1st Midterm Exam

استعن بالله وكن على يقين بأن كل ما ورد في هذه الورقة تعرفه جيدا وقد تدربت عليه بما فيه الكفاية

Student's Name	Girls Section.	**Student's Number**	

Notes for the Exam:

1. *Each question shouldn't take from you more than 20 minutes*
2. *Answers on the same papers*
3. *Use the back of the pages for more space make sure you write the question number.*

Question #1: Answer the following with *True* or *False*: 20

F	1. It is always good to use simulation model even when problem is simple and can be solved by mathematical model.
T	2. Simulation modeling is not good if there is less data or no estimates available.
F	3. Validation step and Verification step in simulation modeling are the same.
T	4. It is always best to use simulation models to evaluate alternatives for complex or highly expensive systems.
F	5. Simulating flight distance for an airplane is a discrete system simulation.
T	6. In Bank simulation, the variable (the amount of money a customer has) is an attribute for each entity.
F	7. Results in simulation is always ***exact***
F	8. After running simulation model the result we get is optimal and best result.
T	9. We always need to do output analysis for the results of the simulation model.
T	10. The sequence of random numbers generated from a given seed is called a random number a *Stream*.

F	11. LCG has full period if and only if we get exactly $(m-1)$ random numbers.
F	12. Always, if the LCG repeat the starting value R_0 then the function has a full period.
F	13. Inverse transform is use the PDF of the distribution to write the inverse function
F	14. If the three conditions are not satisfied for the LCG then, we cannot have all m random numbers from this LCG.
F	15. It is possible to use inverse transform to generate from Normal distribution.
F	16. Every time you run the simulation model you get the same output data.
F	17. The LCG ($X_n = (aX_{n-1} + c) \bmod(m)$) it is possible to find values for: X_0, a, c and m to generate more than m different random numbers.
T	18. The LCG is always used to generate pseudo-random numbers betwee 0 and 1.
T	19. Inverse transform method takes the uniform (0,1) numbers to generate random numbers from some distributions using CDF.
F	20. The Kolmogorov-Simernove test divides the interval [0,1] by k equal intervals.

Question #2: 25

Conceder the following LCG generator: $R_n = (5\ R_{n-1} + 3)\ mod\ (24)$

Answer the following:

5 a) By looking to the LCG function, How many total U[0,1] random numbers this LCG can give you (Answer by giving the number ONLY, do not use the function)?

5 b) Without substitution in the LCG function above, What are all U[0,1] random numbers that you could get from this LCG by looking to the function.

5 c) Show that the **three conditions** make the LCG full period or not.

10 d) Find the uniform [0,1] stream resulting from $R_0 = 0$ numbers from the LCG above rounded to three digits (0.- - -) .

Solution:

a) number of random number from LCG = 24 numbers

b) The possible random numbers from the functions are:

$0/24,\ 1/24,\ 2/24,\ 3/24,\ 4/24,\ 5/24,\ 6/24,\ 7/24,\ 8/24,\ 9/24,\ 10/24,$
$11/24,\ 12/24,\ 13/24,\ 14/24,\ 15/24,\ 16/24,\ 17/24,\ 18/24,\ 19/24,\ 20/24,\ 21/24,$
$22/24,\ 23/24$

c) Cond.# ~~common~~ divisors of $m = 24 \in \{1, 2, 3, 4, 6, 8\}$
divisor of $c = 3 \in \{1, 3\}$
$\Rightarrow$ Common divisors of $m, c \in \{1, 3\}$
$\Leftrightarrow$ Cond#1 is not satisfied. $\Leftrightarrow$ LCG has no Full period.

(d)

i	R_i	R_{i+1}	U_i
0	0	3	$3/24 = 0.125$
1	3	18	$18/24 = 0.750$
2	18	21	$21/24 = 0.875$
3	21	12	$12/24 = 0.500$
4	12	15	$15/24 = 0.625$
5	15	6	$6/24 = 0.250$
6	6	9	$9/24 = 0.375$
7	9	0	$0/24 = 0.000$

Question #3:

15

Consider the following set of pseudo-random numbers.

