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Evaluating Uncertainty in Archie's Water Saturation Equation Parameters Determination Methods

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Abstract

Early in the life of reservoirs, it is required to estimate accurately hydrocarbon volumes in place. Modified Archie formula ($S_w = (a\ R_w\ /\ \phi^m\ R_t)^{1/n}$) is the basic equation to calculate water saturation. The exactness of water saturation value for given reservoir conditions depends on the accuracy of Archie parameters a, m and n. The terms of Archie relationship have been subjected to many laboratory investigations and even more speculation. There are many factors affect porosity exponent, m, saturation exponent, n and tortousity factor, a. Therefore, it is very difficult to fix Archie parameters regardless of reservoir characteristics; rock wettability, formation water salinity, permeability, porosity and fluids distribution.

This paper presents a new technique to determine Archie parameters a, m and n. The developed technique is based on the concept of three dimensional- regression (3D) plot of water saturation, formation resistivity and porosity. 3D method provides simultaneous values of Archie parameters. Also, 3D method overcomes the uncertainty problems due to the separate use of formation resistivity factor- porosity and water saturation equations to get a, m and n parameters.

Two field examples are given to show the applicability of 3D method and three other methods: 1) common values of Archie parameters, 2) conventional method and 3) core Archie parameter estimation (CAPE) method. The comparison between the four methods has shown that 3D method provides an accurate and physically meaningful way to get Archie parameters a, m and n for given core samples. Water saturation profiles, using Archie parameters deduced from the four methods, have been produced for the studied section in the wells. These profiles have shown a significant difference in water saturation values. This difference could be mainly attributed to the uncertainty level for Archie parameters from each method.

Introduction

Classic petrophysics holds that Archie's parameters a,m and n are constants for a given sample of a reservoir rock. In effect, this presumed constancy formulates the basis for the determination of hydrocarbon saturation from resistivity measurements for a particular lithology. An increasing number of cases are being encountered where the saturation exponent, n, has been observed to vary from the common value of 2. Field experience has also shown that the cementation factor, m, and the tortousity factor, a, depend on the petrophysical properties of a given rock.

Petroleum literature contains many reports of the results determining Archie's parameters and related water saturation. In quantitative log interpretation, accurate water saturation requires good values of Archie's parameters.[1-10]

In this paper, the authors propose a new technique to determine Archie's parameters, three dimensional regression (3D) technique. which is based on the analytical expression of 3D plot of Rt/Rw vs. Sw and,ϕ. Water saturation profiles were calculated using common values (1,2,2), conventional; CAPE and 3D methods for selected two wells.

Conventional Determination of a,m and n

In 1942 Archie proposed an empirical relationship between rock rsistivity, Rt, with its porosity, ϕ, and water saturation Sw.

$$Sw^n = aRw/\phi^m Rt = 1/\ Ir \qquad (1)$$

Other terms Ir, m, and n represent resistivity index, cementation factor and saturation exponent. He has also shown experimentally that the resistivity of rock fully saturated with brine, Ro, is related to the brine resistivity, Rw, by:

$$Ro = F\,Rw \qquad (2)$$

where F is formation resistivity factor. Winsauer et al (1952) had modified Archie formula F = 1/ ϕ ‘) and introduced tortousity factor, a, to Archie's formula

$$F = a/\phi^{m} \qquad (3)$$

Conventional Determination of a **and m the** conventional determination of a and m is based on Eq. 3 and is rewritten as:

$$\log F = \log a - m \log \phi \qquad (4)$$

Plot of log F vs. log ϕ is used to determine a and m for the core samples, **Fig. 1.**

Cementation factor m, is detemined from the slope of the least square fit straight line of the plotted points, while tortousity factor is given from the intercept of the line where $\phi = 1$. Note that in this plot only points of Sw = 1.0 are used.

