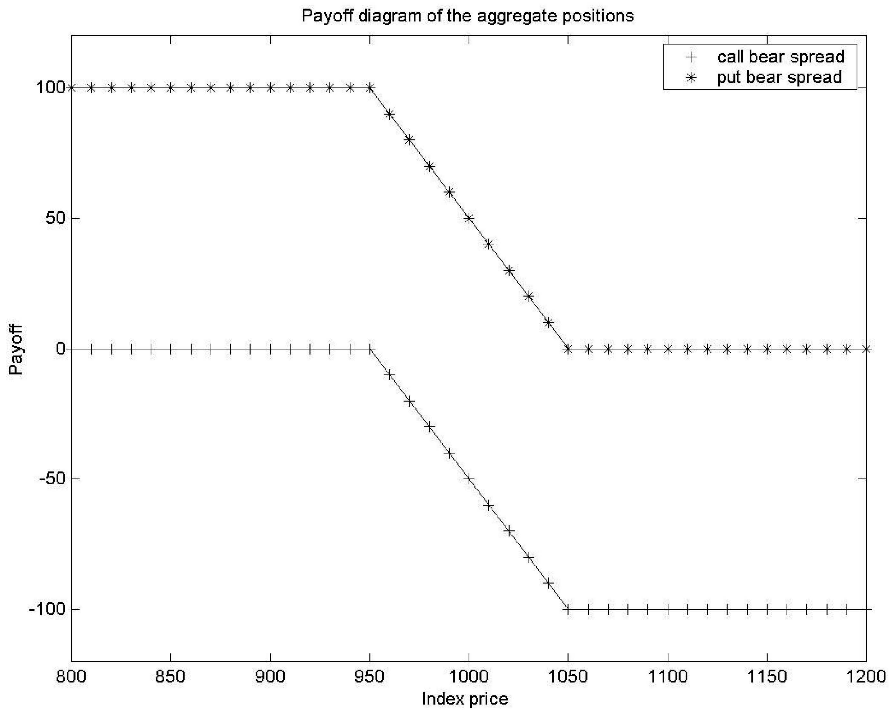
The strategy of selling a call (or put) and buying a call (or put) at a higher strike is called call (put) bear spread. In order to draw the profit diagrams, we need to find the future value of the cost of entering in the bull spread positions. We have:
Cost of call bear spread: $(\$71.802 - \$120.405) \times 1.02 = -\$49.575$
Cost of put bear spread: $(\$101.214 - \$51.777) \times 1.02 = \$50.426$

The payoff diagram shows that the payoff to the call bear spread is uniformly less than the payoffs to the put bear spread. The difference is exactly \$100, equal to the difference in strikes and as well equal to the difference in the future value of the costs of the spreads.

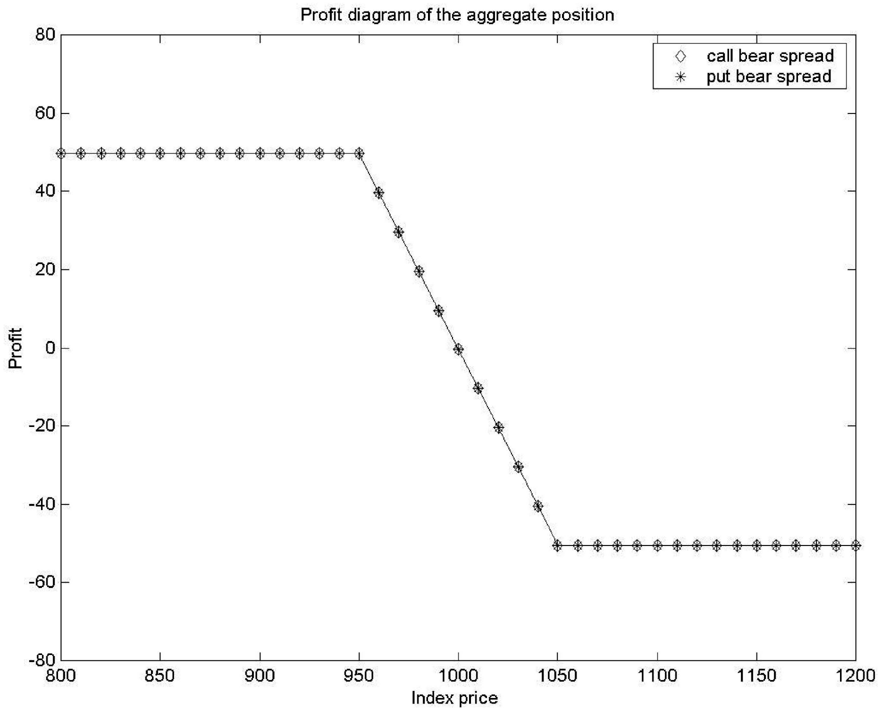
There is a difference, because the call bear spread has a negative initial cost, i.e., we are receiving money if we enter into it.

The higher initial cost for the put bear spread is exactly offset by the higher payoff so that the profits of both strategies turn out to be the same. It is easy to show this with equation (3.1), the put-call-parity.

Payoff-Diagram:



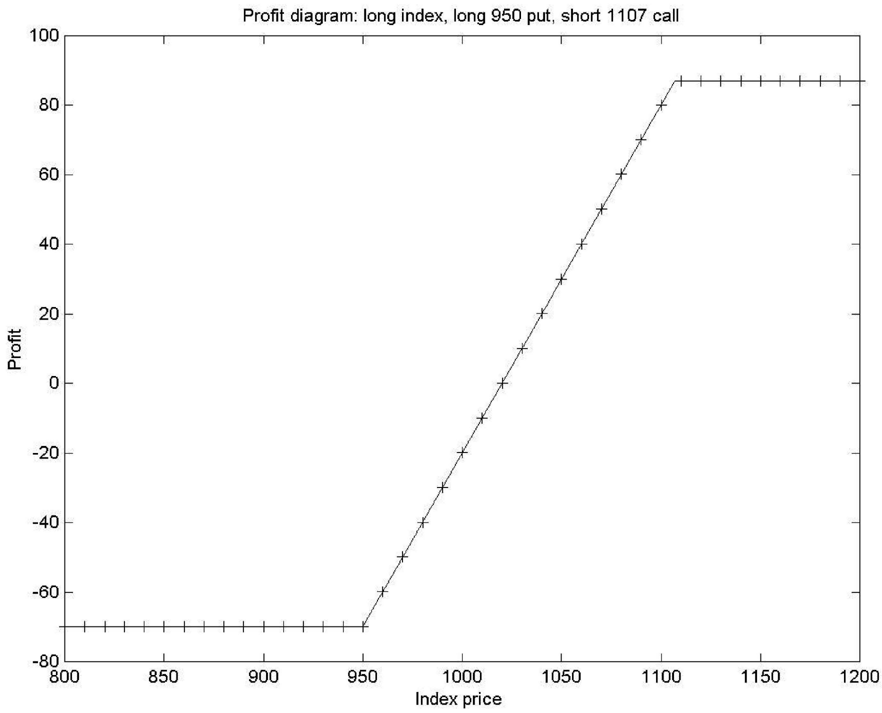
Profit Diagram:



Question 3.12.

Our initial cash required to put on the collar, i.e. the net option premium, is as follows: $-\$51.873 + \$51.777 = -\$0.096$. Therefore, we receive only 10 cents if we enter into this collar. The position is very close to a zero-cost collar.

The profit diagram looks as follows:



If we compare this profit diagram with the profit diagram of the previous exercise (3.11.), we see that we traded in the additional call premium (that limited our losses after index decreases) against more participation on the upside. Please note that both maximum loss and gain are higher than in question 3.11.

Question 3.14.

a. This question deals with the option trading strategy known as Box spread. We saw earlier that if we deal with options and the maximum function, it is convenient to split the future values of the index into different regions. Let us name the final value of the S&R index S_T. We have two strike prices, therefore we will use three regions: One in which $S_T < 950$, one in which $950 \leq S_T < 1{,}000$ and another one in which $S_T \geq 1{,}000$. We then look at each region separately, and hope to be able to see that indeed when we aggregate, there is no S&R risk when we look at the aggregate position.

Instrument	$S_T < 950$	$950 \leq S_T < 1{,}000$	$S_T \geq 1{,}000$
long 950 call	0	$S_T - \$950$	$S_T - \$950$
short 1000 call	0	0	$\$1,000 - S_T$
short 950 put	$S_T - \$950$	0	0
long 1000 put	$\$1{,}000 - S_T$	$\$1{,}000 - S_T$	0
Total	\$50	\$50	\$50

We see that there is no occurrence of the final index value in the row labeled total. The option position does not contain S&R price risk.

b. The initial cost is the sum of the long option premia less the premia we receive for the sold options. We have:

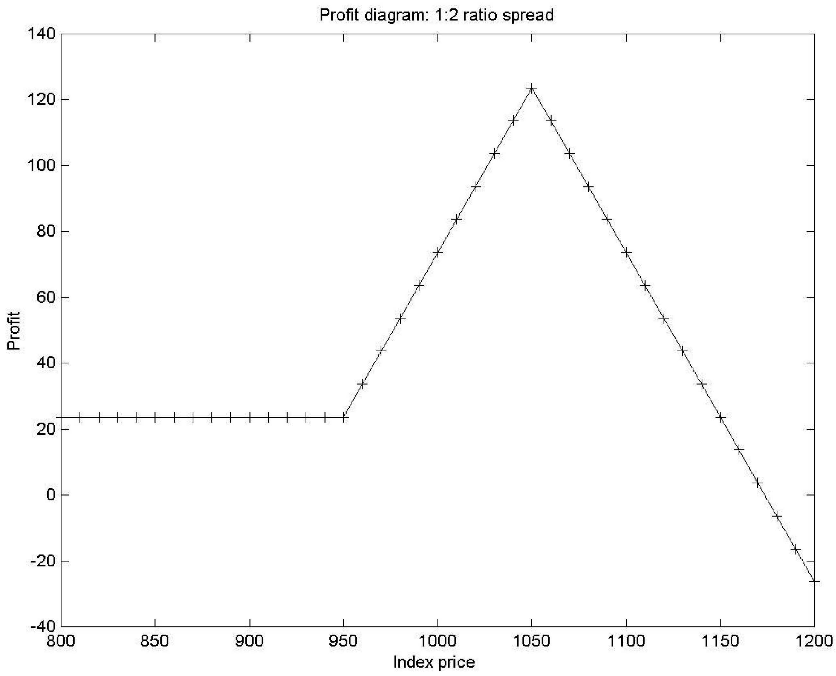
Cost $\$120.405 - \$93.809 - \$51.77 + \$74.201 = \$49.027$

c. The payoff of the position after 6 months is \$50, as we can see from the above table.

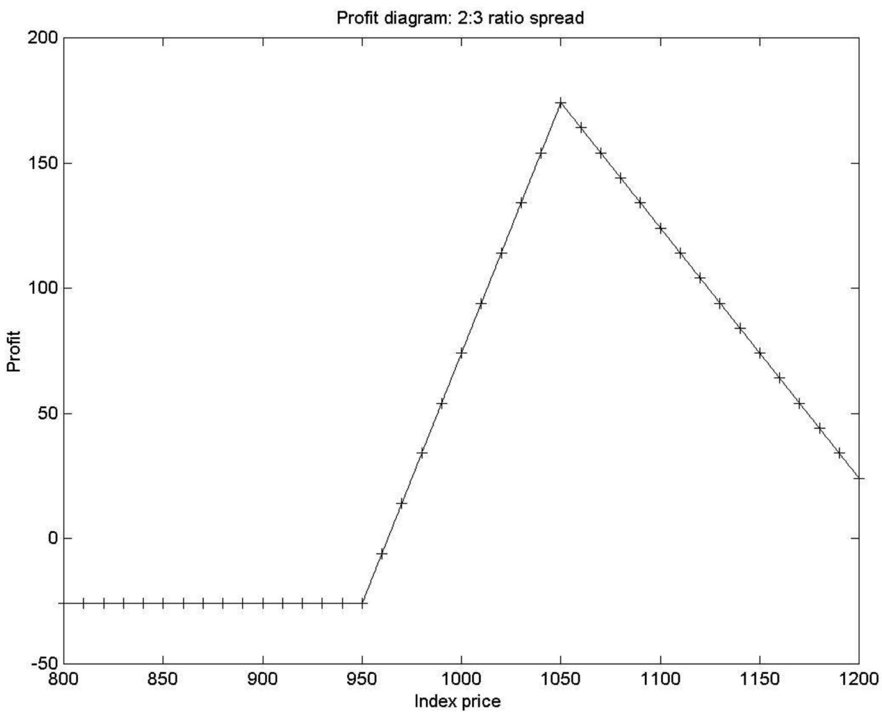
d. The implicit interest rate of the cash flows is: $\$50.00 \div \$49.027 = 1.019 \cong 1.02$. The implicit interest rate is indeed 2 percent.

