OR 441 – Modeling and Simulation
Dr. Khalid Alnowibet

كلية الــعلوم
قسم الإحصــاء وبحوث العمليـات

جــامــعــة
الملك سعود
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2nd Midterm Exam-Solution

استعن بالله وكن على يقين بأن كل ما ورد في هذه الورقة تعرفه جيدا وقد تدربت عليه بما فيه الكفاية

Question #1:

Answer the following with *True* or *False*:

1. The non-parametric input modeling uses Uniform [0,1] numbers to generate random numbers for the rank of the random number from the sample. **TRUE**

2. The empirical input modeling of individual data sorts the data of sample from largest to smallest and uses Uniform [0,1] numbers to generate random numbers for the sample. **FALSE**

3. The empirical input modeling of grouped data gives random values that are not in the sample. **TRUE**

4. It possible that the non-parametric input modeling generates new random numbers that are not in the original sample. **FALSE**

5. In box plot, if the sample has vales that are greater than the lower fence or less than upper fence uses then the values are outliers. **FALSE**

6. For the box plot, the 1st quartile (Q_1) is the point in the sorted data the has 25% of the data that are less than or equal to Q_1. **TRUE**

7. In EXCEL, the function RANDBETWEEN (*a , b*) is used to generate real valued random numbers between *a* and *b*. **FALSE**

8. The KURTUSIS measures the spread of data around the mean of the sample. **TRUE**

9. If the SKEWNESS of the sample is negative, then the data has long tail to the positive values. **FALSE**

10. In ARENA, the block DESCIDE is used to change the direction of the flow of entities in the simulation to choose from two or more directions. **TRUE**

11. In ARENA, the block CREATE is used to simulate processing time to any entity. **FALSE**

12. In ARENA, in the PROCESS block the ACTION (Seize, Delay, Release) means that the server can start a new service before the end of the current customer finish his service. **FALSE**

13. The block CREATE in Arena is used to simulate the arrival of new customers. **TRUE**

14. In moment matching for input modeling number of moment matching equations equals to number of all descriptive statistics of the sample. **FALSE**

15. The P-P plot is used to compare the empirical probabilities of the sample with the theoretical probability from the distribution. **TRUE**

16. In modeling input data, the histogram of the sample data is used to fit a theoretical PDF function to the data. **TRUE**

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17. In modeling input data, the histogram of the sample data is used to fit a theoretical CDF function to the data. **FALSE**

18. In graphical method for input data modeling, the empirical distribution of the sample is used to find the best CDF function for the data. **TRUE**

19. In simulation of ATM system, the average number of customers in the waiting for ATM is computed as a simple mean. **FALSE**

20. In simulation of ATM system, the percentage that there are no customers using the ATM is computed by the time average. **TRUE**

Question #2:

A sample of data of size N = 200 has the following descriptive statistics:

Mean	2.256
Median	1.692
Mode	#N/A
Standard Deviation	1.829
Sample Variance	3.344
Kurtosis	3.199
Skewness	1.816
Range of Data	8.957
Minimum Value	0.523
Maximum Value	9.481
Sum of Data	451.232

1. Model this sample as a **Uniform[*a*,*b*]** using moment matching to estimate the parameters. Write the estimated probability function (pdf) for the sample.
2. Model this sample as an **Exponential(λ)** uniform[*a*,*b*] using moment matching to estimate the parameters. Write the estimated probability function (pdf) for the sample.
3. Model this sample as an **Erlang (α, β)** using moment matching to estimate the parameters. Write the estimated probability function (pdf) for the sample.

Solution

1. Uniform[a,b] → 2 parameters → we need 2 equations from moment matching

Sample mean = 2.256   theoretical mean = (a+b)/2 →   2.256 =(a+b)/2 → a+b = 4.512

Sample variance = 3.344   theoretical mean = (b-a)²/12 →   3.344 = (b-a)²/12 → -a+b = 6.335

Then a= -0.9115 and b=5.4235   → f(x) = 1/6.335   -0.9115 <= x <= 5.4235

2. Exp(λ) → 1 parameter → we need one equations from moment matching

Sample mean = 2.256   theoretical mean = 1/λ → 2.256 = 1/λ → λ = 0.4432

Then, f(x) = (0.4432) $e^{-0.4432\,x}$

3. Erlang (α,β) → 2 parameters → we need 2 equations from moment matching

Sample mean = 2.256   theoretical mean = $\alpha\beta$ →   2.256 = $\alpha\beta$

Sample variance = 3.344   theoretical mean = $\alpha\beta^2$ →   3.344 = $\alpha\beta^2$

Mean/Variance = 1/β = 2.256/3.344 = 0.675   → β = 1.482 → α = 1.522

Question #3:

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Customers arrive to a minimarket according to a random process with arrival rate that is assumed to be constant. After the customer finishes shopping, the arriving customers proceeds to a single server checkout counter. The checkout sever takes a random amount of time to finish the checkout for a customer. Data collected for customers entered the minimarket in the last 40 mints as follows.