Conventional Determination of n The classical process to determine saturation exponent, n is based on Eq. 1. This equation is rewritten as:

$$\log Ir = - n \log Sw \qquad (5)$$

Bi-logaritmic plot of Ir vs. Sw gives a straight line with negative slope n, **Fig.2.**

Sometimes data are plotted as log Rt vs. log Sw. This form is mathematically equivalent to the plot of Fig.2 and provides the same value of n.

It is obvious that the conventional method treats the determination of n as a separate problem from a and m. This separation is not physically correct, thereby, it induces an error in the value of water saturation using Eq. 1.

Core Archie-Parameter Estimation (CAPE)

Ref.5 has presented a data analysis approach to determine Archie's parameters m and n and optionally n from standard resistivity measurements on core samples. The analysis method, Core Archie- Parameters Estimation (CAPE) determines m and n and optionally a by rninimizing the error between computed water and measured water saturations. The mean square saturation error e, , is given by

$$E=\sum j \sum i[swij -(aRwij/\phi i^{m}\, Rt\ ij)]^{2} \qquad (6)$$

where j = core index, i = index for each of the core j measurements, Swij = ith laboratory measured water saturation for core j (fraction), Rtij = ith laboratory measured resistivity for corej and ϕj = corej porosity (fraction). Eq.6calculatestheminimumerror between measured core water saturation and computed water saturation by Archie's formula, this is by adjusting m,n and optionally a in the equation.

Table 1 illustrates typical results from CAPE and Conventional methods. Table 1 shows a, m and n values calculated with the two methods for clean sandstone core samples. It is obvious that the values of a, m and n are different for a given set of points (75 measurement points). Also, note that the saturation error decreases as we go from conventional to CAPE method.

Note that the CAPE method is based on the idea that the two plots shown in Figs. 1 and 2 are not the optimum way of handling the problem. The comparison between the two methods showed that CAPE might not appear as optimal as the conventional method. Instead, we are presenting another approach, the 3D method. In this method, water saturation is treated as an independent variable in 3D plot of electrical resistivity vs. water saturation and porosity.

3D Method

We contend that, so far as Archie's parameters are concerned, the error in the water saturation value should be kept minimum. This is because water saturation quantity is desired and physically meaningful quantity. Here, we have developed a method to determine Archie's parameters a, m and n using standard resistivity measurements on core samples.

Methodology The basis of the 3D method is to view Sw in Archie's formula Eq. 1 as a variable in three-dimensional regression plot of Sw, Rw/Rt and ϕ. The 3D method determines Archie's parameters a, m and n by solving three simultaneous equations of Sw , Rw/Rt and Eq. 1 is rearranged after taking the logarithm of both sides.

$$\log Rw / Rt = - \log a + m \log \phi + n \log Sw \qquad (7)$$

The left hand side of Eq. 7 is a dependent variable of the two independent variables Sw and ϕ. Eq. 7 is an equation of a plane in three dimensional (3D) space of coordinate x, y and z (x = log , y = log Sw and z = log Rw/Rt). The intersection of this plane with the plane (x = 0. 0 gives a straight line of slope m, with the plane (y = 0. 0) giving a straight line with slope n and with the plane (z = 0.0) provides the value of a parameter.

For a given set of data for a core sample, we can obtain an equivalent set of variables x, y and z. Eq. 7 will take the following form for i measurement points:

$$\mathbf{zi = -}\ A + m\,Xi + n\,Yi \qquad (8)$$

After normalizing Eq. 8 for N reading, we can have the following three simultaneous equations

$$\sum Zi = - NA + m \sum Yi + n \sum Yi \qquad (9)$$

$$\sum Zi\,Xi = -A \sum Xi + m\sum Xi^{2} + n \sum Yi\,Xi \qquad (10)$$

$$\sum ZiYi = -A\sum Yi + m\sum XiYi + n\sum Yi^{2} \qquad (11)$$

The solution of Eqs. 9-11 provides the values of Archie's parameters a,m and n for one core sample. For j core samples, running the same analysis for j core samples produces an average value of Archie's parameters. Fig. 3 shows the flow chart of a computer program for 3D method determining a, m and n for j core samples. Also, this program calculates the standard deviation σ(Sw) between the computed and measured water saturations.

