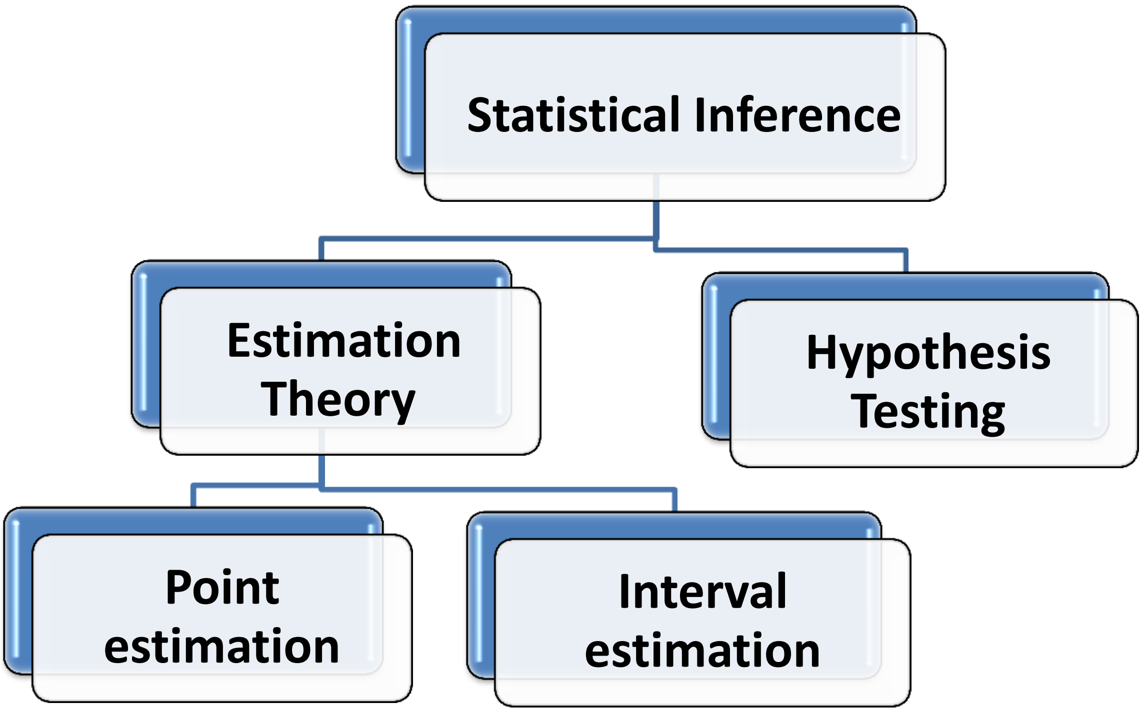
Examples Chapter(9)

Estimation and Confidence Intervals

GOALS

1. Define a *point estimate*.
2. Define *level of confidence*.
3. Construct a confidence interval for the population mean when the population standard deviation is known.
4. Construct a confidence interval for a population mean when the population standard deviation is unknown.
5. Construct a confidence interval for a population proportion.
6. Determine the sample size for attribute and variable sampling.

Definitions



1) Method for making inferences about population parameters fall into one of two categories:

a) Estimation:

Estimating or predicting the value of the parameter.

- A point estimate is a single value (point) derived from a sample and used to estimate a population value.
- A confidence interval estimate is a range of values constructed from sample data so that the population parameter is likely to occur within that range at a specified probability. The specified probability is called the level of confidence.

b) Hypothesis Testing:

Making decision about the value of a parameter based on some preconceived idea about what its value might be.

2) **The parameter θ:** is an index which characterizes the members of a given family of distribution.
3) **A statistic:** any function of the observations of a random sample.

$$\bar{X} = \frac{\sum X_i}{n} \text{ , is a statistic}$$

4) **The estimator:** theestimatorof a parameter θ is a rule, usually expressed as a formula that tells us how to calculate an estimate based on information in the sample.

We denote the estimate of θ by $\hat{\theta}$ (read "theta hat").

Ex .if μ : parameter $\therefore \hat{\mu}$:Estimator

5) **The estimate:** the estimate of θ from a given sample is the value of the estimator $\hat{\theta}$ using the observations of given sample.

parameter	point estimation
μ	$\hat{\mu} = \bar{X} = \frac{\sum X_i}{n}$
σ^2	$\hat{\sigma}^2 = S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$
π	$\hat{\pi} = P = \frac{X}{n}$

6) Constructing confidence interval

The general form of an interval estimate of a populationparameter:

Point Estimate ± Criticalvalue *Standard error

This formula generates two values called the confidence limits;

- Lower confidence limit (LCL).
- Upper confidence limit (UCL).

Confidence Interval for a Population Mean

Case1: Confidence Interval for Population Mean with known variance (normal case):

The confidence limits are:

$$\bar{X} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

Steps for calculating:

1. **Obtain** $Z_{\alpha/2}$ **,from the table of the area under the normal curve.**
2. **Calculate** $Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ **.**
3. **L=** $\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

   U= $\bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$\bar{X}$:The mean estimator

σ : The standard deviation of the population .

$\frac{\sigma}{\sqrt{n}}$: The standard error of the mean $(\sigma_{\bar{x}})$.