Question 3.15.

a. Profit diagram of the 1:2 950-, 1050-strike ratio call spread (the future value of the initial cost of which is calculated as: $(\$120.405 - 2 \times \$71.802) \times 1.02 = -\$23.66$):



b. Profit diagram of the 2:3 950-, 1050-strike ratio call spread (the future value of the initial cost of which is calculated as: $(2 \times \$120.405 - 3 \times \$71.802) \times 1.02 = \$25.91$.



c. We saw that in part a), we were receiving money from our position, and in part b), we had to pay a net option premium to establish the position. This suggests that the true ratio n/m lies between 1:2 and 2:3.

Indeed, we can calculate the ratio n/m as:

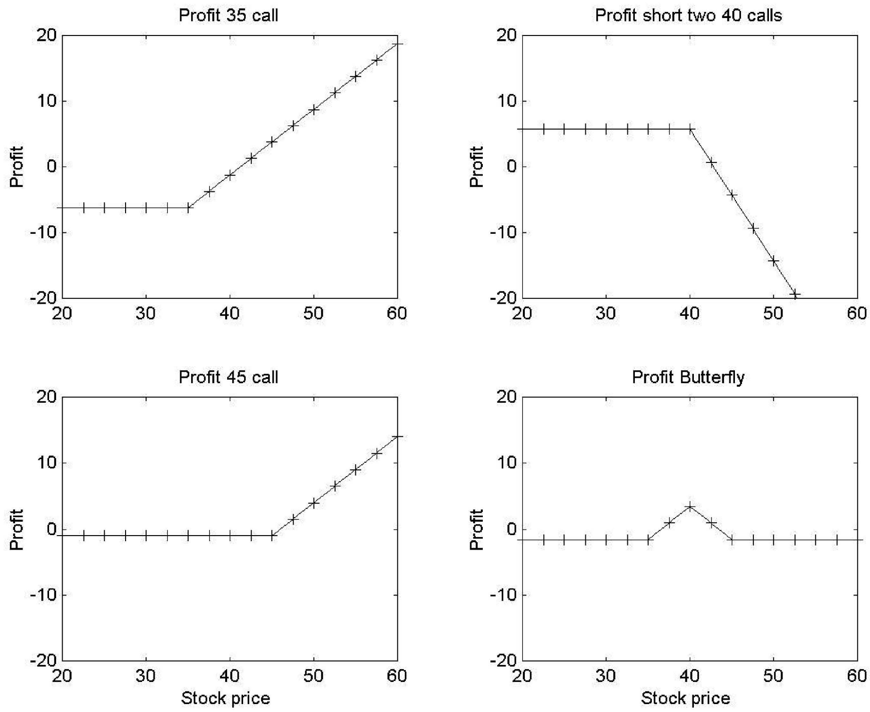
$$\begin{aligned} & n \times \$120.405 - m \times \$71.802 &= 0 \\ \Leftrightarrow\ & n \times \$120.405 &= m \times \$71.802 \\ \Leftrightarrow\ & n/m &= \$71.802/\$120.405 \\ \Leftrightarrow\ & n/m &= 0.596 \end{aligned}$$

which is approximately 3:5.

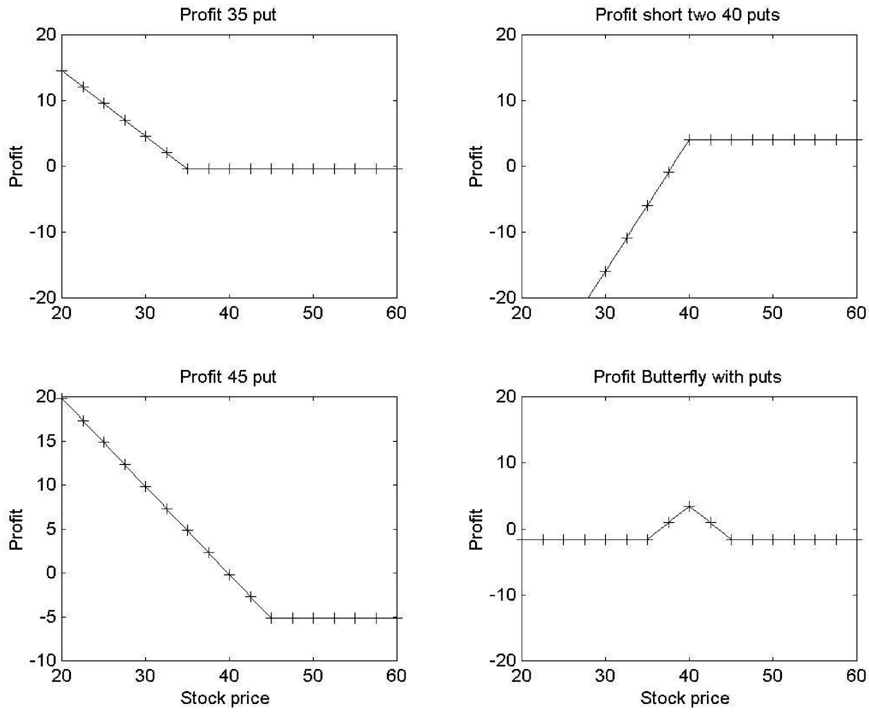
Question 3.18.

The following three figures show the individual legs of each of the three suggested strategies. The last subplot shows the aggregate position. It is evident from the figures that you can indeed use all the suggested strategies to construct the same butterfly spread. Another method to show the claim of 3.18. mathematically would be to establish the equivalence by using the Put-Call-Parity on b) and c) and showing that you can write it in terms of the instruments of a).

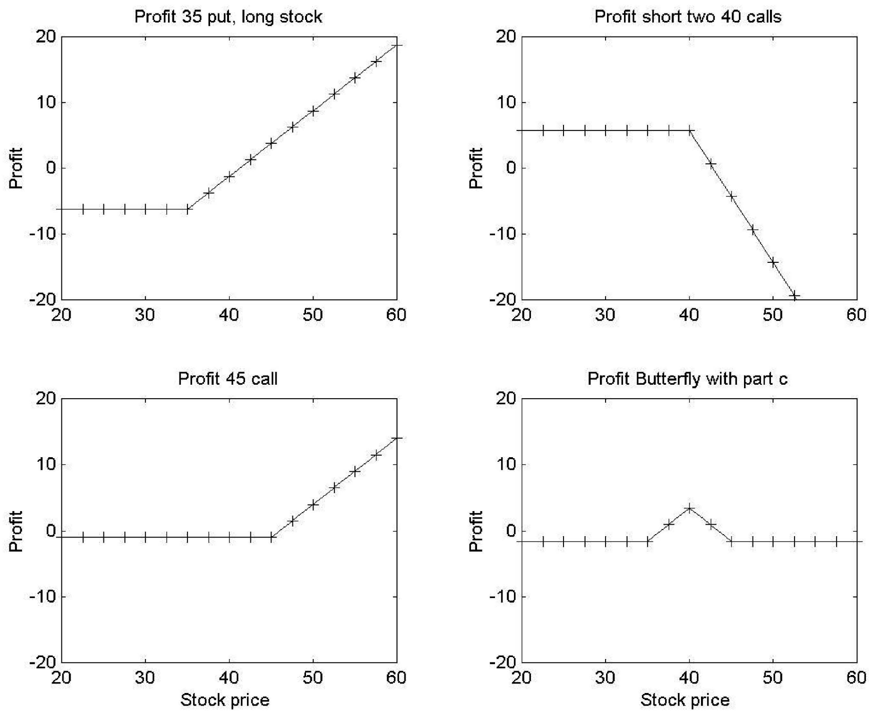
profit diagram part a)



profit diagram part b)



profit diagram part c)



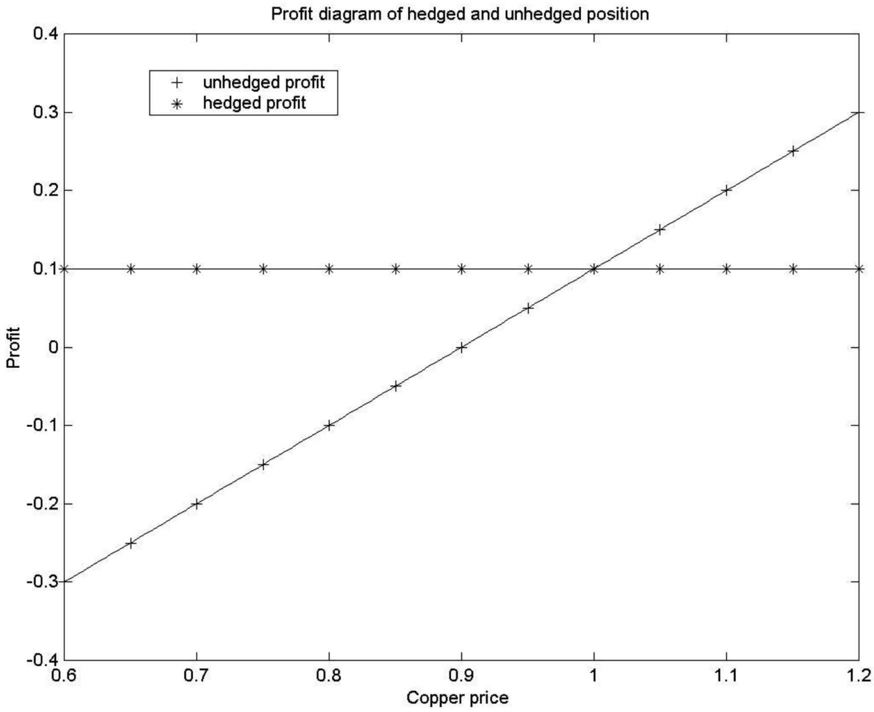
Chapter 4. Introduction to Risk Management

Question 4.1.