Assumptions

First, 3-D method assumes that Archie's formula is applicable to the examined core samples. Also, the core samples represent the zone of interest. For shale sandstone, Archie formula must be modified to account for the presence of shale and its effect on resistivity. measurements. The user is free to select the appropriate clay model, and consequently, the shaly sand water saturation equation, [11-14]. The second assumption might be difficult to satisfy, it is concerned with the accuracy of the laboratory measurements under reservoir conditions, and it is concerned with the accuracy of the laboratory measurements under reservoir conditions. The third assumption deals with the concept of the 3D method, this means that the user must be acquainted with the basis and limitations of each method before using it.

Applications

Now, we develop the 3-D method by considering field examples of effectively clean sandstone. Table 1 shows typical results from the conventional method, the CAPE method, the 3D method, and the common values (1,2,2). Note that for conventional and CAPE methods, cases where a, is fixed at unity and variable are given. In addition to m, n and values, the average error σ(Sw) between measured and calculated water saturations is given.

For wells Al and C1, we note that the values of a, m and n deduced by the three methods, are different. Classic petrophysics holds Archie's parameters constants and commonly taken as 1,2 and 2. In fact. This presumed constancy induces a certain error in the value of water saturation.

Also, note that the saturation error σ(Sw) decreases as we go from the case where (1) common values are used to the cases where the following methods are used: (2) conventional method with, a, fixed at unity, (3) conventional method with, a, variable, (4) CAPE method, a, forced to unity, (5) 3D method, and (6) CAPE with, a, variable. This behavior was expected and it could be attributed to the fact that conventional method tries to optimize the two functions F vs. ϕ. and Rt vs. Sw rather than water saturation, while either CAPE or 3D optimizes water saturation. But 3D method is more credited than CAPE by less computer time consuming and by its optimization technique which is more physically concerned with water saturation and related factors than CAPE method. Therefore it is recommended to use the 3D to get an accurate values of a, m and n required for water saturation equatrion.

Variable Archie's Parameters and Water Saturation Values

Table 2 illustrates typical results of water saturation for different Archie's parameters deduced from conventional method, CAPE, 3D method and common values. Fig. 4 depicts water saturation profiles calculated by the four options against selected interval for wells A1 and C 1. The examination of water saturation profiles has shown that (1) the use of common values yields water saturation values greater than the correct ones, and that (2) Unlike the case of common values the water saturation profiles calculated by conventional, CAPE and 3D methods have shown certain departure from each other. For application where highest possible accuracy in water saturation is desired, it is recommended to leave the conventional method and adopt any of the CAPE or the 3D method. Moreover, the 3D method is more preferred than the CAPE method because of its more physically representation of the data and because it overcomes the dilemma of whether, a, is to be:

Fig. 5 shows the flow chart for the developed computer program to calculate the effective porosity and water saturation for the appropriate Archie's parameters, which are deduced from the selected method. For wells Al and C 1, the 3D method provided an acceptable standard deviation of water saturation with reference to CAPE or conventional methods. Note that the error in hydrocarbon saturation is identical to the error in water saturation because each one might be determined from the other by subtraction from unity.

Conclusion

1. Conventional method optimizes the two functions F vs. ϕ and Rt vs. Sw rather than water saturation values.

2. The CAPE method confirms that the quantity one should optimize is not the two functions but rather the water saturation.

3. The 3D method provides simultaneously the values of Archie's parameters from standard resistivity measurements on core samples.

4. Unlike the conventional method, which ignored the values of Sw < 1.0 in the determination of a and m , the 3D method uses all data of Sw points.

5. The 3D method answer the controversial question of whether tortousity factor a, should be fixed at unity or not. It gives directly a, m and n, and thereby, it is recommended to consider the case of the three variables a, m and n

6. For applications where the highest possible accuracy in hydrocarbon saturation is required, it is recommended to use the 3D method, unless, there are adverse conditions as mentioned in the text.