$\pm Z_{\frac{\alpha}{2}}$: **Critical value.**

Example (1) (exercises "1" p.297-ch9)

A sample of 49 observations is taken from a normal population with a standard deviation of 10.the sample mean is 55,determine the 99 percent confidence interval for the population mean

Solution:

$X \sim N(\mu, \sigma^2)$ $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ σ = 10 , n = 49, $\bar{X} = 55$

Confidence level = 0.99,

$\therefore\ Z_{\frac{0.99}{2}} = Z_{0.495} = 2.58$, The confidence limits are:

$$\bar{X} \pm Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 55 \pm 2.58\left(\frac{10}{\sqrt{49}}\right) = 55 \pm 3.69$$

$51.31 \le \hat{\mu} \le 58.69$

$(51.31\ ,\ 58.68)$

Case (2):

Confidence Interval for a Population Mean with unknown variance (Large Sample n≥30, normal case):

The confidence limits are:

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

S : Sample standard deviation (Estimate of σ)

Example (2)

A scientist interested in monitoring chemical contaminants in food, and thereby the accumulation of contaminants in human diets, selected a random sample of n=50 male adults. it was found that the average daily intake of dairy products was $\bar{X} = 756$ grams per day with a standard deviation of35 grams per day .use this sample information to construct a 95% confidence interval for the mean daily intake of dairy products for men.

Solution:

n =50 S=35 $\bar{X} = 756$, confidence coefficient = 0.95

∴ Z =1.96

Since n≥30 (large), the distribution of the sample mean $\bar{X}$ is approximately normally distributed:

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

The confidence limits are:

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 756 \pm 1.96\left(\frac{35}{\sqrt{50}}\right) = 756 \pm 9.70$$

$$746.3 \le \hat{\mu} \le 765.7$$

$$(746.3 \ , \ 765.7)$$

Hence, the 95% confidence interval for μ is from 746.3 to 756.7 grams per day.

- IF you have (746.3, 765.7). Based on this information, you know that the best point estimate of the population mean $(\hat{\mu})$ is:

$$\hat{\mu} = \frac{upper + lower}{2} = \frac{765.57 + 746.3}{2} = \frac{1512}{2} = 756$$

Example (3)

IF you have (746.3, 765.7). Based on this information, you know that the best point estimate of the population mean $(\hat{\mu})$ is:

$$\hat{\mu} = \frac{upper + lower}{2} = \frac{765.7 + 746.3}{2} = \frac{1512}{2} = 756$$

Case3:

Confidence Interval for a Population Mean with unknown variance (small Sample n<30, normalcase):

In this case, the interval estimate for mean is based on the t distribution; it isa type of probability distribution that is theoretical and resembles a normal distribution .To find confidence interval for a population mean with small samplewe will use the following formula:

$$\bar{X} + t_{\nu,\frac{\alpha}{2}} \quad , \quad \nu = n - 1$$

$t_{\nu,\frac{\alpha}{2}}$ = the"t" value providing an area of α/2in the upper tail of a t distribution with (n– 1) degrees of freedom.

S = the sample standard deviation.

The $t_{\nu,\frac{\alpha}{2}}$read from "t-distribution table"

Example (4) (exercises (11) p.304-ch9)

The owner of Britten's Egg Farm wants to estimate the mean number of eggs laid per chicken. A sample of 20 chickens shows they laid an average of 20 eggs per month with a standard deviation of 2 eggs per month (a sample is taken from a normal population).

i. What is the value of the population mean? What is the best estimate of this value?
ii. Explain why we need to use the t distribution. What assumption do you need to make?
iii. For a 95 percent confidence interval, what is the value of t?
iv. Develop the 95 percent confidence interval for the populationmean.
v. Would it be reasonable to conclude that the population mean is 25 eggs? What about 5 eggs?

Solution:

i. the population mean is unknown, but the best estimate is 20,the sample mean
ii. Use the t distribution as the standard deviation is unknown and n<30.However, assume the population is normally distributed.

iii. $t_{n-1;\frac{\alpha}{2}} = t_{20-1,\frac{0.05}{2}} = t_{19,0.025} = 2.093024$

iv. $\bar{X} \pm t_{n-1;\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 20 \pm 2.093\left(\frac{2}{\sqrt{20}}\right) = 20 \pm 0.936$

$19.064 \le \hat{\mu} \le 20.936$

$(19.064\ ,\ 20.936)$

v. Neither value is reasonable, because they are not inside the interval.