1	0.037
2	0.050
3	0.055
4	0.078
5	0.101
6	0.111
7	0.130
8	0.158
9	0.160
10	0.212

11	0.216
12	0.225
13	0.241
14	0.260
15	0.280
16	0.327
17	0.346
18	0.352
19	0.364
20	0.395

α	$D_{\alpha,6}$	$D_{\alpha,20}$	$\chi^2_{(\alpha,5)}$	$\chi^2_{(\alpha,19)}$
0.10	0.369	0.271	9.23	27.204
0.05	0.409	0.294	11.07	30.144
0.025	--	--	12.83	32.852

Test the hypothesis that these numbers are drawn from a U (0, 1) at a 95% confidence level using the Chi-squared goodness of fit test using **6 intervals**.

Question #4: 15

Consider the following probability function:

$$f(x) = \frac{1}{4}(x+1)^3 \quad ; \quad -1 \le x \le 1$$

10 a) Find the inverse transform of the probability distribution function $f(x)$.

5 b) Generate *three* random numbers from $f(x)$ using the table of U(0,1) random numbers below.

n	1	2	3
$U_n(0,1)$	0.171	0.339	0.980

Solution:

(a) Inverse Transform:

CDF $\rightarrow F(x) = \int_{-1}^{x} \frac{1}{4}(y+1)^3\,dy = \left[\frac{1}{16}(y+1)^4\right]_{-1}^{x} = \frac{1}{16}(x+1)^4$

Let $u \sim U[0,1] \Rightarrow \frac{1}{16}(x+1)^4 = u \iff (x+1)^4 = 16u$

$\therefore \; x+1 = 2\sqrt[4]{u} \iff \boxed{X = 2\sqrt[4]{u} - 1}$

(b)

$u_1 = 0.171 \longrightarrow X = 2\sqrt[4]{0.171} - 1 = 0.28611$

$u_2 = 0.339 \longrightarrow X = 2\sqrt[4]{0.339} - 1 = 0.52608$

$u_3 = 0.980 \longrightarrow X = 2\sqrt[4]{0.980} - 1 = 0.98992$

Question #5: 35

An electronics store sells smartphones. Customers arrive according to the store according to exponential time between arrivals with rate λ= 2 cust./hr between 12:00pm to 3:00pm. From 3:00pm to 6:00pm, the arrival rate increases to λ= 3 cust./hr. When a customer arrives, number of smartphones he buys is between 0 and 3. The owner models the sold smartphones per customer as a Binomial distribution with p=0.35 and n = 3. Store works daily from 3:00am to 6:00 pm.

10 a) Find the inverse transform for number of smartphones sold per customer (Process#1). Explain the process in the back of the paper.

5 b) From your answer in (a), generate the ***number of phones sold*** for 7 customers. Using the following U[0,1] number.

n	1	2	3	4	5	6	7
$U_n(0,1)$	0.305	0.696	0.171	0.023	0.879	0.415	0.901
N	1	1	0	0	2	1	2

10 c) Find the inverse transform for time between arrivals (Process#2). Explain the process in the back of the paper.

10 d) From your answer in (c), generate the ***arrival times*** of 7 customers.

n	1	2	3	4	5	6	7
$U_n(0,1)$	0.815	0.636	0.563	0.923	0.295	0.605	0.971
Ti	0.844 λ_1	0.505 λ_1	0.414 λ_1	1.282 λ_2	0.116 λ_2	0.309 λ_2	0.179 λ_2
AT(i)	0.844 + 12:00 = 12:51	1.349 + 12:00 = 1:21	1.763 + 12:00 = 1:46	3.045 + 12:00 = 3:03	3.161 + 12:00 = 3:10	3.47 + 12:00 3:28	4.65 + 12:00 4:39
	< 3:00	< 3:00	< 3:00	> 3:00	> 3:00	> 3:00	> 3:00

(a) Let N: # of smart phones sold ⇒ $N \sim Bin(n=3, p=0.35)$

Process # 1: Generate random Numbers from $Bin(3, p=0.35)$

1. Get CDF
2. make a table for $u \sim U[0,1]$ inverse function
3. Generate random numbers using the given numbers.

$P_r\{N=x\} = \binom{3}{x} (0.35)^x (0.65)^x$

Solution:

N	0	1	2	3
$P\{N\}$	0.275	0.444	0.239	0.042
$Pr\{N \le n\}$	0.275	0.719	0.958	1

$$\Rightarrow \quad F^{-1}(u) = \begin{cases} 0 & \text{if} \quad 0 \le u \le 0.275 \\ 1 & \text{if} \quad 0.75 < u \le 0.719 \\ 2 & \text{if} \quad 0.719 < u \le 0.958 \\ 3 & \text{if} \quad 0.958 < u \le 1 \end{cases}$$