Nomenclature

a = Tortousity factor
m = Cementation factor
n = Saturation exponent
Sw = Water saturation, fraction
Rt = Resistivity of rock, Ω.m
Rw = Resistivity of brine,- Ω.m
Ro= Resistivity of rock with Sw=1.0, Ω.m
Ir = Resistivity index
F = Formation resistivity factor
ϕ= Formation porosity, fraction
σSw = Standard deviation in water saturation

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TABLE 1 ARACHIE'S PARAMETERS VALUES DETERMINED BY 5 METHODS AND 3- D METHOD

WELL -A1

Method Used	a	m	n	σ_{sw}
Common	1	2	2	0.34
Conventional	1.012	1.67	1.56	0.073
Conventional	1	1.66	1.56	0.0734
Cape (a,m,n)	3.289	1.062	1.62	0.0667
Cape (1,m,n)	1	1.65	1.616	0.0717
3-D	2.937	1.144	1.546	0.0686

WELL –C1

Method Used	a	m	n	σ_{sw}
Common	1	2	2	0.44
Conventional	1.03	1.62	1.76	0.0868
Conventional	1	1.63	1.76	0.0883
Cape (a,m,n)	2.467	1.189	1.78	0.0685
Cape (1,m,n)	1	1.56	1.92	0.0757
3-D	2.59	1.202	1.597	0.0715

TABLE -2 WATER SATURATION VALUES USING ARACHIE'S PARAMETERS DETERMINED BY THE 6 METHODS

WELL -A1

Method Used	a	m	n	S_w
Common	1	2	2	0.287
Conventional	1.012	1.67	1.56	0.217
Conventional	1	1.66	1.56	0.230
Cape (a,m,n)	3.289	1.062	1.62	0.1793
Cape (1,m,n)	1	1.65	1.616	0.1711
3-D	2.937	1.144	1.546	0.173

WELL –C1

Method Used	a	m	n	S_w
Common	1	2	2	0.366
Conventional	1.03	1.62	1.76	0.297
Conventional	1	1.63	1.76	0.2866
Cape (a,m,n)	2.467	1.189	1.78	0.1912
Cape (1,m,n)	1	1.56	1.92	0.197
3-D	2.59	1.202	1.597	0.195