The following table summarizes the unhedged and hedged profit calculations:

Copper price in one year	Total cost	Unhedged profit	Profit on short forward	Net income on hedged profit
\$0.70	\$0.90	−\$0.20	\$0.30	\$0.10
\$0.80	\$0.90	−\$0.10	\$0.20	\$0.10
\$0.90	\$0.90	0	\$0.10	\$0.10
\$1.00	\$0.90	\$0.10	0	\$0.10
\$1.10	\$0.90	\$0.20	−\$0.10	\$0.10
\$1.20	\$0.90	\$0.30	−\$0.20	\$0.10

We obtain the following profit diagram:

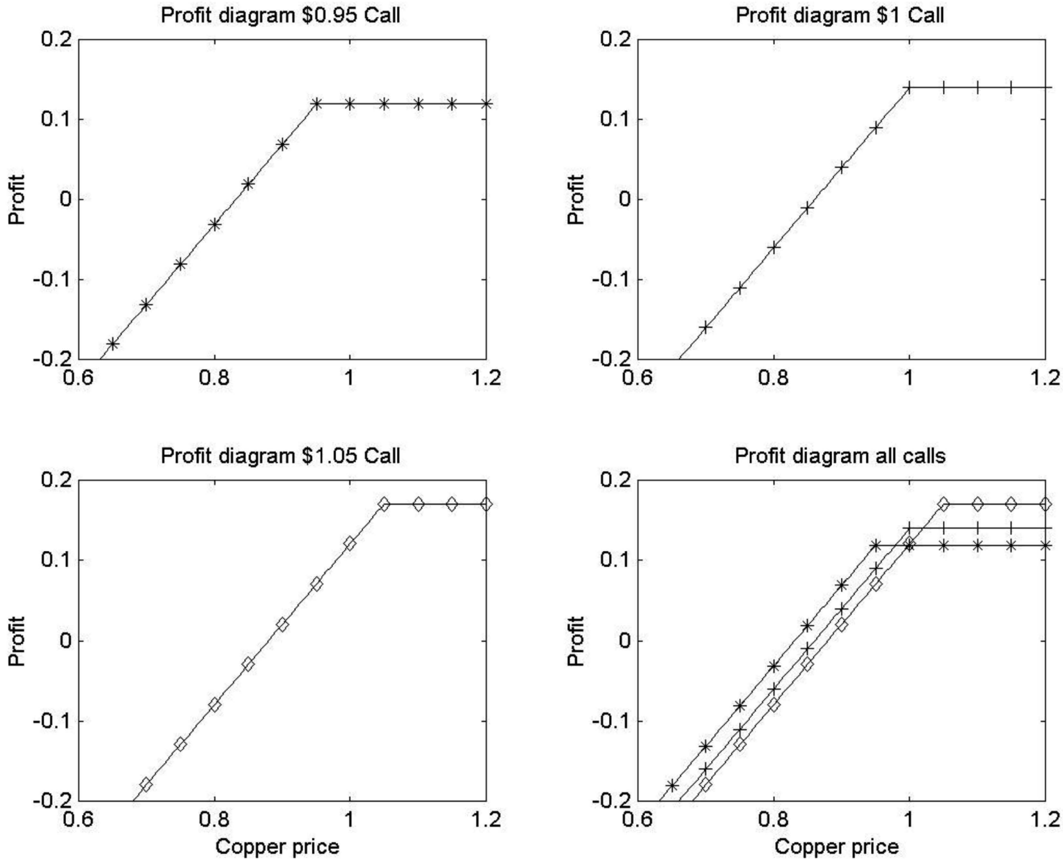


Question 4.4.

We will explicitly calculate the profit for the \$1.00-strike and show figures for all three strikes. The future value of the \$1.00-strike call premium amounts to: $\$0.0376 \times 1.062 = \$0.04$.

Copper price in one year	Total cost	Unhedged profit	Profit on short \$1.00-strike call option	Call premium received	Net income on hedged profit
\$0.70	\$0.90	−\$0.20	0	\$0.04	−\$0.16
\$0.80	\$0.90	−\$0.10	0	\$0.04	−\$0.06
\$0.90	\$0.90	0	0	\$0.04	\$0.04
\$1.00	\$0.90	\$0.10	0	\$0.04	\$0.14
\$1.10	\$0.90	\$0.20	−\$0.10	\$0.04	\$0.14
\$1.20	\$0.90	\$0.30	−\$0.20	\$0.04	\$0.14

We obtain the following payoff graphs:



Question 4.5.

XYZ will buy collars, which means that they buy the put leg and sell the call leg. We have to compute for each case the net option premium position, and find its future value. We have for

a. $(\$0.0178 - \$0.0376) \times 1.062 = -\$0.021$

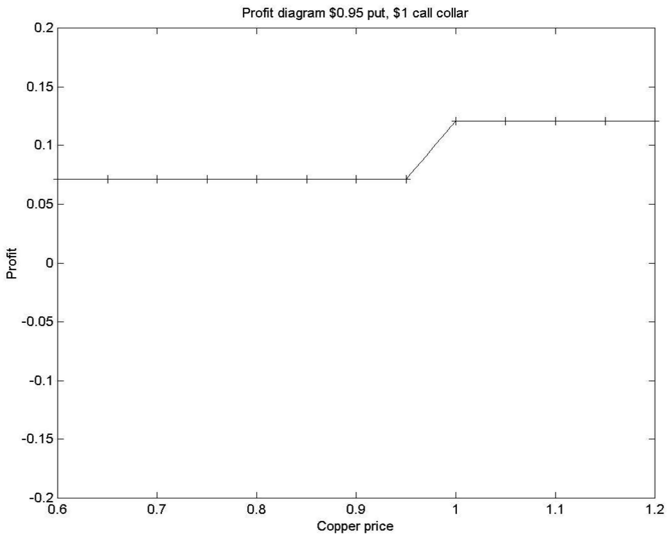
b. $(\$0.0265 - \$0.0274) \times 1.062 = -\$0.001$

c. $(\$0.0665 - \$0.0194) \times 1.062 = \$0.050$

a.

Copper price in one year	Total cost	Profit on .95 put	Profit on short $1.00 call	Net premium	Hedged profit
$0.70	$0.90	$0.25	0	−$0.021	$0.0710
$0.80	$0.90	$0.15	0	−$0.021	$0.0710
$0.90	$0.90	$0.05	0	−$0.021	$0.0710
$1.00	$0.90	$0	0	−$0.021	$0.1210
$1.10	$0.90	0	−$0.10	−$0.021	$0.1210
$1.20	$0.90	0	−$0.20	−$0.021	$0.1210

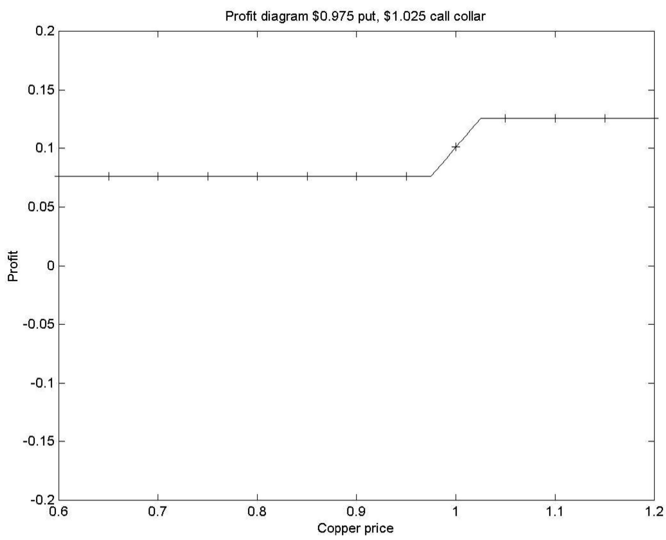
Profit diagram:



b.

Copper price in one year	Total cost	Profit on .975 put	Profit on short $1.025 call	Net premium	Hedged profit
$0.70	$0.90	$0.275	0	−$0.001	$0.0760
$0.80	$0.90	$0.175	0	−$0.001	$0.0760
$0.90	$0.90	$0.075	0	−$0.001	$0.0760
$1.00	$0.90	$0	0	−$0.001	$0.1010
$1.10	$0.90	0	−$0.0750	−$0.001	$0.1260
$1.20	$0.90	0	−$0.1750	−$0.001	$0.1260

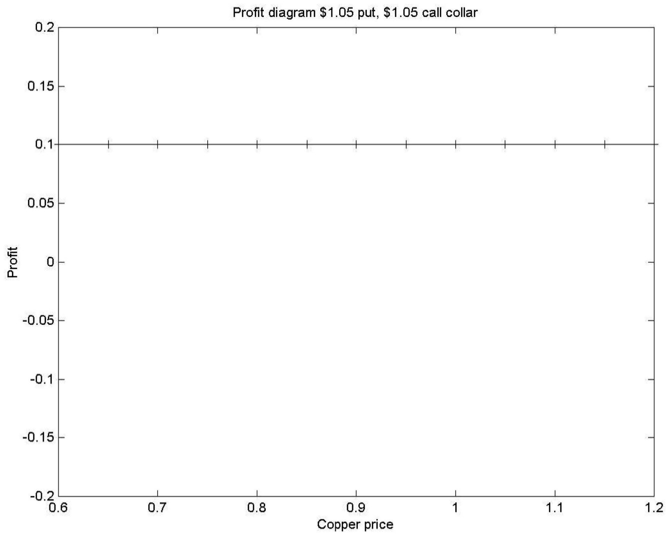
Profit diagram:



c.

Copper price in one year	Total cost	Profit on 1.05 put	Profit on short \$1.05 call	Net premium	Hedged profit
\$0.70	\$0.90	\$0.35	0	\$0.05	\$0.1
\$0.80	\$0.90	\$0.25	0	\$0.05	\$0.1
\$0.90	\$0.90	\$0.15	0	\$0.05	\$0.1
\$1.00	\$0.90	\$0.05	0	\$0.05	\$0.1
\$1.10	\$0.90	0	−\$0.050	\$0.05	\$0.1
\$1.20	\$0.90	0	−\$0.150	\$0.05	\$0.1

We see that we are completely and perfectly hedged. Buying a collar where the put and call leg have equal strike prices perfectly offsets the copper price risk. Profit diagram:



Question 4.15.

If losses are tax deductible (and the company has additional income to which the tax credit can be applied), then each dollar of losses bears a tax credit of \$0.40. Therefore,

		Price = \$9	Price = \$11.20
(1)	Pre-Tax Operating Income	−\$1	\$ 1.20
(2)	Taxable Income	0	\$1.20
(3)	Tax @ 40%	0	\$0.48
(3b)	Tax Credit	\$0.40	0
	After-Tax Income (including Tax credit)	−\$0.60	\$0.72

In particular, this gives an expected after-tax profit of:

$$\text{E[Profit]} = 0.5 \times (-\$0.60) + 0.5 \times (\$0.72) = \$0.06$$

and the inefficiency is removed: We obtain the same payoffs as in the hedged case, Table 4.7.

Question 4.16.

a. Expected pre-tax profit

Firm A: $\text{E[Profit]} = 0.5 \times (\$1,000) + 0.5 \times (-\$600) = \$200$
Firm B: $\text{E[Profit]} = 0.5 \times (\$300) + 0.5 \times (\$100) = \$200$

Both firms have the same pre-tax profit.

b. Expected after tax profit.

Firm A:

		bad state	good state
(1)	Pre-Tax Operating Income	−$600	$1,000
(2)	Taxable Income	$0	$ 1,000
(3)	Tax @ 40%	0	$400
(3b)	Tax Credit	$240	0
	After-Tax Income (including Tax credit)	−$360	$600

This gives an expected after-tax profit for firm A of:

$$\text{E[Profit]} = 0.5 \times (-\$360) + 0.5 \times (\$600) = \$120$$

Firm B:

		bad state	good state
(1)	Pre-Tax Operating Income	$100	$300
(2)	Taxable Income	$100	$300
(3)	Tax @ 40%	$40	$120
(3b)	Tax Credit	0	0
	After-Tax Income (including Tax credit)	$60	$180

This gives an expected after-tax profit for firm B of:

$$\text{E[Profit]} = 0.5 \times (\$60) + 0.5 \times (\$180) = \$120$$

If losses receive full credit for tax losses, the tax code does not have an effect on the expected after-tax profits of firms that have the same expected pre-tax profits, but different cash-flow variability.

Question 4.17.

a. The pre-tax expected profits are the same as in exercise 4.16. a).

b. While the after-tax profits of company B stay the same, those of company A change, because they do not receive tax credit on the loss anymore.

c. We have for firm A:

		bad state	good state
(1)	Pre-Tax Operating Income	−$600	$1,000
(2)	Taxable Income	$0	$1,000
(3)	Tax @ 40%	0	$400
(3b)	Tax Credit	no tax credit	0
	After-Tax Income (including Tax credit)	−$600	$600

And consequently, an expected after-tax return for firm A of:

$$\text{E[Profit]} = 0.5 \times (-\$600) + 0.5 \times (\$600) = \$0$$

Company B would not pay anything, because it makes always positive profits, which means that the lack of a tax credit does not affect them.

Company A would be willing to pay the discounted difference between its after-tax profits calculated in 4.16. b), and its new after-tax profits, \$0 from 4.17. It is thus willing to pay: $\$120 \div 1.1 = \$109.09$.

Chapter 5. Financial Forwards and Futures

Question 5.4.