Cust. #	(Col.1) Arrival time (min)	(Col.2) Service time (min)	(Col.3) Service start (min)	(Col.4) WAITE?	(Col.5) Wait Time (min)	(Col.6) Dep. time (min)	(Col.6) Idle Time (min)	(Col.7) Money Spent (SR)
1	0.24	0.33	0.24	0	0	0.58	0.24	30
2	0.93	2.10	0.93	0	0.00	3.03	0.36	20
3	1.76	9.51	3.03	1	1.27	12.54	0.00	30
4	9.39	4.27	12.54	1	3.15	16.81	0.00	20
5	12.58	4.22	16.81	1	4.23	21.03	0.00	20
6	14.26	1.42	21.03	1	6.77	22.45	0.00	20
7	19.29	0.49	22.45	1	3.16	22.94	0.00	30
8	22.52	1.53	22.94	1	0.42	24.47	0.00	10
9	27.94	2.25	27.94	0	0.00	30.19	3.48	20
10	37.96	1.61	37.96	0	0.00	39.56	7.77	30

Answer The following

1. What is the expected service time?
2. What is the average waiting time?
3. What is the average money spent by any customer?
4. What is the percentage of customers spending at most 20 SR during the simulation run?
5. What is the probability that the cashier is BUSY serving customers during the simulation time?
6. On average what is the expected time that customers spend in the minimarket from the time they enter until the time the leave the minimarket?

Solution

1. E[Service time] = (sum of Col.2)/(# observations)= 27.73/10 = 2.773 min
2. Ave.[waiting time] = (sum of Col.5)/(# observations)= 19/10 = 1.9 min
3. Ave.[money spent by any customer] = (sum of Col.7)/(# observations)= 230/10 = 23 SR
4. Percentage of customers spending at most 20 SR

= (Number of observation<= 20 of Col.7)/(# observations)= 6/10 = 0.6

5. Prob{the cashier is BUSY } = 1 - Prob{ the cashier is IDLE} = 1 – (Sum of idle intervals)/(Total Sim. Time)

= 1 – (0.24+0.36+3.48+7.77)/39.56 = 1 – 0.2995 = 0.7005

6. E[time that customers spend in the minimarket] = Sum (difference Col.6 – Col.1) /10

= (0.34+2.1+10.78+7.42+8.45+8.19+3.65+1.95+2.25+1.6)/10 = 4.673 min

Question #4:

The following table is a snap-shot of a simulation run. This data represents the arrival times and the departure times of customers to a service:

Cust. #	Arrival Time	in	Dep. Time	out	start	end	change	# in queue	interval	(No.Q)*(Interval)
11	48.92	+1	48.96	-1	48	48.92	0	0	0.92	0
12	49.62	+1	52.98	-1	48.92	48.96	1	1	0.04	0.04
13	54.06	+1	55.64	-1	48.96	49.62	-1	0	0.66	0
14	56.24	+1	56.41	-1	49.62	52.98	1	1	3.36	3.36
15	57.03	+1	57.76	-1	52.98	54.06	-1	0	1.08	0
16	69.63	+1	79.75	-1	54.06	55.64	1	1	1.58	1.58
17	70.00	+1	80.32	-1	55.64	56.24	-1	0	0.6	0
18	76.47	+1	83.67	-1	56.24	56.41	1	1	0.17	0.17

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19	79.77	+1	84.59	-1
20	84.48	+1	88.40	-1
21	87.77	+1	90.00	-1
22	92.91	+1	93.82	-1

56.41	57.03	-1	0	0.62	0
57.03	57.76	1	1	0.73	0.73
57.76	69.63	-1	0	11.87	0
69.63	70	1	1	0.37	0.37
70	76.47	1	2	6.47	12.94
76.47	79.75	1	3	3.28	9.84
79.75	79.77	-1	2	0.02	0.04
79.77	80.32	1	3	0.55	1.65
80.32	83.67	-1	2	3.35	6.7
83.67	84.48	-1	1	0.81	0.81
84.48	84.59	1	2	0.11	0.22
84.59	87.77	-1	1	3.18	3.18
87.77	88.4	1	2	0.63	1.26
88.4	90	-1	1	1.6	1.6
90	92.91	-1	0	2.91	0
92.91	93.82	1	1	0.91	0.91
93.82	94	-1	0	0.18	0

Answer The following
1. Compute the table of number of customers in the system during the simulation period.
2. What is the average number of customers in the system during the simulation period?
3. What is the probability that there are 2 customers in the system during simulation period?

