King Saud University
College of Sciences
Department of Mathematics
Math-244 (Linear Algebra); Mid-term Exam; Semester 1 (1442)
Max. Marks: 30 **Time: 2 hours**

Note: ***Attempt all the five questions!***

Question 1: [Marks: 2+3]

a) **Let A** = $\begin{bmatrix} 1 & -1 & 4 & 5 \\ -2 & 1 & -11 & -8 \\ -1 & 2 & 2 & 2 \end{bmatrix}$ and **B** = $\begin{bmatrix} 1 & -1 & 3 & 2 \\ 0 & 1 & 4 & 1 \\ 1 & 0 & 8 & 6 \end{bmatrix}$. Then show that the matrices **A** and **B** are row equivalent to each other.

b) Give any two matrices **A** and **B** that satisfy:

$trace\,(\mathbf{A}+\mathbf{B}) = trace\,(\mathbf{A}) + trace\,(\mathbf{B})$ and $trace\,(\mathbf{AB}) \neq trace\,(\mathbf{A})\, trace\,(\mathbf{B})$.

Question 2: [Marks: 2+3]

a) Let **A**, **B** $\in M_2(\mathbb{R})$ with $|\mathbf{A}| = 3$ and $|\mathbf{B}| = 6$. Then evaluate $\left| |\mathbf{A}| A^t B^2 adj(A^2) \right|$.

b) Let **A** = $\begin{bmatrix} 1 & 0 & \delta \\ 2 & 1 & 2+\delta \\ 2 & 3 & \delta^2 \end{bmatrix}$. Find the values of δ if the matrix **A** is not invertible.

Question 3: [Marks: 2+4]

a) Find the values of x and y if **A** = $\begin{bmatrix} - & 2 & - \\ - & x & - \\ - & y & - \end{bmatrix}$ and $\mathbf{A}^{-1}$ = $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ - & - & - \end{bmatrix}$.

b) Find the value/s of α such that the following linear system:

$$\begin{aligned} x + 2y - z &= 2 \\ x - 2y + 3z &= 1 \\ x + 2y - (\alpha^2 - 3)z &= \alpha \end{aligned}$$

has:

(i) no solution (ii) unique solution (iii) infinitely many solutions.

Question 4: [Marks: 2+3+3]

a) Let $\mathbf{S} = \{(1,1,1,0), (1,2,3,1), (2,0,1,1)\}$ generates the subspace $\mathbf{F}$ of Euclidean space $\mathbb{R}^4$. Show that $(1,1,1,1) \notin \mathbf{F}$.

b) Let $\mathbf{B} = \{(1,0,0), (0,1,0), ((0,0,1)\}$ and $\mathbf{C} = \{(1,1,1), (1,2,2), (1,1,2)\}$ be bases of the Euclidean space $\mathbb{R}^3$ and $[v]_B = [1\ \ 2\ \ 3]^T$. Find the transition matrix ${}_C\mathrm{P}_B$ and $[v]_C$.

c) Let $\mathbf{A}^T = \begin{bmatrix} 1 & 2 & 2 & 0 & 0 \\ 2 & 4 & 5 & 2 & 2 \\ 1 & 2 & 3 & 2 & 2 \\ 3 & 6 & 4 & -3 & -4 \end{bmatrix}$. Then find:

(i) a basis of $col(\mathbf{A})$ (ii) $rank(\mathbf{A})$ (iii) $nullity(\mathbf{A})$.

Question 5: [Marks: 2+1+3]

Let $\mathbf{S} = \{v_1 = (1,-1,0,1), v_2 = (1,1,1,0), v_3 = (0,1,1,1)\}$ generates the subspace $\mathbf{W}$ of the Euclidean space $\mathbb{R}^4$. Then:

a) Show that $\mathbf{S}$ is a basis of $\mathbf{W}$.
b) Find the angle θ between the vectors v_1 and v_2.
c) Apply the Gram-Schmidt process on $\mathbf{S}$ to obtain an orthonormal basis of $\mathbf{W}$.

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