This question asks us to familiarize ourselves with the forward valuation equation.

a. We plug the continuously compounded interest rate and the time to expiration in years into the valuation formula and notice that the time to expiration is 6 months, or 0.5 years. We have:

$$F_{0,T} = S_0 \times e^{r \times T} = \$35 \times e^{0.05 \times 0.5} = \$35 \times 1.0253 = \$35.886$$

b. The annualized forward premium is calculated as:

$$\text{annualized forward premium} = \frac{1}{T} \ln \left(\frac{F_{0,T}}{S_0} \right) = \frac{1}{0.5} \ln \left(\frac{\$35.50}{\$35} \right) = 0.0284$$

c. For the case of continuous dividends, the forward premium is simply the difference between the risk-free rate and the dividend yield:

$$\begin{aligned} \text{annualized forward premium} &= \frac{1}{T} \ln \left(\frac{F_{0,T}}{S_0} \right) = \frac{1}{T} \ln \left(\frac{S_0 \times e^{(r-\delta)T}}{S_0} \right) \\ &= \frac{1}{T} \ln \left(e^{(r-\delta)T} \right) = \frac{1}{T} (r - \delta) T \\ &= r - \delta \end{aligned}$$

Therefore, we can solve:

$$\begin{aligned} & 0.0284 = 0.05 - \delta \\ \Leftrightarrow \ & \delta \quad\quad = 0.0216 \end{aligned}$$

The annualized dividend yield is 2.16 percent.

Question 5.10.

a. We plug the continuously compounded interest rate, the forward price, the initial index level and the time to expiration in years into the valuation formula and solve for the dividend yield:

$$\begin{aligned} & F_{0,T} = S_0 \times e^{(r-\delta) \times T} \\ \Leftrightarrow \ & \tfrac{F_{0,T}}{S_0} = e^{(r-\delta) \times T} \\ \Leftrightarrow \ & \ln \left(\tfrac{F_{0,T}}{S_0} \right) = (r - \delta) \times T \\ \Leftrightarrow \ & \delta = r - \tfrac{1}{T} \ln \left(\tfrac{F_{0,T}}{S_0} \right) \\ \Rightarrow \ & \delta = 0.05 - \tfrac{1}{0.75} \ln \left(\tfrac{1129.257}{1100} \right) = 0.05 - 0.035 = 0.015 \end{aligned}$$

b. With a dividend yield of only 0.005, the fair forward price would be:

$$F_{0,T} = S_0 \times e^{(r-\delta)\times T} = 1{,}100 \times e^{(0.05-0.005)\times 0.75} = 1{,}100 \times 1.0343 = 1{,}137.759$$

Therefore, if we think the dividend yield is 0.005, we consider the observed forward price of 1,129.257 to be too cheap. We will therefore buy the forward and create a synthetic short forward, capturing a certain amount of \$8.502. We engage in a reverse cash and carry arbitrage:

Description	Today	In 9 months
Long forward	0	$S_T - \$1{,}129.257$
Sell short tailed position in index	$\$1{,}100 \times .99626 = \$1{,}095.88$	$-S_T$
Lend \$1,095.88	$-\$1{,}095.88$	\$1,137.759
TOTAL	0	\$8.502

c. With a dividend yield of 0.03, the fair forward price would be:

$$F_{0,T} = S_0 \times e^{(r-\delta)\times T} = 1{,}100 \times e^{(0.05-0.03)\times 0.75} = 1{,}100 \times 1.01511 = 1{,}116.62$$

Therefore, if we think the dividend yield is 0.03, we consider the observed forward price of 1,129.257 to be too expensive. We will therefore sell the forward and create a synthetic long forward, capturing a certain amount of \$12.637. We engage in a cash and carry arbitrage:

Description	Today	In 9 months
Short forward	0	$\$1{,}129.257 - S_T$
Buy tailed position in index	$-\$1{,}100 \times .97775 = -\$1{,}075.526$	S_T
Borrow \$1,075.526	\$1,075.526	\$1,116.62
TOTAL	0	\$12.637

Question 5.12.

a. The notional value of 10 contracts is $10 \times \$250 \times 950 = \$2{,}375{,}000$, because each index point is worth \$250, we buy 10 contracts and the S&P 500 index level is 950.

With an initial margin of 10% of the notional value, this results in an initial dollar margin of $\$2{,}375{,}000 \times 0.10 = \$237{,}500$.

b. We first obtain an approximation. Because we have a 10% initial margin, a 2% decline in the futures price will result in a 20% decline in margin. As we will receive a margin call after a 20% decline in the initial margin, the smallest futures price that avoids the maintenance margin call is $950 \times .98 = 931$. However, this calculation ignores the interest that we are able to earn in our margin account.

Let us now calculate the details. We have the right to earn interest on our initial margin position. As the continuously compounded interest rate is currently 6%, after one week, our initial margin has grown to:

$$\$237{,}500 e^{0.06\times\frac{1}{52}} = \$237{,}774.20$$

We will get a margin call if the initial margin falls by 20%. We calculate 80% of the initial margin as:

$$\$237{,}500 \times 0.8 = \$190{,}000$$

10 long S&P 500 futures contracts obligate us to pay \$2,500 times the forward price at expiration of the futures contract.

Therefore, we have to solve the following equation:

$$\begin{aligned}
& \$237{,}774.20 + (F_{1W} - 950) \times \$2{,}500 && \geq \$190{,}000 \\
\Leftrightarrow\ & \$47774.20 && \geq -(F_{1W} - 950) \times \$2{,}500 \\
\Leftrightarrow\ & 19.10968 - 950 && \geq -F_{1W} \\
\Leftrightarrow\ & F_{1W} && \geq 930.89
\end{aligned}$$

Therefore, the greatest S&P 500 index futures price at which we will receive a margin call is 930.88.

Question 5.15.

a. We use the transaction cost boundary formulas that were developed in the text and in exercise 5.14. In this part, we set k equal to zero. There is no bid-ask spread. Therefore, we have

$$\begin{aligned}
F^{+} &= 800e^{0.055} = 800 \times 1.05654 = 845.23 \\
F^{-} &= 800e^{0.05} = 800 \times 1.051271 = 841.02
\end{aligned}$$

b. Now, we will incur an additional transaction fee of \$1 for going either long or short the forward contract. Stock sales or purchases are unaffected. We calculate:

$$\begin{aligned}
F^{+} &= (800 + 1)\, e^{0.055} = 801 \times 1.05654 = 846.29 \\
F^{-} &= (800 - 1)\, e^{0.05} = 799 \times 1.051271 = 839.97
\end{aligned}$$

c. Now, we will incur an additional transaction fee of \$2.40 for the purchase or sale of the index, making our total initial transaction cost \$3.40. We calculate:

$$\begin{aligned}
F^{+} &= (800 + 3.40)\, e^{0.055} = 803.40 \times 1.05654 = 848.82 \\
F^{-} &= (800 - 3.40)\, e^{0.05} = 796.60 \times 1.051271 = 837.44
\end{aligned}$$

d. We also have to take into account as well the additional cost that we incur at the time of expiration. We can calculate:

$$\begin{aligned}
F^{+} &= (800 + 3.40)\, e^{0.055} + 2.40 = 803.40 \times 1.05654 = 851.22 \\
F^{-} &= (800 - 3.40)\, e^{0.05} - 2.40 = 796.60 \times 1.051271 = 835.04
\end{aligned}$$

e. Let us make use of the hint. In the cash and carry arbitrage, we will buy the index and have thus at expiration time S_T. However, we have to pay a proportional transaction cost of 0.3% on it, so that the position is only worth $0.997 \times S_T$. However, we need S_T to set off the index price risk introduced by the short forward. Therefore, we will initially buy 1.003 units of the index, which leaves us exactly S_T after transaction costs. Additionally, we incur a transaction cost of $0.003 \times S_0$ for buying the index today, and of \$1 for selling the forward contract.

$$F^{+} = (800 \times 1.003 + 800 \times 0.003 + 1)\, e^{0.055} = 805.80 \times 1.05654 = 851.36$$

The boundary is slightly higher, because we must take into account the variable, proportional cash settlement cost we incur at expiration. The difference between part d) and part e) is the interest we have to pay on \$2.40, which is \$.14.

In the reverse cash and carry arbitrage, we will sell the index and have to pay back at expiration $-S_T$. However, we have to pay a proportional transaction cost of 0.3% on it, so that we have exposure of $-1.003 \times S_T$. However, we only need an exposure of $-S_T$ to set off the index price risk introduced by the long forward. Therefore, we will initially only sell 0.997 units of the index, which leaves us exactly with $-S_T$ after transaction costs at expiration. Additionally, we incur a transaction cost of $0.003 \times S_0$ for buying the index today, and \$1 for selling the forward contract. We have as a new lower bound:

$$F^- = (800 \times 0.997 - 800 \times 0.003 - 1)\, e^{0.05} = 794.20 \times 1.051271 = 834.92$$

The boundary is slightly lower, because we forego some interest we could earn on the short sale, because we have to take into account the proportional cash settlement cost we incur at expiration. The difference between part d) and part e) is the interest we are foregoingon \$2.40, which is \$.12 (at the lending rate of 5%).

Question 5.18.

The current exchange rate is 0.02E/Y, which implies 50Y/E. The euro continuously compounded interest rate is 0.04, the yen continuously compounded interest rate 0.01. Time to expiration is 0.5 years. Plug the input variables into the formula to see that:

$$\text{Euro/Yen forward} = 0.02e^{(0.04-0.01)\times 0.5} = 0.02 \times 1.015113 = 0.020302$$
$$\text{Yen/Euro forward} = 50e^{(0.01-0.04)\times 0.5} = 50 \times 0.98511 = 49.2556$$

Chapter 7. Interest Rate Forwards and Futures

Question 7.3.

Maturity	Zero-Coupon Bond Yield	Zero Coupon Bond Price	One-Year Implied Forward Rate	Par Coupon	Cont. Comp. Zero Yield
1	0.03000	0.97087	0.03000	0.03000	0.02956
2	0.03500	0.93351	0.04002	0.03491	0.03440
3	0.04000	0.88900	0.05007	0.03974	0.03922
4	0.04500	0.83856	0.06014	0.04445	0.04402
5	0.05000	0.78353	0.07024	0.04903	0.04879

Question 7.6.

In order to be able to solve this problem, it is best to take equation (7.6) of the main text and solve progressively for all zero-coupon bond prices, starting with year one. This yields the series of zero-coupon bond prices from which we can proceed as usual to determine the yields.

Maturity	Zero-Coupon Bond Yield	Zero Coupon Bond Price	One-Year Implied Forward Rate	Par Coupon	Cont. Comp. Zero Yield
1	0.03000	0.97087	0.03000	0.03000	0.02956
2	0.03500	0.93352	0.04002	0.03491	0.03440
3	0.04000	0.88899	0.05009	0.03974	0.03922
4	0.04700	0.83217	0.06828	0.04629	0.04593
5	0.05300	0.77245	0.07732	0.05174	0.05164

Question 7.8.

a. We have to take into account the interest we (or our counterparty) can earn on the FRA settlement if we settle the loan on initiation day, and not on the actual repayment day. Therefore, we tail the FRA settlement by the prevailing market interest rate of 5%. The dollar settlement is:

$$\frac{(r_{annually} - r_{FRA})}{1 + r_{annually}} \times \text{notional principal} = \frac{(0.05 - 0.06)}{1 + 0.05} \times \$500{,}000.00 = -\$4{,}761.905$$

b. If the FRA is settled on the date the loan is repaid (or settled in arrears), the settlement amount is determined by:

$$(r_{annually} - r_{FRA}) \times \text{notional principal} = (0.05 - 0.06) \times \$500{,}000.00 = -\$5{,}000$$

We have to pay at the settlement, because the interest rate we could borrow at is 5%, but we have agreed via the FRA to a borrowing rate of 6%. Interest rates moved in an unfavorable direction.

Question 7.9.

a. We have to take into account the interest we (or our counterparty) can earn on the FRA settlement if we settle the loan on initiation day, and not on the actual repayment day. Therefore, we tail the FRA settlement by the prevailing market interest rate of 7.5%. The dollar settlement is:

$$\frac{(r_{annually} - r_{FRA})}{1 + r_{annually}} \times \text{notional principal} = \frac{(0.075 - 0.06)}{1 + 0.075} \times \$500{,}000.00 = \$6{,}976.744$$

b. If the FRA is settled on the date the loan is repaid (or settled in arrears), the settlement amount is determined by:

$$(r_{annually} - r_{FRA}) \times \text{notional principal} = (0.05 - 0.06) \times \$500{,}000.00 = -\$5{,}000$$

We receive money at the settlement, because our hedge pays off. The market interest rate has gone up, making borrowing more expensive. We are compensated for this loss through the insurance that the short position in the FRA provides.