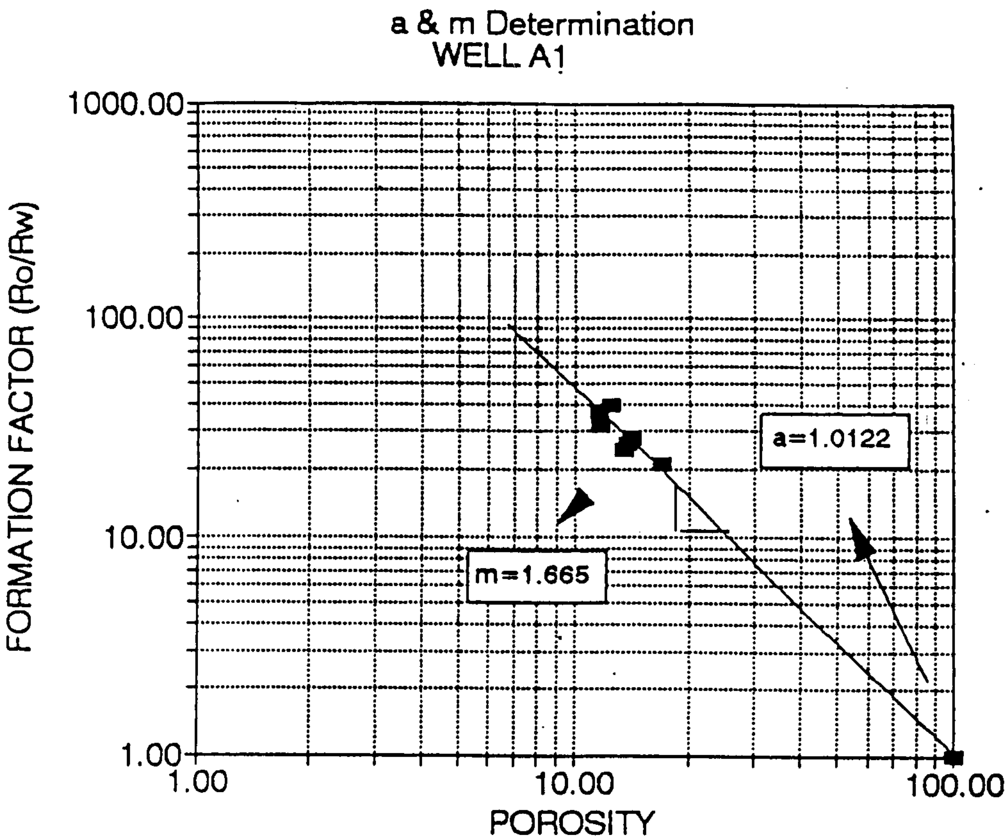

Fig. 1 F vs. ϕ for Conventional Determination of m and a

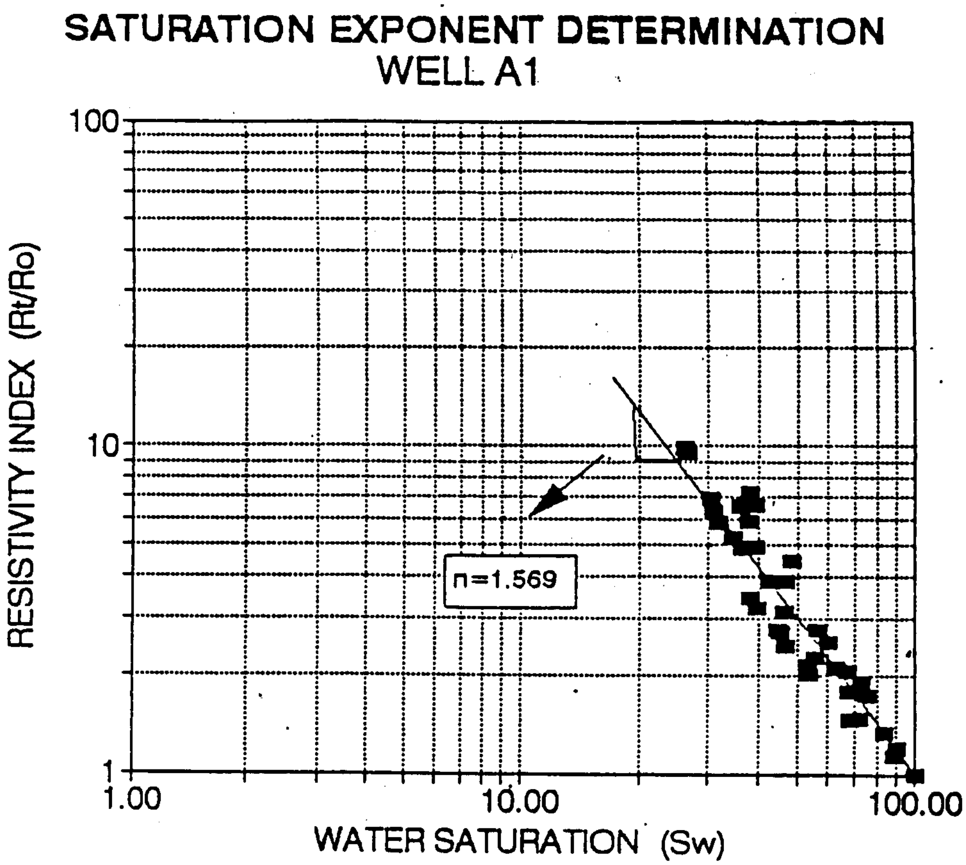

