Grading scheme

Ex 1

a) $\sum_{1}^{10} k^2 - a \sum_{1}^{10} k = 0$ [redacted]

$$\frac{10(10+1)(21)}{6} - a\,\frac{10(10+1)}{2} = 0$$ [redacted]

$$a = \frac{21}{3} = 7$$ [redacted]

b) $F(x) = \cos x^2 \cdot \int_0^{x^2} \cos^4 t\, dt$

[redacted] $F'(x) = -2x \sin x^2 \int_0^{x^2} \cos^4 t\, dt + \cos x^2 \cdot (\cos^4 x^2) \cdot 2x$

$F'(0) = 0$

c) $P = \left\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right\}$ [redacted]

$$S_4 = \frac{2}{3 \cdot 4}\left(f(-1) + 4f\left(-\tfrac{1}{2}\right) + 2f(0) + 4f\left(\tfrac{1}{2}\right) + f(1)\right)$$

$$= \frac{1}{6}\left(\sqrt{2} + 4\sqrt{\frac{17}{16}} + 2 + 4\sqrt{\frac{17}{16}} + \sqrt{2}\right)$$ [redacted]

$$\simeq 2.17911$$ [redacted]

2) a) $u = x^{5/4} - 4$

$du = \frac{5}{4} x^{\frac{1}{4}} dx$

$$\int x^{\frac{1}{4}} (x^{5/4} - 4)^3 dx = \frac{4}{5} \int u^3 du$$

$$= \frac{1}{5} u^4 + C$$

$$= \frac{1}{5} (x^{5/4} - 4)^4 + C$$

b) $u = 1 + \ln x \quad du = \frac{dx}{x}$

$$\int_1^e \frac{\sqrt[3]{1+\ln x}}{x} dx = \int_1^2 u^{\frac{1}{3}} du = \frac{3}{4} \left[u^{\frac{4}{3}} \right]_1^2$$

$$= \frac{3}{4} \left(\sqrt[3]{16} - 1 \right)$$

c) $y = x^{x^2} \qquad \ln y = x^2 \ln x$

$$\frac{y'}{y} = 2x \ln x + x$$

$$y' = (2x \ln x + x) x^{x^2}$$

$$= (2 \ln x + 1) x^{x^2 + 1}$$

3) a) $u = \cos x \quad du = -\sin x\,dx$

$$\int \frac{dx}{\csc x\sqrt{9-\cos^2 x}} = -\int \frac{du}{\sqrt{9-u^2}}$$

$$= -\sin^{-1}\left(\frac{u}{3}\right) + C$$

$$= -\sin^{-1}\left(\frac{\cos x}{3}\right) + C$$

b) $u = e^{5x} \quad du = 5e^{5x}dx$

$$\int \frac{dx}{\sqrt{e^{10x}-25}} = \int \frac{du}{5u\sqrt{u^2-(5)^2}}$$

$$= \frac{1}{25}\sec^{-1}\left(\frac{u}{5}\right) + C$$

$$= \frac{1}{25}\sec^{-1}\left(\frac{e^{5x}}{5}\right) + C$$

c) $u = \sqrt{x} \quad du = \frac{dx}{2\sqrt{x}}$

$$\int \frac{dx}{\sqrt{x}(x+16)} = 2\int \frac{du}{u^2+16}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x}}{4}\right) + C$$