Question 7.12.

We can find the implied forward rate using the following formula:

$$[1 + r_0\,(t, t+s)] = \frac{P(0,t)}{P(0,t+s)}$$

With the numbers of the exercise, this yields:

$$r_0(270, 360) = \frac{0.96525}{0.95238} - 1 = 0.0135135$$

The following table follows the textbook in looking at forward agreements from a borrower's perspective, i.e. a borrower goes long on an FRA to hedge his position, and a lender is thus short the FRA. Since we are the counterparty for a lender, we are in fact the borrower, and thus long the forward rate agreement.

Transaction today	$t = 0$	$t = 270$	$t = 360$
Enter long FRAU		10M	$-10M \times 1.013514$ $= -10.13514M$
Sell 9.6525M Zero Coupons maturing at time $t = 180$	9.6525M	−10M	
Buy $(1 + 0.013514) * 10M * 0.95238$ Zero Coupons maturing at time $t = 360$	$-10M \times 1.013514$ $\times 0.95238 = -9.6525M$		$+10.13514M$
TOTAL	0	0	0

By entering in the above mentioned positions, we are perfectly hedged against the risk of the FRA. Please note that we are making use of the fact that interest rates are perfectly predictable.

Question 7.15.

a. Let us follow the suggestion of the problem and buy the 2-year zero-coupon bond. We will create a synthetic lending opportunity at the zero-coupon implied forward rate of 7.00238% and we will finance it by borrowing at 6.8%, thus creating an arbitrage opportunity. In particular, we will have:

Transaction today	$t = 0$	$t = 1$	$t = 2$
Buy 1.0700237 two-year zero-coupon bonds	−0.881659 *1.0700237 = −0.943396	0	1.0700237
Sell 1 one-year zero coupon bond	+0.943396	−1	
Borrow 1 from year one to year two @ 6.8%		+1	−1.06800
TOTAL	0	0	0.0020237

We see that we have created something out of nothing, without any risk involved. We have found an arbitrage opportunity.

b. Let us follow the suggestion of the problem and sell the 2-year zero-coupon bond. We will create a synthetic borrowing opportunity at the zero-coupon implied forward rate of 7.00238% and we will lend at 7.2%, thus creating an arbitrage opportunity. In particular, we will have:

Transaction today	$t = 0$	$t = 1$	$t = 2$
Sell 1.0700237 two-year zero-coupon bonds	0.881659 *1.0700237 = 0.943396	0	−1.0700237
Buy 1 one-year zero coupon bond	−0.943396	+1	
Lend 1 from year one to year two @ 7.2%		−1	+1.07200
TOTAL	0	0	0.0019763

We see that we have created something out of nothing, without any risk involved. We have indeed found an arbitrage opportunity.

Question 7.16.

a. The implied LIBOR of the September Eurodollar futures of 96.4 is: $\dfrac{100-96.4}{400} = 0.9\%$

b. As we want to borrow money, we want to buy protection against high interest rates, which means low Eurodollar future prices. We will short the Eurodollar contract.

c. One Eurodollar contract is based on a $1 million 3-month deposit. As we want to hedge a loan of $50M, we will enter into 50 short contracts.

d. A true 3-month LIBOR of 1% means an annualized position (annualized by market conventions) of $1\% * 4 = 4\%$. Therefore, our 50 short contracts will pay:

$$[96.4 - (100 - 4) \times 100 \times \$25] \times 50 = \$50{,}000$$

The increase in the interest rate has made our loan more expensive. The futures position that we entered to hedge the interest rate exposure, compensates for this increase. In particular, we pay $\$50{,}000{,}000 \times 0.01 - \textit{payoff futures} = \$500{,}000 - \$50{,}000 = \$450{,}000$, which corresponds to the 0.9% we sought to lock in.

Chapter 8. Swaps

Question 8.3.

Since the dealer is paying fixed and receiving floating, she generates the cash-flows depicted in column 2. Suppose that the dealer enters into three short forward positions, one contract for each year of the active swap. Her payoffs are depicted in columns 3, and the aggregate net cash flow position is in column 4.

Year	Net Swap Payment	Short Forwards	Net Position
1	$S_1 - \$20.9519$	$\$20 - S_1$	-0.9519
2	$S_1 - \$20.9519$	$\$21 - S_1$	$+0.0481$
3	$S_1 - \$20.9519$	$\$22 - S_1$	$+1.0481$

We need to discount the net positions to year zero. We have:

$$PV\,(netCF) = \frac{-0.9519}{1.06} + \frac{0.0481}{(1.065)^2} + \frac{1.0481}{(1.07)^3} = 0.$$

Indeed, the present value of the net cash flow is zero.

Question 8.7.

Using formula 8.13., and plugging in the given zero-coupon prices and the given forward prices, we obtain the following per barrel swap prices:

Quarter	Zero-bond	Swap price
1	0.9852	21.0000
2	0.9701	21.0496
3	0.9546	20.9677
4	0.9388	20.8536
5	0.9231	20.7272
6	0.9075	20.6110
7	0.8919	20.5145
8	0.8763	20.4304

Question 8.10.

We use equation (8.6) of the main text to answer this question:

$$X = \frac{\sum_{i=1}^{8} Q_{t_i} P_0\left(0, t_i\right) F_{0,t_i}}{\sum_{i=1}^{8} Q_{t_i} P_0\left(0, t_i\right)}, \text{where} Q = [1,2,1,2,1,2,1,2]$$

After plugging in the relevant variables given in the exercise, we obtain a value of \$20.4099 for the swap price.

Question 8.13.

From the given zero-coupon bond prices, we can calculate the one-quarter forward interest rates. They are:

Quarter	Forward interest rate
1	1.0150
2	1.0156
3	1.0162
4	1.0168
5	1.0170
6	1.0172
7	1.0175
8	1.0178

Now, we can calculate the deferred swap price according to the formula:

$$X = \frac{\sum_{i=2}^{6} P_0\left(0, t_i\right) r_0\left(t_{i-1}, t_i\right)}{\sum_{i=2}^{6} P_0\left(0, t_i\right)} = 1.66\%$$

Question 8.14.

From the given zero-coupon bond prices, we can calculate the one-quarter forward interest rates. They are:

Quarter	Forward interest rate
1	1.0150
2	1.0156
3	1.0162
4	1.0168
5	1.0170
6	1.0172
7	1.0175
8	1.0178

Now, we can calculate the swap prices for 4 and 8 quarters according to the formula:

$$X = \frac{\sum_{i=1}^{n} P_0\left(0, t_i\right) r_0\left(t_{i-1}, t_i\right)}{\sum_{i=1}^{n} P_0\left(0, t_i\right)}, \text{ where } n = 4 \text{ or } 8$$

This yields the following prices:

4-quarter fixed swap price:	1.59015%
8-quarter fixed swap price:	1.66096%

Question 8.15.

We can calculate the value of an 8-quarter dollar annuity that is equivalent to an 8-quarter Euro annuity by using equation 8.8. of the main text. We have:

$$X = \frac{\sum_{i=1}^{8} P_0\left(0, t_i\right) R^* F_{0,t_i}}{\sum_{i=1}^{8} P_0\left(0, t_i\right),} \text{ where } R^* \text{ is the Euro annuity of 1 Euro.}$$

Plugging in the forward price for one unit of Euros delivered at time t_i, which are given in the price table, yields a dollar annuity value of \$0.9277.

Question 8.17.

We can use the standard swap price formula for this exercise, but we must pay attention to taking the right zero-coupon bonds, and the right Euro-denominated forward interest rates. From the given Euro zero-coupon bond prices, we can calculate the one-quarter forward interest rates. They are:

Quarter	Euro denominated implied forward interest rate
1	0.0088
2	0.0090
3	0.0092
4	0.0095
5	0.0096
6	0.0097
7	0.0098
8	0.0100

Now, we can calculate the swap prices for 4 and 8 quarters according to the formula:

$$X = \frac{\sum_{i=1}^{n} P_0\left(0, t_i\right) r_0\left(t_{i-1}, t_i\right)}{\sum_{i=1}^{n} P_0\left(0, t_i\right),} \text{ where } n = 4 \text{ or } 8$$

This yields the following prices:

4-quarter fixed swap price:	0.91267%
8-quarter fixed swap price:	0.94572%

Chapter 9. Parity and Other Option Relationships

Question 9.9.

Both equations (9.15) and (9.16) of the textbook are violated. We use a call bear spread and a put bull spread to profit from these arbitrage opportunities.

		Expiration or Exercise		
Transaction	$t=0$	$S_T < 50$	$50 \leq S_T \leq 55$	$S_T > 55$
Buy 55 strike call	-10	0	0	$S_T - 55$
Sell 50 strike call	$+16$	0	$50 - S_T$	$50 - S_T$
TOTAL	$+6$	0	$50 - S_T > -5$	-5

		Expiration or Exercise		
Transaction	$t=0$	$S_T < 50$	$50 \leq S_T \leq 55$	$S_T > 55$
Buy 50 strike put	-7	$50 - S_T$	0	0
Sell 55 strike put	14	$S_T - 55$	$S_T - 55$	0
TOTAL	$+7$	-5	$S_T - 55 > -5$	0

Please note that we initially receive more money than our biggest possible exposure in the future. Therefore, we have found an arbitrage possibility, independent of the prevailing interest rate.

Question 9.10.

Both equations (9.17) and (9.18) of the textbook are violated. To see this, let us calculate the values. We have:

$$\frac{C(K_1) - C(K_2)}{K_2 - K_1} = \frac{18 - 14}{55 - 50} = 0.8 \quad \text{and} \quad \frac{C(K_2) - C(K_3)}{K_3 - K_2} = \frac{14 - 9.50}{60 - 55} = 0.9,$$

which violates equation (9.17) and

$$\frac{P(K_2) - P(K_1)}{K_2 - K_1} = \frac{10.75 - 7}{55 - 50} = 0.75 \quad \text{and} \quad \frac{P(K_3) - P(K_2)}{K_3 - K_2} = \frac{14.45 - 10.75}{60 - 55} = 0.74,$$

which violates equation (9.18).

We calculate lambda in order to know how many options to buy and sell when we construct the butterfly spread that exploits this form of mispricing. Because the strike prices are symmetric around 55, lambda is equal to 0.5.

Therefore, we use a call and put butterfly spread to profit from these arbitrage opportunities.

Transaction	$t=0$	$S_T < 50$	$50 \leq S_T \leq 55$	$55 \leq S_T \leq 60$	$S_T > 60$
Buy 1 50 strike call	-18	0	$S_T - 50$	$S_T - 50$	$S_T - 50$
Sell 2 55 strike calls	$+28$	0	0	$110 - 2 \times S_T$	$110 - 2 \times S_T$
Buy 1 60 strike call	-9.50	0	0	0	$S_T - 60$
TOTAL	$+0.50$	0	$S_T - 50 \geq 0$	$60 - S_T \geq 0$	0

Transaction	$t=0$	$S_T < 50$	$50 \leq S_T \leq 55$	$55 \leq S_T \leq 60$	$S_T > 60$
Buy 1 50 strike put	-7	$50 - S_T$	0	0	0
Sell 2 55 strike puts	21.50	$2 \times S_T - 110$	$2 \times S_T - 110$	0	0
Buy 1 60 strike put	-14.45	$60 - S_T$	$60 - S_T$	$60 - S_T$	0
TOTAL	$+0.05$	0	$S_T - 50 \geq 0$	$60 - S_T \geq 0$	0

Please note that we initially receive money and have non-negative future payoffs. Therefore, we have found an arbitrage possibility, independent of the prevailing interest rate.

Question 9.12.

a) Equation (9.15) of the textbook is violated. We use a call bear spread to profit from this arbitrage opportunity.