Example (5)

Find a 90% confidence interval for a population mean μ for these values:

a. $n = 50 \quad , \bar{x} = 21.9 \ , s^2 = 3.44$

b. $n = 14 \quad , \bar{x} = 1258 \ , s^2 = (214)^2$**,** $X \sim N(\mu, \sigma^2)$

Solution:

a. n ≥30

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

$$= 21.9 \pm 1.65 \left(\frac{1.855}{\sqrt{50}} \right)$$

$$= 21.9 \pm 0.4328$$

$$21.47 \le \hat{\mu} \le 22.33$$

$$(21.47 \ , \ 22.33)$$

b. $\bar{X} \pm t_{n-1;\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$

$$1258 \pm 1.77 \left(\frac{214}{\sqrt{14}} \right)$$

$$= 1258 \pm 101.2332$$

$$1156.76 \le \hat{\mu} \le 1359.23$$

$$(1156.76 \ , \ 1359.23)$$

Confidence Interval for a Population Proportion (Large Sample)

When the sample size is large $n \geq 100,\ \ 0.05 \leq \pi \leq 0.95\ \ , n\pi \geq 5, n(1-\pi) \geq 5$, the sample proportion,

$$P = \frac{X}{n} = \frac{Total\ number\ of\ successes}{Total\ number\ of\ trials}$$

$$P \sim N\left(\pi, \frac{\pi(1-\pi)}{n}\right)$$

The confidence interval for a population proportion:

$$\pi = P \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{P(1-P)}{n}}$$

$\sqrt{\frac{P(1-P)}{n}}$, The standard error of the proportion

Example (6) (exercises 15 p.308-ch9)

The owner of the West End credit Kwick Fill Gas Station wishes to determine the proportion of customers who use a credit card or debit card to pay at the pump. He surveys 100 customers and finds that 80 paid at the pump.

a. Estimate the value of the population proportion.
b. Develop a 95 percent confidence interval for the population proportion.
c. Interpret your findings.

Solution:

a.

$$\pi = P = \frac{X}{n} = \frac{80}{100} = 0.8$$

b.

$$P \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{P(1-P)}{n}} = 0.8 \pm 1.96\sqrt{\frac{(0.8)(0.2)}{100}} = 0.8 \pm 1.96\sqrt{0.0016} = 0.8 \pm 1.96(0.04) = 0.8 \pm 0.0784$$

$$0.72 \leq \hat{\pi} \leq 0.88$$

$$(0.72\ \ , 0.88)$$

c .We are reasonably sure the population proportion is between 0.72 and o.88 percent .

Example (7)

The Fox TV network is considering replacing one of its prime-time crime investigation shows with a new family-oriented comedy show. Before a final decision is made, network executives commission a sample of 400 viewers. After viewing the comedy, 0.625 percent indicated they would watch the new show and suggested it replace the crime investigation show.

d. Estimate the value of the population proportion.
e. Develop a 99 percent confidence interval for the population proportion.
f. Interpret your findings.

Solution:

a.

$\pi = P = 0.625$

b.

$$P \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{P(1-P)}{n}} = 0.625 \pm 2.58\sqrt{\frac{(0.625)(0.375)}{400}} = 0.625 \pm 0.06245$$

$0.56 \le \widehat{\pi} \le 0.69$

$(0.56\ ,\ 0.69)$

c .We are reasonably sure the population proportion is between 0.56 and o.69 percent .

Note:

If the value of estimated proportion(p) not mentioned we substitute it by o.5(as studies and reachears recommended)

Choosing an appropriate sample size for the population mean

The determined sample size depends on three factors:

1- The level of confidence desired (according to the level of confidence we select the a value of distribution " Z")

2- The margin of error the researcher will tolerate (is the allowable error. The maximum allowable error designated as E).

$E = \pm Z \frac{\sigma}{\sqrt{n}}$ Or $\quad E = \frac{UCL-LCL}{2}$

The length of confidence interval= UCL –LCL

The length of C.I=

$$\left(\bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) - \left(\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

$$= Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 2Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$E = \frac{2Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}}{2} = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

3- The variability in the population being studied (the population standard deviation " σ ").

Solving "E" equation for "n" yields the following result:

$$E = Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

The sample size for estimating the population mean:

$$n = \left(\frac{Z_{\alpha/2}\,\sigma}{E}\right)^2$$

Note : When the outcome is not a whole number, the usual practice is to **round up any fractional result**.

Example (8)

The registrar wants to estimate the anithmetic mean grade point average (GPA)of all graduating seniors during the past 10 years. GPAs range between 2 and 4.The mean GPA is to be estimated within plus or minus 0.05 of population mean.The standard deviation is estimated to be 0.279.Use the 0.99 percent level of confidence.Will you assist the college registrar in determining how many transcripts to study ?

Solution:

Given in the problem:

- E, the maximum allowable error, is 0.05

- The value of z for a 99 percent level of confidence is 2.58,
- The estimate of the standard deviation is 0.279.

$$n=\left(\frac{\left(Z_{\frac{\alpha}{2}}\right)\sigma}{E}\right)^2=\left(\frac{(2.58)(0.279)}{0.05}\right)^2=207.26\approx 208$$

Example (9)

A population is estimated to have a standard deviation of 10.if a 95 percent confidence interval is used and an interval of ± 2 is desired .How large a sample is required?

Solution:

Given in the problem:

- E, the maximum allowable error, is 2The value of z for a 95 percent level of confidence is 1.96,
- The estimate of the standard deviation is10.

$$n=\left(\frac{\left(Z_{\frac{\alpha}{2}}\right)\sigma}{E}\right)^2=\left(\frac{(1.96)10}{2}\right)^2=96.04\approx 97$$

Example (10)

If a simple random sample of 326 people was used to make a 95% confidence interval of (0.57,0.67), what is the margin of error (E)?