Fig. 2 I_R vs. Sw for Conventional Determination of n

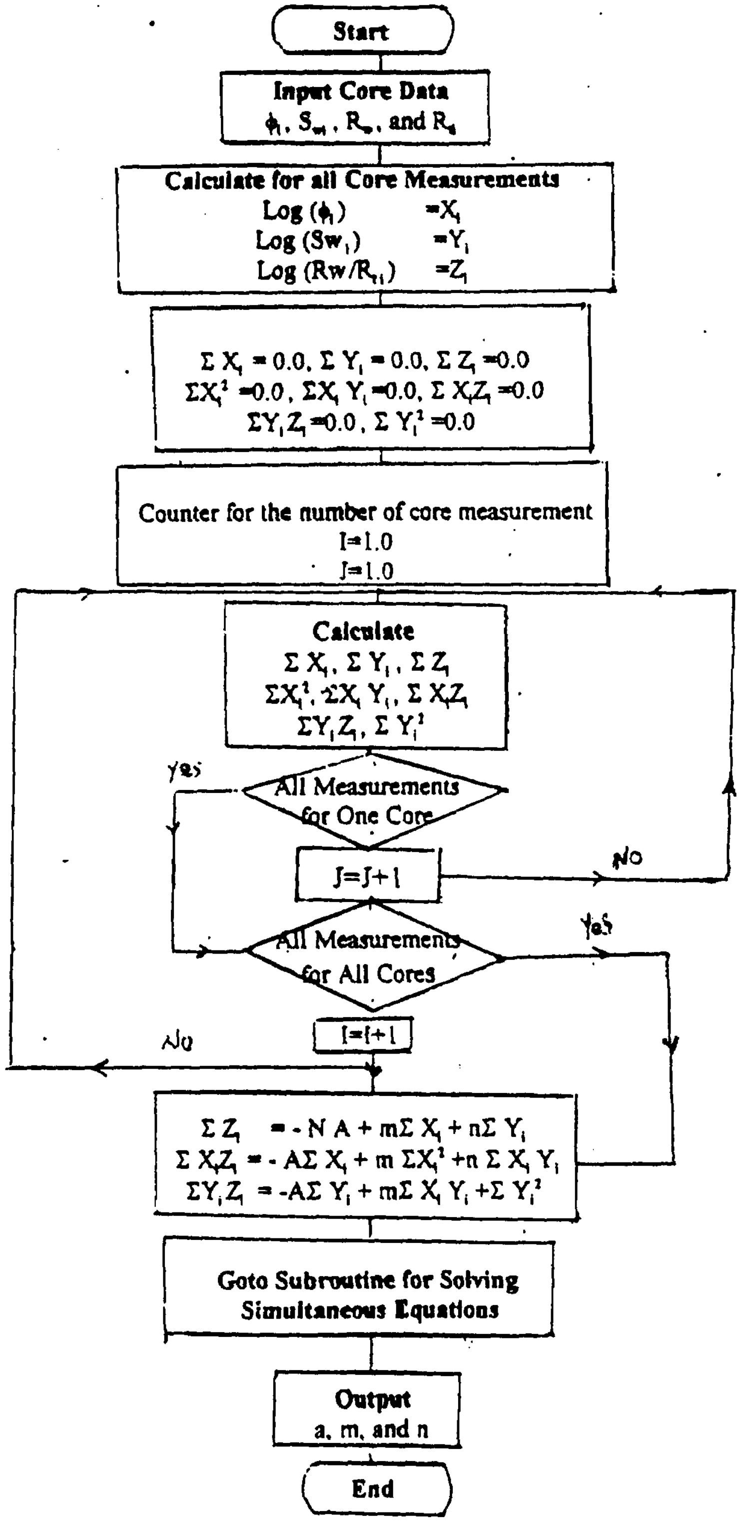

Fig. 3 Flow Chart for a,m and n Determination from 3D Method