		Expiration or Exercise		
Transaction	$t = 0$	$S_T < 90$	$90 \leq S_T \leq 95$	$S_T > 95$
Sell 90 strike call	$+10$	0	$90 - S_T$	$90 - S_T$
Buy 95 strike call	-4	0	0	$S_T - 95$
TOTAL	$+6$	0	$90 - S_T > -5$	-5

Please note that we initially receive more money than our biggest possible exposure in the future. Therefore, we have found an arbitrage possibility, independent of the prevailing interest rate.

b) Now, equation (9.15) is not violated anymore. However, we can still construct an arbitrage opportunity, given the information in the exercise. We continue to sell the 90-strike call and buy the 95-strike call, and we loan our initial positive net balance for two years until expiration. It is important that the options be European, because otherwise we would not be able to tell whether the 90-strike call could be exercised against us sometime (note that we do not have information regarding any dividends).

We have the following arbitrage table:

		Expiration $t = T$		
Transaction	$t = 0$	$S_T < 90$	$90 \leq S_T \leq 95$	$S_T > 95$
Sell 90 strike call	$+10$	0	$90 - S_T$	$90 - S_T$
Buy 95 strike call	-5.25	0	0	$S_T - 95$
Loan 4.75	-4.75	5.80	5.80	5.8
TOTAL	0	5.80	$95.8 - S_T > 0$	$+0.8$

In all possible future states, we have a strictly positive payoff. We have created something out of nothing—we demonstrated arbitrage.

c) We will first verify that equation (9.17) is violated. We have:

$$\frac{C(K_1) - C(K_2)}{K_2 - K_1} = \frac{15 - 10}{100 - 90} = 0.5 \quad \text{and} \quad \frac{C(K_2) - C(K_3)}{K_3 - K_2} = \frac{10 - 6}{105 - 100} = 0.8,$$

which violates equation (9.17).

We calculate lambda in order to know how many options to buy and sell when we construct the butterfly spread that exploits this form of mispricing. Using formula (9.19), we can calculate that lambda is equal to 1/3. To buy and sell round lots, we multiply all the option trades by 3.

We use an asymmetric call and put butterfly spread to profit from these arbitrage opportunities.

Transaction	$t = 0$	$S_T < 90$	$90 \leq S_T \leq 100$	$100 \leq S_T \leq 105$	$S_T > 105$
Buy 1 90 strike calls	-15	0	$S_T - 90$	$S_T - 90$	$S_T - 90$
Sell 3 100 strike calls	$+30$	0	0	$300 - 3 \times S_T$	$300 - 3 \times S_T$
Buy 2 105 strike calls	-12	0	0	0	$2 \times S_T - 210$
TOTAL	$+3$	0	$S_T - 90 \geq 0$	$210 - 2 \times S_T \geq 0$	0

We indeed have an arbitrage opportunity.

Chapter 10. Binomial Option Pricing: I

Question 10.1.

Using the formulas given in the main text, we calculate the following values:

a) for the European call option:

$$\Delta = 0.5$$
$$B = -38.4316$$
$$price = 11.5684$$

b) for the European put option:

$$\Delta = -0.5$$
$$B = 62.4513$$
$$price = 12.4513$$

Question 10.5.

$S(0) = 80$:

	$t = 0,\ S = 80$	$t = 1,\ S = 64$	$t = 1,\ S = 104$
delta	0.4651	0	0.7731
B	-28.5962	0	-61.7980
premium	8.6078	0	18.6020

$S(0) = 90$:

	$t = 0,\ S = 90$	$t = 1,\ S = 72$	$t = 1,\ S = 117$
delta	0.5872	0	0.9761
B	-40.6180	0	-87.7777
premium	12.2266	0	26.4223

$S(0) = 110$:

	$t = 0,\ S = 110$	$t = 1,\ S = 88$	$t = 1,\ S = 143$
delta	0.7772	0.4409	1
B	-57.0897	-29.8229	-91.2750
premium	28.4060	8.9771	51.7250

$S(0) = 120$:

	$t = 0$, $S = 120$	$t = 1$, $S = 96$	$t = 1$, $S = 156$
delta	0.8489	0.6208	1
B	−65.0523	−45.8104	−91.2750
premium	36.8186	13.7896	64.7250

As the initial stock price increases, the 95-strike call option is increasingly in the money. With everything else equal, it is more likely that the option finishes in the money. A hedger, e.g., a market maker, must therefore buy more and more shares initially to be able to cover the obligation she will have to meet at expiration. This number of shares in the replicating portfolio is measured by delta. The initial call delta thus increases when the initial stock price increases.

Question 10.10.

a) We can calculate for the different nodes of the tree:

	node uu	node ud = du	node dd
delta	1	0.8966	0
B	−92.5001	−79.532	0
call premium	56.6441	15.0403	0
value of early exercise	54.1442	10.478	0

Using these values at the previous node and at the initial node yields:

	$t = 0$	node d	node u
delta	0.7400	0.4870	0.9528
B	−55.7190	−35.3748	−83.2073
call premium	18.2826	6.6897	33.1493
value of early exercise	5	0	27.1250

Please note that in all instances the value of immediate exercise is smaller than the continuation value, the (European) call premium. Therefore, the value of the European call and the American call are identical.

b) We can calculate similarly the binomial prices at each node in the tree. We can calculate for the different nodes of the tree:

	node uu	node $ud = du$	node dd
delta	0	−0.1034	−1
B	0	12.968	92.5001
put premium	0	2.0624	17.904
value of early exercise	0	0	20.404

Using these values at the previous node and at the initial node yields:

	$t = 0$	node d	node u
delta	−0.26	−0.513	−0.047
B	31.977	54.691	6.859
put premium	5.979	10.387	1.091
value of early exercise	0	8.6307	0

c) From the previous tables, we can see that at the node dd, it is optimal to early exercise the American put option, because the value of early exercise exceeds the continuation value. Therefore, we must use the value of 20.404 in all relevant previous nodes when we back out the prices of the American put option. We have for nodes d and 0 (the other nodes remain unchanged):

	$t = 0$	node d
delta	-0.297	-0.594
B	36.374	63.005
put premium	6.678	11.709
value of early exercise	0	8.6307

The price of the American put option is indeed 6.678.

Question 10.12.

a) We can calculate u and d as follows:

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.08)\times 0.25+0.3\times\sqrt{0.25}} = 1.1853$$
$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.08)\times 0.25-0.3\times\sqrt{0.25}} = 0.8781$$

b) We need to calculate the values at the relevant nodes in order to price the European call option:

	$t = 0$	node d	node u
delta	0.6074	0.1513	1
B	-20.187	-4.5736	-39.208
call premium	4.110	0.7402	8.204

c) We can calculate at the relevant nodes (or, equivalently, you can use put-call-parity for the European put option):

European put	$t = 0$	node d	node u
delta	-0.3926	-0.8487	0
B	18.245	34.634	0
put premium	2.5414	4.8243	0

For the American put option, we have to compare the premia at each node with the value of early exercise. We see from the following table that at the node d, it is advantageous to exercise the option early; consequently, we use the value of early exercise when we calculate the value of the put option.

American put	$t = 0$	node d	node u
delta	-0.3968	-0.8487	0
B	18.441	34.634	0
put premium	2.5687	4.8243	0
value of early exercise	0	4.8762	0

Question 10.14.

a) We can calculate the price of the call currency option in a very similar way to our previous calculations. We simply replace the dividend yield with the foreign interest rate in our formulas. Thus, we have:

	node uu	node $ud = du$	node dd
delta	0.9925	0.9925	0.1964
B	−0.8415	−0.8415	−0.1314
call premium	0.4734	0.1446	0.0150

Using these call premia at all previous nodes yields:

	$t = 0$	node d	node u
delta	0.7038	0.5181	0.9851
B	−0.5232	−0.3703	−0.8332
call premium	0.1243	0.0587	0.2544

The price of the European call option is \$0.1243.

b) For the American call option, the binomial approach yields:

	node uu	node $ud = du$	node dd
delta	0.9925	0.9925	0.1964
B	−0.8415	−0.8415	−0.1314
call premium	0.4734	0.1446	0.0150
value of early exercise	0.4748	0.1436	0

Using the maximum of the call premium and the value of early exercise at the previous nodes and at the initial node yields:

	$t = 0$	node d	node u
delta	0.7056	0.5181	0.9894
B	−0.5247	−0.3703	−0.8374
call premium	0.1245	0.0587	0.2549
value of early exercise	0.07	0	0.2540

The price of the American call option is: \$0.1245.

Question 10.17.

We have to pay attention when we calculate u and d. We must use the formulas given in the section options on futures contracts of the main text. In particular, we must remember that, while it is possible to calculate a delta, the option price is just the value of B, because it does not cost anything to enter into a futures contract.

We calculate:

$$u = e^{\sigma\sqrt{h}} = e^{0.1\sqrt{1}} = 1.1052$$

$$d = e^{-\sigma\sqrt{h}} = e^{-0.1\sqrt{1}} = 0.9048$$

Now, we are in a position to calculate the option's delta and B, and thus the option price. We have:

delta	0.6914
B	18.5883
premium	18.5883

This example clearly shows that the given argument is not correct. As it costs nothing to enter into the futures contract, we would not have to borrow anything if the statement was correct. We do not borrow to buy the underlying asset.

Rather, we borrow exactly the right amount so that we can, together with the position in the underlying asset, replicate the payoff structure of the call option in the future (remember that we initially solved the system of two equations).

Question 10.18.

a) We have to use the formulas of the textbook to calculate the stock tree and the prices of the options. Remember that while it is possible to calculate a delta, the option price is just the value of B, because it does not cost anything to enter into a futures contract. In particular, this yields the following prices: For the European call and put, we have: *premium* = 122.9537. The prices must be equal due to put-call-parity.

b) We can calculate for the American call option: *premium* = 124.3347 and for the American put option: *premium* = 124.3347.

c) We have the following time 0 replicating portfolios:

For the European call option:

Buy 0.5461 futures contracts.
Lend 122.9537

For the European put option:

Sell 0.4141 futures contracts.
Lend 122.9537

Chapter 11. Binomial Option Pricing: II

Question 11.1.

a) Early exercise occurs only at strike prices of 70 and 80. The value of the one period binomial European 70 strike call is \$23.24, while the value of immediate exercise is $100 - 70 = 30$. The value of the 80 strike European call is \$19.98, while the value of immediate exercise is \$20.

b) From put-call-parity, we observe the following:

$$C = Se^{-\delta} - Ke^{-r} + P = 100 \times 0.92311 - K + P = 92.31164 - K + P$$

Clearly, as long as $100 - K$ is larger than $92.31164 - K + P$ or $P < 7.688$, we will exercise the option early. At a strike of 90, P is already higher than 7.688. Therefore, $100 - K = 10$ is smaller than the continuation value given by the above formula, so we do not exercise early.

c) The value of a put falls when the strike price decreases. From part a), we learned that the decisive criterion was that $P < 7.688$. Therefore, once we cross this threshold, all other calls will be exercised as well.

Question 11.7.

a) We need to find the true probabilities for the stock going up. We will use formula (11.4) of the main text. To find u and d, note that $h = 1/10 = 0.1$.