Solution:

$$E=\frac{upper-lower}{2}=\frac{0.67-0.57}{2}=\frac{0.1}{2}=0.05$$

Example (11)

If n=34, the standard deviation 4.2 (σ), $1-\alpha=95\%$ What is the maximum allowable error (E) ?

Solution:

$$n=\left(\frac{Z_{\frac{\alpha}{2}}\sigma}{E}\right)^2=\frac{\left(Z_{\frac{\alpha}{2}}\sigma\right)^2}{E^2}\ \therefore E^2=\frac{\left(Z_{\frac{\alpha}{2}}\sigma\right)^2}{n}$$

$$E^2=\frac{(1.96\times 4.2)^2}{34}=1.99$$

$$\therefore E=\pm\sqrt{1.99}=\pm 1.41$$

The maximum allowable error (E) = **1.41**

Choosing an appropriate sample size for the population proportion

The margin error for the confidence interval for a population proportion:

$$E = \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}$$

Solving "E" equation for "n" yields the following result:

$$n = \left(\frac{Z_{\frac{\alpha}{2}}\sqrt{p(1-p)}}{E}\right)^2$$

Or

$$n = p(1-p)\left(\frac{Z_{\frac{\alpha}{2}}}{E}\right)^2$$

Example (12)

The estimate of the population proportion is to be within plus or minus o.o5, with a 95 percent level of confidence. The best estimation of the population proportion is o.15.How large a sample is required?

Solution:

$$n = \left(\frac{\left(Z_{\frac{\alpha}{2}}\right)\sqrt{P(1-P)}}{E}\right)^2 = \left(\frac{1.96\sqrt{0.15\times 0.85}}{0.05}\right)^2 = 195.92 \approx 196$$

Example (13)

The estimate of the population proportion is to be within plus or minus o.10, with a 99 percent level of confidence. How large a sample is required?

Solution:

$$n = \left(\frac{\left(Z_{\frac{\alpha}{2}}\right)\sqrt{P(1-P)}}{E}\right)^2 = \left(\frac{2.58\sqrt{0.5\times 0.5}}{0.10}\right)^2 = 166.41 \approx 167$$

Confidence Interval for the Difference between Two Population means.

Case1: Confidence Interval for the Difference between Two Population means with known Variances (independent Samples)
Case2: Confidence Interval for the Difference between Two Population means with unknown Variances the sample sizes are smal (independent Samples) $(\sigma_1^2 = \sigma_2^2)$

Case1: Confidence Interval for the Difference between Two Population means with known Variances (independent Samples):

$$X_1 \sim N(\mu_1, \sigma_1^2) \qquad , X_2 \sim N(\mu_2, \sigma_2^2)$$

$$\bar{X}_1 \sim N\left(\mu_1, \frac{\sigma_1^2}{n_1}\right) \qquad , \bar{X}_2 \sim N\left(\mu_2, \frac{\sigma_2^2}{n_2}\right)$$

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

The confidence limits are:

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$, The standard error of the difference between two population means

Example (14)

A study was conducted to compare the mean numbers of police emergence calls per 8-hour shift in two districts of a large city. If the population variance for region 1 is 2.64 and the population variance for region 2 is1.44. Samples of 100 8-hour shifts were randomly selected from the police record for each of the two regions, and the number of emergency calls was recorded for each shift .the sample statistics are listed here:

$n_1 = 100$, $n_2 = 100$

$\bar{X}_1 = 3.1$, $\bar{X}_2 = 2.4$

Find a 90% Confidence Interval for the difference in the mean numbers of police emergence calls emergence calls per shift between the two districts of the city.

Solution:

The confidence limits are:

$$(\bar{X}_1 - \bar{X}_2) \pm Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$= (3.1 - 2.4) \pm 1.65\sqrt{\frac{2.64}{100} + \frac{1.44}{100}} = 0.7 \pm 1.65\sqrt{0.0264 + 0.0144} = 0.7 \pm 0.33328$$

$0.36672 \le \hat{\mu}_1 - \hat{\mu}_2 \le 1.03328$

$(0.36672 \ , \ 1.03328)$

Case2: Confidence Interval for the Difference between Two Population means with unknown Variances the sample sizes are small (independent Samples) $\sigma_1^2 = \sigma_2^2$

$\sigma_1^2 = \sigma_2^2 = \sigma^2$

$X_1 \sim N(\mu_1, \sigma_1^2)$, $X_2 \sim N(\mu_2, \sigma_2^2)$

The confidence limits are:

$$(\bar{X}_1 - \bar{X}_2) \pm t_{(\frac{\alpha}{2}, n_1 + n_2 - 2)}, S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Example (16)

An experiment was conducted to compare two diets A and B designed for weight reduction. Two group of 15 overweight dieters each were randomly selected .one group was placed on diet A and the other on diet B ,and their weight losses were recorded over a30-day period. The means and standard deviation of the weight-loss measurements for the two groups are shown in the table. Find a 95% Confidence Interval for the difference in the mean weight loss for the two diets.**(**$\sigma_1^2 = \sigma_2^2$**)**