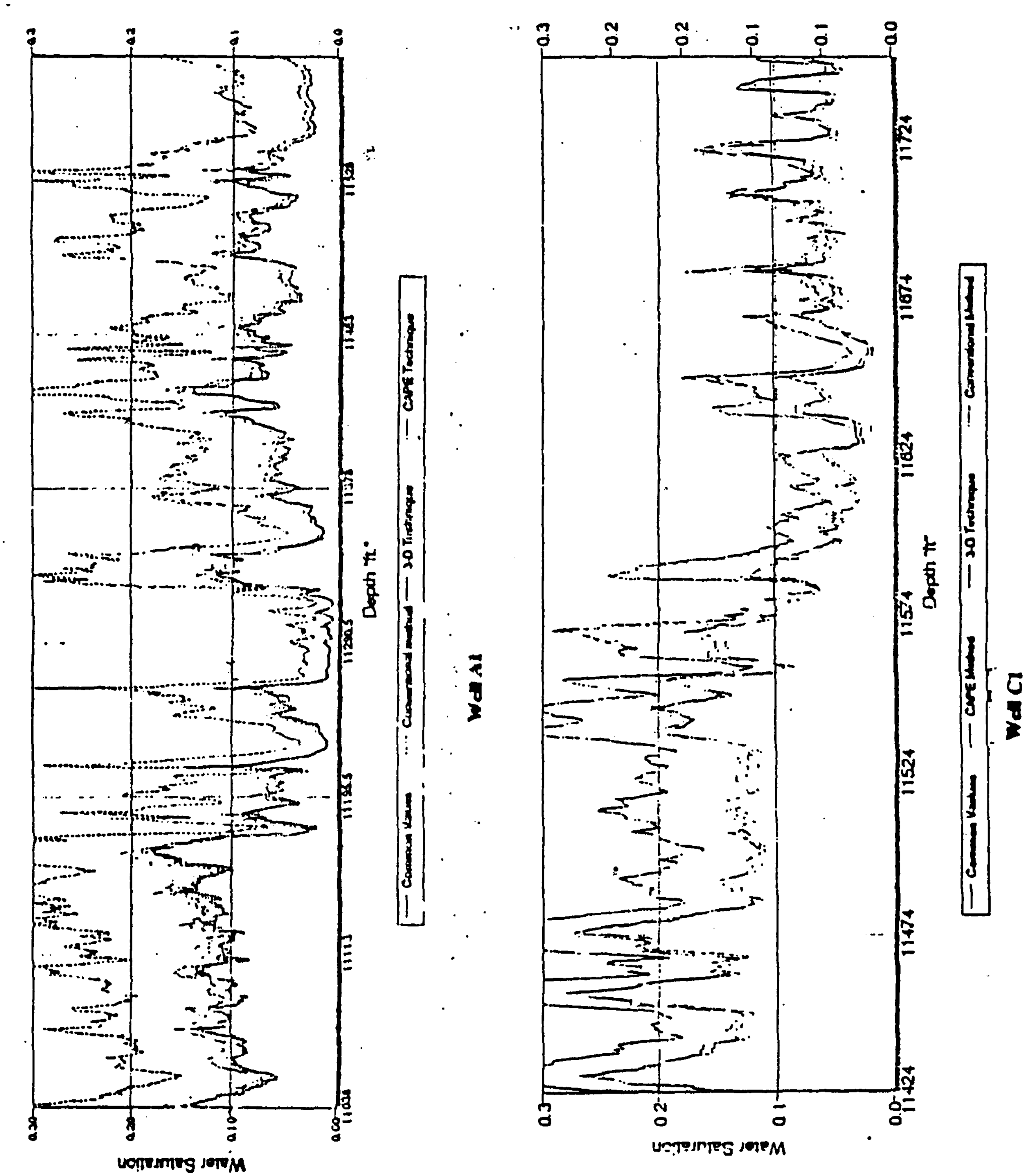

Fig. 4 Water Saturation Profiles Calculated by Common Values, Conventional, CAPE and 3D Methods