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = e^{(0.08)\times 0.1+0.3\times\sqrt{0.1}} = 1.10835$$
$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = e^{(0.08)\times 0.1-0.3\times\sqrt{0.1}} = 0.91680$$
$$p = \frac{e^{ah}-d}{u-d} = \frac{1.015113-0.91680}{1.10835-0.91680} = 0.51325$$

Now, we can calculate the binomial tree (as in figure 11.4) and back out the option prices and the relevant discount rates (= required expected returns of the option holder). We obtain for the European Call (the first entry is the stock price, the second the option price and the third is the required rate of return):

										279.74
										179.74
									252.39	
									153.19	
								227.72	0.20	231.39
								129.31		131.39
							205.46	0.20	208.77	
							107.83		109.57	
						185.37	0.21	188.36	0.21	191.40
						88.52		89.95		91.40
					167.25	0.23	169.95	0.23	172.69	
					71.17		72.32		73.49	
				150.90	0.24	153.34	0.24	155.81	0.24	158.32
				55.79		56.49		57.40		58.32
			136.15	0.27	138.35	0.27	140.58	0.27	142.85	
			42.54		42.66		42.95		43.64	
		122.84	0.29	124.82	0.30	126.84	0.31	128.88	0.31	130.96
		31.52		31.13		30.74		30.47		30.96
	110.83	0.32	112.62	0.33	114.44	0.35	116.28	0.37	118.16	
	22.73		21.99		21.13		20.11		18.96	
100.00	0.34	101.61	0.36	103.25	0.39	104.92	0.43	106.61	0.51	108.33
15.96		15.07		14.01		12.71		11.00		8.33
0.37	91.68	0.39	93.16	0.42	94.66	0.47	96.19	0.57	97.74	
	10.06		9.01		7.75		6.16		3.94	
	0.42	84.05	0.46	85.41	0.51	86.78	0.61	88.18	0.83	89.61
		5.65		4.60		3.37		1.86		0.00
		0.48	77.06	0.54	78.30	0.64	79.56	0.83	80.85	
			2.67		1.81		0.88		0.00	
			0.57	70.65	0.66	71.79	0.83	72.94	0.00	74.12
				0.96		0.42		0.00		0.00
				0.68	64.77	0.83	65.81	0.00	66.88	
					0.20		0.00		0.00	
					0.83	59.38	0.00	60.34	0.00	61.31
						0.00		0.00		0.00
						0.00	54.44	0.00	55.32	
							0.00		0.00	
							0.00	49.91	0.00	50.72
								0.00		0.00
								0.00	45.76	
									0.00	
									0.00	41.95
										0.00

Note that the price is identical to the price we get via risk-neutral pricing with the BinomCall function.

For the European Put option, we have:

										279.74
										0.00
									252.39	
									0.00	
								227.72	0.00	231.39
								0.00		0.00
							205.46	0.00	208.77	
							0.00		0.00	
						185.37	0.00	188.36	0.00	191.40
						0.00		0.00		0.00
					167.25	0.00	169.95	0.00	172.69	
					0.00		0.00		0.00	
				150.90	0.00	153.34	0.00	155.81	0.00	158.32
				0.20		0.00		0.00		0.00
			136.15	−0.65	138.35	0.00	140.58	0.00	142.85	
			0.94		0.39		0.00		0.00	
		122.84	−0.49	124.82	−0.65	126.84	0.00	128.88	0.00	130.96
		2.48		1.62		0.76		0.00		0.00
	110.83	−0.37	112.62	−0.47	114.44	−0.65	116.28	0.00	118.16	
	4.94		3.92		2.77		1.46		0.00	
100.00	−0.28	101.61	−0.34	103.25	−0.45	104.92	−0.65	106.61	0.00	108.33
8.27		7.26		6.07		4.64		2.81		0.00
−0.21	91.68	−0.25	93.16	−0.31	94.66	−0.42	96.19	−0.65	97.74	
	11.43		10.41		9.17		7.61		5.40	
	−0.18	84.05	−0.22	85.41	−0.28	86.78	−0.38	88.18	−0.65	89.61
		15.39		14.51		13.43		12.09		10.39
		−0.16	77.06	−0.19	78.30	−0.23	79.56	−0.32	80.85	
			20.17		19.59		18.94		18.36	
			−0.12	70.65	−0.15	71.79	−0.18	72.94	−0.23	74.12
				25.62		25.48		25.47		25.88
				−0.09	64.77	−0.11	65.81	−0.12	66.88	
					31.51		31.81		32.33	
					−0.06	59.38	−0.07	60.34	−0.07	61.31
						37.47		38.07		38.69
						−0.03	54.44	−0.03	55.32	
							43.19		43.89	
							−0.01	49.91	−0.01	50.72
								48.50		49.28
								0.01	45.76	
									53.45	
									0.02	41.95
										58.05

b) If we increase the standard deviation of the stock return, we decrease the required return of the call option and increase the required (negative) return of the put option. The following table shows the required returns for the first two of the 10 nodes:

For the call option:		and for the put option:	
	118.07		118.07
	34.84		9.82
100.00	0.26	100.00	−0.12
23.28		15.59	
0.28	86.06	−0.07	86.06
	13.75		20.74
	0.31		−0.06

Question 11.16.

Stock Tree									233.621
								210.110	
							188.966		188.966
						169.949		169.949	
					152.847		152.847		152.847
				137.465		137.465		137.465	
			123.631		123.631		123.631		123.631
		111.190		111.190		111.190		111.190	
	100.000		95.000		100.000		100.000		100.000
		89.937		89.937		89.937		89.937	
			80.886		80.886		80.886		80.886
				72.746		72.746		72.746	
					65.425		65.425		65.425
						58.841		58.841	
							52.920		52.920
								47.594	
									42.804

European Call									138.621
								114.746	
							93.500		93.966
						74.600		74.836	
					57.798		57.829		57.847
				43.327		42.719		42.554	
			31.432		30.213		28.976		28.631
		22.118		20.571		18.736		16.442	
	15.140		13.563		11.670		9.221		5.000
		8.703		7.065		5.080		2.431	
			4.183		2.760		1.182		0.000
				1.483		0.575		0.000	
					0.280		0.000		0.000
						0.000		0.000	
							0.000		0.000
								0.000	
									0.000

American Call									138.621
								115.110	
							93.966		93.966
						74.949		74.949	
					57.981		57.884		57.847
				43.422		42.746		42.554	
			31.482		30.226		28.976		28.631
		22.144		20.578		18.736		16.442	
	15.153		13.566		11.670		9.221		5.000
		8.704		7.065		5.080		2.431	
			4.183		2.760		1.182		0.000
				1.483		0.575		0.000	
					0.280		0.000		0.000
						0.000		0.000	
							0.000		0.000
								0.000	
									0.000
European Put									0.000
								0.000	
							0.000		0.000
						0.000		0.000	
					0.000		0.000		0.000
				0.458		0.000		0.000	
			1.819		0.909		0.000		0.000
		4.265		3.169		1.804		0.000	
	7.713		6.711		5.414		3.582		0.000
		11.194		10.263		9.006		7.110	
			15.742		15.146		14.420		14.114
				21.342		21.373		21.762	
					27.745		28.506		29.575
						34.444		35.580	
							40.857		42.080
								46.757	
									52.196

American Put								0.000
							0.000	
						0.000		0.000
					0.000		0.000	
				0.000		0.000		0.000
			0.458		0.000		0.000	
		1.835		0.909		0.000		0.000
	4.353		3.201		1.804		0.000	
7.979		6.870		5.477		3.582		0.000
	11.637		10.548		9.131		7.110	
		16.468		15.651		14.668		14.114
			22.509		22.254		22.254	
				29.575		29.575		29.575
					36.159		36.159	
						42.080		42.080
							47.406	
								52.196

Question 11.17.

Stock Tree								230.142
							207.371	
						186.852		186.153
					168.363		167.733	
				151.704		151.137		150.571
			136.694		136.182		135.672	
		123.168		122.707		122.248		121.790
	110.981		110.566		110.152		109.740	
100.000		99.626		99.253		98.881		98.511
	89.768		89.432		89.097		88.764	
		80.583		80.281		79.981		79.682
			72.338		72.067		71.797	
				64.936		64.693		64.451
					58.292		58.074	
						52.328		52.132
							46.973	
								42.167

European Call

								135.142
							112.024	
						91.412		91.153
					73.044		72.634	
				56.684		56.140		55.571
			42.576		41.460		40.772	
		30.969		29.322		27.611		26.790
	21.865		19.981		17.771		15.001	
15.029		13.198		11.040		8.287		3.511
	8.494		6.680		4.529		1.738	
		3.960		2.453		0.861		0.000
			1.319		0.426		0.000	
				0.211		0.000		0.000
					0.000		0.000	
						0.000		0.000
							0.000	
								0.000

American Call

								135.142
							112.371	
						91.852		91.153
					73.363		72.733	
				56.854		56.190		55.571
			42.667		41.485		40.772	
		31.016		29.334		27.611		26.790
	21.890		19.987		17.771		15.001	
15.042		13.201		11.040		8.287		3.511
	8.496		6.680		4.529		1.738	
		3.960		2.453		0.861		0.000
			1.319		0.426		0.000	
				0.211		0.000		0.000
					0.000		0.000	
						0.000		0.000
							0.000	
								0.000

European Put								0.000
							0.000	
						0.000		0.000
					0.000		0.000	
				0.000		0.000		0.000
			0.455		0.000		0.000	
		1.801		0.919		0.000		0.000
	4.212		3.184		1.858		0.000	
7.602		6.707		5.512		3.753		0.000
	11.146		10.366		9.279		7.582	
		15.810		15.429		14.992		15.318
			21.573		21.890		22.705	
				28.153		29.229		30.549
					34.983		36.343	
						41.441		42.868
							47.374	
								52.833

American Put								0.000
							0.000	
						0.000		0.000
					0.000		0.000	
				0.000		0.000		0.000
			0.455		0.000		0.000	
		1.816		0.919		0.000		0.000
	4.303		3.214		1.858		0.000	
7.877		6.878		5.573		3.753		0.000
	11.610		10.681		9.401		7.582	
		16.576		16.005		15.239		15.318
			22.805		22.933		23.203	
				30.064		30.307		30.549
					36.708		36.926	
						42.672		42.868
							48.027	
								52.833

Question 11.20.

We will use the methodology introduced by Hull, which is described in the main textbook. We can calculate:

$u = 1.2005$	$S = 50.0000$	$K = 45.00$
$d = 0.8670$	$F = 46.0792$	dividend $= 4.0000$
$p = 0.4594$	$t = 1.0000$	$r = 0.0800$
$n = 4.0000$	$h = 0.2500$	sigma $= 0.3255$
		time to div 0.2500

American Call				
				95.7188
				50.7188
			79.7304	
			35.6215	
		66.4127		69.1230
		23.1772		24.1230
	59.3195		57.5771	
	14.3195		13.4682	
50.0000		47.9597		49.9170
8.4551		7.2380		4.9170
	43.3480		41.5791	
	3.7876		2.2141	
		34.6340		36.0474
		0.9970		0.0000
			30.0263	
			0.0000	
				26.0315
				0.0000

European Call				
				95.7188
				50.7188
			79.7304	
			35.6215	
		66.4127		69.1230
		23.1772		24.1230
	59.3195		57.5771	
	14.2721		13.4682	
50.0000		47.9597		49.9170
8.4338		7.2380		4.9170
	43.3480		41.5791	
	3.7876		2.2141	
		34.6340		36.0474
		0.9970		0.0000
			30.0263	
			0.0000	
				26.0315
				0.0000

Chapter 12. The Black-Scholes Formula

Question 12.3.

a)

T	Call-Price
1	7.8966
2	15.8837
5	34.6653
10	56.2377
50	98.0959
100	99.9631
500	100.0000

As T approaches infinity, the call approaches the value of the underlying stock price, signifying that over very long time horizons the call option is not distinguishable from the stock.

b) With a constant dividend yield of 0.001 we get:

T	Call-Price
1	7.8542
2	15.7714
5	34.2942
10	55.3733
50	93.2296
100	90.4471
500	60.6531

The owner of the call option is not entitled to receive the dividends paid on the underlying stock during the life of the option. We see that for short-term options, the small dividend yield does not play a large role. However, for the long term options, the continuous lack of the dividend payment hurts the option holder significantly, and the option value is not approaching the value of the underlying.

Question 12.4.

a)

T	Call Price
1	18.6705
2	18.1410
5	15.1037
10	10.1571
50	0.2938
100	0.0034
500	0.0000

The benefit to holding the call option is that we do not have to pay the strike price and that we continue to earn interest on the strike. On the other hand, the owner of the call option foregoes the dividend payments he could receive if he owned the stock. As the interest rate is zero and the dividend yield is positive, the cost of holding the call outweighs the benefits.

b)

T	Call Price
1	18.7281
2	18.2284
5	15.2313
10	10.2878
50	0.3045
100	0.0036
500	0.0000

Although the call option is worth marginally more when we introduce the interest rate of 0.001, it is still not enough to outweigh the cost of not receiving the huge dividend yield.

Question 12.5.

a) $P(95, 90, 0.1, 0.015, 0.5, 0.035) = 1.0483$

b) $C(1/95, 1/90, 0.1, 0.035, 0.5, 0.015) = 0.000122604$

c) The relation is easiest to see when we look at terminal payoffs. Denote the exchange rate at time t as $X_t = \frac{Y}{E}$.