Diet A	**Diet B**
$\bar{X}_A = 21.3$	$\bar{X}_B = 13.4$
$S_A = 2.6$	$S_B = 1.9$

Solution:

$$\left(\overline{X}_1-\overline{X}_2\right)\pm t_{n_1+n_2-2;\frac{\alpha}{2}}\,S_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

S_p^2 Is the pooled (the common) variance is...

$$S_p^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}=\frac{(15-1)2.6^2+(15-1)1.9^2}{15+15-2}$$

$$\frac{14(2.6)^2+14(1.9)^2}{15+15-2}=\frac{94.64+50.54}{28}=\frac{145.18}{28}=5.185$$

$$S_p=2.277$$

$$\left(\overline{X}_1-\overline{X}_2\right)\pm t_{n_1+n_2-2;\frac{\alpha}{2}}\,S_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$=(21.3-13.4)\pm t_{28;0.025}(2.277)\sqrt{\frac{1}{15}+\frac{1}{15}}$$

$$=7.9\pm 2.04807(2.277)\sqrt{0.0667+0.0667}$$

$$=7.9\pm 2.04807(2.277)\sqrt{0.1334}$$

$$=7.9\pm 2.04807(2.277)(0.3652)$$

$$=7.9\pm 1.7031$$

$$6.2\le \hat{\mu}_1-\hat{\mu}_2\le 9.6$$

$$(6.2\ ,\ \ 9.6)$$

Confidence Interval for the Difference between Two Population Proportions (Large samples)

Assume that independent random samples of n_1 and n_2 observations have been selected form binomial population with parameters p_1 and p_2 , respectively.

The sampling distribution of the difference between sample proportions:

$$p_1 - p_2 = \left(\frac{X_1}{n_1} - \frac{X_2}{n_2}\right)$$

$p_1 - p_2$,is the estimated value for $\pi_1 - \pi_2$
And the standard error is estimated as :

$$Se = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Under the assumption n_1 and n_2 must be sufficiently large so that the sampling distribution of (p_1 - p_2) can be approximated by a normal distribution, if

$n_i \geq 100$, $0.05 \leq \pi_i \leq 0.95$, $n_i\,\pi_i > 5$ and $n_i\,(1-\pi_i) > 5$ for $i = 1, 2.$

$$P_1 - P_2 \dot{\sim} N\left(\pi_1 - \pi_2, \frac{\pi_1(1-\pi_1)}{n_1} + \frac{\pi_2(1-\pi_2)}{n_2}\right)$$

The confidence interval for the deference between two population proportions is:

$$(P_1 - P_2) \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{P_1(1-P_1)_1}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$Se = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$, The standard error of the deference between two population proportions

Example (17)

A bond proposal for school construction will be submitted to the voters at the next municipal election .A major portion of the money derived from this bond issue will be used to build schools in a rapidly developing section of the city , and the remainder will be used to renovate and update school building in the rest of the city. To assess the viability of the bond proposal, a random sample of $n_1 = 119$ Residents in the developing section and $n_2 = 100$ residents from the other parts of the city were asked whether they plan to vote for the proposal. The results are tabulated as following:

	Developing Section	Rest of the City
Sample size	119	100
Number of favoring proposal	90	65

Estimate the difference in the true proportion favoring the bond proposal with a 99% confidence interval.

Solution:

$$P_1 = \frac{X_1}{n} = \frac{90}{119} = 0.76 \quad , \quad P_2 = \frac{X_2}{n_2} = \frac{65}{100} = 0.65$$

$$(P_1 - P)_2 \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}} = 0.76 - 0.65 \pm 2.58\sqrt{\frac{0.76\times 0.24}{119} + \frac{0.65\times 0.35}{100}} = 0.11 \pm 0.159$$

$$-0.05 \le \hat{\pi}_1 - \hat{\pi}_2 \le 0.27$$

$$(-0.05 \ , \ 0.27)$$

Since this interval contains the value $(\pi_1 - \pi_2) = 0$,it is possible that $\pi_1 = \pi_2$,which implies that there may be no difference in the proportion favoring the bond issue in the two section of the city.

Confidence interval for the Population Variance (Normal Case)

If we find the confidence interval fora **population variance**we must to begin with the statistic:

$$\boldsymbol{\chi^2 \sim \frac{(n-1)S^2}{\sigma^2}}$$

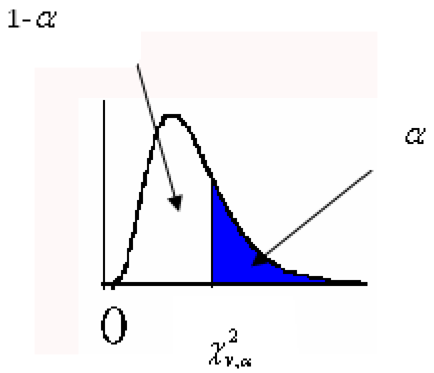
Is called a **chi-square variable** and has a sampling distribution called the **chi-square probability distribution**, with (n-1) degree of freedom.
It is not important to know the complex equation of the density function for χ^2; only to use the well-tabulated critical values of χ^2 given in χ^2 table.
Under assumption the sample is randomly selected from a normal population:

$$X \sim N(\mu\,, \sigma^2)$$

The confidence interval for a **population variance ($\boldsymbol{\sigma^2}$)** is :

$$\frac{(n-1)S^2}{\chi^2_{n-1;\frac{\alpha}{2}}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{n-1;1-\frac{\alpha}{2}}}$$

Where $\chi^2_{\alpha/2}$&$\chi^2_{(1-\alpha/2)}$are the upper and lower χ^2value, which locate one – half ofα in each tail of the chi-square distribution.