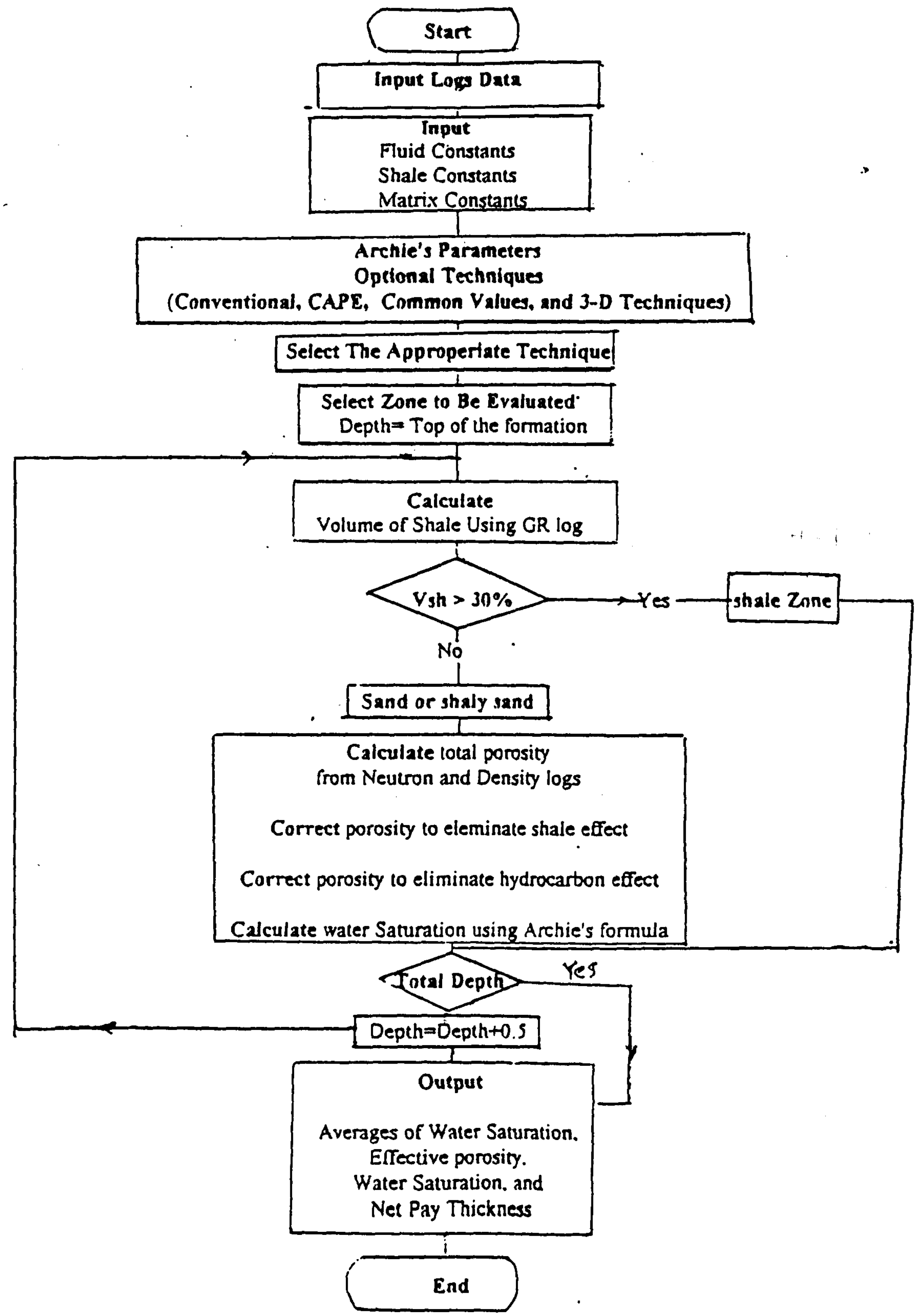

Fig. 5 Flow Chart To Calculate Water Saturation For Optimal Archie's Parameters Determination Technique