Solution:
1. See the table
2. average number of customers in the system during the simulation period = **(No.Q)*(Interval)/Sim Time**
   = 45.4/(94-48) = 0.987 customers
3. probability that there are 2 customers in the system = (Total intervals #queue=2)/(Simulation Time)
   = (6.47+0.02+3.35+0.11+0.63)/(94-48) =10.58/46 = 0.23

Question #5:

Students in College of Science arrive to a *Mr. Cafe* coffee shop to get their beverages and sandwiches during break time. It is estimated that the time between students' arrival is an integer uniform distribution between 3 and 8 minutes. After students get their order they may choose either to DINE-IN the coffee shop or take their orders for TO-GO and leave the coffee shop. From past experience, it is known that 60% of the students who get their orders choose to DINE-IN the coffee shop and 40% choose to TO-GO with their orders. The students who choose to DINE-IN the coffee shop spend an integer uniform distribution between 5 and 15 minute on the table of the coffee shop.
Random Seeds U[0,1]:

1	2	3	4	5	6	7	8	9	10
0.909	0.635	0.077	0.309	0.114	0.277	0.887	0.698	0.394	0.823
0.228	0.809	0.456	0.590	0.767	0.063	0.099	0.116	0.270	0.882
0.787	0.724	0.458	0.254	0.127	0.272	0.707	0.013	0.611	0.577
0.140	0.135	0.153	0.536	0.126	0.271	0.362	0.179	0.934	0.316

1. Use the above random seeds to simulate the first 10 students arrived to *Mr. Cafe* coffee shop. Make a table of results that compute the following:
   - Student number = 1, 2, ... , 10
   - Time between student's arrival (in minutes)

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- Arrival time of student # *j* (in minutes)
- The order type of the students (DINE-IN or TO-GO)
- The time that student stay on table for DINE-IN (in minutes)
- The time at which the student leave the coffee shop after DINE-IN (in minutes)
2. What is the average number of TO-GO students?
3. What is the average time that students stay in the coffee shop for DINE-IN?
4. simulation blocks of ARENA and the inputs values of each block.
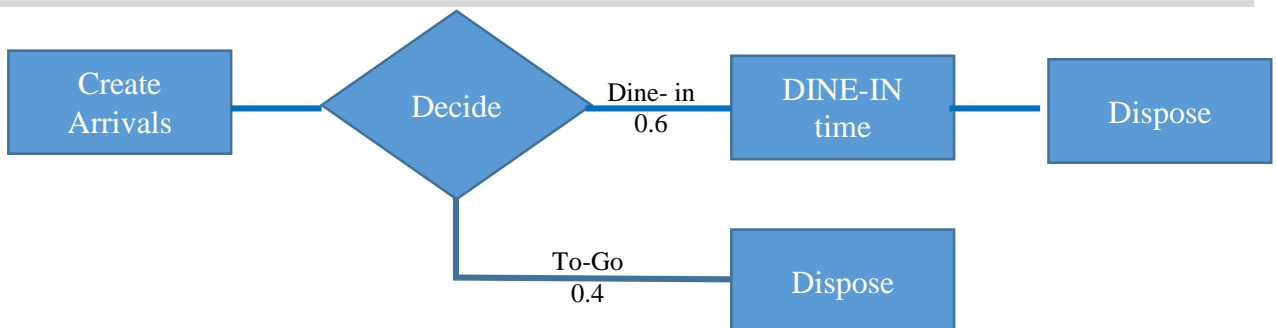
Solution

1. simulate the first 10 students arrived to *Mr. Cafe* coffee shop

ST#	U[0,1] seed row1	Time Bet. Arrivals Int U[3,8] min	Arrivl time min	U[0,1] seed row2	Order type Ber.(0.6,0.4)	U[0,1] seed row3	DINE_IN time Int U[5,15] min
1	0.909	8.0	8.0	0.228	DINE-IN	0.787	13.0
2	0.635	6.0	14	0.809	TO-GO	0.724	0.0
3	0.077	3.0	17	0.456	DINE-IN	0.458	10.0
4	0.309	4.0	21	0.59	DINE-IN	0.254	7.0
5	0.114	3.0	24	0.767	TO-GO	0.127	0.0
6	0.277	4.0	28	0.063	DINE-IN	0.272	7.0
7	0.887	8.0	36	0.099	DINE-IN	0.707	12.0
8	0.698	7.0	43	0.116	DINE-IN	0.013	5.0
9	0.394	5.0	48	0.27	DINE-IN	0.611	11.0
10	0.823	7.0	55	0.882	TO-GO	0.577	0.0

2. average number of TO-GO students = (no. of TO-GO stud.)/Sim. time = 3 / 55 = 0.055 To-go std/min

3. average time that students stay in the coffee shop for DINE-IN
= (Total DINE-IN time)/(No. DINE-IN STD) = 65 / 7 = 9.286 min

4. blocks of ARENA

دعواتنا لكم بالتوفيق والسداد