The properties of the chi square distribution.

- χ^2_v Is continuous distribution.
- χ^2_v Positive skewed curve (skewed to the right curve).
- χ^2_v It is not symmetric curve.

Example (18)

The standard deviation of the lifetimes of 10 electric light bulbs manufactured by a company is 120 hours. Find 95% confidence limits for the standard deviation of all bulbs manufactured by the company.

Solution:

$$\frac{(n-1)S^2}{\chi^2_{n-1;\frac{\alpha}{2}}} \le \hat{\sigma}^2 \le \frac{(n-1)S^2}{\chi^2_{n-1;1-\frac{\alpha}{2}}}$$

$$\frac{(9)120^2}{19.023} \le \hat{\sigma}^2 \le \frac{(9)120^2}{2.7004}$$

$$6812.81 \le \hat{\sigma}^2 \le 47992.89$$

$$82.54 \le \hat{\sigma} \le 219.07$$

Example (19)

Suppose that $X \sim N(20, 4)$ and a random sample of size $n = 17$ is selected, prove that $4S^2 \sim \chi^2_{16}$...

Solution:

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_v$$

$$\frac{(17-1)S^2}{4} \sim \chi^2_{17-1}$$

$$\frac{16S^2}{4} \sim \chi^2_{16}$$

Confidence interval for the Ratio of two Population Variances (Normal Case)

Under the assumption the samples (n_1 and n_2) are randomly and independently selected from normally distributed populations,calculate the confidence interval forthe ratio of two **population variances** according tothe steps:

- Begin from the statistic:

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F_{v_1,v_2}$$

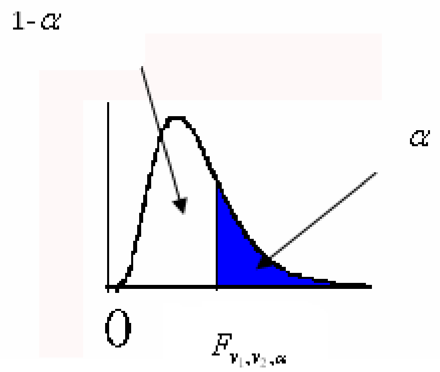
It's called a **F- variable** and has a sampling distribution called the **Fisher's probability distribution** , depends on the number of degrees of freedom associated withS_1^2&S_2^2 , represented as$\nu_1 = n_1 - 1$ & $\nu_2 = n_2 - 1$respectively.
It is not important to know the complex equation of the density function for F; only to use the well-tabulated critical values of F given in F table.

- The confidence interval for the ratio of two population variances :

$$\frac{S_2^2}{S_1^2} F_{v_1,v_2,1-\frac{\alpha}{2}} \le \frac{\sigma_2^2}{\sigma_1^2} \le \frac{S_2^2}{S_1^2} F_{v_1,v_2,\frac{\alpha}{2}}$$

$$\because F_{\nu_1,\nu_2,1-\frac{\alpha}{2}} = \frac{1}{F_{\nu_2,\nu_1,\frac{\alpha}{2}}}$$

$$\therefore \frac{S_2^2}{S_1^2} \frac{1}{F_{v_2,v_1;\frac{\alpha}{2}}} \le \frac{\sigma_2^2}{\sigma_1^2} \le \frac{S_2^2}{S_1^2} F_{v_1,v_2,\frac{\alpha}{2}}$$



The properties of the F distribution (F_{v_1,v_2})

- Is continuous distribution.
- Positive skewed curve (skewed to the right curve).
- It is not symmetric curve.

Example (20)

Two samples of size 16 and 10 respectively are drawn at random from two normal populations. If their variations are found to be 24 and 18 respectively find 98% confidence limits for the ratio of the variances.

Solution:

$$F_{v_1,v_2;\frac{\alpha}{2}} = F_{15,9;0.01} = 4.96$$

$$F_{v_1,v_2;1-\frac{\alpha}{2}} = F_{15,9;0.99} = \frac{1}{F_{v_2,v_1;\frac{\alpha}{2}}} = \frac{1}{F_{v_2,v_1,\frac{0.02}{2}}} = \frac{1}{F_{9,15.0.01}} = \frac{1}{3.90} = 0.2564$$

$$\frac{S_2^2}{S_1^2} F_{v_1,v_2,1-\frac{\alpha}{2}} \le \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} \le \frac{S_2^2}{S_1^2} F_{v_1,v_2,\frac{\alpha}{2}}$$

$$\frac{S_2^2}{S_1^2} \frac{1}{F_{v_2,v_1;\frac{\alpha}{2}}} \le \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} \le \frac{S_2^2}{S_1^2} F_{v_1,v_2,\frac{\alpha}{2}}$$

$$\frac{18}{24}(0.2564) \le \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} \le \frac{18}{24}(4.96)$$

$$0.1923 \le \frac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} \le 3.72$$

$$(0.19 \ , \ 3.72)$$

Example (21)