Then the call option in b) pays (in Euro): $C = \max\left(\frac{1}{X_T} - \frac{1}{90\frac{Y}{E}}, 0\right)$. Let us convert this into yen:

$$C(\text{ in Yen}) = X_T \times \max\left(\frac{1}{X_T} - \frac{1}{90\frac{Y}{E}}, 0\right)$$

$$= \max\left(1 - \frac{X_T}{90\frac{Y}{E}}, 0\right) = \max\left(\frac{90 - X_T}{90\frac{Y}{E}}, 0\right) = \frac{1}{90\frac{Y}{E}} \times \max\left(90 - X_T, 0\right)$$

Therefore, the relationship between a) and b) at any time t should be:

$$P(95, \ldots) = X_t * 90 * C(1/95, \ldots).$$

Indeed, we have: $X_t * 90 * C(1/95, \ldots) = 0.000122604 * 95 * 90 = 1.0483 = P(95, \ldots)$

We conclude that a yen-denominated euro put has a one to one relation with a euro-denominated yen call.

Question 12.7.

a) $C(100, 95, 0.3, 0.08, 0.75, 0.03) = \$14.3863$

b) $S(\text{new}) = 100 * \exp(-0.03 * 0.75) = \$97.7751$
$K(\text{new}) = 95 * \exp(-0.08 * 0.75) = \$89.4676$
$C(97.7751, 89.4676, 0.3, 0, 0.75, 0) = \$14.3863$

This is a direct application of equation (12.5) of the main text. As the dividend yield enters the formula only to discount the stock price, we can take care of it by adapting the stock price before we plug it into the Black-Scholes formula. Similarly, the interest rate is only used to discount the strike price, which we did when we calculated K(new). Therefore, we can calculate the Black-Scholes call price by using S(new) and K(new) and by setting the interest rate and the dividend yield to zero.

Question 12.14.

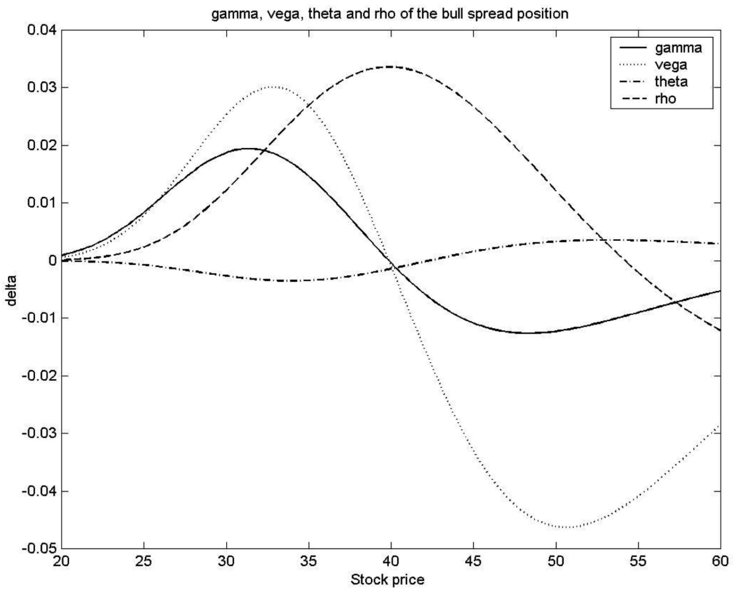
a) The greeks of the bull spread are simply the sum of the greeks of the individual options. The greeks of the call with a strike of 45 enter with a negative sign because this option was sold.

	Bought Call(40)	Sold Call(45)	Bull Spread
Price	4.1553	−2.1304	2.0249
Delta	0.6159	−0.3972	0.2187
Gamma	0.0450	−0.0454	−0.0004
Vega	0.1081	−0.1091	−0.0010
Theta	−0.0136	0.0121	−0.0014
Rho	0.1024	−0.0688	0.0336

b)

	Bought Call(40)	Sold Call(45)	Bull Spread
Price	7.7342	−4.6747	3.0596
Delta	0.8023	−0.6159	0.1864
Gamma	0.0291	−0.0400	−0.0109
Vega	0.0885	−0.0122	−0.0331
Theta	−0.0137	0.0152	0.0016
Rho	0.1418	−0.1152	0.0267

c) Because we simultaneously buy and sell an option, the graphs of gamma, vega and theta have inflection points (see figures below). Therefore, the initial intuition one may have had—that the greeks should be symmetric at $S = \$40$ and $S = \$45$—is not correct.



Question 12.20.

a) $C(100, 90, 0.3, 0.08, 1, 0.05) = 17.6988$

b) $P(90, 100, 0.3, 0.05, 1, 0.08) = 17.6988$

c) The prices are equal. This is a result of the mathematical equivalence of the pricing formulas. To see this, we need some algebra. We start from equation (12.3) of the text, the formula for the European put option:

$$P(\bullet) = \overline{K} \times \exp\left(-\overline{r}T\right) \times N\left(-\frac{\ln\left(\frac{\overline{S}}{\overline{K}}\right)+\left(\overline{r}-\overline{\delta}-0.5\sigma^2\right)T}{\sigma\sqrt{T}}\right) - \overline{S} \times \exp\left(-\overline{\delta}T\right)$$

$$\times N\left(-\frac{\ln\left(\frac{\overline{S}}{\overline{K}}\right)+\left(\overline{r}-\overline{\delta}+0.5\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

Now we replace:

$$\overline{K} = S, \overline{r} = \delta, \overline{\delta} = r, \overline{S} = K$$

Then:

$$= S \times \exp\left(-\delta T\right) \times N\left(-\frac{\ln\left(\frac{K}{S}\right)+\left(\delta-r-0.5\sigma^2\right)T}{\sigma\sqrt{T}}\right) - K$$

$$\times \exp\left(-rT\right) \times N\left(-\frac{\ln\left(\frac{K}{S}\right)+\left(\delta-r+0.5\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

Since $\ln\left(\frac{K}{S}\right) = -\ln\left(\frac{S}{K}\right)$

$$= S \times \exp\left(-\delta T\right) \times N\left(\frac{\ln\left(\frac{S}{K}\right)-\left(\delta-r-0.5\sigma^2\right)T}{\sigma\sqrt{T}}\right) - K \times \exp\left(-rT\right)$$

$$\times N\left(\frac{\ln\left(\frac{S}{K}\right)-\left(\delta-r+0.5\sigma^2\right)T}{\sigma\sqrt{T}}\right)$$

$$= S \times \exp\left(-\delta T\right) \times N\left(d_1\right) - K \times \exp\left(-rT\right) \times N\left(d_2\right) = C(\bullet)$$

Chapter 13. Market-Making and Delta-Hedging

Question 13.1.

The delta of the option is .2815. To delta hedge writing 100 options we must purchase 28.15 shares for a delta hedge. The total value of this position is 1028.9 which is the amount we will initially borrow. If the next day's stock price is 39,

$$-28.15 + 26.56 - .23 = -1.82. \tag{1}$$

If S rises to 40.50, the change in stock value and option value will be the total profit:

$$14.08 - 13.36 - .23 = .49. \tag{2}$$

Question 13.3.

The unhedged delta will be 30.09 hence we have to short 30.09 shares of stock, receiving \$30.09 (40) = \$1203.60. The cost of taking the spread position is 100(2.7804 − 0.9710) = \$180.94. We can lend \$1203.60 − \$180.94 = \$1022.66. This implies will we earn interest (in one day) of

$$1022.66\left(e^{.08/365} - 1\right) = .2242 \approx .22$$

In the two scenarios, we have a profit of

$$30.09 - 30.04 + .22 = .27$$

if S falls to 39 and a profit of

$$-15.04 + 14.81 + .22 = -.01$$

if S rises to 40.5.

Question 13.14.

Using the given parameters, a six month 45-strike put has a price and Greeks of $P = 5.3659$, $\Delta = -.6028$, $\Gamma = .045446$, and $\Theta_{\text{per day}} = -.0025$. Note that Θ, as given in the software is a per day. Equation (13.9) uses annualized rates (i.e. Θh is in the equation. Hence for equation (13.9) we should use −.9139. For equation (13.9) we have a market-maker profit of

$$-\left(\frac{.09}{2}40^2\,(.045446) - .9139 + .08\left((-.6028)\,40 - 5.3659\right)\right)h \tag{3}$$
$$= -\left(3.2721 - .9139 - 2.3582\right)h = 0. \tag{4}$$

Chapter 14. Exotic Options: I

Question 14.6

a) A standard call is worth 4.1293.

b) A knock in call will also be worth 4.1293 (you can verify this with the software). In order for the standard call to ever be in the money, it must pass through the barrier. They therefore give identical payoffs.

c) Similar reasoning, implies the knock-out will be worthless since in order for $S_T > 45$, the barrier must have been hit making knocking out the option.

Question 14.11

a) 9.61
b) In one year, the option will be worth more than \$2 if $S_1 > 31.723$.
c) 7.95
d) If we buy the compound call in part b) and sell the compound option in this question for x we will be receiving the standard call in one year for \$2 regardless of S_1. Hence, our total cost is $7.95 - x + 2e^{-.08} = 9.61$, which implies $x = .18623$. Without rounding errors it would be .18453.

Question 14.12

a) 3.6956

b) In one year, the put option will be worth more than \$2 if $S_1 < 44.35$.

c) 2.2978

d) If we buy the standard put from part a) as well as this compound option for x we will keep the standard put if $S_1 < 44.35$ and sell it for \$2 otherwise. This identical to putting $2e^{-.08}$ in the risk free bond and buying the compound option in part c). The total costs must be identical implying $3.6956 + x = 2.2978 + 2e^{-.08}$, implying $x = .448$.

Chapter 15. Financial Engineering and Security Design

Question 15.1.

Let $R = e^{.06}$. The present value of the dividends is

$$R^{-1} + (1.50)\,R^{-2} + 2R^{-3} + (2.50)\,R^{-4} + 3R^{-5} = 8.1317. \tag{5}$$

The note originally sells for $100 - 8.1317 = 91.868$. With the 50 cent permanent increase, the present value of dividends rises by

$$\frac{R^{-1} + R^{-2} + R^{-3} + R^{-4} + R^{-5}}{2} = 2.0957 \tag{6}$$

to 10.2274 leading the note value to fall to $100 - 10.2274 = 89.773$.

Question 15.3.

a) $S_0 e^{-\delta T} = 1200e^{-.015(2)} = 1164.5$.

b) As in equation (15.5),

$$c = \frac{S_0 - F_T^P}{\sum_{i=1}^{8} P_{t_i}} = \frac{1200\left(1 - e^{-.015(2)}\right)}{7.4475} = 4.762. \tag{7}$$

c) As in the problem 15.2c, letting $D = e^{-.015/4}$,

$$c^* \left(\sum_{i=1}^{8} D^i\right) 1200 + 1200 D^8 = 1200 \Longrightarrow c^* = \frac{1 - D^8}{\sum_{i=1}^{8} D^i} = .003757 \text{ shares}, \tag{8}$$

which is currently worth $.003757\,(\$1200) = \$4.5084$.

Question 15.4.

The relevant 2 year interest rate is $\ln(1/.8763)/2 = 6.6\%$.

a) The embedded option is worth 247.88. The prepaid forward is worth $1200e^{-.015(2)} = 1164.53$. The bond price is worth the sum $1164.53 + 247.88 = 1412.41$.

b) λ must solve $1164.53 + \lambda 247.88 = 1200 \Longrightarrow \lambda = 35.47/247.88 = .1431$.

Question 15.6.

We continue to use 6.6% as the relevant 2 year interest rate.

a) The out of the money option (i.e. $K = 1500$) is worth 141.54, making the bond have a value of $1164.53 + 247.88 - 141.54 = 1270.9$.

b) We must solve $1164.53 + \lambda(247.88 - 141.54) = 1200$ for a solution of $\lambda = .3336$.

c) If $\lambda = 1$, we have to adjust the strike (from part a, we know we have to lower K) to make the out of the option worth $C(K) = 1164.53 + 247.88 - 1200 = 212.41 \Longrightarrow K \approx 1284$.