(b) → See the table

(c) Process # 2 Arrival Times (AT)

$$AT(i) = AT(i-1) + T_i$$

$$T_i \sim Exp(\lambda=2) \rightarrow f_T(t) = 2e^{-2t}$$

or

$$T_i \sim Exp(\lambda=3) \rightarrow f_T(t) = 3e^{-3t}$$

Inverse: T_i : 1. get CDF $\rightarrow F_T(t) = 1 - e^{-\lambda t}$

2. get Inverse $\rightarrow T_i = \frac{-1}{\lambda} e^{-\lambda t}$

We know
$$\begin{cases} T \sim Exp(\lambda=2) & \text{if} \quad AT \in [12, 3] \\ T \sim Exp(\lambda=3) & \text{if} \quad AT \in [3, 6] \end{cases}$$

$$\therefore \quad AT(i) \begin{cases} AT(i-1) + \left(\frac{-1}{2} \ln(1-u)\right) & \text{if } AT(i-1) \in [12, 3] \\ AT(i-1) + \left(\frac{-1}{3} \ln(1-u)\right) & \text{if } AT(i-1) \in [3, 6] \end{cases}$$

Question #6: 15

Customers arrive to a minimarket according to a random process. The arriving customers come to a single server checkout counter after they finish shopping. The checkout sever takes a random amount of time to finish the checkout for a customer. The server calculated that customers purchase from the market with an amount between 10 SR and 50 SR following the probability function The following table is the output of the simulation run. Answer the following from the table:

3 1. What is the average waiting time for the customers who wait in line?
3 2. From the simulation data, what is the average money spent by any customer?
3 3. What is the percentage of customers spending money between 20 SR and 45 SR?
3 4. What is the average service time for the customers who spend more than 20 SR?
3 5. What is the probability that the cashier is NOT WORKING during the *simulation Time*?

Col#1	Col#2	Col#3	Col#4	Col#5	Col#6	Col#7	Col#8	Col#9
Cust. #	Arrival time (min)	Service time (min)	Service start (min)	Cust. WITE?	Wait Time (min)	Departure time (min)	Cashire Idle Time (min)	Money Spent (SR)
1	0.24	0.33	0.24	0	0.00	0.58	0.24	30
2	0.93	2.10	0.93	0	0.00	3.03	0.36	20
3	1.76	9.51	3.03	1	1.27	12.54	0.00	17
4	9.39	4.27	12.54	1	3.15	16.81	0.00	22
5	12.58	4.22	16.81	1	4.23	21.03	0.00	12
6	14.26	1.42	21.03	1	6.77	22.45	0.00	20
7	19.29	0.49	22.45	1	3.16	22.94	0.00	30
8	22.52	1.53	22.94	1	0.42	24.47	0.00	10
9	27.94	2.25	27.94	0	0.00	30.19	3.48	27
10	37.96	1.61	37.96	0	0.00	39.56	7.77	33

Solution

① Ave. [waiting time | cust. wait]

$$= \frac{\sum \text{waiting times}}{\#\ \text{cust. wait}} = \frac{1.27+3.15+4.23+6.77+3.16+0.42}{6}$$

$$= 3.155 \text{ min.}$$

② Ave. [money spent] $= \frac{\sum \text{money spent}}{10} = \frac{221}{10} = 22.1$ SR

③ $Pr\{$Cust. spending between $[20, 45]\}$

$$= \frac{\#\ \text{Cust. spending between } [20, 45]\ \text{SR}}{10} = \frac{7}{10}$$

④ $E[$Service time | Customer spend $> 20^{SR}]$

$$= \frac{\sum \text{service times cust. spends} > 20}{\#\ \text{Cust spend} > 20} = \frac{0.33+4.27+0.49+2.25+1.61}{5}$$

$$= \frac{8.95}{5} = 1.79 \text{ min.}$$

⑤ $Pr\{$Cashire is Idle$\} = \frac{\sum \text{idle times}}{\text{Total Sim Time}} = \frac{0.24+0.36+3.48+7.77}{39.56}$

$$= \frac{11.85}{39.56} = 0.2995$$