Find

$F_{30,24;0.025}$, $F_{30,24;0.975}$

$F_{5,9;0.05}$, $F_{5,9;0.95}$

Solution:

$$F_{30,24;0.025} = 2.21$$

$$F_{30,24;0.975} = \frac{1}{F_{24,30.0.025}} = \frac{1}{2.14} = 0.4673$$

$$F_{5,9;0.05} = 3.48$$

$$F_{5,9;0.95} = \frac{1}{F_{9,5,0.05}} = \frac{1}{4.77} = 0.21$$

Example (22)

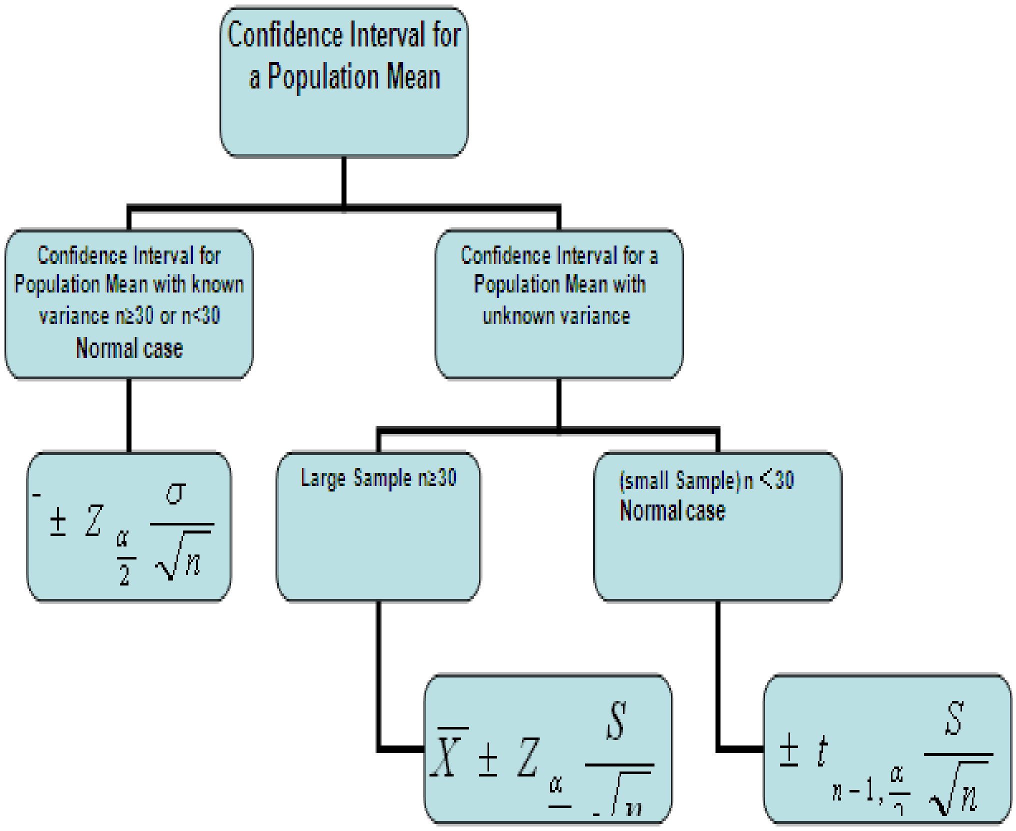
If $F_{30,24,0.25}$**=2.21, Find** $F_{24,30,0.975}$

Solution:

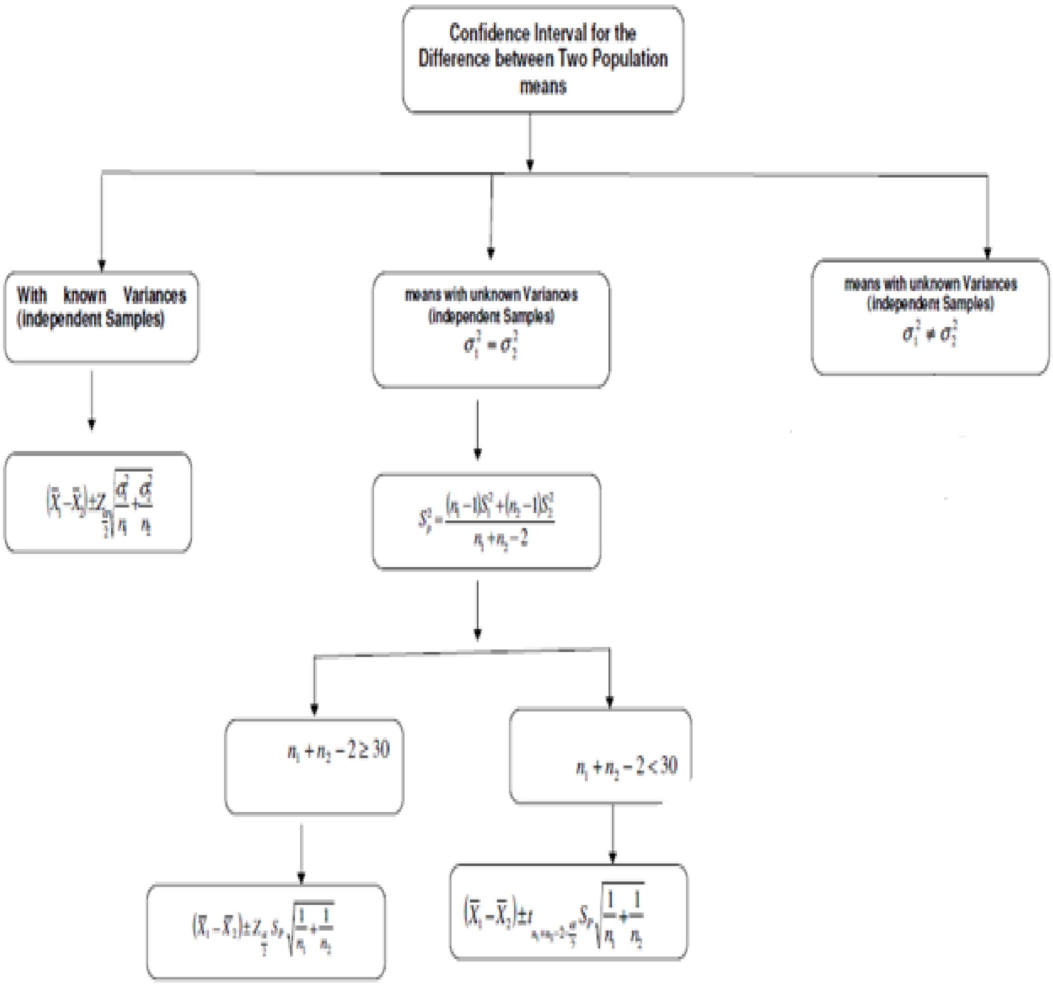
$$F_{24,30,975} = \frac{1}{F_{30,24,0.025}} = \frac{1}{2.21} = 0.4525$$

Summary

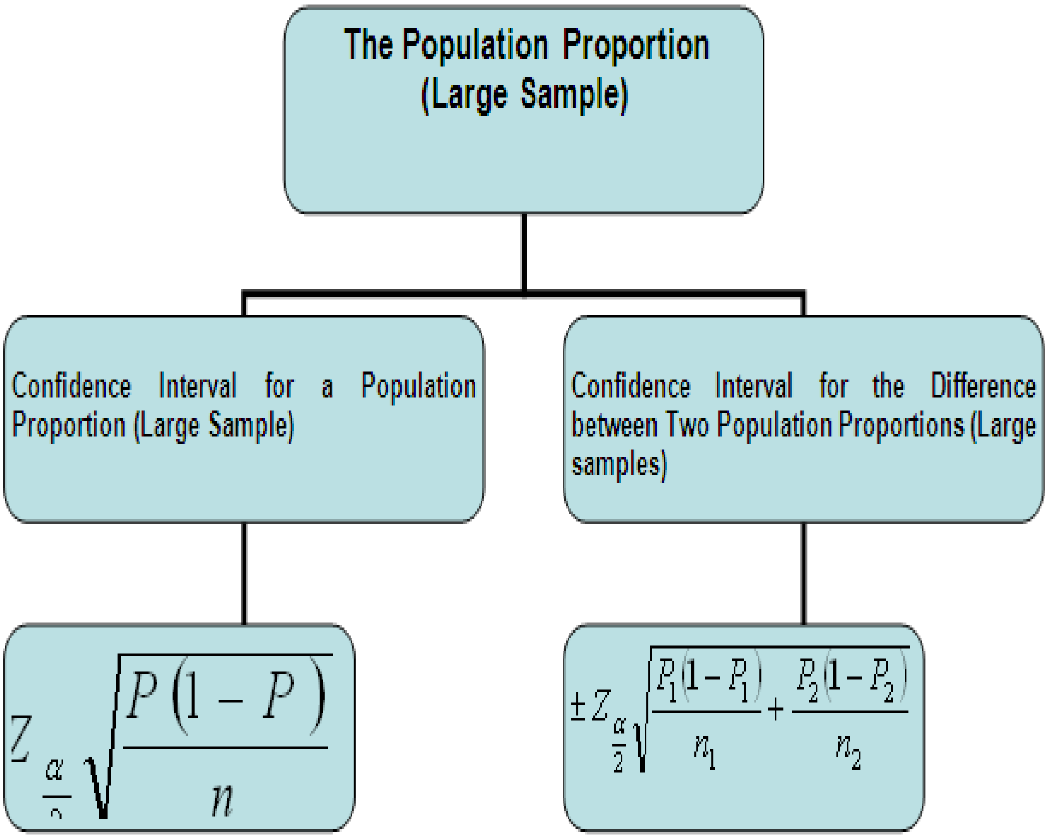
1. Confidence Interval for a Population Mean



2. Confidence Interval for the Difference between Two Population means



3. The Population Proportion (Large Sample)



4. The Population Variance (Normal Case)